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Abstract

This paper examines fiscal sustainability in an inflationary environment, particularly the interrelation between government debt and inflation. A model that explicitly incorporates the political utility/objective function of government is constructed. The government's borrowing behavior and inflation are determined through the simultaneous optimization of government and households. The sustainable fiscal debt in an inflationary environment was found to equal the present value of primary balances discounted by the time preference rate of government, not by the interest rate. This result raises the question of whether it is appropriate to apply the fiscal sustainability test of Hamilton and Flavin to high inflation countries.

JEL Classification code: H63, E31

Keywords: Fiscal sustainability; Inflation; The present-value of primary balances; The fiscal theory of the price level; Leviathan

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I. INTRODUCTION

The argument that inflation will eventually accelerate if the government budget deficit increases greatly has an intuitive appeal. Many economists might accept the notion that unrestrained government borrowing will increase prices, a concept which implies that fiscal sustainability and inflation interact with one another. Hence, it appears that careful consideration must be given to the interrelation between government debt and inflation when analyzing fiscal sustainability. However, much of the literature on fiscal sustainability has not sufficiently considered the interrelation between them and has instead directed attention only to economic activities in the real term (e.g., Hamilton and Flavin, 1986; Trehan and Walsh, 1988; Wilcox, 1989; Blanchard et. al., 1990; Hakkio and Rush, 1991; Haug, 1991; Ahmed and Rogers, 1995; Bohn, 1995). Hamilton and Flavin (1986) and Bohn (1995), two of the most prominent papers in this field, are not exceptions: they hardly mention the interrelation between government debt and inflation.

On the other hand, the fiscal theory of the price level (FTPL) directly examines the interrelation between government debt and inflation — more correctly, the interrelation between government debt and the price level (e.g., Leeper, 1991; Sims, 1994, 1998, 2001; Woodford, 1995, 2001; Cochrane, 1998a, 1998b, 2000). Although the focal point of the FTPL is not fiscal sustainability but price level, the FTPL also has an important implication on fiscal sustainability. According to the FTPL, fiscal sustainability can always be held because a government behaves so as to hold it in case of the Ricardian regime and households adjust prices so as to hold it in case of the non-Ricardian regime. Thus, the FTPL implies that any fiscal policy can be sustainable. Buiter (2002, 2004) criticizes the FTPL on this very point. He has denounced the FTPL as false because if default is ruled out, budget constraints must always be satisfied by any economic agent. This problem seems to be rooted in the very nature of the FTPL such that the concept of non-Ricardian fiscal policy is too general and allows too many fiscal policies. The

FTPL implicitly assumes that, in case of the non-Ricardian regime, households are totally passive and obey any fiscal policy. In any case, households will surely buy the bonds issued by the government and adjust prices accordingly. Hence, any fiscal policy can be sustainable. In actuality, households do not appear so passive as to obey a government, buy the bonds issued by the government, and adjust prices accordingly. As a result, the FTPL has been regarded as a useless gimmick which vaguely argues a curious possibility of fiscal sustainability.

The purpose of my paper is to solve the aforementioned problems with the conventional theory of fiscal sustainability and the FTPL and to present an explanation for fiscal sustainability in an inflationary environment. The drawbacks of both theories suggest that it is necessary to construct a model of government's borrowing behavior to analyze fiscal sustainability in an inflationary environment. I construct such a model in this paper. Several important results are obtained by the model. First, the sustainable fiscal debt in an inflationary environment is equal to the present value of primary balances discounted by the time preference rate of government, less than the value discounted by the interest rate in Hamilton and Flavin (1986). Secondly, the model indicates the relation between the level of government debt and the inflation rate is not linear. In addition, the model indicates that a government gains by deliberately making inflation accelerate because steady state primary balance becomes smaller.

The paper is organized as follows. A model that explicitly incorporates the government's borrowing behavior is constructed in section II. The model shows that the behavior of government is neither Ricardian nor non-Ricardian, but that the government behavior is optimal and consistent with both the budget constraint and the transversality condition. In section III, the model is used to show that the sustainable fiscal debt in an inflationary environment is equal to the present value of primary balances discounted by the time preference rate of government. In section IV, the appropriateness of the fiscal sustainability test developed by Hamilton and Flavin (1986) is questioned, particularly as it applies to some developing countries where high inflation is still endemic. Concluding remarks are offered in section V.

II. THE MODEL

1. An economically Leviathan government

The model assumes a Leviathan government.[1] As is known well, there are two extremely different views regarding government behavior—the Leviathan view and the benevolent view. In the Leviathan view, a government gives priority to pursuing its objectives. In the benevolent view, a government maximizes utility the same as a representative household does. Because the fiscal and monetary policies of a benevolent government are practically under the control of the representative household, the optimal behavior of a benevolent government is to supply money to the representative household's saturation point and keep the deflation rate equal to the real interest rate (the Friedman rule) (Friedman, 1969). In the benevolent view, therefore, inflation is basically unrelated to government fiscal behavior. Hence, a model based on the benevolent view appears inappropriate for the purpose of an analysis of fiscal sustainability in an inflationary environment that focuses on the interrelation between a government's borrowing behavior and inflation. On the other hand, it is not necessarily guaranteed that the Leviathan government's behavior has no influence on the development of inflation because the fiscal and monetary policies of a Leviathan government are not perfectly under the control of the representative household. I therefore assume a Leviathan government in this model.

From an economic point of view, a benevolent government maximizes the expected utility of the representative household, but a Leviathan government does not. Unlike a benevolent government, a Leviathan government is therefore not managed by politically neutral bureaucrats who are obligated to mechanically maximize the expected economic utility of the representative household at any time and under any political party that forms a government. It is

[1] The most prominent reference of Leviathan governments is Brennan and Buchanan (1980).

instead managed by politicians who have strong political wills to achieve their own political objectives by all means.[2] Hence, while the expenditure of a benevolent government is a tool to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool to achieve the government's policy objectives. For instance, if a Leviathan government considers national security to be the most important political issue, the expenditure on defense will be increased greatly. If the improvement of social welfare is the top priority, however, the expenditure on social welfare will be increased dramatically.

Is it possible, however, for such a Leviathan government to hold office for a long period? It is possible if both economic and political points of view are considered. The majority of people will support a Leviathan government even though they know that the government does not necessarily pursue only the economic objectives of the representative household because people choose a government for both economic and political reasons. Households are not necessarily represented, from a political point of view, by the same representative household usually presumed in the economics literature. A government is generally chosen by the median of households under a proportional representation system, but the representative household usually presumed in the literature on economics is basically the mean household.[3] Therefore, the economically representative household is not usually identical to the politically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people whereas the conventional economically benevolent government maximizes the economic utility of people.

The Leviathan view generally requires the explicit inclusion of government expenditure,

[2] The government behavior assumed in the FTPL reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies contemplated in the Ramsey literature, in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

[3] See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

tax revenue, or related government activities in the political utility function of government (e.g., Edwards and Keen, 1996). A Leviathan government derives political utility from expenditure for its political purposes. Hence, the larger the expenditure is, the happier the Leviathan government will be. On the other hand, the Leviathan government knows that raising tax rates will provoke people's antipathy and reduce the probability of being reelected, which makes the Leviathan government uncomfortable because it expects that it cannot expend money to achieve its purposes if it loses power. The Leviathan government may regard taxes as necessary costs to obtain freedom of expenditure for its own purposes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. Hence, the political utility function of government can be expressed as $u_G(g_t, x_t)$,[4] where $g_t = \frac{G_t}{p_t}$ is the real government expenditure, $x_t = \frac{X_t}{p_t}$ is the real tax revenue of government at time *t*, and G_t is nominal government expenditure, X_t is nominal tax revenue, and p_t is the price level at time *t*. All variables are expressed in per capita terms. In addition, it can be assumed based on the previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^2} < 0$, and $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} > 0$.[5]

[4] It may be possible to assume that governments are partially benevolent. In this case the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

[5] Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} < 0$. Nevertheless, how they should be assumed

A Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate.

A Leviathan government pursues political objectives under the constraint of deficit financing. Even a Leviathan government must obey the budget constraint at any time. As a whole, the problem an economically Leviathan government should solve is a maximization problem of its expected political utility subject to the budget constraint.

2. The model

The utility function of an economically Leviathan government is u_G and is a constant relative risk aversion utility function. The government's rate of time preference is θ_G. The tax is assumed to be lump sum. The budget constraint of the government is

$$\dot{B}_t = B_t R_t + G_t - X_t - S_t$$

where B_t is the accumulated nominal government bonds, R_t is the nominal interest rate for government bonds, and S_t is the nominal amount of seigniorage at time t. The government bonds are long-term bonds and the returns on government bonds R_t are realized only after holding the bonds during a unit of period, say a year. Government bonds are redeemed in a unit of period and the government successively refinances them by issuing new bonds at each time. R_t is composed of the real interest rate r_t and the expected change of bonds' price by inflation $\pi_{b,t}^e$ such that $R_t = r_t + \pi_{b,t}^e$.

Let $\pi_t = \frac{\dot{p}_t}{p_t}$ be the inflation rate at time t. Because the returns on government bonds

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is not an important issue here because $\dfrac{x_t \dfrac{\partial^2 u_G(g_t, x_t)}{\partial x_t^{\,2}}}{\dfrac{\partial u_G(g_t, x_t)}{\partial x_t}} \dfrac{\dot{x}_t}{x_t} = 0$ at the steady state as will be shown in the proof of the following proposition 1 and thus the results in the paper are not affected by how they are assumed.

are realized only after holding the bonds during a unit of period, investors buy the bonds if $\overline{R}_t \geq E_t \int_t^{t+1} (\pi_s + r_t) ds$ at time t where $\overline{R}_t$ is the nominal interest rate for bonds bought at t. Hence, by arbitrage, $\overline{R}_t = E_t \int_t^{t+1} (\pi_s + r_t) ds$ and $\overline{R}_t = E_t \int_t^{t+1} \pi_s \, ds + r_t$ if r_t is constant, e.g. if at a steady state. This equation means that during a sufficiently small period between t and $t + dt$, the obligation of government to pay for the return on the bonds in future increases not by $dt \, \pi_t$ but by $dt \, E_t \int_t^{t+1} \pi_s \, ds$. Because $\overline{B}_{t,t+1} - \overline{B}_{t,t} = \overline{R}_t \overline{B}_{t,t} = \left(E_t \int_t^{t+1} \pi_s \, ds + r_t \right) \overline{B}_{t,t}$ where $\overline{B}_{t,t+1}$ is the value of bonds at time $t + 1$, which was issued at time t, then $\dot{\overline{B}}_{t,t} = \left(E_t \int_t^{t+1} \pi_s \, ds + r_t \right) \overline{B}_{t,t}$.

If π_t is constant, then $\dot{\overline{B}}_{t,t} = \left(E_t \int_t^{t+1} \pi_s \, ds + r_t \right) \overline{B}_{t,t} \Leftrightarrow \dot{\overline{B}}_{t,t} = (\pi_t + r_t) \overline{B}_{t,t}$, but if π_t is not constant, they are not necessarily equivalent.[6]

Because bonds are redeemed in a unit of period and successively refinanced, the bonds the government is holding at t are composed of bonds issued during between t - 1 and t. Hence, under the perfect foresight, the average nominal interest rate for the total government bonds at time t is the weighted sum of $\overline{R}_t$ such that $R_t = \int_{t-1}^{t} \overline{R}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r_t$. If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ between t - 1 and t are not so different each other, then approximately $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \; ds + r_t$.[7] In addition, if

[6] $\dot{\overline{B}}_{t,t} = (\pi_t + r_t) \overline{B}_{t,t}$ has been used for many analyses because π_t has been usually assumed to be constant.

[7] More precisely, if π_t is constant, then $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \; ds + r_t = \pi_t + r_t$ for any set of weights. If π_t is increasing, then $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r_t > \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \; ds + r_t$ in general because if

$\int_t^{t+1} \pi_s \, ds = \pi_{t+w}$ for some constant $w\left(0 \le w \le 1\right)$ for any t (i.e., if $\int_t^{t+1} \pi_s \, ds$ is represented by π_{t+w} for any t), then $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \, ds + r_t = \int_{t-1+w}^{t+w} \pi_s \, ds + r_t$. The average nominal interest rate for the total government bonds, therefore, develops by $R_t = \int_{t-1+w}^{t+w} \pi_s \, ds + r_t$ and thus $\pi_{b,t}^e$ indicates a total price change by inflation during a unit of period such that $\pi_{b,t}^e = \int_{t-1+w}^{t+w} \pi_s \, ds$.

Let $b_t = \dfrac{B_t}{p_t}$ and $s_t = \dfrac{S_t}{p_t}$. By dividing by p_t, the budget constraint is transformed to

$$\frac{\dot{B}_t}{p_t} = b_t R_t + g_t - x_t - s_t ,$$

which is equivalent to

$$\dot{b}_t = b_t R_t + g_t - x_t - s_t - b_t \pi_t = b_t \left(R_t - \pi_t\right) + g_t - x_t - s_t .$$

Hence, the optimization problem of the government is

$$Max \; E_0 \int_0^{\infty} u_G\left(g_t, x_t\right) \exp\left(-\theta_G t\right) dt$$

subject to

$$\dot{b}_t = b_t \left(R_t - \pi_t\right) + g_t - x_t - s_t .$$

The government maximizes its expected political utility considering the behavior of the representative household reflected in R_t in its budget constraint.

On the other hand, a representative household maximizes the following expected economic utility:

$$Max \; E_0 \int_0^{\infty} u_P\left(c_t\right) \exp\left(-\theta_P \, t\right) dt$$

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new bonds are issued at t only for refinancing the redeemed bonds, then $\overline{B}_{t,t} = \left(1 + \overline{R}_{t-1}\right) \overline{B}_{t-1,t-1}$. In addition, if π_t is increasing, $\int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \;\; ds > \pi_t$ and thus $R_t > \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \;\; ds + r_t > \pi_t + r_t$. Nevertheless, if weights are nearly equal, then approximately $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \;\; ds + r_t$.

where u_P and θ_P are the economic utility function and the rate of time preference of the representative household, subject to the following constraint:

$$\dot{k}_t = f(k_t) - c_t - g_t,$$

where $f(\cdot)$ is the production function, k_t is the real capital per capita, and c_t is the real consumption per capita.[8] The constraint means that the output $f(k_t)$ is demanded for private consumption c_t, private investment $\dot{k}_t$, and government expenditure g_t. Government expenditure g_t is an exogenous variable for the representative household because the government is Leviathan. The representative household maximizes its expected economic utility considering the behavior of government reflected in g_t in its budget constraint. It is assumed that $u_P' > 0$ and $u_P'' < 0$, and the population is constant..

Note that the time preference rate of government θ_G is not necessarily identical to the time preference rate of the representative household θ_P. This property of heterogeneity plays an important role later in this study. The reasons why the rates of time preference are different between government and the representative household can be summed up as follows: (i) a government is chosen from among many political parties not only from an economic point of view but also from a political one while the time preference rate of the representative household is related only to economic activities and not to political activities; (ii) a government is usually chosen by the median of households under a proportional representation system and thus the converged policy reflects the median voter—not the mean voter—while a representative household is basically the mean household;[9] (iii) even though people want to choose a party that has the same time preference rate as the representative household, those of the chosen party

[8] The constraint is equivalent to $\dot{k}_t = f(k_t) - c_t - \dot{b}_t - x_t - s_t + b_t(R_t - \pi_t)$.

[9] See the literature on the median voter theorem (e.g., also Downs 1957), and also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

may differ from those of the representative household owing to errors in expectations (e.g., Alesina and Cukierman, 1990); and (iv) current voters cannot bind the choices of future voters and thus if current voters are aware of this possibility, they may vote more myopically compared to their own rates of impatience in private economic activities (e.g., Tabellini and Alesina, 1990). Hence, it seems that the rates of time preference of government and the representative household are usually heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, a Leviathan government behaves based only on its own time preference rate without hesitation

3. Neither Ricardian nor non-Ricardian fiscal regime

Before examining fiscal sustainability with the model, an important aspect of the model must be examined to help understand the analyses on fiscal sustainability presented in the following sections. A unique feature of the model is that it explicitly includes the political utility function of government. The FTPL and the quantity theory of money on which the conventional theory of fiscal sustainability is based do not explicitly assume the political utility function of government. Nevertheless, it is easily shown that these theories implicitly assume a common special political utility function of government such that $E_0 \int_0^\infty u_G(g_t, x_t) \exp(-\theta_G t) dt$ = constant for any g_t and x_t; thus u_G is constant. Let Hamiltonian H_1 be

$$H_1 = u_G(g_t, x_t) \exp(-\theta_G t) + \lambda_{1t} [b_t (R_t - \pi_t) + g_t - x_t - s_t]$$

where λ_{1t} is a costate variable. The optimality conditions are

(1) $\frac{\partial H_1}{\partial g_t} = 0$,

(2) $\frac{\partial H_1}{\partial x_t} = 0$,

(3) $\frac{d\lambda_{1t}}{dt} = -\frac{\partial H_1}{\partial b_t}$,

(4) $\frac{db_t}{dt} = -\frac{\partial H_1}{\partial \lambda_{1t}}$,

(5) $\lim_{t\to\infty} \lambda_{1t} b_t = 0$.

If the utility function of the government is that u_G is constant, then conditions (1) and (2) are $\frac{\partial H_1}{\partial g_t} = -\lambda_{1t} = 0$ and $\frac{\partial H_1}{\partial x_t} = \lambda_{1t} = 0$ thus $\lambda_{1t} = 0$. Thereby, conditions (1) and (2) hold for any $\pi^e_{b,t}$, π_t, g_t, x_t, and s_t in any period. In addition, in case of $\lambda_{1t} = 0$, condition (3) $\frac{d\lambda_{1t}}{dt} = -\lambda_{1t}(R_t - \pi_t) = 0$ holds for any $\pi^e_{b,t}$, π_t, g_t, x_t, and s_t in any period. Hence, the optimality conditions are condition (4) and the transversality condition (5). Here, condition (4) is equivalent to the budget constraint $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$. As a result, if the political utility function of the government is a special one such that u_G is constant, then the optimality conditions are (i) the budget constraint $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$ and (ii) the transversality condition. Needless to say, both the FTPL and the quantity theory of money are commonly based upon (i) the budget constraint and (ii) the transversality condition.

Hence, the difference between FTPL and the quantity theory of money is merely the difference between interpretations of (i) the budget constraint and (ii) the transversality condition. As is known well, two extremely different interpretations are possible. Because conditions (1) and (2) hold for any $\pi^e_{b,t}$, π_t, g_t, and x_t in any period and thus $\pi^e_{b,t}$, π_t, g_t, and x_t are indeterminate, exogenously setting either the values on prices $\pi^e_{b,t}$ and π_t or the values on government behavior g_t and x_t is necessary for completing a model based on the FTPL or the quantity theory of money. The former option is called Ricardian, and the latter option is called non-Ricardian.[10] Theoretically both options are equally possible, and it is difficult to judge *a*

[10] Kocherlakota and Phelan (1999) argue that, in the Ricardian regime, the control of money supply on the assumption of the quantity theory of money is not sufficient to fix the time path of inflation rate. Traditionally a

priori which option is more consistent with the real world.

The above result highlights the fundamental difference between the model in this paper and the models based on the FTPL or the quantity theory of money. In this model, neither fiscal policy nor inflation is indeterminate and must be given ad hoc and exogenously but, as will be shown in the following section, both are determined through the simultaneous optimization of the government and the representative household. This is in sharp contrast to the FTPL as well as the quantity theory of money, which presume that either the Ricardian or the non-Ricardian regime is given ad hoc and exogenously. Contrarily, it does not matter whether the fiscal regime is Ricardian or non-Ricardian in my model because political utility $u_G(g_t, x_t)$ changes as the government maneuvers control variables g_t and x_t in its optimization.

III. FISCAL SUSTAINABILITY

1. Inflation

Because the purpose of this paper is to examine fiscal sustainability in an inflationary environment, the nature of inflation in the model is examined before analyzing fiscal sustainability. Let Hamiltonian H_2 be $H_2 = u_G(g_t, x_t)\exp(-\theta_G t) + \lambda_{2t}\left[b_t(R_t - \pi_t) + g_t - x_t - s_t\right]$ where λ_{2t} is a costate variable. The optimality conditions of the government's optimization problem shown in II. 2. are

(6) $\dfrac{\partial u_G(g_t, x_t)}{\partial g_t}\exp(-\theta_G\, t) = -\lambda_{2t}$,

(7) $\dfrac{\partial u_G(g_t, x_t)}{\partial x_t}\exp(-\theta_G\, t) = \lambda_{2t}$,

(8) $\dot{\lambda}_{2t} = -\lambda_{2t}(R_t - \pi_t)$,

(9) $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$,

monetarist type rule (e.g., purely speculative time trends in velocity) has been often assumed implicitly.

(10) $\lim_{t\to\infty} \lambda_{2t} b_t = 0$.

Combining conditions (6), (7), and (8) yields the following equations:

$$\frac{g_t \frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}}{\frac{\partial u_G(g_t,x_t)}{\partial g_t}} \frac{\dot{g}_t}{g_t} + \theta_G = R_t - \pi_t = r_t + \pi_{b,t}^e - \pi_t \quad \text{and} \quad -\frac{x_t \frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^{\ 2}}}{\frac{\partial u_G(g_t,x_t)}{\partial x_t}} \frac{\dot{x}_t}{x_t} + \theta_G = R_t - \pi_t = r_t + \pi_{b,t}^e - \pi_t .$$

Because $\frac{g_t \frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}}{\frac{\partial u_G(g_t,x_t)}{\partial g_t}} \frac{\dot{g}_t}{g_t} = 0$ and $\frac{x_t \frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^{\ 2}}}{\frac{\partial u_G(g_t,x_t)}{\partial x_t}} \frac{\dot{x}_t}{x_t} = 0$ at steady state such that $\dot{g}_t = 0$ and $\dot{x}_t = 0$, then $\theta_G = r_t + \pi_{b,t}^e - \pi_t$. Here, by the optimality conditions of the representative household, $r_t = \theta_P$ at steady state such that $\dot{c}_t = 0$, $\dot{k}_t = 0$ and $\dot{g}_t = 0$. Hence $\theta_G = \theta_P + \pi_{b,t}^e - \pi_t$ and thus

(11) $\pi_{b,t}^e = \pi_t + \theta_G - \theta_P$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.

Equation (11) is a natural consequence of simultaneous optimization by a Leviathan government and the representative household. What should be stressed is that $\pi_{b,t}^e \neq \pi_t$ if the rates of time preference are heterogeneous between the government and the representative household. Some may be surprised by the possibility that $\pi_{b,t}^e \neq \pi_t$ because it has been naturally conjectured that $\pi_{b,t}^e = \pi_t$. However, this conjecture is a simple misunderstanding because, as was explained above, $\pi_{b,t}^e$ indicates a total price change by inflation during a unit of period such that $\pi_{b,t}^e = \int_{t-1+w}^{t+w} \pi_s \, ds$. On the other hand, π_t indicates the instantaneous rate of inflation at a point such that $\pi_t = \frac{\dot{p}_t}{p_t} = \frac{\lim_{h\to 0} \frac{p_{t+h} - p_t}{h}}{p_t}$. Equation (11) therefore indicates that π_t

develops according to the integral equation $\pi_t = \int_{t-1+w}^{t+w} \pi_s \, ds - \theta_G + \theta_P$. The conjecture $\pi_{b,t}^e = \pi_t$ is true in case of constant π_t. Because of $\pi_{b,t}^e = \int_{t-1+w}^{t+w} \pi_s \, ds$, if π_t is constant, then the equation $\pi_{b,t}^e = \pi_t$ holds, but if π_t is not constant, the equation $\pi_{b,t}^e = \pi_t$ does not necessarily hold. Equation (11) indicates that the equation $\pi_{b,t}^e = \pi_t$ holds only in a special case such that $\theta_G = \theta_P$ (i.e., a homogeneous rate of time preference). Probably because the homogeneous rate of time preference such that $\theta_G = \theta_P$ has been regarded as naturally prevailing, the equation $\pi_{b,t}^e = \pi_t$ has not generally been questioned. However, as was argued above, a homogeneous rate of time preference is not usually guaranteed.

What does equation (11) (or the integral equation $\pi_t = \int_{t-1+w}^{t+w} \pi_s \, ds - \theta_G + \theta_P$) indicate? It indicates that inflation accelerates or decelerates when the rates of time preference are heterogeneous.[11] If π_t is constant, the equation $\pi_t = \pi_{b,t}^e = \int_{t-1+w}^{t+w} \pi_s \, ds$ holds, and conversely if $\pi_t \neq \pi_{b,t}^e = \int_{t-1+w}^{t+w} \pi_s \, ds$, then π_t is not constant. Without the acceleration or deceleration of inflation, therefore, equation (11) cannot hold in an economy with $\theta_G \neq \theta_P$. That is, inflation accelerates or decelerates as a result of reconciling the contradiction in heterogeneous rates of time preference.

2. The sustainable fiscal debt

Much of the sustainability literature since Hamilton and Flavin (1986) defines fiscal sustainability as the implementation of a fiscal policy by which the transversality condition is satisfied. As in the literature, this paper defines fiscal sustainability also as implementing a fiscal policy by which the transversality condition (10) is satisfied. Hamilton and Flavin (1986)

[11] The model can be used to analyze inflation. See, Harashima (2004, 2005).

show that the sustainable fiscal debt is equal to the present value of primary balances discounted by the interest rate. On the other hand, Bohn (1995) argues that, in a stochastic environment, the sustainable fiscal debt is equal to the present value of primary balances discounted by the marginal rate of substitution. In this subsection, I examine the sustainable fiscal debt in an inflationary environment.

First, the return on government bonds is examined. By equation (11), $\pi_{b,t}^{e} - \pi_t = R_t - r_t - \pi_t = \theta_G - \theta_P$ at steady state. Hence,

(12) $R_t - \pi_t = \theta_G$

at steady state because $R_t = r_t + \pi_{b,t}^{e}$ and $r_t = \theta_P$. Equation (12) indicates that the real return on government bonds $r_{G,t} = R_t - \pi_t$ is equal to the time preference rate of government θ_G at steady state, (i.e., $r_{G,t} = \theta_G$). Intuitively the equation $r_{G,t} = \theta_G$ appears quite reasonable because the equation $r_{G,t} = \theta_G$ is analogous to the well-known steady state condition $r_t = \theta_P$ in the private sector in the Ramsey model.

By equations (11) and (12), the requirement for satisfying the transversality condition (10) is obtained. Substituting equations (11) and (12) into conditions (8) and (9) and solving both differential equations yields the equation: $\lambda_{2t} b_t = -\exp\left[\left(g_t - x_t - s_t \right) \int \frac{1}{b_t} dt + C^{\#} \right]$ at steady state where $C^{\#}$ is a certain constant. Thereby, it is necessary to satisfy $g_t - x_t - s_t < 0$ and $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$ for the transversality condition (10) to be held. Here, by condition (9), $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t}$ at steady state. Hence if $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t} = 0$ at steady state, then b_t is constant and thus $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$. Thereby, the transversality condition holds. However, if $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t} < 0$ at steady state, then b_t diminishes to zero and the transversality condition (10) cannot hold because $g_t - x_t - s_t < 0$. If $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t} > 0$ at steady state, then $\lim_{t\to\infty} \frac{\dot{b}_t}{b_t} = \theta_G$ and thus b_t increases as time passes and $\lim_{t\to\infty} \int \frac{1}{b_t} dt = \frac{C^{\#\#}}{\theta_G}$ where $C^{\#\#}$ is a certain constant. The transversality condition (10) therefore also cannot hold and thus, if and only if $\theta_G = -\frac{g_t - x_t - s_t}{b_t}$ at steady state can the transversality condition (10) $\lim_{t\to\infty} \lambda_{2t} b_t = 0$ hold. The requirement $\theta_G = -\frac{g_t - x_t - s_t}{b_t}$ indicates that the increase of government debt $\theta^G b_t$ (i.e., the real return on government bonds times accumulated debts) should be equal to the primary surplus $-(g_t - x_t - s_t)$ at steady state.

The requirement $\theta_G = -\frac{g_t - x_t - s_t}{b_t}$ also implies that the sustainable fiscal debt in an inflationary environment is different from that in a non-inflationary environment that is argued in Hamilton and Flavin (1986). The sustainable fiscal debt in Hamilton and Flavin (1986) is equal to the present value of primary balances discounted by the interest rate. The present value of primary balances at steady state in Hamilton and Flavin (1986) is $-\int_0^\infty (1+r)^{-j} (g_{t+j} - x_{t+j} - s_{t+j}) dj = -(g_t - x_t - s_t) \int_0^\infty (1+r)^{-j} dj = -\frac{g_t - x_t - s_t}{r} = -\frac{g_t - x_t - s_t}{\theta_P}$. However, the requirement $\theta_G = -\frac{g_t - x_t - s_t}{b_t}$ indicates that the sustainable fiscal debt b_t^* must satisfy the condition

(13) $b_t^* = -\frac{g_t - x_t - s_t}{\theta_G}$.

Hence, if $\theta_G > \theta_P$, then $b_t^* = -\frac{g_t - x_t - s_t}{\theta_G} < -\frac{g_t - x_t - s_t}{\theta_P}$ and the sustainable fiscal debt b_t^* is less than the present value of primary balances discounted by the interest rate

$-\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj$, i.e., $b_t^* < -\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj$.

IV. DISCUSSION

1. The problem of discount factor

Equation (13) indicates that, in a deterministic but inflationary environment (i.e., $\theta_G > \theta_P$), the sustainable fiscal debt needs to be less than the one discounted by the interest rate.[12] The sustainable fiscal debt is therefore quite different from that in Hamilton and Flavin (1986). An intuitive explanation of this result is that, because the inequality $\theta_G > \theta_P$ means that the real return on government bonds is larger than the interest rate, government debts grow more rapidly and thus the sustainable fiscal debt must be smaller. As equation (12) indicates, the real return on government bonds $r_{G,t} = R_t - \pi_t$ is equal to the time preference rate of government at steady state (i.e., $r_{G,t} = \theta_G$), and thus the real return on government bonds $r_{G,t} = \theta_G$ is larger than the interest rate $r_t = \theta_P$ if $\theta_G > \theta_P$. Nevertheless, if $\theta_G = \theta_P$ (i.e, if in a non-inflationary environment), then equation (13) also indicates that

$$b_t^* = -\frac{g_t - x_t - s_t}{\theta_G} = -\frac{g_t - x_t - s_t}{\theta_P} = -\frac{g_t - x_t - s_t}{r} = -\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj$$

as in the model in Hamilton and Flavin (1986). In other words, the conventional model implicitly assumes a non-inflationary environment such that $\theta_G = \theta_P$.

Equation (13) questions the appropriateness of Hamilton and Flavin's (1986) fiscal sustainability test. This kind of test may be valid if in a non-inflationary environment, but equation (13) indicates that in an inflationary environment, satisfying the equation

[12] In a deflationary environment (i.e., $\theta_G < \theta_P$), the sustainable fiscal debt is more than the present value of primary balances discounted by the interest rate such that $b_t^* = -\frac{g_t - x_t - s_t}{\theta_G} > -\frac{g_t - x_t - s_t}{\theta_P}$.

$b_t^* = -\int_0^\infty (1+r)^{-j}(g_{t+j} - x_{t+j} - s_{t+j})\,dj$ does not guarantee fiscal sustainability. To claim fiscal sustainability in an inflationary environment, the equation $b_t^* = -\int_0^\infty (1+\theta_G)^{-j}(g_{t+j} - x_{t+j} - s_{t+j})\,dj$ instead needs to be satisfied. This discount factor problem may not be serious when this kind of test is applied to most developed countries where inflation is currently very low, but it may be more important when this kind of test is applied to some developing countries where even now high inflation is endemic. In those countries, even if fiscal sustainability is validated by Hamilton and Flavin's (1986) test, debts may not be sustainable in reality.

The discount factor problem in Hamilton and Flavin's test has also been raised from another point of view. Bohn (1995) criticizes it for not considering stochastic environments and argues that, in a stochastic environment, the discount factor cannot be represented by the real interest rate but rather by the marginal rate of substitution. Tests using arbitrarily selected real interest rates are therefore inappropriate. This paper raises another important problem regarding the choice of discount factor. Even in a deterministic environment, the real interest rate is not the appropriate discount factor if the environment is deterministic and inflationary.

2. The interrelation between debt and inflation

Many empirical studies analyzing the relation between government debt and inflation assume a simple linear relation between them (e.g., Karras, 1994; Darrat, 2000; Fischer, Sahay, and Végh, 2002). My model indicates, however, that the relation between the level of government debt and the inflation rate is not linear and is much more complex because the level of government debt and the acceleration of inflation depend commonly on θ_G. For example, equations (11) and (13) indicate that a situation such that $\dot{b}_t = 0$ while $\dot{\pi}_t \neq 0$ is possible. Many empirical studies indicate that the relation between the level of government debt and the inflation rate is unclear and inconclusive. This inconclusiveness may be due to the incorrect assumption that the relation between the two is linear.

Equation (13) also suggests an interesting aspect of the interrelation between government debt and inflation. Assume that initially $\theta_G = \theta_P$ but θ_G is unexpectedly raised to be $\theta_G^{\#}$ at a time and thus, after that time, $\theta_G^{\#} > \theta_P$. This unexpected surprise upward shift of the time preference rate of government has interesting consequences. First, inflation starts to accelerate by equation (11). Secondly, the real value of sustainable government bonds $b_t^* = -\frac{g_t - x_t - s_t}{\theta_G}$ is shifted to be $b_t^{*\#} = -\frac{g_t^{\#} - x_t^{\#} - s_t^{\#}}{\theta_G^{\#}}$ where both the sustainable fiscal debt and steady state primary balance are smaller than before such that $b_t^{*\#} < b_t^*$ and $-\left(g_t^{\#} - x_t^{\#} - s_t^{\#}\right) < -\left(g_t - x_t - s_t\right)$. The downward shifts of the sustainable fiscal debt and steady state primary balance are analogous to those of capital stock and consumption in the Ramsey model on the private economy. Government debt b_t^* corresponds to the capital stock in the Ramsey model. The primary balance $-\left(g_t - x_t - s_t\right)$ corresponds to consumption in the Ramsey model. Finally, the time preference rate of government θ_G that equals the real return on government bonds at steady state as was shown in equation (12) corresponds to the time preference rate of the representative household that equals the real interest rate at steady state in the Ramsey model. As both steady state capital and consumption shift downwards in the Ramsey model if the time preference rate of the representative household shifts upwards, both steady state government debt and primary balance shift downwards if the time preference rate of government shifts upwards. As a result, $b_t^{*\#} < b_t^*$ and $-\left(g_t^{\#} - x_t^{\#} - s_t^{\#}\right) < -\left(g_t - x_t - s_t\right)$ when θ_G shifts upwards such that $\theta_G^{\#} > \theta^P$. Because market participants know this mechanism and thus nobody buy these bonds in markets unless the real value of government bonds has sufficiently fallen in this environment of accelerating inflation, the real value of already issued government bonds soon falls from b_t^* to $b_t^{*\#}$.

If the time preference rate of government is unexpectedly raised, therefore, households will experience double suffering, namely, from accelerating inflation and from the loss of the value of government bonds they hold. On the other hand, the government gains by the unexpected upward shift of θ_G because steady state primary balance that the government is obligated to achieve in the future becomes smaller. This mechanism may tempt a government into raising θ_G to lessen the burden of debts, although this action also accelerates inflation.

V. CONCLUDING REMARKS

An important contribution of this paper is that the concept of fiscal sustainability is extended to an inflationary environment. Fiscal policies and inflation rates are determined simultaneously in the model. The model in this paper is therefore fundamentally different from models based on the FTPL or the quantity theory of money. The main findings of the paper are firstly that the sustainable fiscal debt is equal to the present value of primary balances discounted by the time preference rate of government. The sustainable fiscal debt is therefore less than the present value of primary balances discounted by the interest rate in an inflationary environment. This result appears very important, particularly when studying fiscal sustainability in developing countries where high inflation is still endemic. Secondly, the model indicates that the relation between the level of government debt and the inflation rate is not linear. The relation between them is unclear and inconclusive in empirical studies, possibly because the relation is wrongly assumed to be linear. In addition, the model indicates that a government gains by deliberately making inflation accelerate because the steady state primary balance decreases.

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