Online education and the Great Convergence

Wada, Shuhei

Graduate School of Economics, Doshisha University

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Abstract

In this study, we extended Acemoglu et al. (2014) in the following two ways. First, we used a constant elasticity of substitution human capital production function to show that in the short run, Internet technologies such as online education are likely to be advantageous for middle-income countries. Second, to examine whether one country voluntarily supplies online education to other countries, we changed the static model to a dynamic model. We found that despite it being a public good, developed countries voluntarily supply online education to developing countries. This is because when online education is provided, the level of human capital is higher in both transitional dynamics and the steady state than otherwise.

Keywords: Human Capital; Online Education; Leapfrogging; Limit Cycle

Classification Codes: JEL F63, H41, I24, I25

1. Introduction
With the ongoing spread and advancement of the Internet, the characteristics of globalization are changing. Baldwin (2016) unraveled the history of globalization by focusing on the changing costs of three factors: goods, ideas, and people. The advent of railroads and steamships in the 19th century led to lower transportation costs, which in turn stimulated trade and the concentration of industry in a few countries. The spread of the Internet in the 1990s made it easier to manage production processes and operations. The formation of international value chains led to the rise of emerging economies. Baldwin predicted that the advent of face-to-face technologies and remote technologies, which substantially lower the cost of moving people, will further narrow the income gap between countries.

In this study, we examined online education to consider the impact of decreasing face-to-face costs on the inequality between developed and developing countries, with the aim of answering the following questions. First, which countries experience the greatest benefits from online education? Second, what is the impact of online education on long-term human capital accumulation? Third, is online education voluntarily provided by developed countries to developing countries?

Our approach involved extending Acemoglu et al. (2014) in the following two ways. First, we changed the human capital production function from a Cobb–Douglas
function to a constant elasticity of substitution (CES) function. Second, we extended their model to a long-term model, assuming that teachers’ human capital is determined by students’ human capital after they are educated.

The results revealed the following three findings. First, by extending the human capital production function proposed by Acemoglu et al. (2014), we developed a model in which middle-income countries experience the greatest short-term growth in human capital as a result of the adoption of online education. This is because if the quality of teachers in developing countries is low, the benefits of local teachers specializing in online lectures (benefits of specialization) decrease, and if it is high, the benefits of attending lectures presented by the world’s top teachers (benefits of superstar teacher) decrease.

Second, our long-term dynamics showed that developing countries always catch up with developed countries when online education is available, after which developed and developing countries alternate as leaders. This is because developing countries grow more rapidly than developed countries in terms of human capital and eventually catch up, as in addition to education by quality teachers in developed countries, the educational resources of their own teachers are reallocated to complementary educational activities. Leapfrogging occurs when the gap in human capital between leader and follower
countries is sufficiently small. Because of the small human capital gap between the new leader and the new follower countries, the leader and follower countries will continue to be replaced.

Third, although online education is a public good, a leader country will voluntarily provide online education to a follower country. This is because long-term human capital increases through supplying online education because the leader country receives the benefits of higher-quality education and specialization when it is overtaken by the follower country.

Broadly speaking, our study is related to two bodies of literature: that on online education and that on cross-country inequality. Particularly, our study is closely related to those of Acemoglu et al. (2014) and Li et al. (2016). Acemoglu et al. (2014) presented the first quantitative model of the international effects of online education by comparing the pre-Internet regime (without online education) with the post-Internet regime (with online education). They identified two effects on students’ human capital: a “technological windfall,” that is, an increase in students’ human capital resulting from access to lectures by world-leading teachers and specialization in the supply of hands-on instruction by local faculty, and a “democratizing effect,” which narrows the cross-country human capital gap as a result of greater technological windfalls in countries with
lower levels of human capital. We showed that online education is a technological change that is more likely to benefit middle-income countries than to produce a democratizing effect.

Li et al. (2016), noting that free riding is a dominant strategy in the supply of public goods, modified the model presented by Acemoglu et al. (2014) to a trade model for two types of educational goods and argued that establishing online educational clubs is effective in preventing free riding. By extending the model of Acemoglu et al. (2014) to include long-term dynamics, we show that free riding is not the dominant strategy despite online education being an international public good.

There have also been numerous theoretical and empirical studies on the impact of online education on human capital, although their conclusions have been mixed. Posner (2012) pointed out that one benefit of online education is that it allows students to either rewind or fast-forward their learning, while Perri (2016) based his model on the assumption that the benefits are greater for lower-ability students. Studies that have empirically examined the effect of the introduction of online education on the academic achievement gap include those of Coates et al. (2004), Banerjee and Duflo (2014), and Cacault et al. (2019). For example, Cacault et al. (2019), using a utility function that included academic performance and the cost of attending school, found that online
education improved the grades of high-performing students while lowering those of low-performing students. We use the human capital production function to explain why the benefits of online education are greater for students with lower initial ability, an assumption made by Perri (2016).

Studies on cross-country inequality include those on the human capital Kuznets curve, leapfrogging, and the impact of globalization and the Internet. The human capital Kuznets curve, in which human capital inequality increases in the early stages of economic development and decreases in the later stages of development, was found in empirical studies including those by De Gregorio and Lee (2002), Castello and Domenech (2002), and Lim and Tang (2008). Studies presenting a theoretical explanation of the mechanism include those of Glomm and Ravikumar (1998) and Matsuo and Tomoda (2012). Glomm and Ravikumar (1998) showed that increasing short-term returns to scale is a necessary condition for obtaining Kuznets’ inverted U-shaped curve. Matsuo and Tomoda (2012) introduced the concept of the subsistence consumption level and explained that the human capital gap widens in the early stages of development because only rich households can invest in education, whereas once economic development is sufficiently advanced, the human capital gap shrinks because the return on human capital investment declines. While Acemoglu et al (2014) present a model in which countries
with lower teacher quality benefit more from online education, we present a model that is consistent with the human capital Kuznets curve.

Acemoglu et al. (2014) suggested that online education can contribute to narrowing the human capital gap between developed and developing countries, and that in some cases, developing countries may even eventually exceed the human capital of developed countries. We extended the model of Acemoglu et al. (2014) to include the long-term impact of globalization and the Internet on cross-country inequality. Baldwin (2016) explained why the gap between developed and developing countries widened prior to the 1990s as a result of globalization but then narrowed from the 1990s onwards by focusing on the cost of moving ideas. He also pointed out that service workers in developed countries might compete with service workers in developing countries in the future because online technology advances will lower the cost of face-to-face communication. Our long-term model supports this prediction. Our study is also related to the literature on leapfrogging, when developing countries surpass developed countries, such as the study by Brezis, Krugman, and Tsiddon (1993). It has been shown that a country that is overtaken in terms of technology can re-overtake the country that overtook it, and we explain this mechanism in terms of human capital. In contrast, our dynamics of the human capital gap between the two countries is a limit cycle. In our model, the
country that is overtaken will re-overtake the country that has overtaken it, but this is not a probabilistic phenomenon.

Our model is similar to that of Tamura (1991), where per capita income converges as a result of the spillover effect of human capital. However, Tamura (1991) treats how spillover occurs as a given, whereas in our model, the spillover mechanism itself is determined endogenously.

The remainder of the paper is organized as follows. Section 2 presents the short-term model, Section 3 presents the long-term model, and Section 4 concludes.

2. Short-term Economy

Following Acemoglu et al. (2014) and Cacault et al. (2019), we consider two human capital regimes, the pre- and post-Internet regimes, to enable us to compare the case where online education is available with that when it is not available.

There are two asymmetric countries, and each island \( j \ (j = 1, 2) \) is inhabited by a continuum 1 of students and a continuum 1 of teachers. A student’s post-schooling human capital, a student’s pre-schooling human capital, and the quality of teachers in island \( j \) are denoted by \( y_j \), \( e_j \), and \( h_j \), respectively. The teachers’ time endowment is normalized to 1. Because developed countries tend to have more
developed preschool education than developing countries and the quality of teachers is higher, we also assume that $e_1 \geq e_2$, $h_1 \geq h_2$, and $h_j \geq e_j$.

Let $i \in [0,1]$ be the index of education. The amount of educational service available to students on island $j$, $x_j(i)$, is equal to the composite educational goods supplied on island $j$, $X_j$, and is expressed as follows:

$$\ln X_j = \int_0^1 \ln x_j(i) di.$$  (1)

The resource constraint in relation to education on island $j$ is given by:

$$\int_0^1 x_j(i) di = \int_0^1 H_j(i) di,$$  (2)

where $H_j(i)$ is the supply of education on island $j$ indexed by $i$. Now, we consider two types of education. Following Acemoglu et al. (2014), “lecture” represents the education services supplied by a superstar teacher on island 1 via web-based technologies and broadcast to both island 1 and island 2, and thus there is no limit on the number of students who are able to “attend.” The remaining education services are “hands-on instruction” provided by local teachers, that is, teachers on island $j$ provide hands-on instruction to the students on island $j$. Students receive lectures representing education services indexed by $0 \leq \beta < 1$ and hands-on instruction indexed by $1 - \beta$. In the following explanation, we use variables with a prime (e.g., $y_j^\prime$) to denote pre-Internet values.
We assume that the total supply of effective education services can be assigned
to various educational tasks in any way. We also assume that education services are
competitively priced, and that teachers choose to allocate their time and skills to these
services based on market prices. This allocation can be alternatively calculated as the
solution to the maximization problem of post-schooling human capital. Thus, $X_1^c$, $X_1$, and $X_2$ are given by equations (3), (4), and (5), respectively, as follows:

$$X_1^c = h_1; \quad (3)$$

$$X_1 = h_1; \quad (4)$$

$$X_2 = h_1^\beta \left( \frac{h_2}{1 - \beta} \right)^{1-\beta}. \quad (5)$$

Equation (5) suggests that for island 2, the total supply of effective education
services increases because of both lectures by the world’s top teachers (benefits of
superstar teacher) and local teachers’ specialization in supplementing the online lectures
(benefits of specialization).

Instead of the Cobb–Douglas human capital production function proposed by
Acemoglu et al. (2014), we use post-schooling human capital, $y_j$, which is given by the
CES human capital production function as follows:

$$y_j = \left[ (1 - \alpha)e_j^\sigma + \alpha X_j^\sigma \right]^{\frac{1}{\sigma}}, \quad (6)$$

where $\alpha \in [0, 1]$ and $\sigma \in [0, 1]$. From (3), (4), (5), and (6), $y_j^c$, $y_1$, and $y_2$ are given
by equations (7), (8), and (9), respectively, as follows:

\[
y_j^c = [(1 - \alpha) e_j^\sigma + \alpha h_j^\sigma]^{1/\sigma}; \tag{7}
\]

\[
y_1 = [(1 - \alpha) e_1^\sigma + \alpha h_1^\sigma]^{1/\sigma}; \tag{8}
\]

\[
y_2 = [(1 - \alpha) e_2^\sigma + \alpha h_1^\beta \left(\frac{h_2}{1 - \beta}\right)^{(1-\beta)\sigma}]^{1/\sigma}. \tag{9}
\]

PROPOSITION 1: (Benefits of Online Education for Developing Countries) Countries with lower preschool human capital benefit more from the introduction of online education. An inverse U-shaped relationship is found between the quality of teachers in developing countries and the benefits of online education. The more educational goods that are supplied online, the greater the growth in human capital.

PROOF: The benefit of the introduction of online education, expressed as \(y_2/y_2^c\), satisfies equation (10) as follows:

\[
\left(\frac{y_2}{y_2^c}\right)^\sigma - 1 = \frac{\alpha \gamma^\sigma \left[y^{-\beta\sigma} \left(\frac{1}{1 - \beta}\right)^{(1-\beta)\sigma} - 1\right]}{(1 - \alpha)\delta^\sigma + \alpha \gamma^\sigma}, \tag{10}
\]

where \(h_2 \equiv \gamma h_1\) and \(e_2 \equiv \delta h_1\). Since \(0 \leq \gamma \leq 1\) and \(0 \leq \delta \leq 1\) holds, the left-hand side of equation (10) decreases with \(\delta\) and increases with \(\beta\). Partial differentiation of equation (10) with respect to \(\delta\) yields:
\[
\frac{\partial}{\partial \gamma} \left( \frac{y_2^\sigma}{y_2^c} - 1 \right) \geq 0 \text{ if } \gamma \leq (1 - \beta)^{1+\frac{1-\sigma}{\beta \sigma}}. \tag{11}
\]

Equation (11) can be intuitively explained as follows. The lower the quality of teachers, the smaller the benefit from specialization, and thus up to a certain point, the benefit of online education increases as the quality of teachers increases. However, beyond that point, the decrease in benefit of superstar exceeds the increase in the benefit of specialization, and thus it becomes smaller. The results for \( \beta \) also suggest that the human capital of students in developing countries will rise by providing more classes online.

In summary, it is suggested that the introduction of online education is most beneficial for middle-income countries. The introduction of online education in the least-developed countries (LDCs) is very costly because the Internet is not widely available, which means that \( \beta \) is small in these countries. The quality of teachers, \( \gamma \), is also low in the LDCs. Thus, if the cost of adopting online education is high, the LDCs may not adopt it.

3. Long-term Economy

In this section, we examine the long-term economy in an effort to determine
whether there is an incentive for one country to voluntarily supply online education to another country. The condition to do so is, after a certain period, the human capital of developed countries in the post-Internet regime is higher than that in the pre-Internet regime.

Consider an economy in which there are individuals who live for two periods, receiving schooling in the first period and working as teachers in the second period. In each period $t$, there is a teacher and a student on island $j$. In the following analysis, $e_j$ does not affect the results, and thus we set $\sigma \to 0$ and $e_j = 1$. We also assume that $y_{j,0}$ and $h_{j,0}$ are given. To turn a static model into a dynamic model, we assume that the quality of teachers in period $t + 1$ is determined by the post-school human capital $y_{j,t}$ of students in period $t$. This can be expressed as follows:

$$h_{j,t+1} = f(y_{j,t}) = \theta y_{j,t}, \quad \theta \in (0, 1).$$  \hspace{1cm} (12)

First, let us examine the steady-state human capital of developed and developing countries in the pre-Internet regime. To account for the deviation between the period when the student receives their education and the post-school period, equation (3) can be rewritten as:

$$y_{j,t} = (h_{j,t})^\alpha = \theta^\alpha y_{j,t}^\alpha.$$

If we define $z_{j,t} \equiv \ln y_{j,t}$, $z_{j,t+1}^\alpha$ can be written as:
\[ z_{j,t+1}^c = a z_{j,t}^c + \alpha \ln \theta. \] (14)

**LEMMA 1:** *(Pre-Internet Steady State)* Under the steady state, the levels of human capital of island 1 and island 2 are equal.

**PROOF:** If we define the steady-state human capital \( z_j^c \) such that it satisfies \( z_j^c = z_{j,t+1}^c = z_{j,t}^c, \) \( z_j^* \) is given by:

\[ z_j^* = \frac{\alpha}{1 - \alpha} \ln \theta. \] (15)

Because \( z_j^c \) is determined independently of \( y_{j,0} \) and \( h_{j,0} \), \( z_1^* = z_2^* \) is satisfied.

3.1. Catch-up

In the post-Internet regime, there are two possible dynamics: catch-up and leapfrogging. Catch-up occurs when \( z_{1,t} > z_{2,t} \) and \( z_{1,t+1} > z_{2,t+1} \) or \( z_{1,t} < z_{2,t} \) and \( z_{1,t+1} < z_{2,t+1} \) are satisfied, while leapfrogging occurs when \( z_{1,t} > z_{2,t} \) and \( z_{1,t+1} < z_{2,t+1} \) or \( z_{1,t+1} < z_{2,t+1} \) and \( z_{1,t+1} > z_{2,t+1} \). \( z_{1,t+1} \) and \( z_{2,t+1} \) can be expressed as follows:

\[ z_{1,t+1} = \alpha z_{1,t} + \alpha \ln \theta, \] (16)

\[ z_{2,t+1} = \alpha \beta z_{1,t} + \alpha (1 - \beta) z_{2,t} + \alpha \ln \theta - \alpha (1 - \beta) \ln (1 - \beta). \] (17)
Let period \( s \) be the smallest \( t \) such that \( z_{1,t} > z_{2,t} \) and \( z_{1,t+1} < z_{2,t+1} \) is satisfied. The human capital of island 1 and island 2 is determined by catch-up dynamics until period \( s \). Defining \( a_t \equiv z_{1,t} - z_{2,t} \), the gap in human capital between island 1 and island 2 is expressed as:

\[
a_t = \left[ a_0 - \frac{\alpha(1 - \beta)}{1 - \alpha(1 - \beta)} \ln(1 - \beta) \right] \alpha^t (1 - \beta)^t + \frac{\alpha(1 - \beta)}{1 - \alpha(1 - \beta)} \ln(1 - \beta),
\]

\( t \leq s \).

**PROPOSITION 2:** *(The Transition from Catch-up to Leapfrogging)* Leapfrogging occurs at least once in the post-Internet regime.

**PROOF:** Assuming that leapfrogging does not occur, \( a_t > 0 \) in all periods, and \( 0 < \alpha^t (1 - \beta)^t \leq 1 \) decreases monotonically as \( t \) increases. When \( t = 0 \), \( a_t > 0 \), and when \( t \to \infty \), \( a_t < 0 \). This is a contradiction.

3.2. Leapfrogging

The above discussion shows that the dynamics before period \( s \) is determined by catch-up behavior. We now discuss the dynamics after period \( s + 1 \). When island 1 is a follower and island 2 is a leader in period \( t \), the human capital of island 1 and
island 2 in period $t + 1$ is represented by equations (19) and (20), respectively, as follows:

$$z_{1,t+1} = \alpha(1 - \beta)z_{1,t} + \alpha\beta z_{2,t} + \alpha\ln\theta - \alpha(1 - \beta)\ln(1 - \beta),$$  \hspace{1cm} (19)

$$z_{2,t+1} = \alpha z_{2,t} + \alpha\ln\theta.$$  \hspace{1cm} (20)

**PROPOSITION 3:** *(Change in Leadership)* Island 1 and island 2 alternate as the leader after period $s + 1$.

**PROOF:** We use mathematical induction to prove Proposition 3. Let $k$ be a non-negative integer. We only have to show that island 1 is the leader when $t = s + 2k$, that is, $a_{s+2k} > 0$, and island 2 is the leader when $t = s + 2k + 1$, that is, $a_{s+2k+1} < 0$.

[i] When $k = 0$, by our definition of $s$, $a_s > 0$ and $a_{s+1} < 0$ are satisfied.

[ii] When $k = m$, assume that $a_{s+2m} > 0$ and $a_{s+2m+1} < 0$ are satisfied. Then, when $k = m + 1$, $a_{s+2(m+1)}$ and $a_{s+2(m+1)+1}$ are given by equation (21) and equation (22), respectively, as follows:

$$a_{s+2(m+1)} = \alpha^2(1 - \beta)^2 a_{s+2m} - \alpha(1 - \beta)(1 - \alpha + \alpha\beta)\ln(1 - \beta) > 0,$$  \hspace{1cm} (21)

$$a_{s+2(m+1)+1} = \alpha^2(1 - \beta)^2 a_{s+2m+1} + \alpha(1 - \beta)(1 - \alpha + \alpha\beta)\ln(1 - \beta) < 0.$$  \hspace{1cm} (22)

Because we assumed that $a_{s+2m} > 0$ and $a_{s+2m+1} < 0$, from equations (21) and (22),
\(a_{s+2(m+1)} > 0\) and \(a_{s+2(m+1)+1} < 0\) are also satisfied. From [i] and [ii], island 1 and island 2 will alternate as the leader after period \(s\). ■

From Proposition 3, we obtain equations (23) and (24) as follows:

\[
a_{s+2k+1} = \alpha (1 - \beta) a_{s+2k} + \alpha (1 - \beta) \ln(1 - \beta),
\]

(23)

\[
a_{s+2(k+1)} = -\alpha \beta a_{s+2k+1} - \alpha (1 - \beta) \ln(1 - \beta).
\]

(24)

If we define \(a_E^*\) and \(a_O^*\) as steady-state conditions satisfying \(a_E^* = a_{s+2(k+1)} = a_{s+2k}\) and \(a_O^* = a_{s+2(k+1)+1} = a_{s+2k+1}\), respectively, using equations (21) and (22), we obtain equations (25) and (26) as follows:

\[
a_E^* = -\frac{\alpha(1 - \beta)(1 + \alpha \beta)}{1 + \alpha^2 \beta (1 - \beta)} \ln(1 - \beta),
\]

(25)

\[
a_O^* = \frac{\alpha(1 - \beta)(1 - \alpha + \alpha \beta)}{1 + \alpha^2 \beta (1 - \beta)} \ln(1 - \beta).
\]

(26)

**PROPOSITION 4:** (Limit Cycle) After period \(s\), the dynamics of human capital gap between island 1 and island 2 is a limit cycle.

**PROOF:** Let \(\Delta a_{O,s+2k+1} = |a_{s+2k+1} - a_O^*|\) and \(\Delta a_{E,s+2k} = |a_{s+2k} - a_E^*|\). Then, from equations (23), (24), (25), and (26), \(\Delta a_{O,s+2k+1}\) can be expressed using \(\Delta a_{E,s+2k}\) and \(\Delta a_{E,s+2(k+1)}\) can be expressed using \(\Delta a_{O,s+2k+1}\) as follows:
\[
\Delta a_{0,s+2k+1} = a(1 - \beta)\Delta a_{E,s+2k}, \tag{27}
\]
\[
\Delta a_{E,s+2(k+1)} = -\alpha \beta \Delta a_{0,s+2k+1}. \tag{28}
\]

Thus, it holds that \( \Delta a_{E,s+2(k+1)} < \Delta a_{0,s+2k+1} < \Delta a_{E,s+2k} \). Therefore, after period \( s \), the dynamics of human capital between island 1 and island 2 is a limit cycle. ■

Figure 1 shows a phase diagram of the human capital gap between island 1 and island 2.

Finally, we obtain the human capital of island 1 and island 2 in the steady state. The human capital of island 1 in period \( s + 2(k + 1) \) and period \( s + 2(k + 1) + \)
is given by:

\[
\begin{align*}
&z_{1,s+2(k+1)} = \alpha^2 (1 - \beta + \beta^2) z_{1,s+2k} + \alpha^2 \beta (1 - \beta) z_{2,s+2k} + \alpha (1 + \alpha) \ln \theta \\
&\quad - \alpha (1 + \alpha \beta) (1 - \beta) \ln (1 - \beta), \\
&z_{1,s+2(k+1)+1} = \alpha^2 (1 - \beta) z_{1,s+2k+1} + \alpha^2 \beta z_{2,s+2k+1} + \alpha (\alpha + 1) \ln \theta \\
&\quad - \alpha^2 (1 - \beta) \ln (1 - \beta).
\end{align*}
\] (30)

Let \( z_{1,E}^* \) and \( z_{2,E}^* \) be the steady-state human capital of island 1 and island 2, respectively, in period \( s + 2k \). Similarly, let \( z_{1,O}^* \) and \( z_{2,O}^* \) be the steady-state human capital of island 1 and island 2, respectively, in period \( s + 2k + 1 \). Since \( z_{2,E}^* = z_{1,E}^* - a_E^* \) and \( z_{2,O}^* = z_{1,O}^* - a_O^* \), \( z_{1,E}^*, z_{2,E}^*, z_{1,O}^*, \) and \( z_{2,O}^* \) are given by equations (32), (33), (34), and (35), respectively, as follows:

\[
\begin{align*}
&z_{1,E}^* = -\alpha (1 + \alpha \beta) (1 - \beta) \\
&\quad \frac{(1 + \alpha)(1 - \alpha)[1 + \alpha^2 \beta (1 - \beta)]}{(1 + \alpha)(1 - \alpha)[1 + \alpha^2 \beta (1 - \beta)]} \ln (1 - \beta) + \frac{\alpha}{1 - \alpha}, \quad (32) \\
&z_{2,E}^* = -\frac{\alpha^2 (1 + \alpha \beta) (1 - \beta)}{(1 + \alpha)(1 - \alpha)[1 + \alpha^2 \beta (1 - \beta)]} \ln (1 - \beta) + \frac{\alpha}{1 - \alpha}, \quad (33) \\
&z_{1,O}^* = -\frac{\alpha^2 (1 + \alpha \beta) (1 - \beta)}{(1 + \alpha)(1 - \alpha)[1 + \alpha^2 \beta (1 - \beta)]} \ln (1 - \beta) + \frac{\alpha}{1 - \alpha}, \quad (34) \\
&z_{2,O}^* = -\frac{\alpha [1 + \alpha \beta - \alpha^2 (1 - \alpha) (1 - \beta)]}{(1 + \alpha)(1 - \alpha)[1 + \alpha^2 \beta (1 - \beta)]} \ln (1 - \beta) + \frac{\alpha}{1 - \alpha} \ln \theta. \quad (35)
\end{align*}
\]

From equations (16), (32), (33), (34), and (35), \( z_{1,E}^* > z_{1,O}^* > z_{1}^* \) and \( z_{2,O}^* > z_{2,E}^* > z_{2}^* \) hold (see also Figure 2, Figure 3, and Figure 4). Summarizing the above, we obtain Proposition 5.
PROPOSITION 5: (Voluntary Provision of Online Education) Developed countries have incentives to voluntarily supply online education to developing countries.

As shown in Proposition 5, a leader country will voluntarily provide online education to a follower country, despite the externality. Developed countries will invest in the human capital of developing countries by providing online education to them so that they will be able to receive higher-quality education in the future when their human capital exceeds that of the developed countries. Figure 2 and Figure 3 show the dynamics of human capital on each island. Figure 4 shows the results of a simulation with $\alpha = 0.75$, $\beta = 0.2$, $\theta = 0.8$, $z_{1,0} = -1$, $z_{2,0} = -9$. 

Figure 2
Human capital on island 1

Figure 3

Human capital on island 2
4. Conclusion

In this study, we propose a model generalizing the human capital production function of Acemoglu et al. (2014). The results show that first, the lower the preschool human capital, the greater the benefit from the introduction of online education, and second, the lower the quality of teachers in developing countries, the greater the benefit from the introduction of online education only if the quality of teachers in developed countries is not too much higher than the quality of teachers in developing countries.
We also provide a model of the impact of online education on long-term human capital growth. Developing countries are sure to leapfrog developed countries in terms of human capital in the long term because their human capital increases more rapidly than that of developed countries. Thereafter, the former developed countries and the former developing countries alternate as leaders in terms of human capital. Because the introduction of online education raises the level of human capital in both developed and developing countries, developed countries have an incentive to voluntarily provide online education to developing countries.

In this study, we did not discuss changes in teachers’ wages, which are also likely to influence the number of teachers in an economy, so it may be worth changing the labor market to a two-sector model with a goods sector and an education sector. It would also be worthwhile introducing the supply cost of online education (considered in Li et al. (2016)) or the cost of Internet infrastructure development to the model and comparing the results with those obtained in this study.

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References


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