Invisible hand equilibrium in family: the gravitation between men and women in marriage markets

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"Happy families are all alike; every unhappy family is unhappy in its own way."

Leo Tolstoy "Anna Karenina"

Abstract

The marginal analysis of consumer and producer behavior under wage and price dispersion discovers some previously unknown properties of optimality. Both parts in transaction are making satisficing choices, which unintentionally optimize their allocation of time, for a seller between production and services and for a buyer between labor, search, and leisure. This implicit optimization process results in a particular equilibrium, which can be regarded as the proof for the Invisible hand. Marriage markets can also produce the Invisible hand equilibrium close to the mating of unlikes but with some important differences. On commodity market corner solutions simply limit consumer behavior, but in marriage market they actively work and result in disequilibria of unhappy families, where either a husband or a wife starts to search leisure time with another partner and create the implicit polygamy. But Becker's conclusion on polygamy as a substitution of one superior mate by several inferiors is true only for female polygamy or polyandry. For male polygamy, the optimal choice represents the total of opposite corner solutions, when an unattractive woman is complemented by an attractive lady, who cannot be chosen alone. While the ambiguous substitution effect of dowry can escape from corner solution, the substitution effect of bride price moves the family to the corner solution and results in the phenomenon of Femme Fatale Fate. Wealthy men and beautiful women really attract each other due to strong gravitational fields of both parts in transaction. But at the equilibrium a beautiful woman reduces the attractiveness of her partner's wealth and cuts the time horizon of happy family. If the husband tries to support his attractiveness by gifts, the equilibrium fails, and a beautiful wife comes to the corner solution and searches for another partner. It is therefore not surprising that the common family position

for a beautiful woman is the role of 'beloved wife' in the polygamy, either explicit or implicit, where she can efficiently use her strong gravitational field.

Key words: invisible hand, marriage markets, mating of unlikes, gravitation

JEL Classification: D11,D13,D82,D83.

Introduction

During last decades the economics of marriage has become one of the most dynamic domains in social sciences. The field of study by its nature enables the communication of economics with various disciplines like sociology, psychology, biology, anthropology, and history. And the economics of marriage itself represent a real methodological mosaic, leaving a space for different approaches, from neoclassical to institutional and behavioral ones, that have been thoroughly reviewed once by Chiaporri (2020).

However, there is some ambiguity in the focus of analysis, when the notions of family, household, and marriage are using interchangeably, especially when studies are concerned with household behavior and the allocation of time in the family (Del Boca and Flinn 2013, Goussé et al. 2017). Nevertheless, the reasonings of household behavior revolve around the marital choice or the act of marriage itself. And here the prominent paper of Gary Becker on mating in marriage markets (Becker 1991) stays as one of the most influential study of the market workings in family decision-making.

Becker successfully provided a methodological basis for the analysis of non-market values. The concept of imputed prices gave him an opportunity to analyze the output of the non-market productivity - of intelligence, education, health, religion, personality and even love, that maximize the aggregate output of the marriage market. Becker's belief in market was so strong that he directly addressed to its invisible hand, which was leading selfish brides and grooms to the general equilibrium.

This paper also represents the demonstration of the inner market mechanism, which produces the optimal choices, that can be treated as the workings of the Invisible hand. This approach is based on the proof of the Invisible hand on commodities markets (Malakhov 2020, Malakhov 2021a, Malakhov 2021b). But there, buyers and sellers are not selfish maximizers. They're making under wage and price dispersion explicit satisficing choices that unintentionally and implicitly maximize both the output and utility. Optimal matchings, that mean under price dispersion successful meetings of buyers and sellers, who are spending time to search each other, occur automatically without explicit exchange of information. Moreover, while the equilibrium of the Invisible hand stays as the market phenomenon, wages and prices are constructing only the framework of the model, while the decision-making is provided by natural values of time and quantity. This feature of Invisible hand equilibrium looks very important especially for the analysis of marriage markets because in pre-historical hunter-gatherer societies when market didn't yet exist, the marital matchmaking could be the first sapling – a wife for a bag - of the phenomena of trade and exchange, and the bride price was the first price paid in the history of the civilization. The cliometrics of the family tells us, that "understanding of the households in the past is therefore important for understanding the way in which persistence of culture comes about. Moreover, it means that the behaviours observed when studying households are particularly likely to be transmitted from one generation to next, resulting in persistence of behaviour." (Diebolt et al. 2019). The emotional Victorian Women Writers Project on marriage market (Corelli 1898) provides convincing evidence that in spite of the progress of civilization modern marriage markets have not went far away from the Babylonian Marriage Market, described by Herodotus almost five centuries BC. The brides' market as the open-air trade and exchange exists even today, not only in Rome families in Bulgaria (Charlton 2015) but also in Shanghai, China. And being presented in its shadow form, brides' trade – beauty for wealth – still exist. Even in England marriage markets can be locally identified with respect to indirect attributes of beauty and wealth (Bhroichain et al. 2002).

In this way the equilibrium of the Invisible hand, which provides optimal trade between buyer and seller with respect to natural of values of time and quantity looks like an efficient tool for the understanding of modern marriage markets.

The paper is organized as follows. Part 1 briefly presents the equilibrium of the Invisible hand on commodity market. While it has been already described in published papers, it will be suitable for readers of this article to get it hand. There, the particular attention is paid to the limits of consumer behavior, i.e., to corner solutions.

Part 2 applies the results from the Part 1 to the analysis of the model of the primitive exchange, where a hunter gets a woman's beauty for his quarry, measured in some units, that give the difference between a small game and a big game along his production possibility frontier.

Part 3 comes back to the corner solutions that result in marriage markets like unhappy families where either a husband or a wife gets a chance to spend some leisure time with another partner.

Corner solution can be produced by bride price and dowry. In addition, corner solutions provide a basis for polygamy, which is described in Part 4.

Part 5 formulates the hypothesis that the successful matching both on commodity and in marriage markets take place due to the gravitation either between buyer and seller or between a man and a woman. This hypothesis has been already presented in published papers on commodity markets. But there it doesn't look very convincing, whereas here, in the analysis of marital choice, the mathematics of the equilibrium of the Invisible hand provides the striking illustration of the gravitational force, which confirms on the one hand the Becker's justification of the "popular belief that more beautiful, charming and talented women tend to marry wealthier and more successful men" (Becker, op.cit.p.117), but on the other hand it disproves his conclusion that such an alliance maximizes the aggregate output, because it gives little chances for a beautiful women to be happy.

Part 1. The equilibrium of Invisible hand on commodity market

The consumer reserves labor income wL_0 and starts to search the satisficing price for the given quantity Q under price dispersion $QP(S)$. He chooses the first price P_p below the reservation level, or $wL=QP_p<wL_0$. The purchase results in the value of total efforts of labor L and search S, which is equal to the consumer's willingness to accept or to sell $WTA=w(L+S)$. It is equal to the willingness to pay of *shoppers*, consumers with zero costs at the equilibrium price QP_e, where the equilibrium price P_e is equal to both marginal and average costs on purchase of the *searcher*, the consumer with positive search costs. Divided by the value of marginal savings on purchase $(-Q\partial P/\partial S)$, the equilibrium price gives the value of the time horizon until next purchase $T=L+S+H$, where H is leisure time to enjoy the purchased item:

$$(1)\quad w(L+S) = -Q\frac{\partial P}{\partial S}T = QP_e$$

Equation (1) represents the budget constraint for the utility function $U(Q;H)$, which is maximized at the following MRS $(H$ *for* $Q)$:

$$(2)\quad \frac{\frac{\partial U}{\partial H}}{\frac{\partial U}{\partial Q}} = \frac{w}{P_e} = \frac{Q}{L+S} = MRS(H\ for\ Q)$$

As a result, the explicit satisficing purchase represents the implicit optimal consumption-leisure choice with respect to the equilibrium price.

L'Hopital rule discovers stable preferences of spending total efforts (L+S) on consumption Q. The consumption Q is unit elastic with respect to total efforts (L+S) from the moment of intention to buy $(L, S, Q, \to 0)$ until the moment of purchase QP_p. It means that the purchase of any quantity equalizes marginal benefit of search or its marginal savings with the marginal loss in labor income, and the transformation of the budget constraint (Eq.1) proves this consideration:

$$(3)\quad Q\frac{\partial P}{\partial S} = -w\frac{L+S}{T} = w\frac{\partial L}{\partial S}$$

Equation (3) discovers the limits of consumer choice. If the absolute value of marginal savings on purchase is greater than the absolute value of marginal loss in labor income, this inequality represents the corner solution, when at the moment of intention to buy the consumer gives up the purchase because it is not worth efforts on it:

$$(4)\quad Q\frac{\partial P}{\partial S} < w\frac{\partial L}{\partial S} = -w\frac{L+S}{T} \Longrightarrow \frac{Q}{L+S} > \frac{w}{P_e}$$

If the absolute value of marginal savings on purchase is less than the absolute value of marginal loss in labor income, this inequality represents the opposite corner solution, when at the moment of intention to buy the consumer gives up the purchase because it is not worth leisure time to spend on it:

$$(5)\quad Q\frac{\partial P}{\partial S} > w\frac{\partial L}{\partial S} = -w\frac{L+S}{T} \Longrightarrow \frac{Q}{L+S} < \frac{w}{P_e}$$

Equation (4) leaves space for the arbitrage process. If the *searcher* unexpectedly finds very interesting price, he can re-sell the purchased item for a *shopper* at a price below the equilibrium level. As a result, the arbitrage starts, and it cleans the market from unexpectedly low prices by new equilibrium price. Equation (5) doesn't leave space for the arbitrage because there the willingness to accept or to sell is greater than the equilibrium level, and the consumer cannot re-sell the purchased item.

The producer knows nothing about the consumer. He allocates his time between production, farming, for example, T_f and delivery T_d under constant working time $T_f + T_d = T_{const}$. His total costs *TC* are constant as well as his returns to scale. He calculates his average costs *AC*, which gives him the price *P* equal to his marginal costs *MC*. He chooses the output *Q* along his production possibility frontier *TC=QP*, where production and delivery take time in different proportions with respect to the target quantity *Q*. While both average AC_f and marginal costs MC_f on production stay constant for any quantity to produced, both average AC_d and marginal costs MC_d on delivery are changing with respect to target quantity *Q*. So, the price $P=MC_f+MC_d$ is also changing. It rises with the cut in production in favor of delivery. The total differential represents the ratio of quantity reduced in production dQ_f in favor of quantity to be delivered dQ_d, because the time for delivery T_d reduces the time for production T_f:

(6) $dQ_f MC_f + dQ_d MC_d = 0$

As a result, the producer's choices are limited by the sale on the site, where he devotes almost all his time for production Q_{max}, leaving some time for some service, packaging, for example, and the sale at the doors the quantity Q_{min}, demanded by *shoppers*, consumers with zero search costs.

But under constant returns to scale the rate of product transformation *RPT* comes to the 'time in delivery – time in production' ratio:

(7) $$RPT_{production\ for\ delivery} = -\frac{dQ_f}{dQ_d} = \frac{MC_d}{MC_f} = \frac{AC_d}{AC_f} = \frac{TC_d}{TC_f} = \frac{T_d}{T_f}$$

We see that in any way the producer is selling not only goods but also some leisure time. It means that any buyer-and-seller meeting means some intersection of the producer's frontier curve and the consumer's utility curve.

Equation (7) produces the meeting rule. The consumer knows nothing about the producer's allocation of time, but their meeting means that the consumer definitely encounters some 'time in delivery – time in production' ratio. And when his *MRS (H for Q)* is equal to this ratio, it means that the seller's price *P* satisfies him:

(8.1) $$RPT = \frac{T_d}{T_f} = \frac{Q}{L+S} = MRS$$

(8.2) $$P = MC_f + MC_d = MC_f\left(1 + \frac{T_d}{T_f}\right) = MC_f\left(1 + \frac{Q}{L+S}\right)$$

If *MRS (H for Q) ≠ RPT*, this inequality means that under the meeting rule the seller's price is not equal to the buyer's price, and the frontier curve intersects the utility curve. And it means that under constant costs *TC=QP* the producer has made a wrong choice of the quantity demanded, and the Pareto improvement doesn't take place. However, if the buyer's marginal rate of substitution of leisure for consumption is equal to the seller's 'time in delivery – time in farming ratio', the seller's price becomes equal to the buyer's price. But it means that the utility curve is tangent to the frontier curve, and the satisficing price becomes optimal for both parts in transaction.

Under constant returns to scale the quantity to be produced uniquely defines the allocation of time between production and delivery. And under unit elastic consumption the frontier-utility tangency tells us that the quantity produced is equal to the quantity demanded. It means that the producer's price unintentionally optimizes the consumer's allocation of time and maximizes his utility. The producer knows neither consumer's willingness to pay nor his time spent on labor and search. When he chooses the quantity, it gives him the time for production, because his average costs on production are constant for any quantity. Then he subtracts the time for production from his total working time and gets the time of delivery, i.e., the way he should work up to the consumer. And if he correctly chooses the quantity to be produced, it might be the result of his experience, he comes with this quantity Q and its price P to the meeting point, where the consumer, who has spent labor time L and search time S, is waiting for him.

Part 2. The equilibrium of Invisible hand in marriage market.

The great advantage of the Invisible hand equilibrium is the opportunity to reduce the analysis to natural values of time and quantity. When we start the analysis of marriage market, we should use the concept of imputed prices. But this concept stays less apparent than natural values and sometimes, like it takes place with the wage rate, this concept is deliberately obscured because any family faces not the wage rate itself but its marginal utility, which is very subjective.

The idea to escape from imputed values turns the optics of the analysis to the primitive societies. However, as it was told in the Introduction, it is quite possible that modern marriage markets have inherited features from the history.

Let's take a hunter who is choosing a wife. It seems that the hunter is a buyer, and the bride is a seller. But the question what the bride can sell, immediately comes to her willingness to accept or to sell. She's selling her beauty and her household skills. The homemaking needs time, but the beauty also needs time. Even today women can spend an average of 55 minutes every day primping, that comes out to almost 335 hours or 14 days every year (Women's Health, Mar.3, 2014). And the history of cosmetics spans at least 7,000 years (Wikipedia, History of Cosmetics). Let's suppose that bride's natural beauty and the time of make-up at the equilibrium are inversely related. Less attractive bride needs more time for make-up, while very attractive bride can do without make-up at all. In addition, a wife can spend time on sewing and broidery. In general, the value L gives us the time of woman's work, which cannot be redistributed to the husband. Of course, make-up and sewing can give her some pleasure, but it is no different from the pleasure of labor on a workplace. This is the price she's paying to share the quarry. The voluntary or 'net' pleasure, which provides the utility, is produced by leisure time H also shared in the equilibrium family as well as quarry with her husband. Finally, the time horizon of the family $T=L+S+H$ leaves for a wife some time S for homeworking, which can be redistributed, in contrast to the woman's work during time L, to the husband.

As a result, the woman's willingness to accept or to sell equals to the total of her working and homemaking time, multiplied by some imputed unit value w, or $WTA_{wife}=w(L+S)$. The w value really is imputed. It might be esteemed as a ***unit beauty***. The beauty like wage rate is dispersed on the market. Moreover, the history of marriage market gives an idea that there is some equilibrium beauty. For example, Babylonian marriage market was organized as the bride price – dowry auction, which started with the sale of the most beautiful maiden and progressed to the least. Swains paid the bride price for attractive maidens and got dowry for unattractive ones. It means that some beauty with 'zero bride price – zero dowry' existed.[1] Bride price and dowry will be analyzed later. For the moment, we accept another attribute for the equilibrium beauty:

The equilibrium unit beauty w_e is the value that gives to a woman a possibility to redistribute all allocatable homemaking to the husband and to get the quarry at its equilibrium price P_e.

On commodity market the P_e value gives to a customer a chance to buy at his doors. He becomes a *shopper*, the consumer with zero search costs. While other shoppers might have higher wage rates, they also pay the same competitive price at their doors. So, the price P_e is the lowest willingness to pay at the zero search level or the equilibrium price. Of course, if the commodity market works like an auction, consumers can offer prices over the equilibrium level. And this surplus is working like a bride price at the marriage auction.

As a result, at the P_e level the husband takes all allocatable homemaking, first of all, the transformation of the quarry – butchering, salting, and cooking; all things that he has done as a bachelor and that he can do now in family without a loss of his reputation as a hunter.

All these activities reduce quarry Q itself and rise its imputed price P until P_e by the added value. But, when hunter's working time is constant, his total costs also stay constant as well as the price for the quarry QP. It means that there is some production possibility frontier, which represents the different trade-offs between the quantity and the price. If the hunter has some preferences, he can choose a target beauty, because for the given equilibrium price P_e the unit beauty w uniquely defines the quarry Q. By this way he gives a chance to both attractive and unattractive maiden to marry him. Both of them can maximize their utility with respect to their unit beauty. The attractive wife takes only woman's work L, while the unattractive wife spends some time L on woman's work and also takes all allocatable homemaking S. Got

[1] This ancient logic is nor too archaic, and it was confirmed once in modern times by Molière in 'The Miser' (S.M.)

married with unattractive woman, the hunter spends all his working time in the forest and maximizes the quarry, while with attractive woman he cuts the quarry and makes all allocatable homemaking (Fig.1):

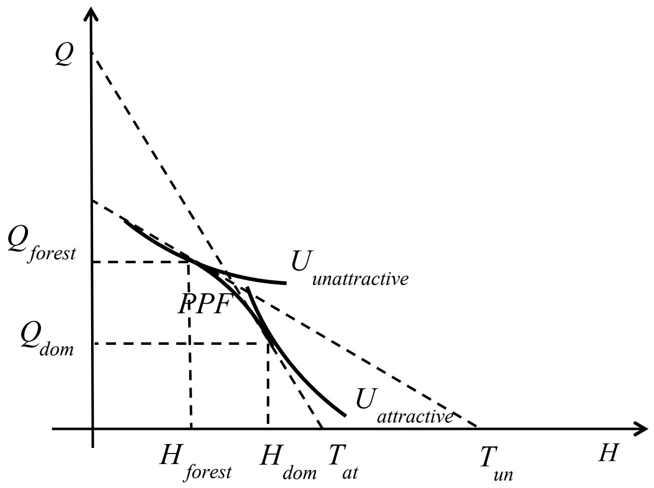

Fig.1.Multiple equilibria for a hunter

where Q_{forest} – maximum output; Q_{dom} – minimum output; H_{forest} – leisure time for maximum output; H_{dom} – leisure time for minimum output; $U_{unattractive}$ – utility of unattractive wife; $U_{attractive}$ – utility of attractive wife; T_{at} – family time horizon of attractive wife; T_{un} – time horizon for unattractive wife.

Fig. 1 illustrates another important assumption. The quarry Q and leisure H are shared in the family. While we don't know exact proportions of individual consumption, this simplification doesn't look excessive. At the equilibrium, the wife and the husband consume together and spend leisure also together.

Fig. 1 also precises the roles in the family. The hunter is 'buying' a wife, but when he does it, he brings in family the quarry. He becomes a real breadwinner, or a 'producer'. The lady is 'selling' her beauty and skills, but she consumes both quarry and leisure.

There is some simple mathematics, which underlies these multiple equilibria. The hunter cannot get more female attractiveness above his *PPF*. It means that whatever choice he makes, he gets the same value:

(9) $QP = wL$

where Q – the quarry; P – the imputed price of the unit of quarry; w – woman's unit beauty; L – time of woman's work.[2]

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[2] Eq.9 resolves very important problem in spite of its simplicity. It presents implicit prices, w and P, for both parts in transaction. But any imputed price should be transformed into some measurable value. Here we can rely on Becker's methodology. If brides and grooms are really selfish, at equilibrium their exchange in the family should

When the wife spends her time on butchering, salting, and grilling, all these activities reduce the imputed price *P* of the quarry *Q*. But she makes it because she is not attractive. The value of her unit beauty *w* is low, and she should also spend more time *L* on make-up and woman's work at home.

Of course, the wife would like to 'sell herself' at her total costs *w(L+S)*. But her *WTA* cannot be greater than *WTA* of an attractive maiden. The *WTA* of attractive maiden is equal to her willingness to pay *WTP*. And her willingness to pay is based on the equilibrium unit beauty w_e, which gives her a chance to reallocate homemaking to the hunter and to get the quarry at the equilibrium price P_e. It means, that for the given quarry *Q* the *WTA* of unattractive wife is equal to the quarry times the equilibrium price P_e:

$$(10) \quad WTA_{unattractive} = w(L+S) = WTA_{attractive} = WTP_{attractive} = QP_e$$

We see that the marriage market reproduces Eq. 1 and 2, and any point of the *PPF* represents some optimal trade-off w/P_e with respect to the individual unit beauty *w* and equilibrium price of quarry P_e.

The hunter provides the quarry *Q* to be shared and allocates his working time between hunting (T_{forest}) and homemaking (T_{dom}), while the wife, allocating her working time between woman's work *L* and homemaking *S*, provides leisure *H* to be shared. This framework exonerates the model from imputed prices, and we come to the sufficient condition, that any happy family represents some Invisible hand equilibrium, which can be described by the meeting rule we have derived from the analysis of the commodity market, but here we put the imputed prices into curve brackets, reducing the analysis to the level of natural values:

$$(11) \quad RPT = \frac{T_d}{T_f} = \left\{\frac{w}{P_e}\right\} = \frac{Q}{L+S} = MRS$$

where *RPT* – rate of product (quarry) transformation; *MRS* – marginal rate of substitution of leisure *H* for consumption *Q*; w/P_e – relative attractiveness.

The graphic exposition of happy family looks as follows (Fig.2):

be fair. Unequal contributions appear only in corner solutions. Thus, Eq.9 appears and resolves the problem of the calculation of imputed prices (S.M.)

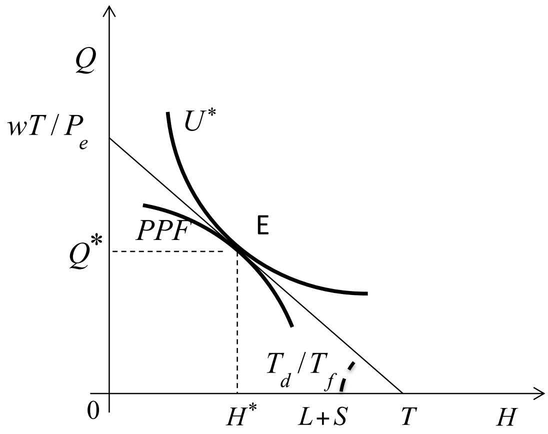

Fig.2.Equilibrium of happy family

We see that it is very difficult to analyze the Invisible hand equilibrium in the marriage market with the straightforward Becker's methodology. On one hand, it looks like the mating of unlikes: a superior woman decreases the hunter's productivity as well as inferior woman raises it. Quarry and beauty are inversely related on the *PPF*. But on the other hand, attractive wife raises husband's productivity at home as well as lucky hunter also raises the household productivity of his wife. In addition, it is very difficult to evaluate here the household productivity, because for both parts in transaction it is taken on basis of opportunity costs – man's household activity equals to the quarry lost, and woman's household activity equals to the time lost for woman's work.

There is an important difference between commodity market and marriage market. On commodity market, the producer doesn't know consumer's wage rate. The only information he needs is the quantity demanded. This is the quantity demanded, which allocates his time. But in marriage market the hunter gets the visible information about woman's beauty. But like the value of quantity demanded Q on commodity market, the value of unit beauty w uniquely defines the allocation of his working time with respect to his physical capacities. He's evaluating the beauty of a bride and decides how much time he should spend at home and in the forest. And the quarry Q becomes the residual value of his choice with respect to his hunting skills. If his evaluation is correct, the family will be happy. If he makes a mistake, his choice results in the disequilibrium.

Here the imputed value of ***relative female attractiveness*** $\boldsymbol{w/P_e}$ plays an important role. We can re-write the meeting rule from Eq.8.2 in the following form:

$$(12)\quad P = MC_f + MC_d = MC_f\left(1+\frac{T_d}{T_f}\right) = MC_f\left(1+\frac{w}{P_e}\right)$$ [3]

At the equilibrium the relative attractiveness uniquely defines both the hunter's allocation of time and respective imputed price of the quarry. If the hunter underestimates the unit beauty w of his wife, he will spend on hunting more time, the T_d/T_f ratio is low, than the wife needs. She is ready to pay high imputed price for the quarry with high added value of husband's home activities. On the other hand, if the hunter overestimates the unit beauty of his wife, he will try to make more work at home, the T_d/T_f ratio is high, while his wife is ready to do it herself. We see, that in any case hunting efficiency and beauty are inversely related, while they are directly related with homemaking, i.e., the care.

However, it is too early to write off Becker's methodology. If we pragmatically look at the constant hunter's costs by the optics of the quantity-price relationship, we understand that *PPF* exposes only the substitution effect with zero income effect. Real marriage market depends on specific income effects, produced by bride price and dowry, which take in the modern society such forms as pre-marital wealth and parental support. But before we start to analyze these income effects, we should understand the limits of Invisible hand equilibrium, when income effect equals to zero.

Part 3. Corner solutions and disequilibria in marriage markets.

Consumer's corner solutions both on commodity and marriage markets that limit the Invisible hand equilibrium don't represent complex calculations. They're based on simple reasonings – either the quantity demanded is not worth efforts, or it is not worth leisure. However, people often disregard even commonsense reasoning. It seems that this mistake is more common on commodity market, where mistakes are not so painful. But it is not so. All the history of marital choice tells us that people are making this mistake in family.

Let's take an unattractive bride who wants to marry a lucky hunter 'at any costs'. She chooses the target quarry to be shared in the family and starts to search. But when she is unaware of a groom's *PPF*, she can make a mistake. Her motivation to get married at any costs can play a dirty trick. She is ready to give up leisure and to please her husband; she is also ready to take over the household. But it might be not enough for a lucky hunter. If his

[3] The family case precises the MC_f value. On commodity market, where there is no explicit exchange of information, it can be derived only on competitive basis under the division of labor, when a trader is buying goods for re-sale on the farm at competitive price. But in primitive society there is some social norm, how much quarry the 'respectable' hunter should bring in a day, which gives the MC_f value for both parts in transaction (S.M.)

possibilities are great, there he is left to endure suboptimal leisure H_{sub}. The value T_d/T_h in the family is too low for him; he can please his wife by suboptimal leisure H_{sub} and to get some more leisure time until H^*. And he simply goes out to play cards or to search an attractive partner with high unit beauty w (Fig.3):

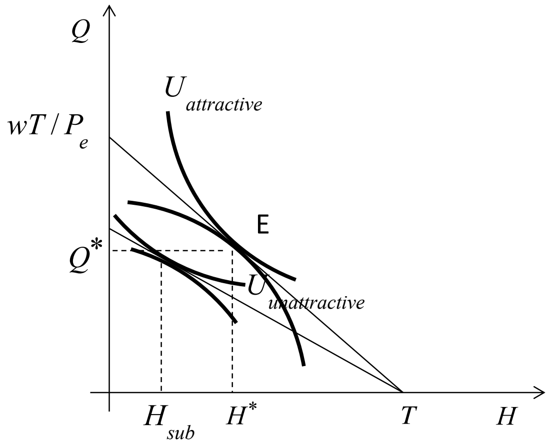

Fig.3. Husband's excess leisure

It happens because according to Equation (5), this choice represents for unattractive wife the corner solution. Her unit beauty is less than the equilibrium beauty w of attractive maiden. The unattractive woman accepts the quarry offered but it takes too much her working and homemaking time, that cuts her leisure to spend with her husband. This is a result of the decision to get married at any costs:

$$(13)\ \frac{Q}{L+S} < \frac{T_d}{T_f} = \left\{\frac{w}{P_e}\right\} \Rightarrow w(L+S) > QP_e$$

Here, the unattractive bride is trying to 'sell' herself at a price greater than the price of the beautiful brides at the local market. But her choice might be optimal, if she finds a hunter with a lower *PPF* – the consideration, which confirms Becker's conclusion that inferior woman chooses inferior man.

The commodity market is more organized than the marriage market. Its Invisible hand doesn't go beyond *Wealth of Nations*, and an optimistic searcher cannot re-sell the purchased item at the door for a shopper at the price greater than the equilibrium price. This is not for marriage market, because its Invisible hand also depends there on *Moral Sentiments*. The predatory female motivation to get lucky hunter to marry her makes him unhappy and he will suffer from suboptimal leisure. But sometimes this decision is taken by the lucky hunter himself. He chooses unattractive wife to please him and to make home; then he gets spare time for a 'comfort woman'.

The opposite corner solution also falls under reasonings of *Moral Sentiments*. If a beautiful bride chooses a lucky hunter, who is not so thoughtful and who has no intention to spend much time at home, she gets the corner solution because his $RPT = T_d/T_f$ is less than her marginal rate of substitution of leisure for consumption. As a result, she gets only 'corner' amount of leisure, and she starts to look for another partner, who can provide her more leisure time (Fig.4):

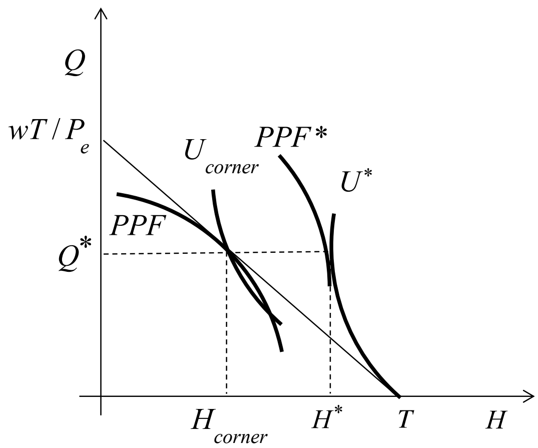

Fig.4. Wife's excess leisure

Here the corner solution is described as follows:

$$(14)\, \frac{Q}{L+S} > \frac{T_d}{T_f} = \left\{\frac{w}{P_e}\right\} \Rightarrow w(L+S) < QP_e$$

This corner solution is produced by high female self-esteem, or by high imputed value of unit beauty *w*. This is a real problem for the family because Figure 4 looks again like the visual confirmation of Gary Becker's assumption that more beautiful women are choosing more productive men as well as his understanding of the implicit polygamy, when "polygamy can match the total resources of a superior man or woman by substituting several inferior for one superior mate" (Becker, op.cit., p.125). Really, if we take into account the total husband's productivity, both in forest and at home, we can say that a beautiful woman makes the equilibrium choice at the upper *PPF*, which can be provided by one superior male or by several inferior males.

We see that both corner solutions can be settled down by some shift of the production possibility frontier. Marriage market produces *PPF* shifts by its specific income effects of bride price and dowry. And these income effects discover the difference between male and female polygamy.

Part 4. Bride price, dowry, and polygamy.

We see that corner solutions are produced by woman's self-esteem, which results in disequilibrium unit beauty *w*. But there are more fundamental inner workings of corner solutions, which can be derived from the male allocation of time.

Let's take a hunter who is ready to pay the bride price. We can suppose that the bride price needs hunting to start before marriage, and hunter comes in the marriage market with some bag. Get married, he continues hunting and starts homemaking. (Fig.5):

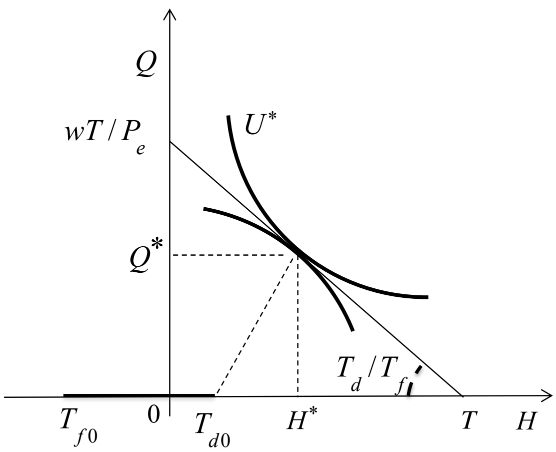

Fig.5. Hunting for bride price

But we see that whenever hunting starts, in any way it results in some *PPF*, described by the allocation of time ratio. But with regard to the equilibrium (Eq.11), the bride price increases the time on hunting and decreases the *RPT*. As a result, the *MRS* should also fall to meet the lower *RPT*.

The fall in *MRS* for the given quarry *Q* can be produced either by low unit beauty *w* or by more total efforts of woman' work *L* and homemaking *S*. While all the history of marriage markets tells us that bride price and beauty correlate, the first assumption of the low unit beauty is less reasonable. But it means that both parts in transaction have different views on the bride price. For a maiden or for her family the bride price means the price for the beauty. For a lucky hunter, who has brought to the marriage market his bag, the bride price means more woman's work and homemaking. It means that the corner solution (Eq.13) appears at the level of expectations, like it happens on commodity market, where corner solution appears not at the moment of purchase but at the moment of intention to buy. In marriage market a woman, a 'consumer', should understand that the quarry demanded is not worth her efforts. But accepting the bride price, she voluntarily comes to the corner in the family. Literally, she becomes a hostage of the bride price.

Let's take a hunter, who intends to marry an attractive woman at the equilibrium level, where he is ready to take all homemaking, leaving for his wife only woman's work *L*. But the bride price makes him to bring to the market some reserves of his quarry. The bride price income effect shifts *PPF* upwards to the right. But the bride price changes his allocation of time. For the new *PPF* the hunter should spend more time on hunting. As a result, the *RPT* falls; the bride price substitution effect shits the optimal choice to the left. There, the hunter expects to get from his wife not only working time, but also homemaking time *S*. But for an attractive lady this husband's choice represents the corner solution (Fig.6):

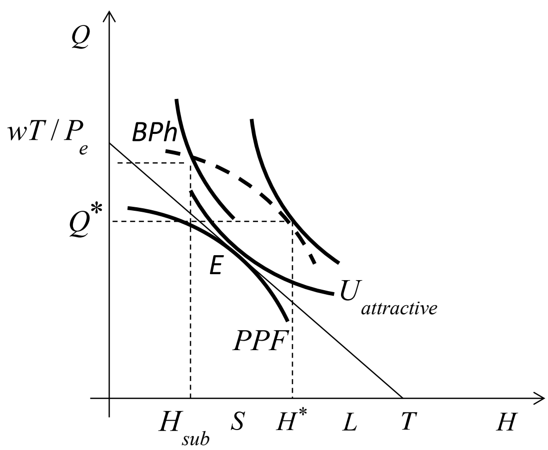

Fig.6. Income and substitution effects of bride price

It seems, that the settlement of the corner solution can take place, if the hunter finds resources to cut wife's both working time *L* and homemaking time *S*. He can surround her by maids. But he should pay for them. And the only source at his disposal is again the hunting.

Maids consume some quarry; they start to do not only the woman's work but also homemaking *S* and shift lady's choice along the utility curve down to the right. But the hunter stays at the hunting level of bride price *BPh*. The family disappears. The couple share the quarry Q^*, which is equal to the quarry at *BPh* less maid's consumption, but not leisure. The wife takes some *L* time on make-up and her optimal leisure H^*. But the husband spends with her only H_{sub} time. The question is how the lady spends the (H^*-H_{sub}) time? It might be a company with a male with low hunting and high homemaking skills, say, a cook or a gardener.

The dowry effect is less clear because it depends on the tangible form of the dowry. The dowry income effect shits the *PPF* inward. But there the substitution effect is ambiguous. If it reduces the costs involved in setting up a new household, the dowry reduces the T_d value for the husband; the T_d/T_h ratio falls, and the optimal hunter's choice moves to the left. As a result, the dowry substitution effect provides the equilibrium solution for the marriage with unattractive wife. But in monetary form, if money already exist, the dowry cuts hunting time. The optimal hunter's choice shifts down to the right. By this way the dowry raises the T_d/T_h ratio, and it results in the corner solution (Eq.12). The husband comes to equilibrium at E where he gets his optimal leisure H^* (Fig.7). But his unattractive wife can give him only suboptimal leisure H_{sub}. And he starts go to football and to play cards, or to search another lady for leisure. For that he can use the part of dowry and to move along the *PPF* or to increase hunting time in order to share both optimal quarry and leisure with two women.

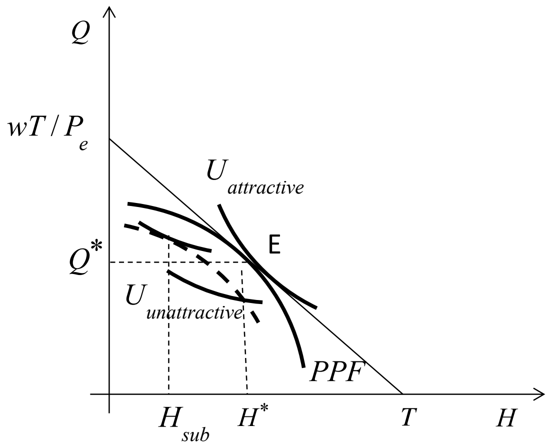

Fig.7. Income and substitution effects of dowry

The analysis of corner solutions, bride price and dowry give an idea that with respect to the equilibrium (Eq.11) the polygamy doesn't represent the substitution of one superior mate by several inferiors. It might be some combination, when one corner solution is complemented by the opposite corner solution in order to produce in total the optimal choice.

Here the question of mathematical model arises. In its simple form the equilibrium solution (Eq.11) is transformed either into the polygamy equilibrium:

$$(15)\quad \frac{T_d}{T_f} = \sum \frac{Q_i}{(L_i + S_i)}$$

or into the polyandry equilibrium:

$$(16)\quad \sum \frac{T_d}{T_f} = \frac{Q}{L + S}$$

By this way Eq.15 and 16 confirm the substitution of one superior mate by several inferior. But behind the back of natural values there are imputed values of the equilibrium price P_e and unit beauty w. And we get the problem. While the equilibrium price stays the same along the given *PPF*, the unit beauty doesn't. And it is very difficult to summarize unit beauties. The beauty cannot be summarized like nine pregnant women cannot give birth for a child in a month. Here we need some mean value. This consideration doesn't change Eq.16, and polyandry stays as the total of inferior choices. But polygamy doesn't. The polygamy equilibrium (Eq.15) should be re-written in some other form. The simplest way is to present it as follows:

$$(17)\quad \frac{T_d}{T_f} = \left\{\frac{\overline{w}}{P_e}\right\} = \overline{\left(\frac{Q_\iota}{(L_\iota + S_\iota)}\right)}$$

But it means that got married with unattractive woman under corner solution from Eq.13, in order to make the optimal choice the hunter should complement it by the corner solution from Eq.14. Then he can share both quarry and leisure with many women like it is presented at Fg.7.[4]

There, the equilibrium beauty $\overline{w_\iota}$ is a mean value. It means that when one wife is unattractive, the other wife should be more attractive even than an attractive maiden at the equilibrium level. The hunter cannot get very attractive woman alone because this choice produces the corner solution (Eq.14) for the wife. And the polygamy, either explicit or implicit, gives him a chance to get very beautiful woman.

The interest of very beautiful woman in polygamy is quate obvious. Both explicit and implicit forms redistribute the allocation of time (L+S) between wives even when their shares in quarry q_i, like it is prescribed by sharia, are equal. In general, the unattractive wife takes all the homework and a big part of women's work. And the greater is the beauty of 'beloved' wife w_i, the higher is her $MRS = w_i/P_e = q_i/L_i$ with respective high share in quarry q_i and low share in woman's work L_i with zero homemaking $S_i=0$.

The last consideration confirms the idea that the optics of commodity market can be used for the analysis of the family, because polygamy represents an equilibrium where a producer sells his output to many buyers. And it also confirms the hypothesis that there is some force, which attracts beautiful women to lucky hunters, and vice versa. The Invisible hand equilibrium provides some grounds for this hypothesis. It seems, that there is gravitation between men and women, which depends on quarry and allocation of time like it takes place between buyers and sellers on commodity market.

Part 5. The gravitation between men and women.

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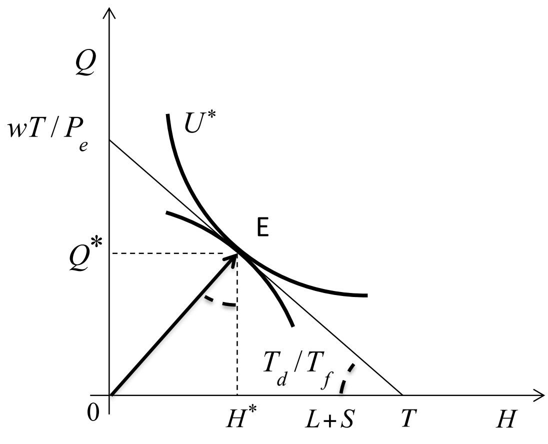
[4] It looks like the general rule for both parts in transaction: when partner's trait is measurable, say, a natural value, the substitution of one superior mate by several inferiors takes place by the direct summation. If the partner's trait is not measurable, say, an imputed value, the derivation of mean value and the total of opposite corner solutions takes place. (S.M.).

The mathematics of the gravitation between men and women is the same as for sellers and buyers. But here it becomes more perceptible because corner solutions, which are taken as simple constraints on commodity market, stay active in marriage market, because unhappy families cannot be written off from the analysis of the equilibrium.

We start with the assumption that a bride before she makes a choice doesn't know the total hunter's costs of time. Moreover, it doesn't interest her. She has some preferences how much quarry Q and how much leisure H should be shared. It means that for any quarry there is some transformation rate δ of total hunter's time (T_f+T_d) into family leisure time H, or

(18) $H = \delta(T_f + T_d)$

The hunter also has some preferences how his time should be allocated between hunting and homemaking. But whatever allocation he makes, the total time stays constant with regard to his physical abilities. And husband's constant total costs result with regard to Eq.9 in constant wife's cost *wL* for any unit beauty *w*. This consideration gives a possibility to assume that for any optimal choice there is a normal from the origin of family woman's allocation of time. Hunter's *PPF* has a particular attribute – whatever change in hunting skills takes place, the *PPF* doesn't change its shape like it happens under technological progress on commodity market; it simply shifts outward because the progress in hunting skills changes both quantities lost in hunting and got for consumption in the same proportion. As a result, the normal from the origin to the equilibrium choice produces the geometry of happy family (Fig.8):



The geometry confirms the assumption that the equilibrium doesn't need imputed monetary values, and it can be derived with the help of natural values. And the geometry of happy family results in the following equation:

(19) $H = Q\dfrac{Q}{L + S}$

However, when we speak about optimal woman's choice, we understand that man's optimal choice is implicitly presented with its optimal allocation of time:

$$(20.1)\quad H = Q^2 \frac{T_d}{T_f}$$

$$(20.2)\quad Q^2 = H(L+S) = \delta(T_f + T_d)(L+S)$$

$$(20.3)\quad \delta = \frac{Q^2}{(T_f + T_d)(L+S)}$$

$$(20.4)\quad \delta_i = \frac{Q_{polygamy}}{(T_f + T_d)} \frac{q_i}{(L_i + S_i)}$$

We see that the transformation rate δ represents the product of quantities supplied and consumed, divided by the product of time that both parts in transaction have spent to meet each other, i.e., to spend leisure together. Eq.20.3 and 20.4 really look like specific forms of Newton's law of universal gravitation, where quantities represent 'masses', and the distance is substituted by time. It means, that that the transformation rate δ can measure a gravitational force between a man and a woman.[5] Its physical nature becomes more evident, if we re-write Eq.19.4 in the general form:

$$(21)\quad \delta = \frac{Q_j}{T_j} \frac{w_i}{P_e}$$

[5] The hypothesis of gravitational force specifies the production possibility frontier. Let's take a simple example of a hunter, who can get *20* units of quarry in *10* days. The allocation of his time between T_f and T_d produces the straight-line $Q=20-H$, where H is equal to a half a day lost for hunting. If he really is indifferent to the allocation of his time, he can make happy any woman along this straight-line frontier. But for a chosen unit beauty w he should allocate his time with respect to the meeting rule (Eq.8.2 and Eq.12). As a result, the value of leisure time H, as well as the following $(L+S)$ and T values, appears here with the help of Eq.20.1:

TC	*Q*	*P*	*-dQf/dQd*	*MCf=ACf*	*MCd=ACd*	*Td/Tf*	*H*	*L+S*	*T*	*Ó*
10	20	0,500	0,000	0,500	0,000	0,000	0	n/a	∞	0
10	19	0,526	0,053	0,500	0,026	0,053	1	361,000	362,000	0,1
10	18	0,556	0,111	0,500	0,056	0,111	2	162,000	164,000	0,2
10	17	0,588	0,176	0,500	0,088	0,176	3	96,333	99,333	0,3
10	16	0,625	0,250	0,500	0,125	0,250	4	64,000	68,000	0,4
10	15	0,667	0,333	0,500	0,167	0,333	5	45,000	50,000	0,5
10	14	0,714	0,429	0,500	0,214	0,429	6	32,667	38,667	0,6
10	13	0,769	0,538	0,500	0,269	0,538	7	24,143	31,143	0,7
10	12	0,833	0,667	0,500	0,333	0,667	8	18,000	26,000	0,8
10	11	0,909	0,818	0,500	0,409	0,818	9	13,444	22,444	0,9
10	10	1,000	1,000	0,500	0,500	1,000	10	10,000	20,000	1
10	9	1,111	1,222	0,500	0,611	1,222	11	7,364	18,364	1,1
10	8	1,250	1,500	0,500	0,750	1,500	12	5,333	17,333	1,2
10	7	1,429	1,857	0,500	0,929	1,857	13	3,769	16,769	1,3
10	6	1,667	2,333	0,500	1,167	2,333	14	2,571	16,571	1,4
10	5	2,000	3,000	0,500	1,500	3,000	15	1,667	16,667	1,5
10	4	2,500	4,000	0,500	2,000	4,000	16	1,000	17,000	1,6
10	3	3,333	5,667	0,500	2,833	5,667	17	0,529	17,529	1,7
10	2	5,000	9,000	0,500	4,500	9,000	18	0,222	18,222	1,8
10	1	10,000	19,000	0,500	9,500	19,000	19	0,053	19,053	1,9

or the ***gravitational force equals to the product of male productivity Q_j/T_j and female relative attractiveness w_i/P_e.***

While in reality we know neither the production possibility frontier nor the utility curve, the last consideration looks like the basis for the understanding of relationship between men and women because with respect to the theory of gravitation ***the productivity exhibits the male gravitational field***, and ***the attractiveness exhibits the female gravitational field***. This reasoning gives the understanding why wealthy men and beautiful women really attract each other, but they have little chances to create a happy family.

This consideration results in a specific phenomenon that can be called as Femme Fatale Fate. At the equilibrium a beautiful woman reduces a gravitational field of her partner. When total working time (T_f+T_d) is constant along the frontier, the value Q is falling. But the value (L+S) is falling more rapidly than the quarry, and the beautiful woman increases her own gravitational field. The growth of her gravitational field can save the family because $\partial\delta/\partial Q$ is negative along the frontier. But the cost of the gravitational force's growth is high. A beautiful wife enforces her husband to make her life comfortable but with that he becomes less attractive in her eyes with respect to Eq.22. As a result, multiple family equilibria, presented at Fig.1, propose the short time horizon for the family with a beautiful wife.

The husband can sense the fall of his gravitational field. However, it is difficult to imagine that a hunter, once got married with a beautiful lady, changes his preferences and starts to sweet her off her feet. It is more natural for him to compensate the fall of his attractiveness by gifts – clothing, jewelry, yachts. But with that he should increase his hunting time. And the fragile equilibrium of the family with the beautiful lady immediately fails. Rising hunting time, the husband decreases his T_d/T_f time allocation ratio and produces for his wife the corner solution. And she starts to look for another partner. It is therefore not surprising that the common family position for a beautiful woman is the role of 'beloved wife' in the polygamy, either explicit or implicit, where she can efficiently use her strong gravitational field.

Conclusion

While the gravitation is more evident in marriage market than on commodity market, it adds some understanding for sales. Producers with strong gravitational field like Coca-Cola don't need consumers' attractiveness and high purchasing power. The economy on scale needs many low-income consumers, while Chateau Lafitte Rothschild needs few high-income consumers. We see that the hypothesis of gravitation, presented here, and the initial Adam Smith's gravitation metaphor, associated with the natural price, might overlap. From the very beginning the Invisible hand equilibrium discovers the gravity of the equilibrium price, which equalizes the willingness to accept of consumers with positive search costs. But on the other hand, this is a purchase price, which attracts consumers. As a result, the producer's gravitational field and the purchase price are inversely related, and Coca-Cola' field is stronger that that one of Chateau Lafitte Rothschild. But the total gravitational force between Chateau Lafitte Rothschild and its customers is much stronger that between soft drinks' lovers and Coca-Cola.

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