Work of invisible hand: the gravitation between sellers and buyers on the consumption-leisure production possibility frontier

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Abstract

If the equilibrium price is equal to the lowest willingness to pay of consumers with zero search costs, it accumulates under price dispersion the willingness to sell of consumers with positive search costs. The satisficing searcher buys at a low price, which unintentionally equalizes marginal costs of his search with its marginal benefit and maximizes his consumption-leisure utility with regard to the equilibrium price. Labor and search costs are unit elastic with respect to the quantity demanded. Suboptimal choices reproduce initial corner solutions; satisficing purchases become optimal; consumers either buy optimally or quit the market.

The producer's knowledge is limited by the quantity demanded, but he meets the consumer with a price, like he knows in advance his willingness to pay and the time spent on search. The consumption-leisure production possibility frontier optimally allocates his time between production and delivery; it determines not only the price for the quantity demanded, but also the meeting point, where the producer stops consumer's search and sells him goods with leisure. Being unaware of how much the consumer has spent on labor and search, the producer unintentionally optimizes his consumption-leisure choice.

The transformation of producer's time into consumer's leisure time discovers the rate of their mutual interest or their gravitation, where its force is directly proportional to the product of quantity supplied and quantity demanded, and inversely proportional to the product of times the parties to the transaction have spent on it.

Key words: invisible hand, consumption-leisure choice, production possibility frontier, search, price dispersion, gravitation.
JEL classification: D11, D83.

This paper upgrades both the article 'Force of Invisible Hand, Labor, and Just Price: basic principles of sorting and matching under wage and price dispersion', written in Russian for *The Journal of Institutional Studies* ((2021) 13 (1), 86-99, DOI: 10.17835/2076-6297.2021.13.1.086-099) and the report "Invisible Hand at Consumption-Leisure Production Possibility Frontier: the allocation of time between goods and services under wage and price dispersion", presented in June 2021 at 69th Congress of Association Française des Sciences Economiques. (S.M.)

Introduction

The voluminous literature on the Invisible hand can be divided into three general threads: the skeptical, regarding this famous notion as the metaphor (Stiglitz 2002, Schlefer 2012); the enthusiastic, recognizing its role in the analysis of the self-interested individual behavior (Stigler 1976, Sen 2009); and theological, based on Adam Smith's religious background (Macfie 2003, Oslington 2012).

The enthusiastic approach to the most famous allegory of the economic thought is owing to the continued interest in inner workings of the market in itself. Indeed, "*the view that competitive equilibria have some special optimality properties is at least as old as Adam Smith's invisible hand…*" (Arrow 1985, p.110).

The transformation of the classical consumer labor-leisure choice into the labor-search-leisure choice discovers some optimality properties of imperfect markets and their potentials to the self-organization under price dispersion (Malakhov 2018). This methodological comeback to the basic principles of microeconomics is explained by the recent trends in the economics of search. Its fundamental results had been successfully documented by two comprehensive overviews (Baye *et al*. 2006, McCall and McCall 2008). And during last years the economics of search has been developed in the very promising direction of matching modeling; the issue that seems to be very important in the understanding of the inner coherence of the economy. However, sometimes the outcomes of this research thread look to a large extent instrumental, paying attention to particular attributes of the matching process like matching stability (Liu *et al*. 2014), meeting technologies (Lester *et al*. 2014), or sorting through search (Chade *et al*. 2017).

The instrumental approach to the search and matching pays more attention to active buyers and sellers, and less attention to the potentials of the market in resources allocation where time remains the most important input. The labor-search-leisure model demonstrates how the search is rewarded by the purchase price, which provides the optimal allocation of consumers' time between labor, search, and leisure. However, the efficient search doesn't mean that active consumers calculate marginal values of his efforts. When they follow the simple 'it's enough to spend time' rule, the marginal values of search become automatically equated. It looks like the producer who is unaware of consumers' allocation of time comes to the right place at the right time with the 'just price', which unintentionally maximizes the consumption-leisure utility function (Malakhov 2020b). However, the question how the producer invariably comes there remains the open issue for qualitative assessment of the inner market mechanism, presented literarily by Adam Smith: "*he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention*" (Smith 1976, *Wealth of Nations*, p.456).

This paper tries to answer to this question. The analysis of the Invisible hand is organized as follows.

Part I starts with the presentation of the behavioral labor-search-leisure choice. The behavioral explicit model of satisficing choice is transformed into the implicit consumption-leisure utility maximization model. The paper tries to minimize the cumbersome examination of marginal values of search, used earlier in the labour-search-leisure model. Here, the analysis of the consumption-leisure utility is levelled up to absolute values, which facilitate the presentation of the satisficing optimal choice.

Part II presents the particular consumption-leisure production possibility frontier. The paper argues that producers, delivering goods to some point of sale, provide not only consumption but also leisure time, and their simple self-interested decisions result in the maximization of customers' consumption-leisure utility.

The analysis of the consumption-leisure equilibrium, presented in Part III, results in the hypothesis that the rate of mutual interest or the gravitation between the seller and the buyer is directly proportional to the product of quantity supplied and quantity demanded, and inversely proportional to the product of producer's time and consumer's non-leisure time.

The Conclusion pays attention to the methodology of the analysis of quantity demanded because if the gravitation between producers and consumers exists, it depends more on consumption units than on trade units.

Part I. Labor-search-leisure model

If we start with the traditional problem of search for the fixed quantity demanded Q (Stigler 1961), we get the intersection of $QP(S)$ curve and labour income $wL(S)$ curve with regard to the time of search S when the consumer chooses the first offer QP_P below his willingness to pay wL_0:

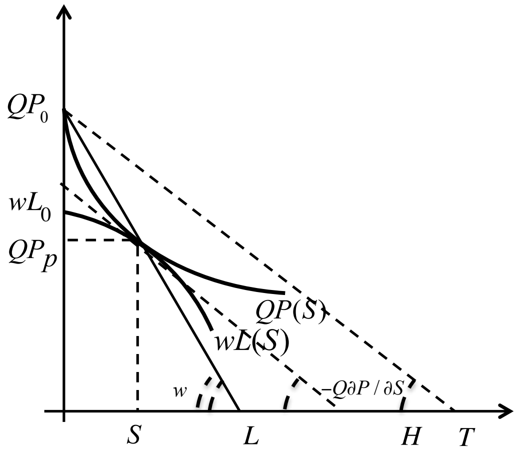

Fig.1. Behavioral satisficing choice

where S – the search; L – labor; H – leisure; T – time horizon until next purchase; Q – quantity demanded; w – wage rate; wL_0 – willingness to pay; P_P – purchase price.

The straight line with the slope w, passing the intersection point, i.e., the purchase, gives us the QP_0 value on the $0Y$ axis and $(S+L)$ value on the $0X$ axis. The straight dotted line from the point QP_0 with the slope ($-Q\partial P/\partial S$), i.e., the tangent to the moment of purchase, gives us the value of the time horizon T on the $0X$ axis.

These considerations result in the following equation:

$$w(L+S) = -Q\frac{\partial P}{\partial S}T = QP_0 \tag{1}$$

The behavioral model of the optimal search uses the assumption of the diminishing efficiency of the search or $\partial P/\partial S<0;\partial^2 P/\partial S^2>0$. However, the shape of the labor cost curve $\partial wL/\partial S<0;\partial^2 wL/\partial S^2<0$ can and should be proved.

Let's take Eq.1 as the budget constraint to some consumption-leisure utility function $U(Q,H)$, keeping in mind that for the given time horizon $T=L+S+H$ the value $\partial L/\partial H+\partial S/\partial H=-1$:

$$\mathcal{L} = U(Q,H) + \lambda(w(L+S) - QP_0) \tag{2.1}$$

$$\frac{\partial \mathcal{L}}{\partial Q} = \frac{\partial U}{\partial Q} - \lambda P_0 = 0 \tag{2.2}$$

$$\frac{\partial \mathcal{L}}{\partial H} = \frac{\partial U}{\partial H} + \lambda w\left(\frac{\partial L}{\partial H} + \frac{\partial S}{\partial H}\right) = \frac{\partial U}{\partial H} - \lambda w = 0 \tag{2.3}$$

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(H\ for\ Q) = \frac{w}{P_0} \tag{2.4}$$

For the moment this utility function looks implicit because here the value of consumption Q becomes a variable, and value of price reduction $\partial P/\partial S$ stays constant (Fig.2):

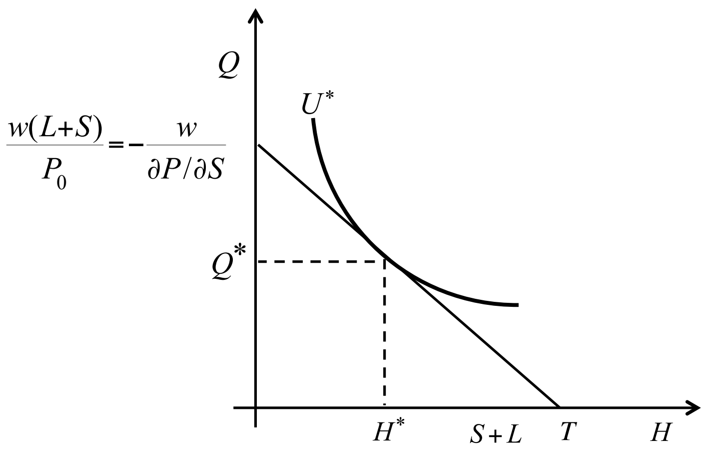

Fig.2. Consumption-leisure utility

where S – search; L – labor; H^* – leisure; T – time horizon until next purchase; Q^* – quantity purchased; U^* - consumption-leisure utility.

However, its implicit optimal solution, where the fixed *∂P/∂S* value displays the given place of purchase, matches the explicit satisficing, i.e., behavioral choice when the variable *∂P/∂S* value exhibits a sequential search. There, the satisficing search really becomes optimal because Eq.1 also provides the equality of its marginal values at the purchase price level:[1]

$$QP_0 = -Q\frac{\partial P}{\partial S}T \ = w(L+S) \tag{3.1}$$

$$Q\frac{\partial P}{\partial S} = -w\frac{L+S}{T} = w\ \frac{\partial L}{\partial S} \tag{3.2}$$

While the value of the marginal benefit on purchase *Q∂P/∂S* is widely used in economics either with respect to the units of search (Stigler 1961) or to the time of search (Aguiar and Hurst 2007b), the value of the marginal costs looks rather unusual. Here, it is equal to the wage rate times the *propensity to search ∂L/∂S*. This is the key variable of the labor-search-leisure model. It is negative because labor and search represent different sources of income. The propensity to search provides the optimal allocation of time in the explicit behavioral model and maximizes the implicit consumption-leisure utility.[2] But its value comes from the simple natural reasoning. When we fill the glass (*time horizon*) with whiskey (*labor*) and soda (*leisure*), an ice cube (*search*) spills the drink over the edge of the glass. The volume of the spilled soda (*leisure*) is

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[1] The equation of the marginal values of search (3.2) is used as the constraint in the basic labor-search-leisure model where it produces very specific values of the marginal utility, which nevertheless give the same results (Malakhov 2018).

[2] The analysis of the propensity to search *∂L/∂S* explains why the labor-search-leisure model doesn't consider the utility of the search *S* itself. The utility, i.e., the degree of pleasure of search, changes leisure time; it appears indirectly in different *∂L/∂S* values from the corresponding leisure-search substitutability *∂H/∂S*. For example, pleasurable search significantly cuts leisure time to low *H/T=-∂H/∂S* values and results in great absolute values of |*∂L/∂S*| (Malakhov 2019).

equal to the volume of the ice cube (*search*) times the soda's (*leisure*) share in the glass (*time horizon*) or

$$dH(S) = -dS\frac{H}{T} = dS\frac{\partial H}{\partial S} \tag{4.1}$$

$$L + S + H = T_{const;} \frac{\partial L}{\partial S} + 1 + \frac{\partial H}{\partial S} = 0 \tag{4.2}$$

$$\frac{\partial L}{\partial S} = -1 + \frac{H}{T} = -\frac{L+S}{T} \tag{4.3}$$

$$\frac{\partial^2 L}{\partial S^2} = -\frac{\frac{\partial L}{\partial S} + 1}{T} \tag{4.4}$$

Eq.4.3 confirms the equality of the marginal values of search in Eq.3.2; and Eq.4.4 proves the shape of *wL(S)* curve at Fig.1 because the derivative of the value of the propensity to search $\partial L/\partial S$, when $1<\partial L/\partial S<0$, is negative, or $\partial^2 L/\partial S^2<0$.[3]

The logic of the behavioral model takes the value of time horizon as the time until next purchase. And it really leaves to the consumer some leisure time to enjoy the item. However, the time horizon is often pre-determined by some calendar. If it takes place, the Eq.1 is ceasing to be unconditioned.

However, the confirmation of the reliability of the budget constraint (Eq.1) for the given time horizon results in some unexpected conclusions. First, we rearrange the budget constraint to get the physic trade-off between leisure and consumption for the given time horizon:

$$w(L+S) = QP_0 \Rightarrow \frac{w}{P_0} = \frac{Q}{L+S} = MRS\ (H\ for\ Q) \tag{5}$$

If the consumer early stops the search, its marginal costs will be less than its marginal benefit. The inequality of the marginal values of search at the purchase price level results in the corner solution at the equilibrium price level:

$$-w\frac{L+S}{T_{const}} > Q\frac{\partial P}{\partial S}; \left\lfloor -w\frac{L+S}{T_{const}} \right\rfloor < \left\lfloor Q\frac{\partial P}{\partial S} \right\rfloor \tag{6.1}$$

$$w(L+S) < Q\frac{\partial P}{\partial S}T_{const} = QP_0 \tag{6.2}$$

$$\frac{w}{P_0} < \frac{Q}{L+S} \tag{6.3}$$

[3] While the value $\partial L/\partial S$ is always negative because labor and search represent alternative sources of income, it might come to $\partial L/\partial S<-1$. Here the consumption-leisure choice occurs under the leisure model of behavior where the positive leisure-search relationship $\partial H/\partial S>0$ results in positive $\partial Q/\partial H$ trade-off, which produces negative marginal utilities and anomalies like Veblen effect. But when it happens, the natural analogy with the whiskey, soda, and ice doesn't work. The economic choice loses its natural grounds, and the Invisible hand becomes helpless (Malakhov 2018a,b).

Theoretically, this situation cannot take place at the moment of purchase. If we come back to the moment of the intention to buy when the fridge is empty ($Q, L, S \to 0; T_{\text{const}}$), l'Hôpital rule gives us the following result:

$$\lim_{H\to T} Q(H) = \lim_{H\to T}(L+S)(H) = 0; \frac{\partial(L+S)}{\partial H}|_{T_{const}} = -1 \qquad (7.1)$$

$$\lim_{H\to T}\frac{\partial Q/\partial H}{\partial(L+S)/\partial H} = -\frac{\partial Q}{\partial H} = \lim_{H\to T}\frac{Q}{L+S} \qquad (7.2)$$

And we get the unit elasticity of cost on purchase with respect to consumption:

$$e_{w(L+S),Q} = \frac{\partial(L+S)}{\partial Q}\frac{Q}{L+S}|_{L_0;S_0;Q_0\to 0;T_{const}} = \frac{\partial(T-H)}{\partial Q}\frac{Q}{L+S} = \left(-\frac{\partial H}{\partial Q}\right)\left(-\frac{\partial Q}{\partial H}\right) = 1 \quad (8)$$

We see that the consumer doesn't make cumbersome calculations of marginal values. He needs from the very beginning only the realistic evaluation of his efforts *Q/L+S*. If the consumer overestimates the value of his efforts from the very beginning, finally he finds himself 'in the corner' (Fig.3):

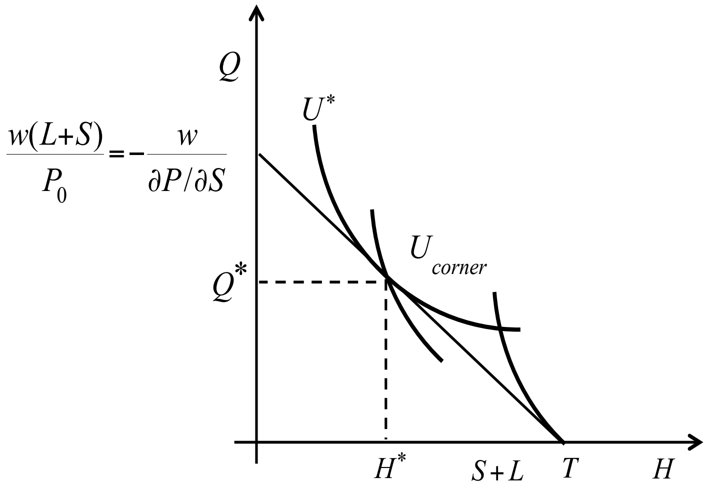

Fig.3. Corner solution.

And he should either quit the market or accept its rules; he adjusts his aspiration level and spends more efforts on purchase. But if this starting evaluation is realistic, he buys optimally with respect to Eq.8 any quantity, which automatically equalizes the marginal values of his search (Malakhov 2020b). The consumer follows the 'it's enough to spend time' rule and stops the search when total efforts on purchase, both on labour and search, correspond to the quantity purchased. He looks satisficing but his purchase is optimal because the purchase price optimizes his allocation of time with respect to consumption.

This unit elasticity rule illustrates the stability of preferences. But in our case it means that Eq.6.3 cannot take place at the moment of purchase. The inequality of the marginal values of search, i.e., the corner solution, appears at the moment of the intention to buy when the consumer doesn't even start to work and to search because the quantity demanded isn't worth money and efforts.

Nevertheless, the comparative statics can produce such inequality. It happens when the arbitrage at the zero search level takes place.

The consumer starts the search with the willingness to pay $WTP=wL_0$. When he buys at a low price, he gets an option either to consume or to re-sell the item. If he sells it, the resale price will be equal to his costs $w(L+S)$. And we can consider this value as his willingness to accept or to sell $w(L+S)=QP_0=WTA$.

This particular *WTP-WTA* relationship becomes more clear when we take the home production as a specific form of the search. Indeed, "the opportunity cost of time of the shopper is the same as that of the person undertaking home production." (Aguiar and Hurst 2007b, p.1594). While the time horizon is divided between labor, search, and leisure, the search represents any activity, which decreases the purchase price. The consumer can buy the grilled steak in the restaurant, or he can make it at home. There, his *WTP* is limited by the price of inputs, while his *WTA* goes up with the market price of grilled steak. Theoretically, and sometimes it happens in real life, the skilled consumer can sell the output of home production to his neighbor who hasn't time for home production because his opportunity costs are higher.

Here we come to the understanding of the nature of QP_0 value. The economics of search had successfully developed for a long time the concept of consumers' heterogeneity (Diamond 1987). It describes *shoppers*, consumers with zero search costs, and *searchers*, consumers with positive search costs (Stahl 1989).

Now we understand that the QP_0 value represents the willingness to pay of *shoppers* who have no time to search. But the zero search level has a specific attribute – even if *shoppers* have different opportunity costs of time, i.e., different willingness to pay, there is no price dispersion at this level, since otherwise some *shoppers* become *searchers*. It means that the QP_0 value is equal to the lowest willingness to pay among *shoppers*. The P_0 value is the equilibrium price or $P_0=P_e$.

And the consumer can use it. If he decides to re-sell the bought item, he becomes a 'producer' who bears some costs. And his costs, both average and marginal, really come to the equilibrium level:

$$w(L+S) = QP_0 = WTA \tag{9.1}$$

$$\frac{w(L+S)}{Q} = AC = \frac{\partial w(L+S)}{\partial Q} = MC = P_0 = P_e \tag{9.2}$$

However, the consumer can unexpectedly find a low price. As a result, consumer's costs might be lower than the equilibrium level $WTA=w(L+S)<QP_e$. It means, that the low purchase price creates the low price $P_0'<P_e$ at the zero search level. If the time horizon is variable, the consumer cuts it to the T' value; the buyer consumes the item without delay to enhance the enjoyment, the $T'<T$ value equalizes marginal values of search in Eq. 6.1; the $wL(S)$ curve becomes steeper at the moment of purchase, and the purchase itself becomes optimal (Fig.4):

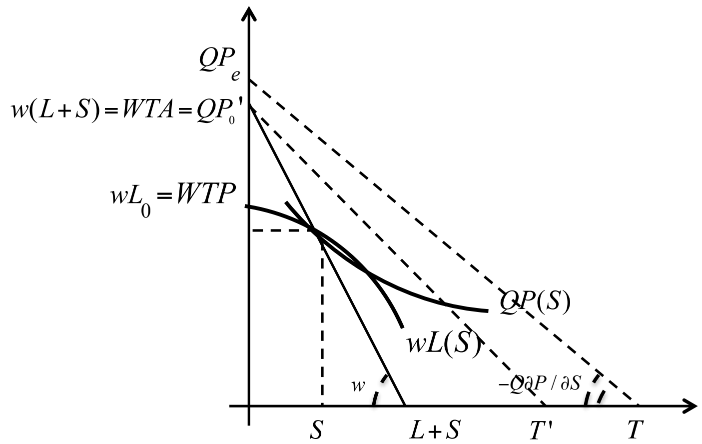

Fig.4.Suboptimal purchase and arbitrage process

But if the time horizon is constant, the purchase stays suboptimal but the *searcher*, i.e., the consumer with positive search costs, gets an option to re-sell the bought item to his neighbor, the *shopper* with zero search costs. The arbitrage process starts; it changes not only prices but also the corresponding price reductions $\partial P/\partial S$ with respect for the given time horizon T. And the arbitrage finishes with the new equilibrium price $P_e=P_0'$ under the $WTA=w(L+S)=QP_0'$ condition.

Part II. Consumption-leisure production possibility frontier

The unit elasticity rule (Eq.8) tells us that if the consumer realistically estimates the efficiency of his efforts at the moment of the intention to buy ($MRS_0\ (H\ for\ Q)=Q/L+S=w/P_e$) , he optimally buys any quantity. It means that the producer gets some degree of freedom. And he appears on the market with the quantity and price that being summarized with consumer's search costs produce the optimal purchase. The producer comes to the right place at the right time with the 'just price' and optimizes intentionally by $QP_p=wL$ value the allocation of consumer's time horizon and maximizes his consumption-leisure utility. It looks like the producer knows in what manner the consumer allocates his time. But if the producer is unaware of it, all the composition becomes really enigmatic.

Let's take a simplified example of the producer who allocates his time between farming and delivery. There are two extreme cases where he is not concerned about the consumer's allocation of time - when he sells "door to door", to a high-income *shopper* with zero search costs, and at the farm to a low-income *searcher* coming there from the downtown. But between these two extremes there is a middle-income customer who is not ready to pay the price at 'the door' and to go to 'the farm.'

Now we can construct some virtual production possibility frontier, limited by two extremes – by 'the farm' and 'the door' (Fig.5):

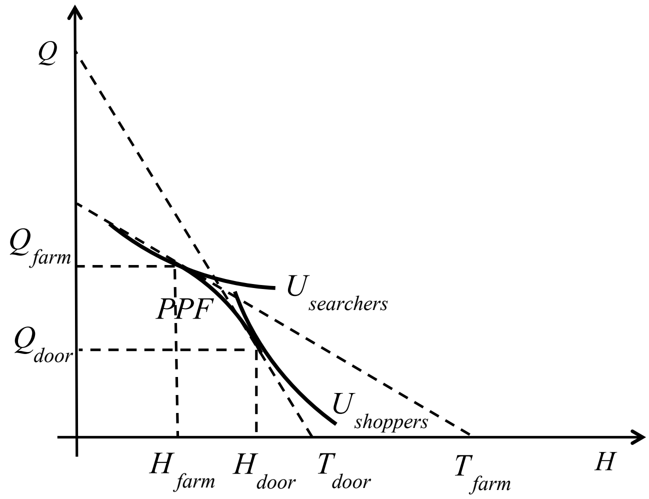

Fig.5. Consumption-leisure production possibility frontier

Along this virtual frontier the farmer produces and sells not only goods. He also trades consumers' leisure. For consumers, any point on this frontier represents the w/P_e ratio with respect to their wage rates. It is low at 'the farm' and high at 'the door.' Low-income consumers spend much time on search and the high-income consumers don't search at all. But for the producer, who knows nothing about consumer's allocation of time, any point of the *PPF* represents a particular combination of his time spent on the farm and on his way to the point of sale.

This *PPF* is very specific. When the producer's total working time is constant, his total costs *TC(Q)* are also constant. The producer chooses the target quantity demanded *Q* and gets the price *P* on the basis of his average costs *AC*. His total costs become proportional to output. It means that his production function exhibits constant return to scale both for farming and delivery. And average and marginal costs become equal to the price:

$$TC(Q) = aQ; a = AC = MC = P \tag{10}$$

As a result, the set of production functions with respect to different output levels looks as follows (Fig.6):

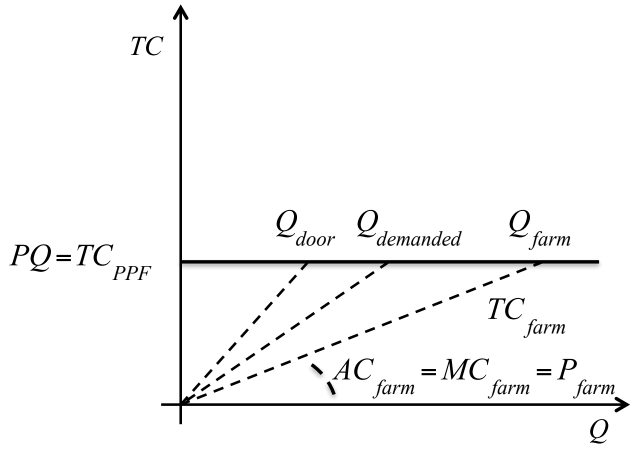

Fig.6. The set of production functions under constant total costs of the production possibility frontier

We can also assume that both average and marginal costs on farming stay constant for any output. But it is not for average and marginal costs on delivery. The producer chooses the quantity to be produced and delivered Q and determines his average costs AC on the basis of his constant total costs TC_{const}. But if he divides his costs between farming and delivery, he gets two values – his average costs in farming AC_f, which are constant for any output, or $AC_f = MC_f$, and his average costs in delivery AC_d, which vary with the output along the frontier but stay constant for the chosen output Q, or $AC_d = MC_d$. So, we get the following set of equations:

$$P = AC = AC_f + AC_d = MC = MC_f + MC_d \qquad (11.1)$$

$$AC_f = MC_f; AC_d = MC_d \qquad (11.2)$$

While total costs stay constant, their total differential is equal to zero. But this differential is also specific. Here the logic 'one unit less in farming – one unit more in delivery' is not working because the quantity delivered is always equal to quantity produced. But the differentials always exist, here with respect to the maximum quantity the farmer can produce without cleaning:

$$dQ_f = Q_{produced\&delivered} - Q_{max} \qquad (12.1)$$

$$dQ_d = Q_{produced\&delivered} \qquad (12.2)$$

When customers are not interesting in dirty harvest the dQ_f value becomes equal to zero. So, on the farm the producer should harvest and clean vegetables in order to sell them to the low-income customers. Cleaning reduce his output by the value $dQ_f \neq 0$. And the real output 'on the farm" becomes equal to the value dQ_d of the quantity produced and delivered with respect to the $Q=0$ value when the farmer is not working at all.[4]

If we apply this reasoning for any output along the *PPF*, we get its concave shape:

$$dTC(Q) = dQ_f \frac{\partial TC}{\partial Q_f} + dQ_d \frac{\partial TC}{\partial Q_d} = 0 \qquad (13.1)$$

$$-\frac{dQ_f}{dQ_d} = \frac{\partial TC/\partial Q_d}{\partial TC/\partial Q_f} = \frac{MC_d}{MC_f} = RPT \qquad (13.2)$$

where *RPT* – is the rate of product transformation.

When the constant total costs' function is convex with respect to the price-quantity trade-off, it becomes concave with respect to the farming-delivery trade-off. The *PPF* is concave because goods in farming and goods in delivery exhibit constant returns to scale but only with respect to the target consumption level *Q*. Farming and delivery use time in different proportions but these proportions change over the frontier.

So, the choice of the point of sale changes both the output and its price. The sale in the downtown with regard to the farm cuts the output; its marginal costs of farming MC_f stay constant but its marginal costs of delivery MC_d rise. So, the *RPT* value also rises.[5]

From the academic point of view, when the producer faces the consumer's allocation of time, we should take the consumption-leisure trade-off with its corresponding MC_H/MC_Q values also for the frontier. But the farmer knows nothing about the consumer's allocation of time. He

[4] For better understanding, suppose that the farmer can produce *20* units in *10* hours without services. As a result, the price per unit is $P=AC=MC=0.5$. But nobody comes for dirty vegetables. So, the farmer makes cleaning&packaing, and the output falls to *16*. The price rises to *0.625*. But average costs on farming $AC_f=MC_f$ stay at *0.5*. It means that both average and marginal costs on delivery $AC_d=MC_d$, i.e, cleaning&packaging, rise from *0* to 0*.125*. That gives us the MC_d/MC_f ratio, which equals to '*0.125/0.5=4* units lost in farming */16* units produced for sales' and '2 hours of cleaning for 8 hours production', or to *1/4*^th^. The total allocation of 10 hours between production and services can be illustrated as follows:

TC	*Q*	*P*	*-dQf/dQd*	*MCf=ACf*	*MCd=ACd*	*Td/Tf*
10	20	0,500	0,000	0,500	0,000	0,000
10	19	0,526	0,053	0,500	0,026	0,053
10	18	0,556	0,111	0,500	0,056	0,111
10	17	0,588	0,176	0,500	0,088	0,176
10	16	0,625	0,250	0,500	0,125	0,250
10	15	0,667	0,333	0,500	0,167	0,333
10	14	0,714	0,429	0,500	0,214	0,429
10	13	0,769	0,538	0,500	0,269	0,538
10	12	0,833	0,667	0,500	0,333	0,667
10	11	0,909	0,818	0,500	0,409	0,818
10	10	1,000	1,000	0,500	0,500	1,000
10	9	1,111	1,222	0,500	0,611	1,222
10	8	1,250	1,500	0,500	0,750	1,500
10	7	1,429	1,857	0,500	0,929	1,857
10	6	1,667	2,333	0,500	1,167	2,333
10	5	2,000	3,000	0,500	1,500	3,000
10	4	2,500	4,000	0,500	2,000	4,000
10	3	3,333	5,667	0,500	2,833	5,667
10	2	5,000	9,000	0,500	4,500	9,000
10	1	10,000	19,000	0,500	9,500	19,000

[5] As we can see at Fig.4, the high wage rate reduces the time horizon until next purchase. This consideration corresponds to the statistics on the shopping frequency with respect to income (Kunst 2019).

knows only his allocation of time between farming T_f and delivery T_d with respect to the target quantity Q to be produced and delivered. And he really gets it:

$$-\frac{dQ_f}{dQ_d} = \frac{MC_d}{MC_f} = \frac{AC_d}{AC_f} = \frac{TC_d}{TC_f} = \frac{T_d}{T_f} \tag{14}$$

When the total producer's time is limited, the frontier of trade-offs between farming and delivery should exist. The producer is not concerned at all about consumer's leisure time. He simply gives to the consumer a set of options – to go to 'the farm'; to wait him at 'the door'; or to search for him between 'the farm' and 'the door'.

It is clear that any optimal consumer's choice represents some allocation of producer's time between farming and delivery. And this meeting is optimal for both parts in transaction when:

$$-\frac{\partial Q}{\partial H} = \frac{Q}{L+S} = \frac{w}{P_e} = -\frac{dQ_f}{dQ_d} = \frac{MC_d}{MC_f} = \frac{T_d}{T_f} \tag{15}$$

The Invisible hand works as follows. The farmer knows his productivity on the farm; he takes the target quantity Q and gets the time for farming T_f. And the time for delivery T_d appears as the residual value with respect to the total working time.

When the direction from the farm to the town is given, the producer spends time for delivery T_d and comes with the quantity Q and the price P to the meeting point where he finds the consumer who has spent time on search S and who is satisfied by QP offer.

We see that this matching occurs almost automatically. And it really looks like the work of the Invisible hand. The 'just price' adds to its mystique by meeting the consumer's wishes. The producer sets the price, which unintentionally optimizes the buyer's allocation of time and maximizes the utility of his consumption-leisure choice. When the total output is sold, sales are equal to the labor income spent on the purchase where the price is the same for both parts:

$$PQ = wL \Rightarrow P = \frac{wL}{Q} \tag{16}$$

This 'just price' confirms the successful matching because it stays from the very beginning on the production possibility frontier. When the producer is going to the meeting point, he can be stopped by some consumers with a proposal to sell them his output. But this occasional meeting cannot be optimal to the producer because consumers who have spent more time on search don't give him the price he is expecting. And he rejects the proposal and continues his way to 'his' consumer (Fig.7), shifting up the T_d/T_f ratio to the target level:

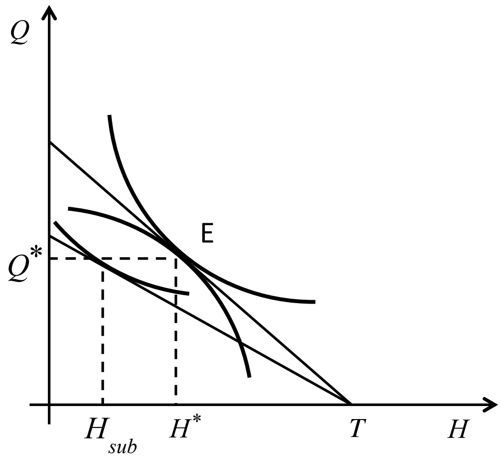

Fig.7. Suboptimal and optimal sale

The suboptimal proposal means that consumers have spent more search time *S* on purchase that decreases their labor time *L* and their readiness to pay *wL*. But under the $\partial^2L/\partial S^2<0$ rule the total time on purchase (*L*+*S*) is increasing. It means that the marginal rate of substitution of leisure for consumption in suboptimal proposals is less than the target rate of product transformation or $Q/(L+S) = MRS\ (H\ for\ Q) < RPT = T_d/T_f$. These relative values become equal only in the end of producer's way at the level of the target price *P*.

The price *P* for the given quantity *Q* confirms the optimal allocation of time for both parts. It definitely states that the producer spends the time T_d on delivery when the consumer spends the time *S* on search. A consumer with a greater propensity to search rejects the offer because the price seems to him high, and a consumer with a lower propensity to search, waiting for the producer somewhere nearby with a high price, simply doesn't come to the meeting point.

The price *P* for the given quantity *Q* also uniquely identifies the allocation ($T_d;T_f$) of total farmer's on the actual production possibility frontier, i.e., for the given productivity. If we come back from Eq. 15 and 16 to the Eq.10.2, we can get the visual confirmation of the tangency of utility curve and the *PPF* curve:

$$P = MC_f + MC_d = MC_f\left(1 + \frac{T_d}{T_f}\right) = MC_f\left(1 + \frac{Q}{L+S}\right) \tag{17}$$

While the consumer buys optimally, his utility curve cannot intersect the *PPF* curve. Both curves are tangent at the *P* level. If they are not tangent, $T_d/T_f \neq Q(L+S)$ and $P_{seller} \neq P_{buyer}$.

The intersection of the utility curve with the *PPF* curve means that the producer made a mistake about the quantity demanded and the corresponding allocation of time between farming and delivery. However, the Pareto-improvement along the production possibility frontier doesn't take place because any adjustment he makes results in a loss, either in price or in quantity.

The producer's self-evaluation also plays an important role because it results, as Eq.17 demonstrates, in the price setting from the very beginning, at the level of the marginal costs of farming MC_f. If the producer finds MC_f value to be too high, he accepts it like the corner solution from the very beginning, and he doesn't start farming for not to find himself 'in the corner' after delivery. However, high MC_f value might be realistic, when the farmer is more productive. Then, he will spend less time on the farm but more on delivery. And he will come closer to the potential buyer. The production possibility frontier will change its shape and establish the new equilibrium trade-off T_d/T_f equal to the consumption-leisure trade-off of the consumer who is waiting in the 'corner' with higher labor time L to be spent on purchase and lower search time S under $\partial^2 L/\partial S^2 < 0$ rule (Fig.8a,b). He is ready to pay higher price, and it really will become higher because of greater productivity and closer delivery.

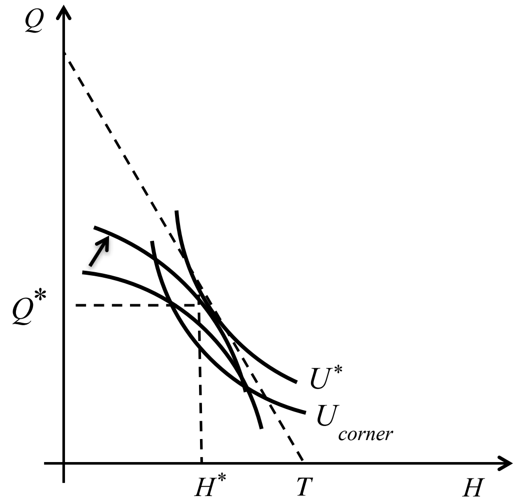

Fig.8a. The shift of PPF under greater productivity

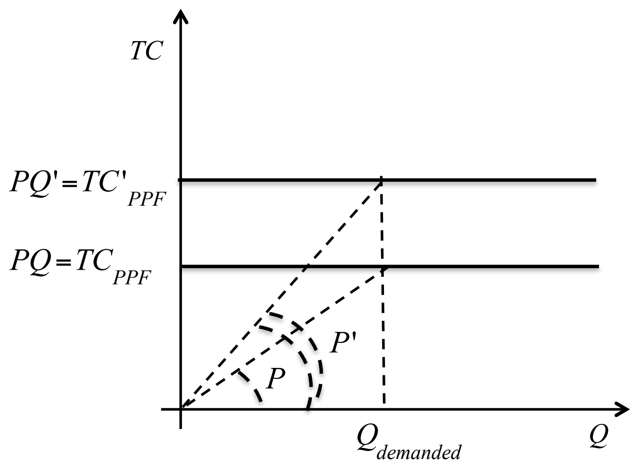

Fig.8b. The growth of total costs under greater productivity

However, the farmer might have too much self-esteem and set a high price groundlessly. But then he should either quit the market or accept its rules, like a consumer does it in 'the corner.' The game theory describes 'satisficing sellers' who adjust their aspiration levels (Berninghaus et al. 2011). However, like consumers' satisficing decisions become optimal (Malakhov 2020b), here the sellers' satisficing decisions become optimal, now with respect to their actual market value of producer's efforts, i.e., to the price. If the producer overestimates the value of his efforts from the very beginning, i.e., the value of MC_f, at the meeting point $Q/(L+S) = MRS\ (H \text{ for } Q) = RPT = T_d/T_f$ his price will be greater than the consumer's readiness to pay $wL=QP$. But the price itself doesn't change consumer's leisure time and doesn't shift the *PPF*.

The wrong self-evaluation results in completely different outcomes. The producer can underestimate his efforts and set a low price. There he launches the arbitrage process because the consumer accepts the low price, and the new equilibrium price appears at the zero search level. But if the producer overestimates his efforts and sets a high price, he has no more time at the *PPF* level to the search for another buyer, and he should either accept the 'just' consumer's price or quit the market.

We see that when the consumer buys optimally, the exact producer's knowledge of the individual demand finally results in the equilibrium transaction; of course, if it takes place. Both parties to the transaction make simple calculation of averages and successfully meet each other for their mutual benefit.

Of course, the set of possible outcomes of this meeting might be larger if we take into account an option for bargaining. But it doesn't change the logic of Eq.17. According to it the Invisible hand will lead the producer to the meeting point in strict compliance with Adam Smith's prediction, leaving the price setting for both parties to the transaction. And even there the Invisible hand helps them and limits the bargaining option by Eq.9.1. Consumer's total costs $w(L+S)$ and his willingness to sell *WTA* cannot exceed the equilibrium level with its zero-search purchase price, i.e., the price 'at the door'.

However, the work of Invisible hand doesn't stop here. It provides grounds not only in the short run for one transaction but also the basis for long-term relationship when transactions take place regularly, under the natural order of a local market with its ordinary rates, both wages and profit Adam Smith was speaking about.

The analysis of long run decisions needs many specifications that go beyond the scope of this paper. But it can contribute to the understanding of the basic principles of long-term relationship. The Eq. (17) can be rewritten in the following way:

$$P = MC_f + MC_d = MC_f\left(1 + \frac{Q}{L+S}\right) = MC_f\left(1 + \frac{w}{P_e}\right) \quad (18.1)$$

$$\frac{w}{P_e} = \frac{P - MC_f}{MC_f} = \frac{P - AC_f}{AC_f} = m \tag{18.2}$$

where m is the sales markup.

The long-term relationship needs the trust. Here the trust is provided by the Eq. (18.2). For example, if the trader buys goods at the farm to resale them on a local market, he cannot earn more than his customers. At the equilibrium his sales markup is equal to the consumers' purchasing power. But the seller doesn't get money for nothing. He is selling his skills and experience like the customer gets wages also for his skills and experience.

We see that for both short and long run decisions the producer's knowledge is very limited. In the long run he needs the information about the purchasing power. And in the short run he doesn't need the information even about the consumer's willingness to pay. The producer should be realistic about the efficiency of his efforts, and the reliable knowledge and information on individual demand provides his get-together with the consumer on the imperfect market where he not only meets buyer's needs but also unintentionally optimizes his allocation of time and maximizes the general consumption-leisure utility at the equilibrium. As a result, the optimal exchange is followed by the optimal social allocation of time.

These considerations confirm the assumption made once by Kenneth Arrow:

"*The notion of the inner coherence of the economy – the way markets and the pursuit of self-interest could in principle achieve a major degree of coordination without any explicit exchange of information, but where the results may diverge significantly from those intended by the individual actors – is surely the most important intellectual contribution that economic thought has made to the general understanding of social processes*." (Arrow, op.cit., p.108).

However, the labor-search-leisure model doesn't stop at the optimal social allocation of time. The general understanding of social processes Arrow told about can go deeper to the level of natural processes.

Part III. Gravitation

The analysis of Fig.8a,b demonstrates that the price setting itself cannot move the production possibility frontier. If the producer overestimates the cost of his working time, it doesn't mean that he automatically shifts the frontier and gets more time. To get it, the manufacturer should be more productive. And the market supports this natural rule, here by consumer's leisure time. The price setting itself doesn't change consumer's leisure time, and the optimistic producer stays on the same frontier even with the high price. His industry is valuable if it is transformed into consumer's leisure time, i.e., the time when the bought item is using.

It has been noticed that both parties to the transaction are unaware of the opposite allocation of time. In that sense the production possibility frontier really is virtual. The production cycle can start with respect to consumption cycle earlier or later. But some relationship between producer's time and consumer's time exists. Producer's efforts finally are transformed into the time when the bought item is being consumed.

This simple consideration results in the same simple conclusion:

$$H = \delta\left(T_f + T_d\right) \tag{19}$$

or consumer's leisure time is proportional to the time of production and delivery.

Although this conclusion is really simple, it keeps some important economic reasoning. The producer tries to be more productive and to cut his working time; the consumer is interesting to use the bought item as long as possible. Even when the time horizon is predetermined either naturally or technologically, the consumer tries to spend less time on labor and search and to increase leisure time.

It is evident that the value δ is low for groceries, and it is high for durables. But this general reasoning can be specified.

If we come back to the consumer's choice when he is unaware of producer's allocation of time but nevertheless, he is agreeing that the producer has tried his best and the item is sold on the production possibility frontier, we can get a normal *0E* for his choice with the corresponding geometrical, i.e., natural proportions (Fig.9):

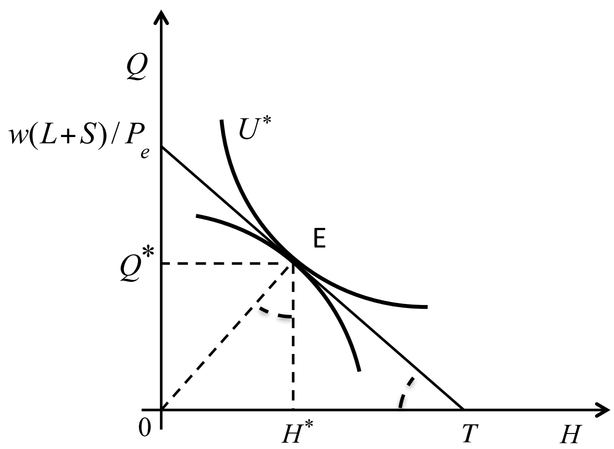

Fig.9. The geometry of market equilibrium

The geometry of the equilibrium results in the following considerations

$$H = Q\frac{Q}{L + S} \tag{20.1}$$

$$\frac{H}{L+S} = \left(\frac{Q}{L+S}\right)^2 = \left(\frac{w}{P_e}\right)^2 \tag{20.2}$$

$$\frac{w}{P_e} = \sqrt{\frac{H}{L+S}} \tag{20.3}$$

or the purchasing power is equal to the square root of leisure time to non-leisure time ratio.[6]

This finding looks very interesting, but it is only the interim conclusion. Being complemented by Eq.19, it gives us the value δ:

$$H(L+S) = Q^2 \tag{21.1}$$

$$H = \delta\left(T_f + T_d\right) \tag{21.2}$$

$$\delta = \frac{Q^2}{(T_f + T_d)(L+S)} \tag{21.3}$$

However, when the individual consumer choice q_i usually represents a part of producer's supply Q, the Eq.21.3 can be rewritten in the following manner:

$$\delta_i = \frac{Q}{T_f + T_d}\frac{q_i}{L_i + S_i} \tag{22}$$

Here the analogy with the Newton's law of universal gravitation is quite obvious. We don't know the distance between a seller and a buyer but we know the time they have spent to meet each other. And their 'masses' represent the quantities supplied and demanded. For example, the Coca-Cola gravitation field is extremely strong due to great Q value but the gravitation itself varies with respect to individual demand.[7]

These considerations clarify the meaning of Eq.19. There the δ value really looks like some force, which transforms the time of production into the time of consumption under the division of labor – the cornerstone of Adam Smith's philosophy. The δ value demonstrates how the time as the most important resource is allocated between sellers and buyers.

The Newton's influence on Adam Smith has been acknowledged by economic science, and his gravitation metaphor on the natural price had inspired many fruitful studies, especially in gravity models of trade. However, the analysis presented in this paper provides an assumption that the gravitation is not only the property of the price mechanism. It also exists between producers and consumers at the level of natural values – quantity and time. At least, the Eq. 22 dispels the well-known marketing myth that all customers are equal. The labor-search-leisure

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[6] The statistical verification of Eq.20.3 should be approached with caution because general data on consumer's allocation of time doesn't make difference between 'the common model' of behavior presented here, and 'the leisure model' of behavior with negative marginal utilities of both money and consumption (Malakhov 2015,2018a).

[7] The δ value can be interpreted on the macroeconomic level as the labor productivity per hour, both of manufacturing and services, measured in natural units and adjusted by the real wage rate $w/P_e=Q/(L+S)$. But the measurement of labor productivity in natural units needs here some methodological efforts (S.M.)

model has successfully described many psychological phenomena by standard economic tools. Here it shows that some customers are 'more equal'. High wage rates raise the $MRS\ (H\ for\ Q) = Q/(L+S)=w/P_e$ ratio that makes some customers more valuable for producers by greater δ values even for the same quantity q_i demanded.

In this way the Eq.22 can contribute to the analysis of sorting and matching. We see that high δ values correspond to the meetings between the most productive manufactures and the most 'productive' customers, while the low δ value matches marginal producers witn marginal customers.

Conclusion

The labour-search-leisure model demonstrates how both buyers and sellers, following simple decision rules, can meet each other on the imperfect market. From the theoretical point of view, the producer fails only if he overestimates the value of his efforts – the same mistake the consumer makes when he comes 'to the corner.' [8]

Actually, if producers and consumers don't make correct estimations of their efforts, then imperfect markets keep them spending more time to meet each other. Their matching is ceasing to be frictionless. However, the labor-search-leisure model doesn't claim to justify some matching rule; it discovers the potentials of the market to enable the matching process. Although this presentation is academic, it underlies many practical decisions like the common greengrocer's choice of the place for his store with respect to the purchasing power in the area where the target income group is living. There, he gets his gains and makes residents happy in a way Adam Smith was speaking about the natural order with its ordinary wages and profits.

The introduction of different productivity on production itself and delivery doesn't change the logic of the model. There, the results would be reconsidered under the assumptions either of the labour advanced or services advanced technological progresses where services present vehicles that provide consumers' leisure (Malakhov 2020a).

The limited scope of this paper leaves many questions in abeyance, first of all the methodology of the analysis of quantity demanded. There are many ways to promote this study, from the economics of attributes (Lancaster 1966) to Eugen Slutsky's ideas on the difference between trade units and consumption units (Slutsky 2010). The choice between synthetic pullover and hand-made Norwegian wool sweater doesn't represent the choice between one unit and another. Consumers compare leisure time they can wear it or how many times they can put it on. And the development of renting clothes' market (Brightwaite 2018) supports the idea of

[8] The possibility of corner solutions that represent the highest payoffs for both parties to the transaction results in the assumption that the equilibrium solution provided by Invisible hand is the Nash equilibrium (S.M.)

reconsideration of quantity demanded on clothes' markets in general, like the analysis of expected mileage explains why sellers of good cars don't quit the market under the theorem of 'lemons'. If we look at the automobile market from the point of view of labor-search-leisure model, we can see that the consumption unit that *shoppers* can buy there without efforts is a mile in the taxicab. Its price becomes the equilibrium price on the market where vehicles are traded with regard to their expected mileage. The options 'to take a taxi or to rent/buy a car' turn the purchase of a car into the acquisition of input when driving becomes a specific form of home production, here the 'production of miles.' As a result, independent taxi drivers can make the efficient arbitrage at the zero search level with respect to their willingness to accept, and buyers of good 'lemons' easily substitute leisure by pleasurable driving, i.e., pleasurable search of miles in home production (Malakhov 2019).

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