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Abstract

We build a theoretical model to explain the complex patterns of income and obesity, accounting for changes in behavior related to exercise. We combine the theory of time allocation with the theory of conspicuous leisure in a growth model, assuming that consumption expenditures connected to exercise time are conspicuous, and that conspicuous behavior changes with economic development. As a result, as economies develop, we show that there is a growing wedge between optimal exercise and consumption choices made by individuals with different income levels. We show that this pattern is connected to a dynamic Kuznets curve linking body weight to economic development over time, and a static Kuznets curve linking different steady state levels of income per worker to body weight. Thus, our model helps explain the rise and slowdown in obesity prevalence in the USA, as well as the positive correlation between obesity and income per worker in developing countries, and the negative correlation between obesity and income per worker in industrialized countries. We supplement our theoretical results with numerical simulations of the static and dynamic obesity Kuznets curves for the USA. We show that while exercise choices have contributed to a slowdown in the rise in obesity prevalence, there is to this date no dynamic Kuznets curve pattern for obesity in the USA. By contrast, we find the existence of a static Kuznets curve: the steady state level of average body weight increases with the per worker stock of capital up to a level of 186.5 pounds, corresponding to a capital stock 25% higher than the current steady state US capital stock, and decreases thereafter. We discuss policy implications of our findings.

Keywords: Obesity, Status, Conspicuous leisure, Inequality, Kuznets Curve, Economic Development.

JEL classification: D11, D30, H31, I15, O41.

1 Introduction

The Covid-19 pandemic forcing more than 90% of US adults to shelter-in-place has been accompanied by an increase in average body weight, which is a stern reminder of the fact that obesity remains a major public policy issue involving significant private and social costs. However, to this date, the theoretical literature has provided limited explanations for the complex link between income and obesity. The literature review by Mathieu-Bolh (2021) shows that the link between income and obesity seems to follow a Kuznets curve pattern. Average obesity rates seem to first increase and then decrease or at least plateau after a certain development threshold. Additionally, the positive link between income and obesity for population

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cross sections in poor countries reverses to a negative link when countries reach a certain level of economic development. In rich countries, the income obesity link is negative at least for women. To explain this complex pattern, the theoretical literature has essentially focused on food consumption choices, assuming exogenous preferences. However, the decrease in physical activity during confinement underlines that calorie expenditure is important to maintain healthy body weights (e.g. Ziegler et al. 2020),[1] and it is likely that preferences are not fixed but change over time (Bowles, 1998). Therefore, we build the first theoretical growth model that combines Becker's (1965) theory of the allocation of time and Veblen's (1899) theory of conspicuous leisure, and we focus on exercise choices and changing preferences to explain complex obesity income patterns. We simulate our model to describe obesity patterns in the US. We discuss the fact that our model also provides a tool for policy analysis.

So far, theoretical explanations of the link between income and obesity are limited and mostly focus on the role of food consumption. Some contributions explain the historical rise in obesity. Dragone (2009) attributes this rise to individual habit formation and Dragone and Ziebarth (2017) attribute it to globalization and habit formation. Burke and Heiland (2007), Levy (2002 and 2009), Dragone and Savorelli (2012), Strulik (2014), and Mathieu-Bolh (2019) connect it to social norms, and Mathieu-Bolh (2021) connects it to a relative price threshold. Other contributions explain the inverse relation between income and obesity for population cross-sections in rich countries. Cutler et al. (2015), Fuchs (1982), Grossman (1972), Cawley and Ruhm (2012), and Mathieu-Bolh (2021) attribute this inverse relation to differences in schooling and health behavior. Only two articles provide insights on the changing link between income and obesity, and focus on food consumption. For Phillipson and Posner (1999), technological change lowers the price of food, which explains the rise in obesity over time. As economies develop, the change in the relation between income and obesity, from positive to negative, relates to the assumption that rich individuals exogenously care more about their health and weight than poor individuals, and to complementarity between consumption and weight. In this literature, Mathieu-Bolh and Wendner (2020) is the only article that includes endogenous preferences with respect to food consumption and explains the changing link between income and obesity as the result of competing income and dynamic status effects. As economies develop, the dominating income effect makes individuals increase calorie consumption. After a certain income threshold, the dominating dynamic status effect makes individuals prefer low-calorie food over high calorie food and decrease their calorie intake. In addition, only a few theoretical contributions explore the role of calorie expenditure in models with exogenous preferences. The static model by Yaniv et al. (2009) analyzes optimal choices made by weight conscious and weight unconscious consumers accounting for interactions between food consumption choices and physical activity. Dynamic models by Lakdawalla et al. (2005) and Lakdawalla et al. (2009) extend Philipson and Posner (1999). They account for the implications of technological progress-driven price changes on labor, as one of the factors explaining the income-obesity relation. While technology-induced changes in food prices plays a central role in their model, they argue that in rich countries, people are heavier than in poor countries because technological progress leads to more sedentary work and raises the cost of physical activity. In rich countries, where work-place technology is more uniform, rich people are thinner than poor people because the authors assume that the demand for thinness is exogenously higher among rich individuals. Therefore, there is to this date no dynamic model with endogenous preferences that incorporates exercise choices to explain the changing link between income and obesity.

[1] The decrease in physical activity relates to gym closures as well as to the lack of normal daily routine that usually provides structured exercise and spontaneous physical activity (Bhutani and Cooper, 2020), and correlates to the increase in the use of television and internet connected devices and applications (Nielsen report, 2020).

Our model directly extends the theoretical literature by combining Becker's (1965) theory of the allocation of time and Veblen's (1899) theory of conspicuous leisure in a growth model. We extend Becker's (1965) theory of time allocation (followed by Gossen, 1983, LeVan et al., 2018, Ha-Hui et al., 2020) in two ways. In the first place, we acknowledge that there are two types of consumption goods, those that do not require time and those that do, and we introduce a distinction between sedentary leisure and exercise. We associate time consuming expenditure to exercise. For example, the purchase of a gym pass is associated to spending time exercising. Exercise choices therefore involve a double cost, which includes the exogenous price paid for consumer goods (such as the cost of a gym pass) and the endogenous opportunity cost of exercise. This assumption reinforces the rising opportunity cost that goes with capital accumulation and pushes individuals to exercise less. This mechanism is important as it can explain the decrease in exercise and calorie expenditure and the rise in obesity that goes with economic development. We will also discuss the fact that this mechanism is important to consider for policy analysis as it reflects that exercise choices can be influenced through two channels, consumer prices and wages.

However, accounting for the rising opportunity cost of exercise also yields a counterfactual result to the observed inverse income obesity relation among population cross sections, or the decline in obesity captured by the empirical obesity Kuznets curve (e.g. Clément, 2016). Thus, in the second place, we introduce positionality in our model with time-consuming exercise expenditure. Specifically, we incorporate Veblen's (1899) theory of conspicuous leisure in the growth model. According to Veblen, conspicuous leisure is visible leisure in which people engage with the objective of displaying and reaching a certain social status. We apply the idea of visible leisure to exercise choices, which seems a perfect candidate to express status as it is both a time consuming leisure and a pricy consumption expenditure. In our study, the taste for exercise relates to the degree of positionality. We specifically extend the literature on endogenous positionality (Dioikitopoulos et al. 2019 and 2020; Mathieu-Bolh and Wendner 2020) as the degree of positionality has an exogenous and an endogenous component. The exogenous component reflects individual rank in society and the endogenous component is tied to capital accumulation. While exogenous differences in taste exist at all times, as the stock of capital accumulates, everyone's taste for exercise increases. As a result of this dynamic positionality effect, individuals exercise more as the stock of capital increases.

The rising opportunity cost and the dynamic positionality effect create a wedge between optimal exercise and consumption choices of individuals with different socioeconomic status as economies develop. As a result, our model explains the changing link between income and obesity as a reflection of dynamic positionality effects (with respect to physical activity choices) competing with the rising opportunity cost of exercise.

Several empirical facts support that exercise choices differ according to income and that individuals become increasingly positional with respect to exercise choices. First, individual's calorie expenditure through labor is tied to economic development. Based on the U.S. National Health and Nutrition Examination Surveys (NHANES), Church et al. (2011) show that in the early 1960's, almost half the jobs in the private industry in the U.S. used to require at least moderate-intensity physical activity, whereas nowadays, less than 20% of those jobs demand this level of energy expenditure. Since 1960, the estimated mean daily energy expenditure due to work-related physical activity has dropped by more than 100 calories for both women and men. While Lakdawalla et al. (2005) and Lakdawalla et al. (2009) have focused on calorie expenditure related to technological progress and work, we are focusing on calorie expenditure related to leisure time. Our model reflects that when individuals do not spend calories at work, they may chose to exercise during leisure time.

Second, there are also important differences in time spent exercising based on income cross-sections.

Shuval et al. (2017) find that compared to those making less than $20,000 per year, those with an annual income of $75,000 or more engage in 4.6 more daily minutes of moderate to vigorous-intensity physical activity. Higher income earners also exhibit more intense, less frequent weekly patterns of physical activity and more daily sedentary time. The 2008 BLS Spotlight on Statistics covering time spent on sports and exercise, provides insights on differences in the practice of exercise between different income groups. The study indicates the percentage of adults 25 and older who engage in those activities between 2003 and 2006, according to their educational attainment (see Figure A1 in the appendix). It shows that 10 percent of people with less than a high-school diploma engage in those activities, while 23 percent of individuals with a bachelor's degree or higher engage in those activities. However, if exercise choices were solely the mirror of calories spent at work, there would be no difference between overall calorie expenditure of individuals with low-income strenuous jobs and high-income sedentary jobs. Thus, exercise choices may also reflect other influences, such as the quest for social status.

Third, while the literature on leisure initially did not find evidence of positionality with respect to overall leisure time (Carlsson, Johansson-Stenman, and Martinsson, 2007), recent findings suggest that there has been a change in positional behavior from goods purchased to how time is spent. Holthoff and Scheiben (2018) find that leisure activities like travel, cultural events, and dining out lead to even higher perceived status than busyness at work. Furthermore, individuals are highly positional with respect to physical attractiveness (Solnick & Hemenway, 1998). Individuals control their appearance through food consumption and physical activity. Individuals work-out to be thin, which in empirical studies is linked to attractiveness, especially for women (Fletcher et al., 2014). Thus, it makes sense to assume that exercise choices are positional in the same way as some healthy food choices are positional. This assumption is further supported by the increase in the portion of the population using social media, as well as the rise in the number of followers of sport and fitness influencers. For example, the number of Instagram users has increased from 100 million in 2010 to 800 million in 2017. The number of followers of the top 20 fitness influencers on Instagram reached almost 100 million individuals in 2017 (Influencer Marketing Hub, 2021).

Our main theoretical results are as follows. We show the existence of both a dynamic and a static Kuznets curve for obesity. Indeed, in the empirical literature, the Kuznets curve is initially presented as a dynamic relation between economic development and obesity. However, empirical studies solely demonstrate the existence of a static Kuznets curve for population cross-sections (Clément, 2017; Grecu and Rotthoff, 2015) or cross-country analysis (Windarti et al., 2019; Deuchert et al.'s, 2014). In order to show the change in the relation between income and obesity over time, Clément (2017), resorts to two separate cross-sectional analyses covering two different time periods in China. By contrast, our theoretical model generates both a dynamic and a static obesity Kuznets curve, which is an important complement to the empirical literature on this topic, and a new concept in the theoretical literature. Our results differ from Mathieu-Bolh and Wendner (2020) who do not describe the two Kuznets curves and ignore the role of exercise in describing obesity patterns.

We also provide a novel explanation for the mechanisms generating the dynamic and static Kuznets curves. First, our dynamic model shows that the difference between the growth rate of consumption and the growth rate of exercise reflects two competing effects. On the one hand, the opportunity cost of exercise increases as the economy develops, which renders the difference between the growth rate of food consumption and exercise larger over time. On the other hand, positional behavior with respect to exercise increases with economic development generating a dynamic positionality effect, which renders the difference between the growth rate of consumption and exercise smaller over time. In our model, economic development is captured by the stock of capital per worker (identical to the per capita stock of capital) which, given technology, fully determines income per worker. Second, we formally demonstrate the existence of a level of capital per worker for which the growth rate of exercise starts exceeding the growth rate of food consumption. This result is consistent with the empirical dynamic pattern of obesity shown, for example, by Clement (2017). Our steady state analysis demonstrates that in a similar way, both the opportunity cost of exercise and positionality increase with the steady-state stock of capital. Accounting for the fact that exercise is a consumption expenditure raises the opportunity cost of exercise further, and attenuates the effect of positionality on the gap between the change in consumption and the change in exercise. There is therefore a steady state level of capital per worker beyond which the the correlation between steady state body weight and stock of capital per worker is negative. This explains the negative correlation between body weight and income for high income levels per worker in cross-sectional analysis of individuals or countries with different income per worker.

Furthermore, we show that in the presence of dynamic positionality, for high levels of economic development, body weight gain becomes negative as the economy develops over time. Ignoring dynamic positionality with respect to exercise choices would yield a positive correlation between economic development and weight gain at all levels of economic development. The static analysis provides an additional result: For high levels of steady-state stock of capital per worker, the link between body weight and steady state capital stock is negative only for high degrees of positionality. These results suggest that the static and dynamic Kuznets curves for obesity may be different.

We supplement the qualitative analysis with a quantitative analysis relying on numerical simulations. We use a standard calibration procedure, which consists in matching the model's steady state equilibrium characteristics with the long-term characteristics of the actual US economy. Once the model steady state is obtained, we derive the optimal simulated paths toward the steady state. Our numerical simulations confirm the existence of two different relations between body weight and the stock of capital per worker. First, our simulated model shows consistency with data on body weight evolution in the USA. Indeed, the simulated economy shows that the dynamic evolution of body weight has been monotonous. In other words, given the current degree of positionality, there is to this date no dynamic Kuznets curve pattern for obesity in the USA. By contrast, we find the existence of a static Kuznets curve for the USA: the steady state level of average body weight increases with the average stock of capital up to a level of 186.5 pounds, corresponding to a stock of capital per worker 25% higher than its the baseline, and decreases thereafter. Those results implies that the US economy is at a level of economic development that yields a steady body weight currently below the tipping point of the Kuznets curve. Additionally, those results suggest that the steady state average body weight starts being inversely related to wealth when individual wealth reaches 25% above the average wealth in the US, and is positively related to wealth below this threshold.

Second, we conduct a sensitivity analysis. Ignoring that exercise is an expenditure would result in significantly shifting the Kuznets curves down as it would decrease the opportunity cost of exercise and increase exercise. While there is no empirical estimate of the elasticity of substitution between food consumption and exercise, we chose a unit elasticity in the baseline scenario and examine how sensitive the results are to different values of this elasticity. The model predicts that with an elasticity of substitution of 0.66, the tipping point of the Kuznets curve would happen at a steady-state stock of capital 67 percent higher than it currently is, and a body weight of 344 pounds, which is a more pessimistic scenario than the baseline scenario. The dynamic relation between weight and the per worker capital stock does not exhibit a Kuznets curve pattern for a wide range of elasticities of substitution between food consumption and effective exercise. For both the static and the dynamic relations, the more effective exercise and food consumption are complements, the more prevalent the positionality effect, the lower the level of steady state body weight. Additionally, when the intertemporal elasticity of substitution becomes smaller than in the baseline scenario that sets it at one, the dynamic relation between weight and the stock of capital per worker becomes flatter, indicating that as the dynamic positionality effect become dominant, individuals postpone net calorie intake to the future. Last, with very high values of κ, we obtain a simulated dynamic obesity Kuznets curve, which confirms the role of the DPE in limiting and potentially inverting body weight growth.

The rest of the paper is structured as follows. In Section 2, we present the model and provide analytical results. In Section 3, we provide numerical results. In Section 4, we conclude and discuss policy implications of our work.

2 Model

2.1 The economy

We consider a continuous time dynamic general equilibrium model. We model a closed economy in which the capital accumulation technology exhibits decreasing returns. There is a large number of firms and households, the respective number of which is normalized to unity. Households derive utility from two types of consumption: fast-food consumption that does not require time and is not positional, and exercise-related consumption that requires time and is positional. Households maximize utility subject to a dynamic capital accumulation constraint, and body weight is the result of optimal food and exercise choices. In what follows, the time index t is suppressed, unless needed for clarity.

2.1.1 Time-consuming consumption

First, we account for the fact that some types of consumption take time. The representative individual distinguishes between two types of goods, those that do not require time, C, and those that require time, X. Specifically, C may be thought of as fast-food or junk-food consumption. This type of consumption requires almost no preparation time, so in our model, we consider that it is a standard consumption good that does not require time. Although our model applies to a broader framework than junk food and exercise, in order to provide clear intuitions in what follows, we refer to C as food consumption and X as the time spent exercising.

Second, we introduce endogenous labor in the model. Individuals are endowed with one unit of time, used for endogenous labor N, endogenous exercise X, and exogenous sedentary leisure $\bar{S}$ (such as sleeping or watching television). As a consequence:

$$1 = N + X + \bar{S}\,. \tag{1}$$

To simplify the notation, we write that the amount of time that is not spent on sedentary leisure is $\bar{L} = 1 - \bar{S}$, such that:

$$\bar{L} = N + X \tag{2}$$

As a consequence, an individual's flow budget constraint accounts for both working time and consumption expenditure related to exercise:

$$\dot{K} = rK + w\left(\bar{L} - X\right) - p_C C - p_X X\,,\;\; p_X, p_C > 0\,, \tag{3}$$

where w is the wage rate, r denotes the interest rate, and p_C and p_X are the respective prices of C and X.

2.1.2 Positional activities

Individuals are positional with respect to exercise. The impact of the reference level of exercise is captured by effective exercise $\hat{X}$, which differs from absolute exercise X according to the standard subtractive specification (Ljungqvist and Uhlig, 2000):[2]

$$\hat{X} = X - \varepsilon\left(k\right)\bar{X}\,, \quad 0 \leq \varepsilon\left(\bar{k}\right) \leq 1, \tag{4}$$

where $k \equiv K/N$ (wealth per worker), and $\bar{X} \equiv X/1$ (average exercise expenses, with the population size equals unity) is the reference level for exercise, and the upper bar indicates that the variable is exogenous from an individual's point of view. Positionality is captured by function $\varepsilon\left(k\right)$ representing the degree of positionality (DOP). When $\varepsilon\left(k\right) = 0$, individuals are not positional and effective exercise equals absolute exercise. When $\varepsilon\left(k\right)$ is high, effective exercise is low, and the marginal utility of X is high. The formulation of positionality is similar to Mathieu-Bolh and Wendner's (2020) in the sense that it includes an exogenous and an endogenous element. We use the following standard functional form to describe the DOP:

$$\varepsilon(k) = 1 - e^{-\kappa k}, \quad \kappa > 0. \tag{5}$$

The static element yields a property called the static positionality effect. In our model, it means that, given the average stock of capital k, the higher the parameter κ, the more an individual cares about exercise. The dynamic element yields a property that we call the dynamic positionality effect. It captures that a higher stock of capital — either built over time due to economic growth, or observed at a given point in time among different countries or population subgroups — endogenously raises positionality :

$$\frac{\partial \varepsilon(k)}{\partial k} > 0\,. \tag{6}$$

As a consequence, increasing development, captured by the average stock of capital k, raises individuals' positionality with respect to their exercise choice. Tying positionality to economic development means that over time, as a country develops, individuals become on average more concerned with exercise. A plausible reason may be that in poor countries, individuals who face tighter budget constraints and work long hours likely relegate conspicuous exercise to a secondary preoccupation.

2.1.3 Body weight

The choices of C and X have an impact on body weight change $\dot{W}$. We connect weight gain to net energy intake, the difference between energy intake and expenditure (described in a general manner by Schofield, 1985). Energy intake is a function of food consumption. We introduce a modification in the Schofield equation to take into consideration the role of exercise. Recall that the usual Schofield equation is: $\dot{W} = \lambda_C C - \lambda_W W$, where the parameter $\lambda_C > 0$ represents the energy density of food (measured in joules per unit of food consumed), and $\lambda_W > 0$ reflects a metabolic rate (measured in joules per unit of weight). In this expression, calorie expenditure is a fixed proportion λ_W of body weight W. Implicitly, calorie expenditure is measured for a unit of time of one, which can be one year or one day, and each type of activity during this unit of time exerts the same amount of calories. By contrast, in our model, we take into consideration that individuals allocate one unit of time to activities that exert different amounts of calories.

[2] This specification of status preferences is prevalent throughout the literature. Formulating it as a multiplicative function (Gali 1994) is also possible and yields essentially equivalent results.

This unit of time is spent in exogenous sedentary leisure $\bar{S}$, endogenous exercise X and labor N. Therefore, we re-write the Schofield equation as:

$$\dot{W} = \lambda_C C - \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right)}{N} W. \tag{7}$$

In this expression, $\lambda_S \bar{S}$ represents the basal metabolic rate (BMR), which is basic energy expenditure to maintain the functioning of a body at rest. The terms $\lambda_X X$ and $\lambda_N N$, with $\lambda_X > 0$ and $\lambda_N > 0$, denote the extent to which time spent on exercise and labor reduces net energy. Note that λ_C is a rate in front of the variable C. In the same way, the term in front of W is expressed as a rate. For that reason, $\lambda_S \bar{S} + \lambda_X X + \lambda_N N$ is divided by N (since W is expressed in total terms). While sedentary leisure $\bar{S}$ is fixed and proportional to weight, calorie expenditure associated to non-sedentary leisure and work $\lambda_X X + \lambda_N N$ vary with individuals' choices. Since $N = 1 - \bar{S} - X$, it is straightforward that when individuals choose to exercise more, they spend relatively fewer calories at work.

2.1.4 Preferences and optimal choices

Individuals face a constrained intertemporal optimization problem that is described as follows. Instantaneous utility is given by the strictly concave and twice continuously differentiable function:

$$U(C, \hat{X}, W) = u(C, \hat{X}) - v\left[(W - W^*)^2\right]. \tag{8}$$

Both sub-utility functions u and v are strictly increasing in their respective arguments. The intertemporal utility function is:

$$\int_{t=0}^{\infty} U(C, \hat{X}, W) e^{-\rho\tau} d\tau\,, \tag{9}$$

where $\rho > 0$ is the constant rate of time preference. Utility positively depends on food consumption and exercise. An exercise reference level $\bar{X}$ increases marginal utility of exercise. We account for the fact that body weight in excess or below the healthy norm, W^*, causes dis-utility. Considering that individual and average weight are equal in equilibrium, this formulation captures obesity related externalities and justifies considering policy interventions discussed in the conclusion. However, weight is not a choice variable in our model, so conspicuous behavior is solely reflected in individuals' food consumption and exercise choices. Given the DOP, $\varepsilon(\bar{k})$, and (7), individuals choose C and $\hat{X}$, to maximize (9), subject to their initial endowment of wealth $K_0 > 0$, their flow budget constraint (combining (4) and (3)):

$$\dot{K} = rK + w\bar{L} - \hat{p}_X \varepsilon\left(\bar{k}\right) \bar{X} - p_C C - \hat{p}_X \hat{X}\,, \tag{10}$$

and a No-Ponzi-Game (NPG) constraint:

$$\lim_{\tau\to\infty} e^{-R(t,\tau)} K \geqslant 0\,, \tag{11}$$

where $R(t,\tau) = \int_t^\tau r(v)dv$ represents the interest factor, and:

$$\hat{p}_X \equiv w + p_X. \tag{12}$$

Price $\hat{p}_X$ represents the total cost of effective exercise that includes the cost of exercise expenditure and the opportunity cost of exercise.

The model is solved in a standard way relying on a current-value Hamiltonian:

$$\mathcal{H} = U(C, \hat{X}, W) + \mu \left[rK + w\bar{L} - \hat{p}_X \varepsilon \left(\bar{k} \right) \bar{X} - p_C C - \hat{p}_X \hat{X} \right] ,$$

where μ is the shadow value of saving expressed in utility units. An interior solution regarding the control variables implies:

$$\frac{\partial \mathcal{H}}{\partial C} = U_C - \mu \, p_C = 0 \, , \tag{13}$$

$$\frac{\partial \mathcal{H}}{\partial \hat{X}} = U_{\hat{X}} - \mu \, \hat{p}_X = 0 \, . \tag{14}$$

The canonical equations regarding the state variable K are:

$$\frac{\partial \mathcal{H}}{\partial K} = \mu r = \rho \mu - \dot{\mu} \, , \tag{15}$$

$$\lim_{\tau \to \infty} \mu(\tau) e^{-\rho \tau} K(\tau) = 0, \tag{16}$$

where (16) is the transversality condition, and (15) yields:

$$\frac{\dot{\mu}}{\mu} = -(r - \rho) \, . \tag{17}$$

We deduce the following expressions for the growth rates of food consumption and exercise. Noticing that in equilibrium, average exercise expenditure equals individual's exercise expenditure, $\bar{X} = X$, and likewise, $\bar{k} = k$, and combining the optimality conditions and the relation between effective and actual exercise (4), we obtain the growth rate of consumption and exercise as (see details in Appendix 7.1):

$$\frac{\dot{C}}{C} = \Omega^C \left(C, \hat{X} \right) (r - \rho) + \Phi^C \left(C, \hat{X} \right) \left(\Delta \frac{\dot{w}}{w} \right) \, , \tag{18}$$

$$\frac{\dot{X}}{X} = \Omega^X \left(C, \hat{X} \right) (r - \rho) + \frac{\dot{\varepsilon}(k)}{1 - \varepsilon(k)} - \Phi^X \left(C, \hat{X} \right) \left(\Delta \frac{\dot{w}}{w} \right) \, . \tag{19}$$

The function Δ is such that $0 < \Delta \equiv \frac{w}{w + p_X} < 1$ (implying that $\frac{\partial \Delta}{\partial w} > 0$, and $\frac{\partial \Delta}{\partial p_X} < 0$). We define the following elasticities: $\Omega^C \left(C, \hat{X} \right) = \frac{e_{\hat{X}\hat{X}} - e_{C\hat{X}}}{-e_{CC} e_{\hat{X}\hat{X}} + e_{C\hat{X}} e_{\hat{X}C}}$, $\Omega^X \left(C, \hat{X} \right) = \frac{e_{CC} - e_{\hat{X}C}}{-e_{CC} e_{\hat{X}\hat{X}} + e_{C\hat{X}} e_{\hat{X}C}}$, $\Phi^C \left(C, \hat{X} \right) = \frac{e_{C\hat{X}}}{-e_{CC} e_{\hat{X}\hat{X}} + e_{C\hat{X}} e_{\hat{X}C}}$, and $\Phi^X \left(C, \hat{X} \right) = \frac{e_{CC}}{-e_{CC} e_{\hat{X}\hat{X}} + e_{C\hat{X}} e_{\hat{X}C}}$, with $e_{CC} = (U_{CC} C / U_C)$, $e_{C\hat{X}} = \left(U_{C\hat{X}} \hat{X} / U_C \right)$, $e_{\hat{X}C} = \left(U_{\hat{X}C} C / U_{\hat{X}} \right)$, $e_{\hat{X}\hat{X}} = \left(U_{\hat{X}\hat{X}} \hat{X} / U_{\hat{X}} \right)$.

The difference between the growth rate of food consumption and the growth rate of exercise expenditure is therefore expressed as:

$$\frac{\dot{C}}{C} - \frac{\dot{X}}{X} = \underbrace{\left[\Omega^C \left(C, \hat{X} \right) - \Omega^X \left(C, \hat{X} \right) \right] (r - \rho)}_{ECE} + \underbrace{\left[\Phi^X \left(C, \hat{X} \right) + \Phi^C \left(C, \hat{X} \right) \right] \left(\Delta \frac{\dot{w}}{w} \right)}_{ROC} - \underbrace{\frac{\dot{\varepsilon}(k)}{1 - \varepsilon(k)}}_{DPE} \tag{20}$$

The growth rate of food consumption differs from the growth rate of exercise expenditure due to three terms. The first term represents Elasticities for Consumption and Exercise (ECE). It includes two elasticities, respectively $\Omega^C \left(C, \hat{X} \right)$ and $\Omega^X \left(C, \hat{X} \right)$. The second term represents the Rising Opportunity Cost of exercise

(ROC) and the third term represents the Dynamic Positionality Effect (DPE). Note that accounting for the fact that exercise is a consumption expenditure influences the ROC through two channels. One channel operates through $\Phi^C\left(C, \hat{X}\right) < 0$, which denotes that exercise is an expenditure using up resources that cannot be allocated to food consumption. Other things equal, it decreases the growth rate of food consumption and limits the ROC. The other channel operates through p_X and lowers $\Delta \equiv \frac{w}{w+p_X}$. Other things equal, it decreases the importance of ROC related to wage growth. The ultimate impact involves general equilibrium effects that will be accounted for in the next sections.

Proposition 1: *For all forms of preferences, the difference between the growth rate of consumption and the growth rate of exercise can be positive or negative as the DPE is an offsetting force to the ROC.*

Proof: Straightforward from (20). The ECE includes two elasticities, respectively $\Omega^C\left(C, \hat{X}\right)$ and $\Omega^X\left(C, \hat{X}\right)$, which are different unless preferences are CES, in which case this term disappears. To illustrate the terms Ω^C, Ω^X, Φ^C, Φ^X, we can consider a constant elasticities of substitution specification for the utility function, such that:

$$U(C, \hat{X}, W) = \frac{\left[\left(\alpha C^{\zeta} + (1-\alpha)\hat{X}^{\zeta}\right)^{1/\zeta}\right]^{1-\gamma}}{1-\gamma} - v\left[(W - W^*)^2\right], \quad 0 < \alpha < 1,\ \gamma > 0,\ \zeta \leq 1\,. \tag{21}$$

where α represents the taste for food consumption relative to effective exercise consumption. The intratemporal elasticity of substitution between C and $\hat{X}$ is given by $1/(1-\zeta)$, while the intertemporal elasticity of substitution (IES) is given by $1/\gamma$. For the whole class of constant elasticities of substitution utility functions, the term $\left[\Phi^X + \Phi^C\right] = 1/(1-\zeta) > 0$ (with $\Phi^C = -(1-\alpha)(1-\gamma)/\gamma < 0$) and $\Omega^C = \Omega^X = 1/\gamma$. That is, in (20), the term ECE equals zero. Therefore, equation (20) indicates that the sign of the rate of change of exercise expenditure is ambiguous, solely depending on the difference between DPE and ROC, which are two positive terms.■

Thus, the term ECE is neither specific to our assumption that exercise is both a consumption and a time expenditure, nor to our assumption that exercise is a status symbol. It is also not present with all functional utility forms. By contrast, the terms DPE and ROC are specific to our model and are present for all utility's functional forms. For that reason, in what follows, we will focus on the roles of DPE and ROC. On their own, those terms cause a wedge between the growth rates of consumption and exercise, and generate the pattern for body weight gain. This mechanism is at the core of our results and holds for any utility function.

2.2 Firms

A unit mass of competitive firms produces a homogeneous output Y. The production process is described by a Cobb-Douglas production function: $Y = F(K, N) = K^{\eta}N^{1-\eta}$, where $0 < \eta < 1$ denotes the capital elasticity of production. Factors are paid their respective marginal products:

$$\frac{\partial F(K, N)}{\partial K} = r + \delta, \quad \frac{\partial F(K, N)}{\partial N} = w\,, \tag{22}$$

where $\delta \geq 0$ denotes the rate of depreciation. We introduce a normalization (per unit of labor): $y \equiv Y/N$. Noting that $k = K/N$, by homogeneity of degree 1, we can write:

$$y = F(k, 1) \equiv f(k) = k^{\eta}\,. \tag{23}$$

The first-order conditions (22) then become:

$$r(k) = f'(k) - \delta\,, \quad w(k) = f(k) - k\,f'(k)\,. \tag{24}$$

A linear production process transforms total consumption into food consumption and exercise consumption. Output can be used for either investment I or consumption such that $Y = p_C C + p_X X + I$.

2.3 Equilibrium

Definition (Equilibrium) A competitive equilibrium is a price vector (r, w, p_C, p_X) and an attainable allocation for all $t \geq 0$, such that:

1. Individuals choose feasible streams of C, X, K, to maximize intertemporal utility, given the stream of price vectors, initial wealth endowments, the individual DOP, and average capital.
2. Firms choose K and N in order to maximize profits, given the price vector.
3. All markets clear. Specifically, $\dot{K} = Y - p_C C - p_X X - \delta K$, the goods market clears, the capital market clears, and $N = \bar{L} - X$, the labor market clears.
4. Reference levels are: $\bar{X} = X$.

To study stability and ultimately enable comparisons between economies or population cross sections, we express the dynamic system in per unit of labor terms, such that $c \equiv C/N$, $x \equiv X/N$, and $\mathrm{w} \equiv W/N$. Furthermore, we express the dynamic system as a function of x and k. Noting that $x = X/N = X/(\bar{L} - X)$, we can re-write $X = x(\bar{L} - X)$, hence, $X = [x/(1+x)]\,\bar{L}$. Thus $X = X(x)$. Next, consider that $\hat{X} = (1 - \varepsilon(k))X(x)$ in equilibrium. Thus, $\hat{X} = \hat{X}(x, k)$. Finally, dividing (13) by (14), the resulting intratemporal first-order condition $U_C(C, \hat{X})/U_{\hat{X}}(C, \hat{X}) = p_C/\hat{p}_X(k)$ implicitly defines a relationship $C(\hat{X}(x, k), k)$. Since the arguments $\left(C, \hat{X}\right)$ can also be written in terms of (x, k), $\Omega^i(C, \hat{X}) = \Omega^i(x, k)$ and $\Phi^i(C, \hat{X}) = \Phi^i(x, k)$, $i = C, X$. Recall that: $\Delta = w/(p_X + w)$ and $\hat{p}_X = p_X + w$. Since w depends on k, we write $\Delta = \Delta\,(k)$. The positionality term $\varepsilon(k)$ is specified according to (5).

The dynamic system in per unit of labor terms becomes (see Appendix 7.2 for details):

$$\frac{\dot{k}}{k} = \left[1 - x\left[\kappa k - \Phi^X(x, k)\,\eta\Delta(k)\right]\right]^{-1}\left[x\,\Omega^X(x, k)\left(f'(k) - \delta - \rho\right) + \frac{f(k)}{k} - \delta - \frac{1}{k}\left(p_C c(x, k) + p_X x\right)\right], \tag{25}$$

$$\frac{\dot{x}}{x} = (1 + x)\left[\Omega^X(x, k)\left(f'(k) - \delta - \rho\right) + \left(\kappa k - \Phi^X(x, k)\,\Delta(k)\,\eta\right)\frac{\dot{k}}{k}\right]. \tag{26}$$

Therefore, the macroeconomic equilibrium gives rise to a dynamic system in two dimensions: k and x. Indeed, the dynamic system is separable in w, since the dynamic variables k and x affect body weight w, but w does not affect k and x, which is to be expected since weight is not a decision variable.[3] The dynamic system in normalized variables is given by (25) – (26): $\dot{k} = \dot{k}(k, x)$; $\dot{x} = \dot{x}(k, x)$.

We separately determine the change in body weight per unit of labor over time and steady state body weight. The normalized Schofield equation, also expressed with normalized variables, reads (see Appendix 7.4):

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C\frac{c(x, k)}{\mathrm{w}} - \left(\lambda_S\bar{s}\,(x) + \lambda_X x + \lambda_N\right) + \Omega^X(x, k)\left(f'(k) - \delta - \rho\right) + \left(\kappa k - \Phi^X(x, k)\,\eta\Delta(k)\right)\frac{\dot{k}}{k}\,, \tag{27}$$

[3]Notice the different fonts for weight (w) and wage (w).

where consumption $c(x,k)$ is derived from the intratemporal optimality condition (ratio of (13) and (14)), and solely depends on k and x, and where $\bar{s} = \frac{\bar{S}}{N}$. Since N is endogenous, and depends on X, which depends on x, $\bar{s} = \bar{s}(x)$.

A steady-state equilibrium is an equilibrium for which $\dot{k} = \dot{x} = 0$. Let k^* denote the steady state value of k, and $x^* = x(k^*)$, $w^* = w(k^*)$, $\hat{p}_X^* = p_X + w^*$, and $c^* = c(x^*, k^*)$. Considering the dynamic system (25) – (26), the steady state is described by the following system (see Appendix 7.3):

$$k^* = f'^{-1}(\delta + \rho)\,; \tag{28}$$

$$x^* = \frac{f(k^*) - \delta k^*}{p_X + p_C c(x^*, k^*)/x^*}\,, \tag{29}$$

where $f'^{-1}(.)$ denotes the inverse function of $f'(.)$. Equation (28) follows from our model-equivalent of the Keynes-Ramsey rule (equation (26)). The term $c(x^*, k^*)/x^*$ is implicitly given by dividing (13) by (14), which gives $U_C(C, \hat{X})/U_{\hat{X}}(C, \hat{X}) = p_C/\hat{p}_X(k)$. In case $u(C, \hat{X})$ is homothetic, the left-hand side is a function of $(C/\hat{X}) = c/[x(1-\varepsilon(k)]$. By strict concavity, an inverse function exists ($U_C/U_{\hat{X}}$ as a function of $C/\hat{X}$ is one to one). In this case, c^*/x^* is a function of k^*, allowing us to express x^* *explicitly.* [4]

As a consequence, neither our static nor our dynamic positionality effects impact the steady state level of k. Equation (29) determines the steady-state share of output to be devoted to exercise, which increases with positionality $\varepsilon(k^*)$. As $f'(k)$ is a strictly monotonous function, there exists only one value k^* satisfying (28). Therefore, the steady state (k^*, x^*) is unique. Similarly to the standard neoclassical growth model, since there is one predetermined and one jump variable, the unique steady state is a saddle point.

Once x^* and c^* are determined, we deduce the steady state body weight per unit of labor w^*:

$$\mathrm{w}^* = \left(\frac{\lambda_C c(x^*, k^*)}{\lambda_S \bar{s}(x^*) + \lambda_X x^* + \lambda_N} \right)\,. \tag{30}$$

3 Main theoretical results

In the theoretical section, in order to derive straightforward analytical results, we employ a CES utility function (such as 21). Considering that the intratemporal of substitution plays no qualitative role specific to the model, we assume $\zeta = 0$. In this case, where $1/\gamma$ is the intertemporal elasticity of substitution and $\alpha > 0$ represents the taste for food consumption relative to effective exercise consumption, $\Omega^C = \Omega^X = 1/\gamma$, $\Phi^C = -(1-\alpha)(1-\gamma)/\gamma$, $\Phi^X = (1 - \alpha(1-\gamma))/\gamma > 0$ and $\Phi^X + \Phi^C = 1$. Recall that it means that the term ECE equals zero and only the effects specific to our model, DPE and ROC, remain in place. In the steady state the intertemporal elasticity of substitution plays no role either, so γ can take any value without altering our results. We complement our theoretical results with a numerical section, in which we present dynamic and steady state quantitative results for a range of intra and intertemporal elasticities of substitution with CES utility (see Section 4).

[4] For example with constant elasticities of substitution in (21), $c^*/x^* = \alpha/(1-\alpha)\,(\hat{p}_X(k^*)/p_C)\,(1 - \varepsilon(k^*))$, and the steady state value of x becomes: $x^* = \frac{f(k^*) - \delta k^*}{p_X + \frac{\alpha}{1-\alpha}\hat{p}_X(k^*)(1-\varepsilon(k^*))}$.

3.1 Dynamic obesity Kuznets curve

In what follows, we show that the two opposing effects, DPE and ROC, suggest a dynamic Kuznets curve pattern for obesity. We also explain the puzzling fact that high income earners may increase exercise expenditure despite its rising opportunity cost.

Using the functional form (5), in normalized variables, the DPE and ROC are re-expressed as:

$$DPE = \frac{\dot{\varepsilon}(k)}{1-\varepsilon(k)} = \kappa \dot{k}\,; \tag{31}$$

$$ROC = \left(\underbrace{\Phi^X + \Phi^C}_{=1}\right)\Delta(k)\frac{\dot{w}}{w} = \frac{1}{\hat{p}_X}\left(-kf''(k)\right)]\dot{k}\,,, \tag{32}$$

Considering (20), the proof of Proposition 1, and that $\frac{\dot{X}}{X} - \frac{\dot{C}}{C} = \frac{\dot{x}}{x} - \frac{\dot{c}}{c}$, we have:

$$\frac{\dot{c}}{c} - \frac{\dot{x}}{x} = \underbrace{\frac{1}{\hat{p}_X}\left(-kf''(k)\right)\dot{k}}_{ROC} - \underbrace{\kappa\dot{k}}_{DPE}. \tag{33}$$

We can now shed light on the difference between the growth rates of food consumption c and exercise x in equilibrium.

Proposition 2: *In a growing economy ($\dot{k} > 0$), for a low level of k, $DPE < ROC$ and $\dot{x}/x < \dot{c}/c$. As k rises, there exists a $\underline{k}(\kappa)$ such that for all $k > \underline{k}(\kappa)$, $DPE > ROC$ and $\dot{x}/x > \dot{c}/c$.*

Proof: See Appendix 7.5.■

Corollary 1: *In a growing economy ($\dot{k} > 0$), for a high level of k, body weight growth eventually becomes negative.*

Proof: See Appendix 7.6■

Those results help us explain the evolution of obesity that goes with economic development as coming from a change in preferences connected to social status. The intuition is that positionality with respect to exercise is higher in the future than in the present. Consequently, ceteris paribus, the marginal utility of X increases over time. In response, individuals shift X from the present to the future, which implies a higher growth rate of exercise than in the absence of positionality. For low levels of economic development, Proposition 2 predicts that the growth rate of c exceeds the growth rate of x (note that $\frac{\dot{X}}{X} - \frac{\dot{C}}{C} = \frac{\dot{x}}{x} - \frac{\dot{c}}{c}$). As a consequence, the ratio c/x and body weight increase. At some point of economic development, as the stock of capital per worker exceeds a certain threshold, Proposition 2 predicts that the growth rate of x exceeds the growth rate of c. As a consequence, the ratio c/x and body weight decrease. Since the evolution of body weight is tied to the evolution of c/x, it is easy to show that the growth rate of body weight eventually decreases. Therefore, Proposition 2 and its corollary provide an explanation for the empirically estimated dynamic obesity Kuznets curve.

Corollary 2: *In a growing economy, if we ignore the DPE ($\kappa = 0$), $\dot{x}/x < \dot{c}/c$, $\forall k$.*

Proof: Straight from ((33)) and Proposition 1, in the absence of the DPE ($\kappa = 0$), the difference between $\frac{\dot{x}}{x} - \frac{\dot{c}}{c}$ is always negative, so that exercise expenses grow at a slower pace than food consumption.■

A consequence of Corollary 2 is that without dynamic positionality (DPE=0), the ratio c/x and body weight would never decrease. This would yield two counterfactual results. It would imply that exercise expenses always grow at a lower pace than food consumption, and that obesity always rises and never exhibits a Kuznets curve.

Additionally, ignoring that exercise is an expenditure would be equivalent to setting $p_X = 0$. This would decrease the relative cost of exercise and the coefficient Δ, directly increasing the ROC ceteris paribus. It would also modify the optimal choice of exercise relative to food consumption and therefore influence capital accumulation, and the marginal product of labor, which indirectly influences both DPE and ROC. The effects of relaxing the assumption of $p_X > 0$ on the dynamic Kuznets curve can therefore not be made explicit but will be studied in the numerical section.

3.2 Static obesity Kuznets curve

In what follows, we draw a parallel between mechanisms operating for the dynamic obesity Kuznets curve and the static obesity Kuznets curve. The following comparative static analysis can be considered a cross-sectional analysis, comparing countries or individuals with different steady state wealths k^*. It can also be considered a comparative static analysis of a single country for which a change in a technology or preference parameter causes a change in k^*.

In the steady state, the DPE and ROC are re-expressed as (see Appendix 7.8 for details):

$$ROC^* = \Delta\left(k^*\right) \frac{dw^*}{w^*} = \frac{\eta(1-\eta)\left(k^*\right)^{\eta-1}}{(1-\eta)\left(k^*\right)^{\eta} + p_X}$$

$$DPE^* = \frac{d\varepsilon(k^*)}{(1-\varepsilon(k^*))} = \kappa$$

The difference between the change of c^* and the change of x^* is:

$$\frac{dc^*}{c^*} - \frac{dx^*}{x^*} = \left[\underbrace{\frac{\eta(1-\eta)\left(k^*\right)^{\eta-1}}{(1-\eta)\left(k^*\right)^{\eta} + p_X}}_{ROC^*} - \underbrace{\kappa}_{DPE^*}\right], \tag{34}$$

where ROC^* represents the higher opportunity cost of time spent on exercise that goes with a higher marginal product of labor associated with a higher steady state capital stock, and DPE^* represents the positionality effect that denotes the increase in positionality associated with a higher steady state capital stock. The terms ROC^* and DPE^* are the steady state equivalents of the terms ROC and DPE presented in the dynamic setting. From this expression, we deduce Proposition 3, which is the steady state equivalent of Proposition 2.

Proposition 3: *For a low level of k^*, $DPE^* < ROC^*$ and $\frac{dx^*}{x^*} < \frac{dc^*}{c^*}$. As k^* is higher, there exists a $\underline{k^*}(\kappa)$ such that for all $k > \underline{k}^*(\kappa)$, $DPE^* > ROC^*$ and $\frac{dx^*}{x^*} > \frac{dc^*}{c^*}$.*

Proof: See Appendix 7.9.■

The comparison of body weight for different wealth levels is obtained by totally deriving body weight

(42) with respect to k^*, which yields:

$$\frac{d\mathrm{w}^*}{dk^*} = \frac{\lambda_C c^*/x^*}{z^*}\left[\frac{\frac{d(c^*/x^*)}{c^*/x^*} - \frac{dz^*}{z^*}}{dk^*}\right] \tag{35}$$

where $z(k^*) = \lambda_S \bar{s}(x^*)/x^* + \lambda_X + \lambda_N/x^*$ represents total calorie expenditure divided by exercise expenditure.

Corollary 3: *If we ignore the fact that exercise is a type of consumption expenditure, ROC^* is larger and it is less likely that $DPE^* > ROC^*$ and $\frac{dx^*}{x^*} > \frac{dc^*}{c^*}$.*

Proof: straightforward from Proposition 3 and the term ROC^* that unambiguously decreases when p_X increases.■

Therefore, acknowledging that exercise is an expenditure, is a mechanism that increases ROC^*, increases the gap between the growth rate of consumption and the growth rate of exercise, and limits the possibility of a static Kuznets curve.

Proposition 4: *For high values of k^*, $\frac{d\mathrm{w}^*}{dk^*} < 0$ if $\kappa > \underbrace{-\frac{dz^*}{z^*}}_{(+)}$, and positive otherwise.*

Proof: See Appendix 7.10.■

As shown in the proof, for high values of k^* the term $\frac{d(c^*/x^*)}{c^*/x^*}$ converges toward $-\kappa$. At the same time, $\frac{dz^*}{dk^*}/z^* < 0$, which implies that the ratio of total calorie expenditure over exercise decreases. For for high values of k^*, the percentage change in c^*/x^* is negative and converges to the negative of the degree of positionality, which has a negative effect on body weight. However, the percentage change in z^*, the ratio of total calorie expenditure over exercise decreases, which has a positive effect on body weight. Equivalently, when the degree of positionality is large enough, the decrease in the ratio of calorie intake over exercise expenditure is large and results in lower body weight as long as it dominates the effect of the decrease in the ratio of total calorie expenditure over exercise.

Corollary 4: *For high values of k^*, if we ignore the DPE ($\kappa = 0$), $\frac{d\mathrm{w}^*}{dk^*} > 0$.*

Proof: straightforward from Proposition 4, $0 < \underbrace{-\frac{dz^*}{z^*}}_{(+)} \Rightarrow \frac{d\mathrm{w}^*}{dk^*} > 0$.■

The DPE^* generates substitution toward exercise and drives the negative correlation between the stock of capital and body weight for high values of steady state capital stock. In the absence of the DPE^*, for high values of steady state capital stock, the main effect is the ROC^* that unambiguously results in a decrease of exercise and a higher steady state body weight.

Therefore, the introduction of dynamic positionality helps explain why, despite a higher opportunity cost of exercise, rich individuals may exercise more than poor individuals. With Proposition 2 and Corollary 1, we showed that the evolution of the stock of capital over time generates ROC and DPE, eventually yielding an obesity Kuznets curve. With Proposition 3, we explain how differences in ROC^* and DPE^* are tied to different steady state capital stocks between poor and rich countries (or individuals). With Corollary 3, we show that acknowledging that exercise is a consumption expenditure constitutes a mechanism that raises the ROC^* and competes with the DPE^* . Corollary 4 is consistent with evidence presented in the introduction that $d\mathrm{w}^*/dk^* > 0$ for poor countries (or individuals) and $d\mathrm{w}^*/dk^* < 0$ for rich countries (or individuals), and suggests that dynamic positionality associated with the consumption of time consuming goods plays a larger role for rich than poor countries (or individuals).

4 Numerical results

We supplement the qualitative analysis with a quantitative analysis relying on numerical simulations. Our numerical simulations illustrate our theoretical results and give a sense of magnitudes regarding the role of positional exercise expenditures regarding the steady state and dynamic Kuznets curves. [5]

4.1 Calibration

First, we set parameters for the Schofield equation. We use data on time use, calories spent exercising, and basal metabolic rate to calibrate the parameters of the Schofield equation.

An individual spends about 2000 hours per year working (40 hours times 50 weeks) out of 8736 total hours in a year (24 hours time 7 days times 52 weeks). The time that is not spent working represents 6736 hours of leisure time (8736 - 2000 = 6736) and is split between sedentary leisure and exercise. With exercise time representing only 122 hours (20 minutes times 365 days divided by 60), sedentary leisure represents 6614 hours (6736 -122 = 6614). As a result, sedentary leisure represents a fraction equal to $\bar{S} = 6614/8736 = 75.7\%$ of total time, and time that is not spent on sedentary leisure represents a fraction of $\bar{L} = 1 - 0.757 = 24.3\%$ of total time. This time is split between exercise, which represents a fraction equal to 122/8736=1.4% of total time, and work, which represents a fraction equal to 2000 / 8736 = 22.9% of total time. The average of men and women average weight in the US is 185 pounds. On average, an individual at rest spends $\lambda_S \bar{S} W = 1577.5$ calories per day. As a result, $\lambda_S = 1577.5/(0.757 * 185) = 11.26$.

Based on the yearly American time use survey by the Bureau of Labor Statistics (BLS), time spent "participating in sports, exercise and recreation", measured since 2003, represents 0.33 hours (20 minutes) a day for the average individual. How many calories do those activities burn? The 2008 BLS Spotlight on Statistics indicates that the three most popular types of exercise are walking, weightlifting, and using cardiovascular equipment (see Figure A2 in the appendix). For a 185 pound individual, we estimate that these activities respectively burn 2.0, 1.4 and 4.7 calories per pound per hour. The calories burnt are taken from the chart provided by Harvard Medical School and activities specifically correspond to walking 4 miles per hour, general weight lifting, and high impact step (or vigorous rowing). We estimate that calories burnt by an individual splitting their exercise time among those three activities, weighted by their popularity, equals 2.5 calories per pound per hour. The details of our calculation are provided in Table A1 in the appendix. Thus, an average individual of 185 pounds, spends $2.5 * 185 = 462.5$ calories per hour. Since individuals exercise 20 minutes a day, they spend $\lambda_X X W$ =154 calories per day. As a result, $\lambda_X = 154/(0.014 * 185) = 59.5$.

Based on the U.S. Department of Health and Human Services (HHS), we use the average of daily caloric expenditure of men and women, which equals ((1600+2400)/2+(2000+3000)/2)/2=2250. With 1577.5 calories spent in sedentary leisure and 154 calories spent exercising, calories spent at work are $\lambda_N N W = 2250 - 1577.5 - 154 = 518.5$. As a result, $\lambda_N = 518.5/(0.23 * 185) = 12.2$.

In order to estimate parameter λ_C (energy density of food), we use the Schofield equation, written for a stationary body weight: $\lambda_C = \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right) W/N}{C} = \frac{2250}{4} = 562.5$ per unit of food expenditure per day. As explained above, the numerator represents total calorie expenditure per day. The denominator represents the quantity of food consumed per day, which is between 3 and 5 pounds. We take 4 pounds for the baseline calibration. As a result, parameter λ_C represents energy intake per unit of food expenditure per day. Note that there is no distinction between men and women or individuals of different ages within households in the per quintile food consumption data.

[5] We rely on Mathematica and use the relaxation algorithm to obtain dynamic results.

We need a consistent estimate for food and exercise, so we use cost per day. To estimate the price of exercise, we use the work by Valero-Elizondo (2016), who estimate the marginal benefit of exercising (which, in equilibrium, equals the marginal cost) equal to \$2500 per year. We divide it by by 365 days and by the time a person spends exercising daily to obtain p_X, the price per day. Note that the marginal benefit (or cost) of exercising is high as it represents more than the sole expenditure on exercise related activities or goods in that case. It also encompasses additional benefits in the form of savings on health care expenditure due to better health outcomes. So it overstates the direct cost of goods and services connected to calorie expenditure.

The average amount spent on food since 2003 is \$2,251 per person year. We obtain this number as follows. Based on the Consumer Expenditure Survey, aggregate food consumption expenditure per year equals 776,647 millions of dollars. The share of aggregate food expenditure coming from middle income households is 17.8%. The average number of consumer units in the middle income category is 24,560 thousands and the number of person per consumer unit is 2.5. As a result, 776,647,000,000 * 17.8% / 24,560,000 / 2.5 = \$2,251 per year per person. We divide this number by 365 days and by the quantity of food a person consumes in a day (4 pounds) to obtain p_C, the price per pounds.

The model is simulated using a production function with a unit elasticity of substitution between consumption and effective exercise ($\zeta = 0$) and a unit intertemporal elasticity of substitution ($\gamma = 1$). We estimate the remaining yearly parameters. They are consistent with the standard range of parameters of theoretical growth models (cf. Turnovsky, 2000; Barro and Sala-i-Martin, 2003) and are adjusted to reproduce some essential features of the US economy. We set the production elasticity of capital η , the rate of depreciation δ , and the rate of time preference ρ to obtain a capital output ratio of 3 (e.g. Burda and Wyplosz, 2017, p.64).

Parameter α, which represent the taste for consumption relative to leisure, and parameter κ, which represent the exogenous component of the degrees of positionality influence steady state positionality, consumption, calorie expenditure and body weight. However, neither are directly observable. We set those parameters to reflect the following empirical observations. Our calibration generates a degree of positionality close to 0.4, which is consistent with experimental estimates,[6] and a ratio of exercise related expenditure over food expenditure close to one.

Additionally, the food consumption-income ratio is 10% in the USA and total consumption in output is about 65%. Because our model has only one good, which is food consumption, if we set parameters to reproduce a consumption output ratio of 10 percent, it would mean that most resources are allocated to investment. The economy and food consumption would grow too fast compared to reality. At the same time, if we set parameters to reproduce a consumption output ratio of 65 percent, food consumption and its effect on body weight would largely be overstated. With our parameters, we obtain a ratio of for food consumption in total output between 10 and 65 percent. Parameters are presented in Table 1. Table 2 shows the fit between actual and simulated steady state economies.

Table 1: Parameters

λ_X	λ_N	λ_S	λ_C	$\bar{S}$	p_X	p_C	ρ	δ	η	α	κ	ζ	γ
59.5	12.2	11.26	562.5	0.757	20.76	1.54	0.05	0.05	0.3	0.6	0.1	0	1

[6]Quasi-experimental research provides estimates in the 0.2–0.6 range (see, e.g., Johansson-Stenman et al., 2002; Clark and Senik, 2010; Carlsson et al., 2007; and the overview by Wendner and Goulder, 2008).

Table 2: Actual and simulated economies				
	k/y	$(\hat{p_X}\ X)/(p_C\ C)$	DOP	C/Y
Actual US economy	3	1.1	0.4	between 0.10 and 0.65
Simulated US economy	3	1	0.4	0.28

4.2 Results

In this section, we simulate steady state body weight for different steady-state levels of the stock of capital per worker to explore the possibility of a static Kuznets curve for obesity in the USA. We also study the evolution of body weight toward its steady state as the stock of capital increases over time to study the possibility of a dynamic obesity Kuznets curve in the USA. Additionally, we conduct sensitivity analysis for the static and dynamic Kuznets curves.

4.2.1 Baseline scenario

First, we build the curve that connects steady-state body weight to the steady-state capital stock. Recall that in our model, positionality has an endogenous component tied to the level of capital per worker. In the steady state, the stock of capital is given by exogenous parameters, which are the rates of time preference and depreciation, and the production elasticity of capital. These parameters have an impact on the steady-state per worker capital stock k* and steady-state weight. We simulate the steady-state weights by directly changing k*, going from the baseline steady-state capital stock to two times its value.[7] Since positionality also has an exogenous component κ related to idiosyncratic differences between individuals or countries, we provide simulations for three different levels of κ, at the baseline value of 0.10 and close to it. The static relation between obesity and the stock of capital per worker is presented on the left graph in Figure 1. Second, we build the curve showing the evolution of body weight over time toward the current steady state. The dynamic relation between obesity and the stock of capital per worker is presented on the right graph in Figure 1.

The current steady state body weight for the US is 185 pounds, which is the starting point for the static relation between body weight and the stock of capital per worker in our baseline scenario. We find the existence of a static Kuznets curve: for the baseline level of κ, the steady state level of average body weight increases with the average stock of capital up to a level of 186.5 pounds, corresponding to a stock of capital per worker 25% higher than its the baseline, and decreases thereafter. The first interpretation of this finding is that the US economy's tipping point of the Kuznets curve is not yet reached. It will be reached once the stock of capital per worker is 25% higher than its baseline in the simulations.

The second interpretation of this result is that the steady state average body weight starts being inversely related to wealth when individual wealth is 25% above the average wealth in the US, and is positively related to wealth below this threshold. Furthermore, when the parameter κ is higher than in the baseline, DPE* is relatively stronger than ROC*. As a consequence, as expected, steady state body weight is lower and the obesity Kuznets curve is even more concave as individuals choose to exercise more. Thus our simulations also show that exogenous differences in the degree of positionality between individuals or countries may yield different Kuznets curves.

By contrast, the dynamic evolution of body weight towards its steady state value of 185 points does not

[7] Note that what we called per worker variables are in fact per hours worked variables and not per worker variables. To obtain meaningful body weight levels, we normalize the steady state body weight at 185 pounds in the initial steady state.

show a dynamic Kuznets curve pattern for body weight in our baseline scenario. It means that given the level of κ, the model shows that the increase in the capital stock has so far generated a DPE that has resulted in slowing down the growth rate of body weight but that it has not been sufficient to produce a dynamic obesity Kuznets curve. For both the static and the dynamic relations, the higher κ, the more prevalent the positionality effect, the lower the level of steady state body weight.

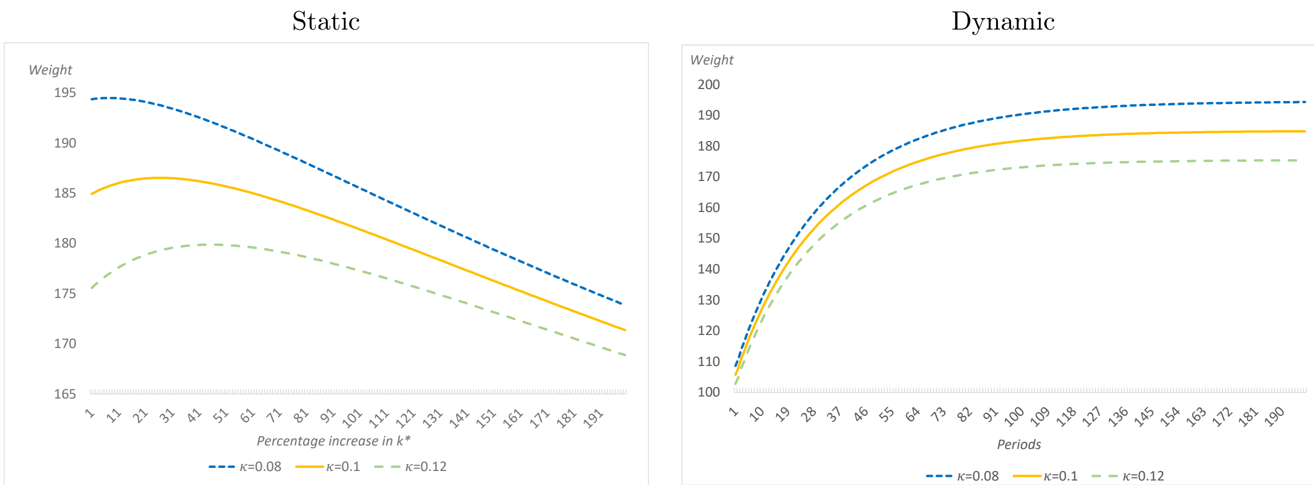

Figure 1: Body weight and capital stock with different degrees of positionality

The existence of a static Kuznets curve is consistent with results of empirical studies for the USA presented in the introduction. Given the exogenous degree of positionality κ, the stock of capital per worker would need to be 25 percent higher for the DPE* to be large enough and steady state body weight to decrease. The absence of the Kuznets curve to this date is also consistent with US data showing that obesity has increased over time and its growth rate has slowed down (see Figure A3 in the appendix). The DPE may explain the slowdown of the evolution of obesity in the USA as positional individuals have allocated more time toward exercise as the economy developed, but has so far been insufficient on its own to produce a decrease in average weight gain.

4.2.2 Sensitivity analysis

In this subsection, we first show the effect of considering exercise as a consumption expenditure on the static and dynamic Kuznets curves by varying the cost of exercise expenditure from $p_X = 0$ (absence of consumption expenditure related to exercise) up to the parametrized cost of 20.76. Furthermore, in our numerical simulations, recall that the baseline scenario corresponds to unit elasticities, with the parameters being set at $\zeta = 0$ for the elasticity of substitution between food consumption and effective exercise, and $\gamma = 1$ for the intertemporal elasticity of substitution. We estimate the static and dynamic relations between body weight and the stock of capital per worker with different elasticities of substitution.

We simulate the static and dynamic relations between body weight and the stock of capital per worker with the price of consumption expenditure ranging from 0 to 20.76. The results are presented in Figure 2.

When the price of exercise expenditure decreases, the static and dynamic Kuznets curves shifts down. The reason is that a lower price of exercise lowers ROC* thereby encouraging exercise. The increase in calorie expenditure results in lower body weight for all levels of the per worker stock of capital.

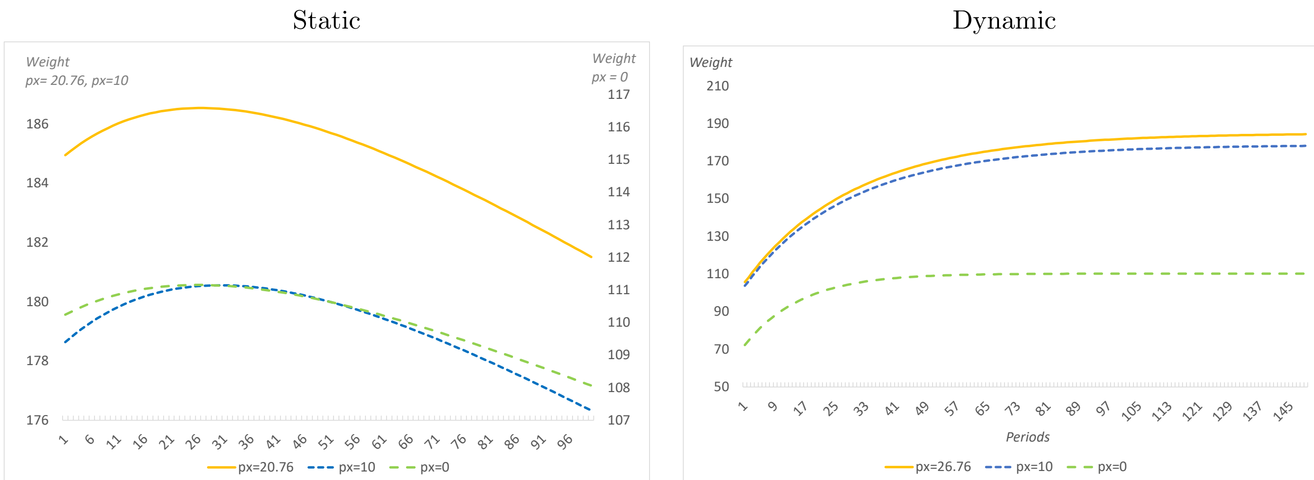

Figure 2: Body weight and capital stock with different consumer prices for exercise

We simulate the static and dynamic relations between body weight and the stock of capital per worker with elasticities of substitution between consumption and effective leisure ranging from 0.66 to 2 (corresponding to ζ ranging from -0.5 to 0.5). The results are presented in Figure 3.

When the elasticity of substitution between food consumption and effective exercise becomes larger than in the baseline scenario and equal to 2, with $\zeta = 0.5$, then, as the stock of capital becomes higher in the steady state, individuals substitute food consumption for exercise more easily than in the baseline with unit elasticity of substitution. As a consequence, the relation between weight and the per worker capital stock is decreasing as the steady-state stock of capital becomes higher. In this case, the relation between steady state weight and the per worker capital stock is not a Kuznets curve. In other words, the DPE* dominates the ROC* for all steady-state levels of per worker capital. In contrast, when the elasticity of substitution between food consumption and effective exercise becomes smaller than in the baseline scenario and equal to 0.66, with $\zeta = -0.5$, individuals substitute food consumption for exercise less easily as the stock of capital becomes higher in the steady state. The relation between steady-state weight and the per worker capital stock is a Kuznets curve. In this case, until a certain level of steady-state stock of capital per worker, the ROC* dominates and weight increases. Beyond this level, the DPE* dominates, and weight decreases as the steady-state stock of capital becomes higher. The static Kuznets curve is also present in the baseline case (solid curve) but less pronounced than for the lower CES with $\zeta = -0.5$.

The dynamic relation between weight and the per worker capital stock does not exhibit a Kuznets curve pattern for a wide range of elasticities of substitution between food consumption and effective exercise. For both the static and dynamic relations, the more food consumption and effective exercise are complements, the more prevalent the positionality effect, the lower the level of steady state body weight.

To our knowledge, there is no empirical estimate of the elasticity of substitution between food consumption and exercise. If food consumption and positional exercise are substitutes, we should see a decrease in steady state weight happening much earlier that in our baseline. If we consider that when people exercise, they also eat more, then food consumption and effective exercise may be complements and the relation between steady state weight and stock of capital per worker could exhibit a Kuznets curve. However, the model predicts that with an elasticity of substitution of 0.66, the tipping point of the Kuznets curve would happen at a steady-state stock of capital 67 percent higher and a body weight of 344 pounds, which is a more pessimistic scenario than the baseline scenario.

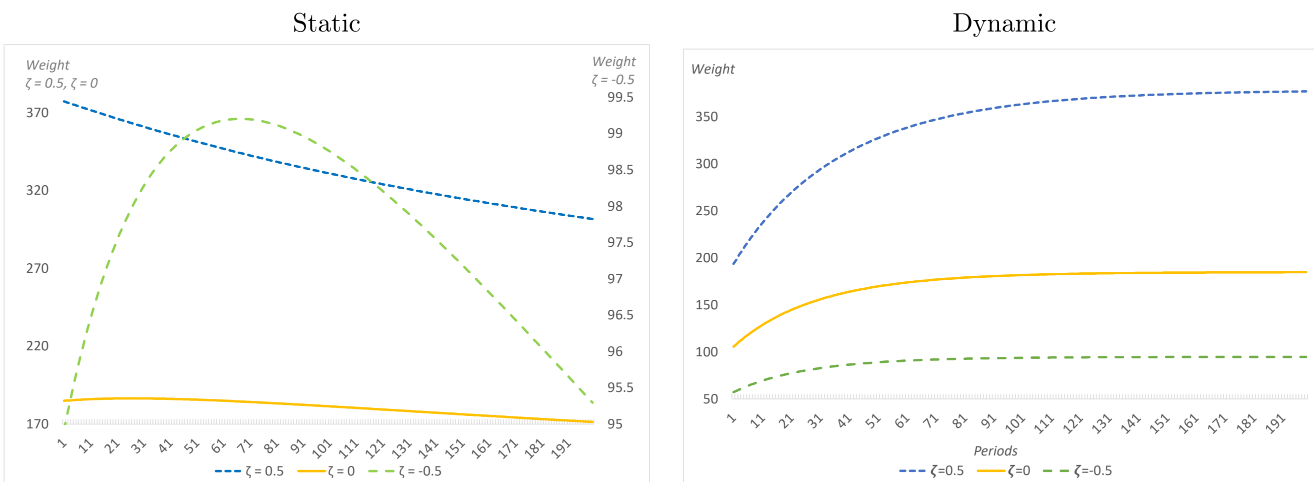

Figure 3: Body weight and capital stock with different elasticities of substitution between food consumption and effective exercise

We simulate the relation between body weight and the stock of capital per worker with intertemporal elasticities of substitution ranging from 0.33 to 1 (corresponding to γ ranging from 3 to one). By definition, the intertemporal elasticity of substitution does not affect the steady state but it modifies the dynamic evolution of weight as the stock of capital builds up. The results are presented in Figure 4.

When the intertemporal elasticity of substitution becomes smaller, from 1 to 0.33 (as γ goes from 1 to 3), the dynamic relation between weight and the stock of capital per worker becomes flatter. As γ increases, the effect of the ROC on the difference between the growth rate of exercise and the growth rate of food consumption, $\frac{\dot{x}}{x} - \frac{\dot{c}}{c}$, becomes smaller, and the effect of the DPE becomes dominant, explaining that the relation between obesity and capital becomes less and less positive. As the DPE become dominant, individuals postpone net calorie intake to the future, which flattens the evolution of body weight.

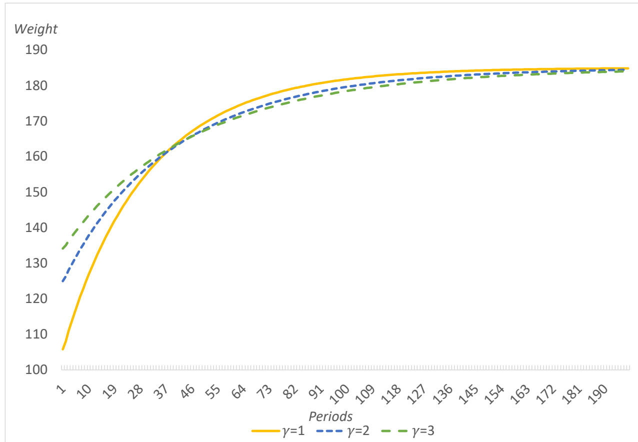

Figure 4: Dynamic of body weight with different intertemporal elasticities of substitution

While in the baseline scenario, the Kuznets curve is generated for low values of κ matching empirical estimates of positionality, we further explore the role of a higher degree of positionality in generating a dynamic Kuznets curve and present it in Figure 5. We simulate the evolution of body weight towards its steady state as the stock of capital increases over time for parameter values of κ ranging from 0.78 to 0.82. In that range of values for κ, there are very large differences in the level of body weight: the higher κ, the lower the body weight. The magnitude of those differences is too large to put the results on the same graph. For that reason, instead of presenting the level of body weight, in Figure 5, we present the percentage change in body weight with respect to its steady state value as the stock of capital increases. With high values of κ, we obtain obesity Kuznets curves: Body weight starts below the steady state value and the percentage change with respect to the steady state becomes less negative, which means that body weight increases toward its steady state value. After a number of periods, body weight overshoots its steady state value, and then decreases to its steady state value. For high values of κ, a dynamic Kuznets curve regularly occurs (and is not sensitive with respect to the specific value of κ, as was the case for the lower values of κ discussed in relation to the baseline case).

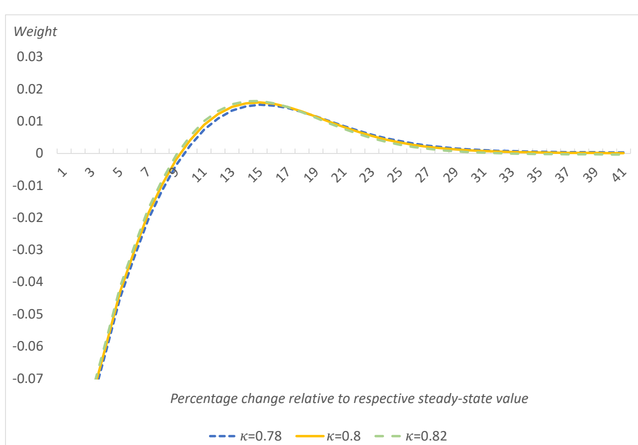

Figure 5: Dynamic of body weight gain with high degrees of positionality

5 Conclusion

Our model expands the theoretical literature which to this date has essentially focused on food consumption choices, and assumed exogenous preferences. We acknowledge the importance of calorie expenditure to maintain healthy body weights and the fact that preferences change over time. We build and simulate the first theoretical growth model that combines Becker's (1965) theory of the allocation of time and Veblen's (1899) theory of conspicuous leisure, and focus on exercise choices and changing preferences to explain obesity-income patterns.

While in some countries, data suggest the existence of a dynamic obesity Kuznets curve, and the empirical literature estimates static obesity Kuznets curves, we show the existence of both a dynamic and a static Kuznets curve for obesity and provide a novel explanation of the mechanisms generating the dynamic and static Kuznets curves. Our dynamic model shows that the difference between the growth rate of consumption and the growth rate of exercise reflects the difference between the rise in the opportunity cost of exercise and the change in positional behavior with respect to exercise. We formally demonstrate the existence of a level stock of capital per worker for which the growth rate of exercise starts exceeding the growth rate of food consumption. These mechanisms apply to a dynamic environment in which the stock of capital per worker builds up over time and a static environment used for cross-sectional analysis of countries or individuals with different income levels, explored through comparative statics.

Furthermore, we show that ignoring dynamic positionality with respect to exercise choices would yield a positive correlation between economic development and weight gain at all levels of economic development. By contrast, in the presence of dynamic positionality with respect to exercise, for high levels of economic development, we show that body weight gain becomes negative as the economy develops over time. The static analysis indicates that for high levels of steady-state stock of capital per worker, the link between body weight and steady state capital stock is negative only for relatively high degrees of positionality.

Our model therefore explains the rise in obesity over time as the result of economic growth and the rise in the opportunity cost of exercise. It also explains the change from a positive correlation between obesity and income for low income earners and low income countries to a negative correlation between obesity and income for high income earners and high income countries as the result of dynamic positionality and the fact that exercise is a type of expenditure. These results also suggest that the static and dynamic Kuznets curve for obesity are typically different.

We supplement the qualitative analysis with a quantitative analysis relying on numerical simulations. We find two different relations between body weight and the stock of capital per worker. To this date, the simulated US economy shows that the dynamic evolution of body weight does not show a dynamic Kuznets curve pattern for obesity in the USA. However, we find a static Kuznets curve for the USA: the steady state level of average body weight increases with the average stock of capital up to a level of 186.5 pounds, corresponding to a stock of capital per worker 25% higher than its current steady state level, and decreases thereafter.

We acknowledge a few caveats in our work. Neither our behavioral model, nor our simulations distinguish between men and women, while the data show differences in the income obesity relation between men and women. This task is limited by two elements. One is the lack of empirical knowledge on behavioral differences with respect to food consumption and exercise which would force us to make arbitrary assumptions in a theoretical model to distinguish between the two types of behaviors. The other is the lack of data that distinguishes between men and women with respect to food consumption and exercise and prevents refining numerical simulations. Additionally, for tractability reasons, we solely account for positionality with respect to exercise and not with respect to other types of consumption such as low-calorie foods, which could increase the possibility of dynamic and static Kuznets curves in the USA. Nevertheless, our model is the first to account for the role of exercise, making it explicit and sheding light on its role in generating two distinct Kuznets curves.

A natural extension of this work is to use our model to study the effect of exercise subsidies. While calorie expenditure is an important factor in weight gain, in the US, government policies aiming at encouraging exercise have been limited. However, Cawley's (2015) literature review underlines that innovative physical exercise programs that challenge and interest youth could be effective in preventing obesity. In the USA, the most recent federal level initiative involving calorie expenditure is the Let's Move program led by former First Lady Michelle Obama. Additionally, a larger and larger number of employers are interested in offering financial incentives for heathy behavior. Mukhopadhyay and Wendel (2013) explain that in the short term, work-place incentives successfully recruited a broad spectrum of participants among employees and improve health behavior. However, wellness programs are not reaching all employees. Small business (employing less than 500 employees) currently employ more than half of the private sector workforce but less than five percent of worksites with 50 to 99 employees offer comprehensive workplace health programs. Goetzel (2016) presents the potential benefits for the government to implement those prevention programs with its own employees and small businesses. It is therefore important to understand the effect of exercise subsidies on obesity, distinguishing between subsidies to consumers as opposed to employers. Our model provides a starting framework for this analysis since it accounts for the fact that the cost of exercise is both an expenditure and an opportunity cost and provides a vehicle to study various types of exercise subsidies. It also suggests that subsidies targeting the price of exercise expenditure (p_X) may have a different quantitative effect from those targeting wages (w) because they affect the ROC in different ways. By lowering p_X to show the role of exercise as an expenditure, we already showed that a subsidy lowering the cost of exercise would result in lower body weight. Exploring the effects of subsidies further should be the focus of future investigations.

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7 Appendix

7.1 Solution to the individual's optimization problem (equations (18) and (19))

Based on the first-order conditions (13), (14), and (15), the intratemporal optimality condition becomes

$$\frac{U_{\hat{X}}(C,\hat{X},W)}{U_C(C,\hat{X},W)} = \frac{u_{\hat{X}}(C,\hat{X})}{u_C(C,\hat{X})} = \frac{p_{\hat{X}}}{p_C},$$

due to separability of the utility function in W. Differentiating the first-order conditions with respect to time yields

$$\frac{\dot{U}_C(C,\hat{X},W)}{U_C(C,\hat{X},W)} = \frac{u_{CC}(C,\hat{X})C}{u_C(C,\hat{X})}\frac{\dot{C}}{C} + \frac{u_{C\hat{X}}(C,\hat{X})\hat{X}}{u_C(C,\hat{X})}\frac{\dot{\hat{X}}}{\hat{X}} = \frac{\dot{\mu}}{\mu} = -(r-\rho),$$

and

$$\frac{\dot{U}_{\hat{X}}(C,\hat{X},W)}{U_{\hat{X}}(C,\hat{X},W)} = \frac{u_{\hat{X}C}(C,\hat{X})C}{u_{\hat{X}}(C,\hat{X})}\frac{\dot{C}}{C} + \frac{u_{\hat{X}\hat{X}}(C,\hat{X})\hat{X}}{u_{\hat{X}}(C,\hat{X})}\frac{\dot{\hat{X}}}{\hat{X}} = \frac{\dot{\mu}}{\mu} + \frac{\dot{\hat{p}}_X}{\hat{p}_X} = -(r-\rho) + \Delta\frac{\dot{w}}{w}.$$

Let $e_{ij},\ i,j \in \left\{C,\hat{X}\right\}$ define the elasticities $u_{ij}(i,j)j/u_i$. Then, the above growth rates can be written as (suppressing the arguments of the elasticity functions):

$$e_{CC}\frac{\dot{C}}{C} + e_{C\hat{X}}\frac{\dot{\hat{X}}}{\hat{X}} = -(r-\rho),$$

and

$$e_{\hat{X}C}\frac{\dot{C}}{C} + e_{\hat{X}\hat{X}}\frac{\dot{\hat{X}}}{\hat{X}} = -(r-\rho) + \Delta\frac{\dot{w}}{w}.$$

Considering both equations and collecting terms yields:

$$\frac{\dot{C}}{C} = \left[\frac{e_{\hat{X}\hat{X}} - e_{C\hat{X}}}{-e_{CC}e_{\hat{X}\hat{X}} + e_{C\hat{X}}e_{\hat{X}C}}\right](r-\rho) + \left[\frac{e_{C\hat{X}}}{-e_{CC}e_{\hat{X}\hat{X}} + e_{C\hat{X}}e_{\hat{X}C}}\right]\Delta\frac{\dot{w}}{w},$$

and

$$\frac{\dot{\hat{X}}}{\hat{X}} = \left[\frac{e_{CC} - e_{\hat{X}C}}{-e_{CC}e_{\hat{X}\hat{X}} + e_{C\hat{X}}e_{\hat{X}C}}\right](r-\rho) - \left[\frac{e_{CC}}{-e_{CC}e_{\hat{X}\hat{X}} + e_{C\hat{X}}e_{\hat{X}C}}\right]\Delta\frac{\dot{w}}{w}.$$

Considering the definitions Ω^C, Ω^X, Φ^C and Φ^X we can re-write the above equations as:

$$\frac{\dot{C}}{C} = \Omega^C\left(C,\hat{X}\right)(r-\rho) + \Phi^C\left(C,\hat{X}\right)\Delta\frac{\dot{w}}{w},$$

and

$$\frac{\dot{\hat{X}}}{\hat{X}} = \Omega^X\left(C,\hat{X}\right)(r-\rho) - \Phi^X\left(C,\hat{X}\right)\Delta\frac{\dot{w}}{w}.$$

In equilibrium, $\hat{X} = X(1-\varepsilon(k))$, that is:

$$\frac{\dot{\hat{X}}}{\hat{X}} = \frac{\dot{X}}{X} - \frac{\dot{\varepsilon}(k)}{1-\varepsilon(k)},$$

so:

$$\frac{\dot{X}}{X} = \Omega^X\left(C,\hat{X}\right)(r-\rho) + \frac{\dot{\varepsilon}(k)}{1-\varepsilon(k)} - \Phi^X\left(C,\hat{X}\right)\Delta\frac{\dot{w}}{w}.$$

7.2 Derivation of the dynamic system in k and x (equations (25) and (26))

Considering our functional specification of $\varepsilon(k)$,

$$\frac{\dot{\varepsilon}(k)}{1-\varepsilon(k)} = \frac{\varepsilon'(k)k}{1-\varepsilon(k)}\frac{\dot{k}}{k} = (\kappa k)\frac{\dot{k}}{k}.$$

As $\dot{w}/w = \eta\,\dot{k}/k$, and considering that $\hat{X} = \hat{X}(x,k)$ and $C = C(x,k)$ – see Subsection 2.3 – we re-write the growth rate of X:

$$\frac{\dot{X}}{X} = \Omega^X(x,k)(r-\rho) + \left[\kappa k - \Phi^X(x,k)\Delta\eta\right]\frac{\dot{k}}{k}.$$

As $x = X/N$, $\dot{x}/x = \dot{X}/X - \dot{N}/N$. We note that $\dot{N}/N = -X/(\bar{L}-X)\left(\dot{X}/X\right) = -x\left(\dot{X}/X\right)$. Thus, $\dot{x}/x = (1+x)\dot{X}/X$:

$$\frac{\dot{x}}{x} = (1+x)\left[\Omega^X(x,k)(f'(k)-\delta-\rho) + \left[\kappa k - \Phi^X(x,k)\Delta\eta\right]\frac{\dot{k}}{k}\right]. \tag{36}$$

Next, $\left(\dot{k}/k\right) = \left(\dot{K}/K\right) - \left(\dot{N}/N\right) = \left(\dot{K}/K\right) + x\left(\dot{X}/X\right)$. We consider the resource constraint $\dot{K} = f(k)N - \delta K - p_C C - p_X X$:

$$\frac{\dot{k}}{k} = \left(\frac{\dot{K}}{K}\right) + x\frac{\dot{X}}{X} = \left(\frac{f(k)}{k} - \delta - p_C\frac{c}{k} - p_X\frac{x}{k}\right) + x\left\{\Omega^X(x,k)(r-\rho) + \left[\kappa k - \Phi^X(x,k)\Delta\eta\right]\frac{\dot{k}}{k}\right\},$$

thus,

$$\frac{\dot{k}}{k}\left\{1 - x\left[\kappa k - \Phi^X(x,k)\Delta\eta\right]\right\} = \left(\frac{f(k)}{k} - \delta - p_C\frac{c}{k} - p_X\frac{x}{k}\right) + x\,\Omega^X(x,k)(r-\rho),$$

and as a result:

$$\frac{\dot{k}}{k} = \left\{1 - x\left[\kappa k - \Phi^X(x,k)\Delta\eta\right]\right\}^{-1}\left[x\,\Omega^X(x,k)(f'(k)-\delta-\rho) + \frac{f(k)}{k} - \delta - p_C\frac{c(x,k)}{k} - p_X\frac{x}{k}\right]. \tag{37}$$

7.3 Derivation of the steady state (equation (29))

By definition, $\dot{k} = 0$. From (36), for $\dot{x} = 0$, $f'(k^*) = \delta + \rho$. By strict concavity of $f(k)$, $f'(k)$ is one-to-one, and an inverse function $f'^{-1}(.)$ exists. Thus, we can express $k^* = f'^{-1}(\delta + \rho)$. Next, from (37), and considering that $f'(k^*) = \delta + \rho$, the requirement for $\dot{k} = 0$ is given by:

$$\left[\frac{f(k^*)}{k*} - \delta - p_C \frac{c(x^*, k^*)}{k^*} - p_X \frac{x^*}{k^*}\right] = 0 \Longleftrightarrow x^* = \frac{f(k^*) - \delta k^*}{p_X + p_C \frac{c(x^*,k^*)}{x^*}} \ ,$$

which implicitly defines x^*.

7.4 Derivation of the Schofield equation with normalized variables (equation (27))

We now derive (27). We rewrite the Schofield equation (38) as:

$$\dot{W} = \lambda_C C - \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right)}{N} W, \tag{38}$$

$$\frac{\dot{W}}{W} = \lambda_C \frac{C}{W} - \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right)}{N} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} + \frac{\dot{N}}{N}. \tag{39}$$

Since:

$$\frac{\dot{N}}{N} = -x\frac{\dot{X}}{X} = -\frac{1}{1+x}\frac{\dot{x}}{x},$$

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C \frac{C}{N}\frac{N}{W} - \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right)}{N} + \frac{1}{1+x}\frac{\dot{x}}{x}.$$

Substituting (36) in this expression yields:

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C \frac{c}{\mathrm{w}} - \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right)}{N} + \Omega^X(x,k)\left(f'(k) - \delta - \rho\right) + \left(\kappa k - \Phi^X(x,k)\,\eta(k)\,\Delta(k)\right)\frac{\dot{k}}{k},$$

which is equivalent to:

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C \frac{c}{\mathrm{w}} - \left(\lambda_S \bar{s} + \lambda_X x + \lambda_N\right) + \Omega^X(x,k)\left(f'(k) - \delta - \rho\right) + \left(\kappa k - \Phi^X(x,k)\,\eta(k)\,\Delta(k)\right)\frac{\dot{k}}{k}.$$

7.5 Proof of Proposition 2

Throughout, we consider a growing economy, $\dot{k} > 0$, so that the sign of the difference $ROC - DPE$ is determined by :

$$\frac{\dot{c}}{c} - \frac{\dot{x}}{x} = \underbrace{\frac{1}{\hat{p}_X}\left(-kf''(k)\right)\dot{k}}_{ROC} - \underbrace{\kappa\dot{k}}_{DPE}. \tag{40}$$

equivalent to:

$$\frac{\frac{\dot{c}}{c} - \frac{\dot{x}}{x}}{\dot{k}} = \frac{ROC - DPE}{\dot{k}} = \frac{1}{p_X + w}\left(-kf''(k)\right) - \kappa\,. \tag{41}$$

In the presence of a DPE, the sign of this expression is ambiguous, as both terms DPE and ROC are positive. However, while coefficient κ, related to the term DPE, is constant, the term related to ROC changes as k increases. Notice that $\lim_{k\to 0}\left[-k\, f''(k)\right] = \infty$, and $\lim_{k\to 0}\left[1/(w+p_X)\right] = 0$. Likewise, $\lim_{k\to\infty}\left[-k\, f''(k)\right] = 0$ and $\lim_{k\to\infty}\left[1/(w+p_X)\right] = 1/p_X > 0$. That is, given an increase in capital over time $\dot{k} > 0$, ROC approaches infinity for a low level of k, and it approaches zero for a high level of k. Moreover, the ROC is monotonously decreasing as k increases, because w increases and $[-k\, f''(k)]$ monotonously decreases: $\partial[-k\, f''(k)]/\partial k < 0$. As a consequence, given $\kappa > 0$, and given an increase in capital over time $\dot{k} > 0$, the sign of $(ROC - DPE)$ is positive for low k and negative for high k.

7.6 Proof of Corollary 1

The normalized Schofield equation is reexpressed as:

$$\frac{\dot{\mathrm{w}}}{x} = \left\{\lambda_C \frac{c}{x} - \left(\lambda_S \bar{s}\,(x) + \lambda_X x + \lambda_N\right)\frac{\mathrm{w}}{x} + \left(f'(k) - \delta - \rho\right)\frac{\mathrm{w}}{x} + \left(\kappa k - \eta(k)\Delta(k)\right)\frac{\dot{k}}{k}\frac{\mathrm{w}}{\mathrm{x}}\right\}. \tag{42}$$

Based on Proposition 2 and its proof, given $\dot{k} > 0$ (that is, $k < k^*$), when k is large, $\frac{\frac{\dot{x}}{x} - \frac{\dot{c}}{c}}{\frac{\dot{k}}{k}} \to \kappa$ and therefore $\frac{c}{x} \to 0$. Furthermore, for k is large, $f'(k) - \delta - \rho \to 0$ and $\left(\kappa k - \eta(k)\Delta(k)\right)\frac{\dot{k}}{k} \to 0$. As a consequence, $\frac{\dot{\mathrm{w}}}{x} \to -\left(\lambda_S \bar{s}\,(x) + \lambda_X x + \lambda_N\right)\frac{\mathrm{w}}{x} < 0$. Indeed, this term represents the negative of per worker calorie expenditure from all activities (sedentary leisure, exercise and work) divided by exercise. As a consequence, the growth rate of body weight becomes negative when k is large.

7.7 Steady-State equilibrium x* and k*

In the steady state $f'(k^*) = \delta + \rho$. As $f(k)$ is a strictly concave function, $f'(k)$ is monotone, and there exists an inverse function $f'^{-1}(.)$. An asterisk denotes a steady state value. Then:

$$k^* = f'^{-1}(\delta + \rho).$$

We express the intratemporal trade-off between exercise and food consumption as:

$$p_C c^* = (p_X + w^*)\frac{\alpha}{1-\alpha}(1 - \varepsilon(k^*))x^*\,.$$

From (25), we know that:

$$f(k^*) - \delta k^* = p_C c^* + p_X x^*\,.$$

Combining both equations yields an implicit steady-state relationship between x and k:

$$f(k^*) - \delta k^* = x^*\left[p_X + \frac{\alpha}{1-\alpha}\left(p_X + w^*\right)\left(1 - \varepsilon(k^*)\right)\right],$$

equivalent to:

$$x^* = \frac{f(k^*) - \delta k^*}{p_X + \frac{\alpha}{1-\alpha}\left(p_X + w^*\right)\left(1 - \varepsilon(k^*)\right)}\,. \tag{43}$$

Furthermore, c^* follows from substituting x^* in the optimality condition:

$$c^* = \frac{f(k^*) - \delta k^* - p_X x^*}{p_C} \,. \tag{44}$$

7.8 Change of c^* with respect to k^*

The ratio of first order conditions (13) by (14) yields:

$$c^* = \alpha/(1-\alpha)\,(\hat{p}^*_X/p_C)\,(1-\varepsilon(k^*))\,x^*$$

Recalling that $\hat{p}^*_X = p_X + w^*$ and that w^* also depends on k^*, we derive c^* with respect to k^* and obtain:

$$\frac{dc^*}{dk^*} = \alpha/(1-\alpha)\left[\frac{dw^*/dk^*}{p_C}\,(1-\varepsilon(k^*))\,x^* - \frac{p_X + w}{p_C}\frac{d\varepsilon(k^*)}{dk^*}x^* + \frac{p_X + w}{p_C}\,(1-\varepsilon(k^*))\,\frac{dx^*}{dk^*}\right]$$

Dividing $\frac{dc^*}{dk^*}$ by $\frac{c^*}{k^*}$, and simplifying, we obtain the elasticity of $c^*$with respect to k^*:

$$\frac{dc^*/c^*}{dk^*/k^*} = \quad -k^*\left[\frac{d\varepsilon(k^*)/dk^*}{(1-\varepsilon(k^*))} - \Delta\,(k^*)\,\frac{dw^*/dk^*}{w^*} - \frac{dx^*/dk^*}{x^*}\right], \tag{45}$$

As a consequence, the difference between the elasticity of $c^*$with respect to k^* and the elasticity of $x^*$with respect to k^* is:

$$\frac{dc^*/c^*}{dk^*/k^*} - \frac{dx^*/dx^*}{dk^*/k^*} = \quad \left[\Delta\,(k^*)\,\frac{dw^*/w^*}{dk^*/k^*} - \frac{d\varepsilon(k^*)/dk^*}{(1-\varepsilon(k^*))/k^*}\right], \tag{46}$$

which yields:

$$\frac{dc^*}{c^*} - \frac{dx^*}{x^*} = \quad \left[\underbrace{\Delta\,(k^*)\,\frac{dw^*}{w^*}}_{ROC^*} - \underbrace{\frac{d\varepsilon(k^*)}{(1-\varepsilon(k^*))}}_{DPE^*}\right], \tag{47}$$

Using the functional forms for utility and positionality, we get:

$$ROC^* = \frac{\eta(1-\eta)k^{*\eta-1}}{(1-\eta)k^{*\eta} + p_X}\,,$$

$$DPE^* = \frac{d\varepsilon(k^*)}{(1-\varepsilon(k^*))} = \kappa\,.$$

which produces equation (34).

7.9 Proof of Proposition 3

The sign of the difference $ROC^* - DPE^*$ is determined by :

$$\frac{\frac{dc^*}{c^*} - \frac{dx^*}{x^*}}{dk^*} = \frac{\underbrace{\frac{\eta(1-\eta)k^{*\eta-1}}{(1-\eta)k^{*\eta} + p_X}}_{ROC^*} - \underbrace{\kappa}_{DPE^*}}{dk^*}, \tag{48}$$

The proof is similar to the proof for Proposition 2. In the presence of DPE*, the sign of this expression is ambiguous, as both terms DPE* and ROC* are positive. However, while coefficient κ, related to the term DPE*, is constant, the term related to ROC* changes as k^* increases. Noticing that $\frac{\eta(1-\eta)k^{*\eta-1}}{(1-\eta)k^{*\eta}+p_X} = \frac{\eta}{k^* + \frac{p_X}{(1-\eta)}k^{*1-\eta}}$ We have $\lim_{k\to 0}[\frac{\eta}{k^* + \frac{p_X}{(1-\eta)}k^{*1-\eta}}] = \infty$, and $\lim_{k\to\infty}[\frac{\eta}{k^* + \frac{p_X}{(1-\eta)}k^{*1-\eta}}] = 0$. That is, ROC* approaches infinity for a low level of k^*, and it approaches zero for a high level of k^*. Moreover, ROC* is monotonously decreasing as k^* increases. As a consequence, given $\kappa > 0$, the sign of $ROC^* - DPE^*$ is positive for low k^* and negative for high k^*.■

7.10 Proof of Proposition 4

We express steady state weight as:

$$\mathrm{w}^* = \left(\frac{\lambda_C c(x^*, k^*)/x^*}{z(k^*)}\right),$$

where $z(k^*) = \lambda_S \bar{s}(x^*)/x^* + \lambda_X + \lambda_N/x^*$ represents total calorie expenditure divided by exercise expenditure. Comparing body weight for different wealth levels is obtained by totally deriving body weight with respect to k^*, which yields:

$$\frac{d\mathrm{w}^*}{dk^*} = \frac{\lambda_C c^*/x^*}{z^*}\left[\frac{\frac{d(c^*/x^*)}{c^*/x^*} - \frac{dz^*}{z^*}}{dk^*}\right] \tag{49}$$

First, we consider the sign of $\frac{d(c^*/x^*)}{c^*/x^*}$. Based on the proof for Proposition 3:

$$\frac{d(c^*/x^*)}{c^*/x^*} = \frac{dc^*}{c^*} - \frac{dx^*}{x^*} = \left[\underbrace{\Delta(k^*)\frac{dw^*}{w^*}}_{ROC^*} - \underbrace{\frac{d\varepsilon(k^*)}{(1-\varepsilon(k^*))}}_{DPE^*}\right], \tag{50}$$

We know from Proposition 3 that this term is negative for high values of k^* and converges towards $-\kappa$.

Second, we consider the impact of dk^* on $\frac{dz^*}{z^*}$:

$$\frac{dz^*}{dk^*}/z^* = \frac{\lambda_S \frac{d[\bar{s}(x^*)/x^*]}{dx^*}\frac{dx^*}{dk^*} - \frac{\lambda_N}{x^*}\frac{dx^*/dk^*}{x^*}}{\lambda_S \bar{s}(x^*)/x^* + \lambda_X + \lambda_N/x^*}$$

We examine the second term of the numerator ($\frac{\lambda_N}{x^*}\frac{dx^*/dk^*}{x^*}$) that denotes the change in exercise that goes with a higher steady state capital stock:

$$\frac{dx^*}{x^*}/dk^* = \frac{1}{p_X + \frac{\alpha}{1-\alpha}\left(1-\varepsilon\left(k^*\right)\right)}\left\{\underbrace{\frac{d\left(f\left(k^*\right)-\delta k^*\right)}{\left(f\left(k^*\right)-\delta k^*\right)}}_{IE^*} + \frac{\alpha}{1-\alpha}\frac{w^*}{\Delta^*}\left(1-\varepsilon\left(k^*\right)\right)\left[\underbrace{\frac{d\varepsilon(k^*)}{(1-\varepsilon(k^*))}}_{DPE^*} - \underbrace{\Delta^*\frac{dw^*}{w^*}}_{ROC^*}\right]\right\}/dk^*$$

This expression indicates that exercise expenditure increases (decrease) when the income effect (IE^*) and the DPE^* are large (small) compared to ROC^*. When k^* is high, we know from proposition that ROC^*-DPE^* is positive and IE^* is positive. Therefore $\frac{dx^*}{x^*}/dk^* > 0$ when k^* is high. We examine the first term of the numerator ($\frac{d[\bar{s}(x^*)/x^*]}{dx^*}$). Based on the definition of $\bar{S}$, we obtain:

$$\bar{s}^* = \frac{1}{N} - 1 - \frac{X^*}{N} = \frac{1}{N} - 1 - x^*$$

Because N is endogenous, we need to make a reasonable assumption: we consider an increase in x^*, representing an increase in X^* given N, or assume that it is not offset by a decrease in N. In that scenario, an increase in x^* produces a decrease in $\bar{s}^*$. In other words, an increase in per worker exercise leads to a decrease in per worker sedentary leisure. Therefore $\frac{d[\bar{s}(x^*)/x^*]}{dx^*} < 0$. As a consequence, since for high values of k^* , x^* increases, and $\frac{dz^*}{dk^*}/z^* < 0$. In other words, the ratio of total calorie expenditure over exercise decreases.

Based on (49), for high values of k^*, $\frac{d\mathrm{w}^*}{dk^*} < 0$ if:

$$\underbrace{\frac{d\left(c^*/x^*\right)}{c^*/x^*}}_{(-)\to-\kappa} < \underbrace{\frac{dz^*}{z^*}}_{(-)} \to \kappa > -\underbrace{\frac{dz^*}{z^*}}_{(+)}$$

8 Figures and tables

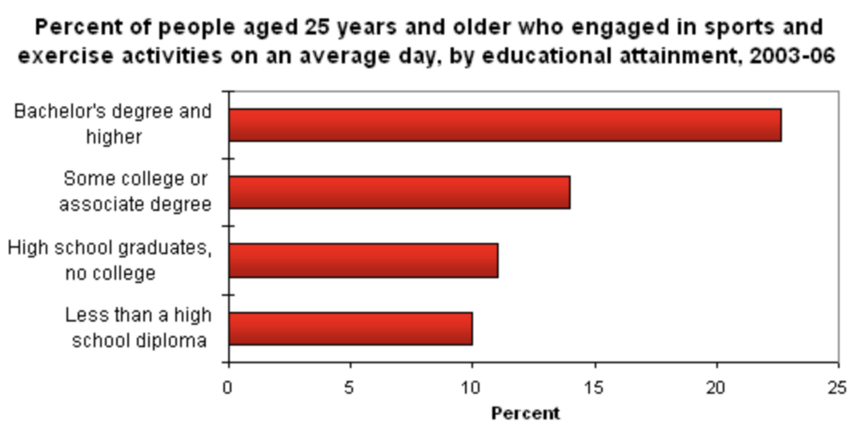

Figure A1: Engagement in sports and educational attainment (Source: American time use survey, 2008)

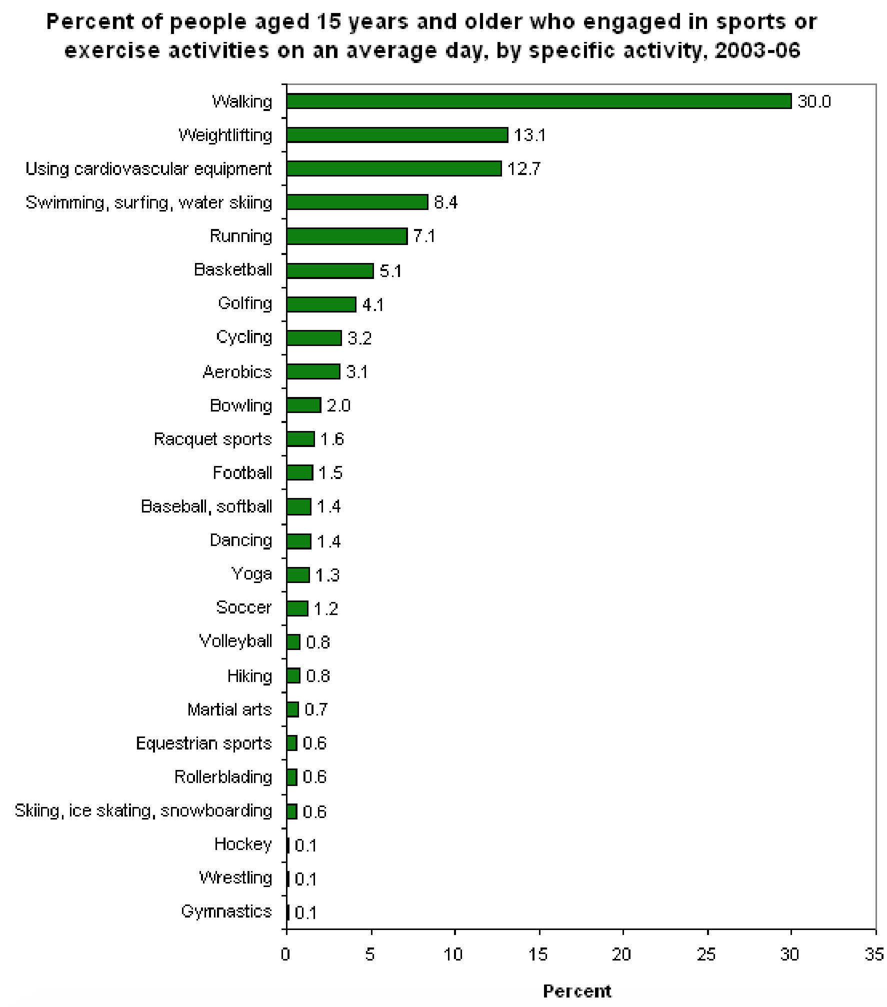

Figure A2: Engagement in sports per type of exercise (Source: American time use survey, 2008)

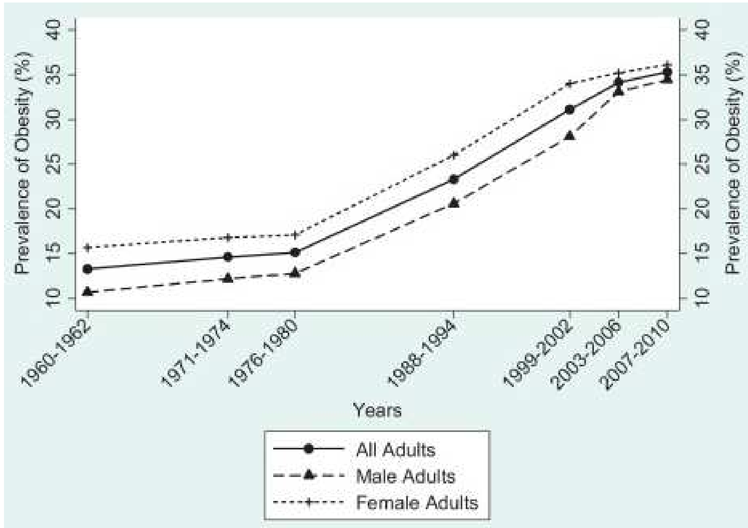

Figure A3: Obesity prevalence over time in the USA (Source: Cawley, 2014)

Table A1: calorie spent per activity

Activity	Percentage of total time	Calorie expenditure per pound per hour
walking	=30/(30+13.1+12.7)=0.54	378/185=2.0
weight lifting	=13.1/(30+13.1+12.7)=0.23	252/185=1.4
cardio	=12.7/(30+13.1+12.7)=0.23	880/185=4.7
Weighted average		2.5

Sources: 2008 BLS Spotlight on Statistics and Harvard Medical School (https://www.health.harvard.edu/diet-and-weight-loss/calories-burned-in-30-minutes-of-leisure-and-routine-activities).