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Donze, Jocelyn and Dubec, Isabelle

Université des Sciences Sociales de Toulouse

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Paying for ATM usage: good for consumers, bad for banks?

Jocelyn Donze* and Isabelle Dubec†

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Abstract

We compare the effects of the three most common ATM pricing regimes on consumers’ welfare and banks’ profits. We consider cases where the ATM usage is free, where customers pay a foreign fee to their bank and where they pay a foreign fee and a surcharge. Paradoxically, when banks set an additional fee profits are decreased. Besides, consumers’ welfare is higher when ATM usage is not free. Surcharges enhance ATM deployment so that consumers prefer paying surcharges when reaching cash is costly. Our results also shed light on the Australian reform that consists in removing the interchange fee.

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In most countries, banks share their automated teller machines (hereafter ATMs): a cardholder affiliated to a bank can use an ATM of another bank and make a “foreign withdrawal”. This transaction generates two types of monetary transfers. At the wholesale level, the cardholder’s bank pays an interchange fee to the ATM-owning bank. It is a compensation for the costs of deploying the ATM and providing the service. This interchange system exists in most places where ATMs are shared.\(^1\) At the retail level, the pricing of ATM usage varies considerably across countries and periods. In the United Kingdom or France, banks do not levy any usage fee. In Australia, consumers pay a “foreign fee” to their bank when they use an ATM of another bank. In the USA, cardholders using an ATM of another bank pay two separate fees: a foreign fee to their bank and a “surcharge” to the ATM-owning bank.

There have been substantial debates about the pricing of ATM networks since the beginning of the 90s. There are two main issues. At the wholesale level, banks choose the level of the interchange fee jointly in most countries. Some economists have argued that this level could be reflected in the ATM usage fees or the account fee so that banks may use the collective setting at the wholesale level to relax price competition at the retail level (see the Cruickshank report (2000) for the UK, Balto (2000) for the USA, Donze & Dubec (2006) for France). Several attempts have been made to limit the possibility of collusion: in Australia and South Africa, regulation authorities have studied the possibility to replace the interchange system by a “direct charging regime” in which each bank charges a single ATM usage fee to non customers using its ATMs (see Reserve Bank of Australia & the Australian Competition and Consumer Commission (2000) ; Competition Commission of South Africa (2007)).

At the retail level, consumers have been reluctant to pay ATM usage fees. It is especially

\(^1\)Banks also pay the network a switch fee per foreign transaction and an annual membership fee to cover its costs.
true in the USA where cardholders generally have to pay a foreign fee and a surcharge for using an ATM not operated by their bank.\(^2\) This “double charging” makes foreign withdrawals quite expensive (around 3$ in 2006, see Hayashi, Sullivan, Weiner (2006)); and consumers’ associations have argued against surcharges, since interchange fees are already charged to compensate banks for processing foreign withdrawals. On the other hand, banks and independent ATM deployers claim that the introduction of surcharging in 1996 has allowed them to deploy more ATMs and thus has facilitated access to cash. Evidence shows that ATM deployment has been much faster from 1996.\(^3\) Nowadays, the USA and Canada have much more ATMs per inhabitant than countries in which surcharging is not applied.\(^4\) Banks’ profits have also been affected by this change. Indeed, the larger number of ATMs and rising surcharges have induced a sharp fall in transactions per machine, notably the foreign acquired transactions that generate revenues in the form of surcharges and interchange fees.\(^5\)

The revenues per ATM have fallen and recent studies show that on average banks tend to

\(^2\)In Great Britain in 1999, consumer anger over an attempt by several banks to introduce ATM surcharges was so strong that the banks have not only abandoned their surcharge plans but also eliminated existing ATM foreign fees.

\(^3\)After growing at an annual rate of 10.2 percent from 1991 to 1996, the number of ATMs increased at an annual rate of 19.7 percent between 1996 and 2001, and 4.1 percent between 2001 and 2006. This made the number of ATMs grow from 139000 in 1996 to 324000 in 2001 and 395000 in 2006 (Hayashi, Sullivan, and Weiner (2006)).

\(^4\)In 2006 in the USA, there are 1312 ATMs per million inhabitants and in Canada: 1666 ATMs per million inhabitants. In both countries, cardholders pay foreign fees and surcharges. These figures have to be compared with the 1007 ATMs per million inhabitant in the United Kingdom or the 741 ATMs per million inhabitant in France. In both countries, usually banks do not charge ATM usage. The difference between France and the UK comes from the existence of independent ATM deployers in the UK (43% of the total ATM network in 2006).

\(^5\)In the USA, the monthly transactions per ATM declined from 6400 in 1996 to 3500 in 2001, and 2130 in 2006.
lose money on their ATMs.\footnote{According to the ATM deployer study 2006 (Dove, 2007), deployers earned an average of $1,104 per month at their on-premise ATMs, and $1,013 at their off-premise ATMs. On the spending side, deployers incurred average monthly expenses of $1,444 per on-premise ATM, and $1,450 per off-premise ATM.}

In this paper we study how the ATM pricing scheme affects the ATM deployment, consumers’ welfare and banks’ profits. We address the following questions: Does the collective setting of the interchange fee favor collusion? Should the interchange fee be abandoned in favor of direct charging as proposed by the Australian Competition Commission? Do ATM usage fees harm or enhance consumers’ welfare?

To answer these questions, we develop a model where two horizontally differentiated banks first choose the interchange fee jointly and then deploy ATMs and compete for depositors non cooperatively. We compare three regimes of retail ATM pricing. In each regime consumers pay a fixed account fee to join a bank. Under regime one, cardholders can freely access to all ATMs of the shared network (ATM usage fees are nil). Under regime two, cardholders pay a foreign fee per foreign withdrawal to their bank. Under regime three, they pay a foreign fee to their bank and a surcharge to the ATM-owning bank.

In our framework, consumers need a fixed amount of cash in a shopping space, that can range from a concentrated to a sprawling area. For a given number of ATMs, the average travel cost to withdraw cash is higher in a wider shopping space and consequently consumers’ valuation of an additional machine is higher. The parameter reflecting the dispersion of the shopping space will play an important role in the comparison of consumers’ welfare across the different regimes.

We find that the size of the shared network is sensitive to the pricing regime. Under regime one, the ATM usage is free and hence all ATMs are identical for consumers. In this case banks deploy ATMs not to attract new depositors but rather to generate interchange revenues. Under regime two and three, foreign withdrawals are not free and the two networks...
are differentiated by usage fees: consumers prefer a bank with a large ATM network in order to make less foreign withdrawals. In this case each bank can increase its share of deposits by deploying more ATMs. We show that for a given interchange fee, this differentiation effect of ATM usage fees leads banks to deploy more ATMs under regime two than under regime one. In general the network is even larger under regime three: for a given interchange fee, surcharging increases the revenue per non-customer’s withdrawal above the interchange fee. Consequently banks are even more eager to open ATMs to attract foreign withdrawals. This result could explain the difference in the number of ATMs per inhabitant observed between the USA and Canada, under regime three, and the United Kingdom and France, under regime one.

Paradoxically, for a given interchange fee consumers’ welfare is larger when they pay foreign fees (regime two) than when ATM usage is free (regime one) while the opposite is true for banks’ profits. As noted before, more ATMs are deployed under regime two which benefits consumers but increases deployment costs. Banks’ pricing strategy reinforces this effect on surpluses and profits. Indeed, when ATM usage is free, cardholders make “many” foreign withdrawals which generates a large gross surplus that banks extract through the account fee. With the unitary foreign fees, consumers adjust the demand for foreign withdrawals downward and pay less to their bank.

Another striking result is that when banks set the interchange fee at the joint-profit maximizing level, profits are highest under regime one, and lowest under regime three. When surcharges are prohibited, equilibrium profits depend on the interchange fee, so that banks can use the interchange fee as a collusive device. This possibility is especially profitable for banks under regime one because they can generate and extract a large consumers’ surplus as explained above. Regime three is the worst for banks for two reasons: first, the deployment costs are very high. Second, we show that the interchange fee does not affect banks’ profits
anymore so that collusion is impossible.\textsuperscript{7} Under regime three banks lose money on average on their ATMs, which is consistent with evidence presented earlier.

We also compare the consumers’ surpluses at the profit-maximizing interchange fee. We show that banks can extract all consumer surplus under regime one. This is not the case when usage fees are levied. As noted before more ATMs are deployed under regime three than under regime two and consequently, consumers benefit from a better but more expensive service. We show that consumers prefer regime three to regime two when the dispersion parameter is high enough. In this case, accessing to a machine is more costly and consumers highly value each additional ATM: they are ready to pay for the large ATM network of regime three. When the shopping space is more concentrated, consumers are satisfied with the smaller but less expensive network of regime two. The importance of travel costs in comparing consumers’ welfare is consistent with evidence. Knittel and Stango (2008a) use the regime change of 1996 (from regime two to three) as a “before and after experiment” to study the impact of the ATM pricing on ATM deployment and consumers’ welfare. They find that “consumers in high travel cost counties experience substantially higher welfare after 1996, while the net effect remains negative for consumers in low travel cost counties” (p 24). Our paper is the first theoretical work justifying the importance of travel cost in the welfare comparison.

At the regulatory level, we show that the “direct charging scheme” where the interchange fee is suppressed and where customers pay a unique ATM usage fee to the ATM-owning bank is equivalent to regime three. This comes from the irrelevance of the interchange fee under regime three. In this case, ATM usage fees distort competition on the deposit market and banks deploy so many ATMs that they lose money on each machine.

Our analysis highlights the interactions between the “withdrawal market” and the deposit

\textsuperscript{7}The “neutrality” of the interchange fee when surcharges are allowed was first informally stated by Salop (1990).
market: in our model consumers’ choice of where to open an account is endogenous and depends on the account fees, on the number of ATMs deployed by each bank and on the ATM usage fees. Empirical works have shown that all three elements matter. Ishii (2006) and Knittel and Stango (2008a) find that when banks levy usage fees, the relative size of banks’ ATM networks has a significant impact on consumers’ decisions where to bank. Massoud, Saunders and Scholnick (2006) find that increasing surcharges give customers some incentives to switch accounts from smaller banks to larger banks in order to avoid high usage fees.

Our work is also related to an extensive theoretical literature on ATM pricing (see McAndrews (2003) for a survey). Donze and Dubec (2006) focus attention to regime one and show the collusive role of the interchange fee. We extend this model by considering a more general demand for withdrawals. Massoud and Bernhardt (2002) consider a framework without interchange fees in which banks impose surcharges on non depositors. ATM deployment is exogenous. They show that banks set high surcharges for non customers in order to rise the cost of foreign withdrawals for these customers and hence to become more attractive. This indirect effect of surcharging on the deposit market also exists in our framework under regime three. Chioveanu, Fauèi-Oller, Sandonis and Santamaria (2006) compare regimes two and three. To keep tractability, they ignore the effect of ATM usage fees on consumers’ decisions where to open an account. They also show that banks’ profits are higher under regime two than under regime three when banks set the interchange fee at its maximizing level. Nevertheless our results differ concerning the effect of surcharges on consumers’ welfare. Chioveanu et al find that consumers are always better off when surcharges are allowed. In their model, the shopping space consists in several malls where banks install machines. Consumers are infinitely sensitive to travel costs to withdraw cash in the sense that they are never willing to go to another mall to find a machine of their bank. Hence, in some way, their model corresponds to the situation in which our dispersion parameter is high.

The paper is organized as follows. Section 2 builds up the model. Section 3 studies the
regime where there is no ATM usage fee (regime one). Sections 4 and 5 examine the regimes with foreign fees (regime two) and foreign fees plus surcharges (regime three). Section 6 compares the welfare across regimes and studies the effect of suppressing the interchange fee. Section 7 concludes.

1 The model

We use the Hotelling framework. Two banks are located at the two ends of a linear product space of length 1. Consumers of banking services are distributed uniformly along this product space. Their number is normalized to one. Consumers do their shopping and withdraw money in another space, which we refer to as the shopping space. We assume that within this space, consumers’ location is uniform at any time when they need of cash. Furthermore, the commercial activity and the total amount of cash withdrawn per cardholder are fixed. However the shopping space can be more or less spread out. It is a shortcut to take into account travel costs to reach cash in the analysis. We will return to this point later.

Banks

They provide two kinds of services: (i) basic banking services (deposit management, possibility to withdraw cash at the bank’s branch office,...) and (ii) access to a network of compatible ATMs. The number of ATMs deployed by bank $i$ is $n_i$ and the total number of ATMs is $n = n_1 + n_2$. As the measure of consumers is one, $n$ is a number of ATMs per consumer and 1 is clearly an upper bound for $n$. We assume that each bank uniformly deploys its ATMs in the shopping space since each bank’s cardholders are uniformly distributed when they need cash. The cost of deploying and operating an ATM is denoted by $c$.\footnote{This cost is annual and includes the purchase costs of the machine (depreciated over approximatively five years), installation, site rental, repairs and maintenance, cleaning, communication costs, cash delivery and replenishment, insurance and security, and the opportunity cost of the cash in the machine.} The
marginal cost of providing the basic services is constant and denoted by $c_b$. The marginal cost of processing a withdrawal is independent from the affiliation of the cardholder and it is normalized to zero. When a cardholder of bank $i$ makes a withdrawal at an ATM of bank $j$, bank $i$ pays an interchange fee, $a$, to bank $j$.

Bank $i$ sets an account fee $p_i$ for its cardholders. There are two possible ATM usage fees $f_i$ and $s_i$: bank $i$ charges its own cardholders a foreign fee $f_i$ for each withdrawal made at an ATM of bank $j$ (foreign withdrawal). Bank $i$ charges cardholders of bank $j$ a usage fee $s_i$ (referred to as “surcharge”) when they use one of its ATMs. As usually observed, banks do not charge their cardholders for domestic withdrawals. We will consider three pricing regimes: under regime one, there is no ATM usage fees: $f_i = s_i = 0$. Under regime two, only surcharges are prohibited: $s_i = 0$. Under regime three, all ATM usage fees are allowed.

**Consumers**

A consumer gets zero surplus when he does not bank. To become a cardholder of bank $i$ located at a distance $\delta_i$ in the product space, a consumer must pay the account fee $p_i$ to the bank. In this case, the consumer obtains a total surplus equal to:

$$w_i = v_b - t\delta_i + v_i - p_i$$

The first term $v_b$ is the fixed surplus from consuming basic services. The second term $t\delta_i$ is a differentiation cost in the product space (where $t > 0$). Parameters $v_b$, $c_b$ and $t$ have the following properties:

**Assumption 1.** (i) $t$ is “sufficiently large” and (ii) $v_b - c_b \geq 3t/2$.

Assumption 1(i) is necessary for the second order conditions of the profit maximization to be verified. Assumption 1(ii) guarantees that consumers want to affiliate to a bank even if there is no ATM. To ensure the existence of equilibria, $v_b$, $c_b$ and $t$ must satisfy extra
conditions described later. The third term of expression (1), \( v_i \), corresponds to the variable net surplus from “consuming” withdrawals. More precisely,

\[
v_i = u_i(n_i, n_j, q^d_i, q^f_i) - (f_i + s_j)q^f_i
\]

where \( q^d_i \) (respectively \( q^f_i \)) is the number of domestic (respectively foreign) withdrawals made by a cardholder of bank \( i \).

To keep the model tractable, we do not explicitly model the micro behavior of consumers when they choose an ATM. Instead, in appendix 1 we specify a surplus function \( u_i(n_i, n_j, q^d_i, q^f_i) \) generating the “reasonable” individual demands for withdrawals that follow:

\[
q^d_i(n_i, n_j, d_i, f_i + s_j) = \alpha \frac{n_i}{n} n^\gamma + \beta' (f_i + s_j)
\]

and

\[
q^f_i(n_i, n_j, d_i, f_i + s_j) = \frac{n_j}{n} n^\gamma - \beta (f_i + s_j)
\]

with \( \alpha > 0, \beta > \beta' \geq 0 \) and \( \gamma \in [0, 0.39] \).\(^9\)

To justify the shape of the demands, let us first consider the case where usage fees, \( f_i, s_j \), are equal to zero. In this case, consumers withdraw cash regardless of the ATM owner. The total number of withdrawals per cardholder, \( q^d_i + q^f_i \), is equal to \( \alpha n^\gamma \). For a given positive \( \gamma \), the total number of withdrawals per cardholder is increasing in \( n \). We have in mind a justification à la Allais (1947) - Baumol (1952) - Tobin (1956): consumers trade off the costs (the forgone interests) and the benefits (avoiding a trip to an ATM) to hold cash. A larger ATM network reduces the distance to reach a machine, benefits to hold cash decrease and consumers make more withdrawals. However each extra machine reduces the distance less and less so that the number of withdrawals increases slower than the number of ATMs: the number of withdrawals per machine is decreasing in the network size.

Parameter \( \alpha \) is a scale parameter. Parameter \( \gamma \) reflects the spread of the shopping space. As \( n < 1 \), the total number of withdrawals \( \alpha n^\gamma \) is decreasing in \( \gamma \). A small \( \gamma \) describes a

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\(^9\)The upper bound on \( \gamma \) is needed for the second order conditions of the profit maximization.
case where the shopping space is concentrated so that the average travel cost to access cash is low. In this case, consumers make many withdrawals but do not attach a high value to an additional ATM. When $\gamma$ is high, the shopping space is spread out and it is more costly to reach cash. Cardholders make fewer withdrawals and each additional ATM is more valued.

Recall that ATMs and cardholders are uniformly located in the shopping space. As $f_i, s_j$ are equal to zero, cardholders withdraw cash at the closest ATM. With probability $n_i/n$ the ATM belongs to bank $i$ so that the individual demand for domestic withdrawals is $\alpha(n_i/n)n^\gamma$ and the individual demand for foreign withdrawals is equal to $\alpha(n_j/n)n^\gamma$.

Introducing a usage fee of one euro increases the number of domestic withdrawals by $\beta'$ and decreases the number of foreign withdrawals by $\beta$. Our demand specification reflects the existence of a substitution effect that can be measured by the ratio $\beta'/\beta < 1$. For tractability, we assume that this ratio does not depend on the ATM market shares. Note that the linearity of demand functions guarantees the finiteness of demands when fees tend to zero. This last property is necessary to compare the three pricing regimes.

In order to obtain a network size lower than one under the different regimes, we need a second assumption:

**Assumption 2.** $c >> \frac{\alpha^2(3 + \gamma)}{12(\beta - \beta')}$.

**Competition and profits**

We deal with cases where the market is entirely covered. Let $\delta$ denote the distance between bank 1 and the consumer who is indifferent between purchasing services from bank 1 or 2:

$$v_1 - t\delta - p_1 = v_2 - t(1 - \delta) - p_2$$  \hspace{1cm} (5)
Bank $i$’s market share of deposits is

$$D_i = \frac{1}{2} + \frac{1}{2t}(v_i - v_j - p_i + p_j)$$

(6)

Note that $D_1 + D_2 = 1$. The profit of bank $i$ is

$$\pi_i = (p_i - c_b)D_i + (a + s_i)q_j^f(1 - D_i) + (f_i - a)q_i^fD_i - cn_i.$$  

(7)

The term $(p_i - c_b)D_i$ is the net revenue from providing accounts. The term $(a + s_i)q_j^f(1 - D_i)$ corresponds to the revenues coming from the foreign withdrawals made by the cardholders of bank $j$. The term $(f_i - a)q_i^fD_i$ corresponds to the net revenues coming from foreign withdrawals made by cardholders of bank $i$. The term $cn_i$ corresponds to the total cost of deploying and operating ATMs.

**Timing of the game**

**Stage one:** banks choose the interchange fee $a$ jointly.

**Stage two:** banks simultaneously and non-cooperatively choose the number of ATMs they deploy, $n_1$ and $n_2$, and the fees, $p_1$, $f_1, s_1$ and $p_2, f_2, s_2$.

**Stage three:** each consumer chooses the bank that provides him with the highest positive surplus.

**Stage four:** consumers make withdrawals in the shopping space.

We deal with three different regimes. Under all regimes, consumers pay the account fee $p_i$. Under regime one, there is no ATM usage fee $f_1 = f_2 = s_1 = s_2 = 0$. Under regime two, there are foreign fees but no surcharge, $s_1 = s_2 = 0$ and under regime three, foreign fees and surcharges are allowed.
We look for the symmetric Nash equilibrium of the model for a given interchange fee. We write the four first order conditions of the complete maximization problem:

\[
\begin{align*}
\frac{\partial \pi_i}{\partial n_i} &= 0 \iff \tilde{p}_i \frac{\partial D_i}{\partial n_i} + (f_i - a) \frac{\partial q_f^i}{\partial n_i} D_i + (a + s_i) \frac{\partial q_f^j}{\partial n_i} (1 - D_i) - c = 0 \quad (i) \\
\frac{\partial \pi_i}{\partial p_i} &= 0 \iff \tilde{p}_i \frac{\partial D_i}{\partial p_i} + D_i = 0 \quad (ii) \\
\frac{\partial \pi_i}{\partial f_i} &= 0 \iff \tilde{p}_i \frac{\partial D_i}{\partial f_i} + (f_i - a) \frac{\partial q_f^i}{\partial f_i} D_i + q_f^i D_i = 0 \quad (iii) \\
\frac{\partial \pi_i}{\partial s_i} &= 0 \iff \tilde{p}_i \frac{\partial D_i}{\partial s_i} + \left( (a + s_i) \frac{\partial q_f^j}{\partial s_i} + q_f^j \right) (1 - D_i) = 0 \quad (iv)
\end{align*}
\]

with \( \tilde{p}_i = \left( p_i - c_b - (a - f_i) q_f^i - (a + s_i) q_f^j \right) \).

In each expression, the variation of the relevant decision variable has two effects on \( \pi_i \): one through the demand for deposits faced by bank \( i \) and the other through the demands for withdrawals. Note that term \( \tilde{p}_i \) corresponds to the net revenue per additional depositor.

We now detail the effects more precisely for each regime.

### 2 Regime one: ATM usage fees are prohibited

We take \( f_1 = f_2 = s_1 = s_2 = 0 \): the account fee is the only available tool to charge consumers.

As withdrawals are free, all machines are equivalent for cardholders and they withdraw cash regardless of the ATM owner.

#### 2.1 The equilibrium for a given interchange fee

We consider a given interchange fee \( a \). To generate a network size smaller than one we assume that \( a \) is smaller than \( 2c/\alpha \). We characterize a Nash equilibrium \( \{n_1^*(a), p_1^*(a), n_2^*(a), p_2^*(a)\} \) where all the market is covered \( (D_1 + D_2 = 1) \). Only conditions (8)(i) and (8)(ii) are
relevant. It is convenient to start with the determination of the account fee. As all ATMs are equivalent for cardholders, cardholders obtain the same net variable surplus from consuming withdrawals whatever their home bank is: \( v_1 = v_2 \) for any \( n_1 \) and \( n_2 \). Hence we have

\[
D_i = \frac{1}{2} + \frac{1}{2t}(p_j - p_i)
\]

Using (8)(ii) and the symmetric condition on bank \( j \), we get

\[
p^*_i(a) = t + c_b + aq^f_i(a) + aq^f_j(a)
\]

The equilibrium account fee is the sum of the differentiation parameter plus the total marginal cost for bank \( i \) of accepting a cardholder. This total marginal cost is composed of three parts: \( c_b \) is a marginal cost of basic services. The term \( aq^f_i \) corresponds to the interchange fees that bank \( i \) will have to pay for the \( q^f_i \) foreign withdrawals of this cardholder. The term \( aq^f_j \) can be interpreted as an opportunity cost: if the cardholder chose to become a cardholder of bank \( j \), he would make \( aq^f_j \) foreign withdrawals at bank \( i \)'s ATMs and bank \( i \) would receive \( aq^f_j \). Hence, by accepting the customer, bank 1 loses \( aq^f_j \), which appears in the equilibrium account fee \( p_i \).

Let us consider the deployment problem by rewriting expression (8)(i) as

\[
i \frac{\delta D_i}{\delta n_i} + a \frac{\partial q^f_i}{\partial n_i} (1 - D_i) + (-a) \left( \frac{\partial q^f_j}{\partial n_i} \right) D_i = c
\]

(A1) (B1) (C1)

Solving (11) with respect to \( n_i \) gives the deployment reaction function. Let us examine the three terms which measure the three effects of deploying an extra ATM on bank \( i \)'s profits.

- **Term A1.** As there is no ATM usage fee, all cardholders benefit the same way from this extra ATM and bank \( i \) does not attract new cardholders thus its deposit market share remains unchanged: \( \partial D_i / \partial n_i = 0 \).
- Term B1. The cardholders of bank $j$ make more foreign withdrawals ($\partial q^f_j / \partial n_i > 0$). Consequently bank $i$ receives more interchange fees.

- Term C1. The cardholders of bank $i$ make less foreign withdrawals ($\partial q^f_i / \partial n_i < 0$) and less interchange fees must be paid to bank $j$.

To sum up: under regime one, a bank does not deploy ATMs to attract new depositors but rather to process withdrawals made by its competitor’s cardholders and limit the foreign withdrawals made by its own cardholders. Associating condition (11) with the symmetric condition on bank $j$ yields the total network size under regime one as a function of $a$:

$$n^*(a) = \left( \frac{\alpha a}{2c} \right)^{\frac{1}{1-\gamma}}$$

At equilibrium $n^*_i(a) = n^*_j(a) = n^*(a)/2$. The total ATM network size is increasing in $a$. A higher interchange fee increases each bank’s incentives to open ATMs in order to attract withdrawals from its own depositors and from non customers. As we have assumed that $a < 2c/\alpha$, the network size is decreasing in $\gamma$. A wider shopping space means that cardholders incur a higher average travel cost to reach an ATM so that they make less withdrawals (for any given network size). Consequently banks’ competition to process withdrawals is less intense and the equilibrium network size is smaller. From expressions (3) and (4) we have

$$q^{d^*_1}(a) = q^{d^*_2}(a) = q^{d^*_2}(a) = q^{f^*_1}(a) = q^{f^*_2}(a) = \alpha n^*(a)/2$$

Interestingly, the total profit can be rewritten as

$$\pi^*_i(a) = \frac{t}{2} + a q^{f^*_1}(a) - c \frac{n^*(a)}{2}$$

At equilibrium, the interchange outflows per cardholder of bank $i$, $aq^f_i$, are entirely recovered through the account fee $p_i$. Consequently $aq^f_i$ does not appear in profit.

For this equilibrium to exist, we must verify two extra conditions:
(i) For the market to be covered, the surplus of the consumer who is indifferent between the two banks cannot be negative:

\[ v_b - \frac{t}{2} + u_i \left( n_i^*(a), n_j^*(a), q_i^{d*}(a), q_j^{f*}(a) \right) - p_i^*(a) \geq 0 \quad (14) \]

In appendix 2, we show that the previous condition is verified for all \( a \) smaller than \( a^* \), where \( a^* \) is the unique positive interchange fee verifying condition (14) with equality. The interchange fee \( a^* \) permits to extract all the surplus of the indifferent consumer and it appears in figure 1. It is not possible to characterize \( a^* \) explicitly. However, we will be able to compare the profits in the different regimes.

(ii) The second order conditions must hold. We show that it is indeed the case if \( v_b - c_b \) is “not too large” in the sense defined precisely in appendix 3.

### 2.2 The effect of the interchange fee on equilibrium profits

For any \( a \leq a^* \), the equilibrium profit of bank \( i \) is

\[ \pi_i^*(a) = \frac{t}{2} + \left( \frac{\alpha a}{2^{2-\gamma} c^2} \right)^{\frac{1}{1-\gamma}} \quad (15) \]

**Proposition 1** Under regime one, equilibrium profits are monotonically increasing in \( a \) on \([0, a^*]\). By setting \( a = a^* \) banks extract all the surplus of the indifferent consumer.

To understand proposition 1, note that raising the interchange fee has two opposite effects on profits (characterized in expression (13)): first, there is a more intense competition to attract domestic and foreign withdrawals. Hence, banks deploy more ATMs and the deployment costs increase. Second, the revenues from interchange inflows \( aq_j^f \) increase. The effect on interchange inflows outweighs the effect on deployment costs so that profits increase.
Proposition 1 shows that setting the interchange fee jointly allows banks to collude and to extract consumers surplus. As profits are monotonically increasing in \( a \), collusion is only limited by the participation constraint of the marginal consumer. The account fee is a lump sum. By raising the interchange fee, the lump sum increases, consumers’ surplus decreases till the participation constraint of the marginal consumer binds.

There could exist equilibria for interchange fees above \( a^* \) yielding higher equilibrium profits. However we do not need to characterize these equilibria for our subsequent analysis.  

3 Regime two: surcharges are prohibited

We take \( s_1 = s_2 = 0 \). Each bank charges its cardholders the two-part tariff \( p_i + f_i q_i^f \).

3.1 The equilibrium for a given interchange fee

We consider a given interchange fee smaller than \( 4c/\alpha(3 + \gamma) \). We characterize a symmetric Nash equilibrium \( \{ n_{1ff}^*, p_{1ff}^*, n_{2ff}^*, p_{2ff}^*, f_1^*, f_2^* \} \). It is convenient to start with the determination of the foreign fee. In appendix 4, we show that in equilibrium, the foreign fee set by bank \( i \) is equal to the marginal cost of a foreign withdrawal, that is, to the interchange fee:

\[
 f_i^* = a 
\]

Doing so, bank \( i \) maximizes its cardholders’ surplus and use the fixed account fee \( p_i \) to recover a part of this surplus in a manner compatible with the competitive intensity.

Let us determine the equilibrium account fee. Using expressions (8)(i), (16) and the fact that \( D_i = 1/2 \) at the symmetric equilibrium, we obtain:

\[ ^{10}\text{See Donze & Dubec (2006) for an exemple of derivation of such equilibria above } a^* \text{ with less general demands for withdrawals.} \]
\[ p^*_{i,ff}(a) = t + c_b + aq^*_{i,ff}(a) \]  \hspace{1cm} (17)

The interpretation of the equilibrium account fee under regime 2 is nearly the same as under regime 1 except that the account fee only recoups the opportunity cost of accepting an extra depositor, \( aq^f \). The second part of the marginal cost of affiliation, \( aq^i \), is now recovered through the foreign fee revenues \( f_iq^f \).

Let us turn to the deployment problem of banks. Using (17), expression (8)/(ii) can be rewritten:

\[
\frac{\delta D_i}{\delta n_i} = (\alpha/2t)an^\gamma - 1; \hspace{1cm} (A2)
\]

\[
\partial q^f_i (1 - D_i) + (f_i - a) \frac{\delta q^f_i}{\delta n_i} D_i = c \hspace{1cm} (B2)
\]

\[
\frac{\delta D_i}{\delta n_i} + a \frac{\delta q^f_i}{\delta n_i} (1 - D_i) + (f_i - a) \frac{\delta q^f_i}{\delta n_i} D_i = c \hspace{1cm} (C2)
\]

Let us examine the three terms \((A2)\), \((B2)\) and \((C2)\) to explain the determinants of ATM deployment under regime two:

- We show in appendix 5 that at the symmetric equilibrium \( \delta D_i/\delta n_i = (\alpha/2t)an^\gamma - 1; \) term \((A2)\) is positive and consequently higher than term \((A1)\) in expression (11). As consumers pay for foreign withdrawals, they take into account the ATM market shares when they decide where to open an account: consumers prefer a bank with a larger network because they can make less costly foreign withdrawals. Hence, the deployment of an extra ATM by bank \( i \) makes this bank more attractive and increases its deposit market share \( (\delta D_i/\delta n_i > 0) \) and its revenues. In some sense, the existence of foreign fees creates a differentiation between the ATM networks of the two banks. For a given interchange fee, this differentiation effect of foreign fees makes banks deploy more ATMs than under regime one.

- Term \((B2)\) is positive and equal to term \((B1)\) of expression (11). As under regime one, deploying an extra ATM increases the interchange inflows of bank \( i \).
Term (C2) is smaller than term (C1) of expression (11). This is the interchange recovery effect of introducing foreign fees: since the foreign fee is equal to the interchange fee, the foreign withdrawals of bank i’s cardholders become costless for this bank. Consequently limiting its cardholders’ foreign withdrawals is no more a reason to deploy ATMs. This interchange recovery effect of foreign fees makes banks deploy fewer ATMs than under regime one.

Hence under regime two, banks deploy ATMs both to attract new depositors and generate interchange inflows. Note that the differentiation effect and the interchange recovery effect on deployment are opposite. In appendix 5, we show using expressions (16) and (18) that the total number of ATMs deployed under regime two for a given interchange fee is

\[ n_{ff}^*(a) = \left( \frac{\alpha(3 + \gamma)a}{4c} \right)^{\frac{1}{1-\gamma}} \]  

(19)

The number of ATMs deployed under regime two is increasing in the interchange fee, decreasing in the deployment cost, and decreasing in \( \gamma \). Comparing expressions (12) and (19) shows that in our framework, banks deploy more ATMs under regime two than under regime one for a given interchange fee: the differentiation effect of foreign fees outweighs the interchange recovery effect. From expressions (3), (4), and (16), we have

\[ q_{1ff}^d(a) = q_{2ff}^d(a) = \alpha n_{ff}^\gamma(a)/2 + \beta' a \]  

(20)

and

\[ q_{1ff}^t(a) = q_{2ff}^t(a) = \alpha n_{ff}^\gamma(a)/2 - \beta a \]  

(21)

The equilibrium profit can be written as

\[ \pi_{i,ff}^*(a) = \frac{t}{2} + aq_{1ff}^*(a) - c \frac{n_{ff}^*(a)}{2} \]  

(22)

This expression is similar to expression (13) we obtained under regime one. However, the interchange outflows per cardholder of bank i, \( aq_i^f \), are now entirely recovered through the
foreign fee $f_i$ and no more through the account fee $p_i$. Using (19), (21) and (22), we can write the equilibrium profit of bank $i$ as a function of $a$:

$$\pi_{i,ff}^*(a) = \frac{t}{2} + \frac{1 - \gamma}{8} \left( \frac{3 + \gamma}{4} \right)^{\frac{\gamma}{1-\gamma}} a^{\frac{1}{1-\gamma}} \frac{a^{\frac{1}{\gamma}}}{e^{\frac{1}{\gamma}}} - \beta a^2$$

(23)

We verify the second order conditions of maximization in appendix 6.

We compare network sizes, banks profits and consumers’ surpluses under regime one and two for a given interchange fee. To compare consumers’ surplus across regimes, we consider the surplus of the indifferent consumer which we denote by $\tilde{w}(a)$. At the symmetric equilibrium: $\tilde{w}(a) = v_b - \frac{t}{2} + v_1(a) - p_1(a)$.

**Proposition 2** For any given interchange fee $a \in [0, a^*]$, switching from regime one to regime two yields

(i) a larger network: $n_{ff}^*(a) > n^*(a)$

(ii) higher profits for both banks: $\pi_{i,ff}^*(a) < \pi_i^*(a), i = 1, 2$.

(iii) a higher consumers’ surplus: $\tilde{w}^*(a) < \tilde{w}_{ff}^*(a)$.

Proof: appendix 7.

As noted before, for a given interchange fee, banks deploy more ATMs under regime two because of the differentiation effect of foreign fees. The increase in deployment costs outweighs the change in revenues so that banks profits are lower under regime two.\textsuperscript{11} Furthermore, consumers are better off under regime two. This is for two reasons: first, as the network size is larger, accessing to cash is easier; second, consumers prefer to pay for their foreign withdrawals through the usage fees of regime two rather than through the lump sum of regime one: in the latter case, cardholders consume (and pay for) “too many” foreign withdrawals.

\textsuperscript{11}Note that the effect of introducing foreign fees on the demand for foreign withdrawals and hence on revenues is ambiguous since more ATMs are available at a higher usage fee.
3.2 The effect of the interchange fee on equilibrium profits

Contrary to what happens under regime one, \( \pi_{i,ff}^*(a) \) has a unique maximum, characterized by

\[
a_{ff}^* = \left( \frac{\alpha(3 + \gamma)^{\gamma}}{24(2\gamma)(3 - \gamma)c} \right)^{\frac{1}{2\gamma}}
\]  

(24)

The profit \( \pi_{i,ff}^*(a) \) is increasing in \( a \) up to the point \( a_{ff}^* \). To understand why, let us consider the effect of increasing the interchange fee in expression (22). In a first time, profits follow the increase in revenues per foreign withdrawals, \( a \), and in a second time, the declining demand for foreign withdrawals and the ever increasing deployment costs make profits fall.

In appendix 7, we verify that when \( a = a_{ff}^* \), the net surplus \( \tilde{w}_{ff}^* \) of the indifferent consumer is positive. We sum up the results in the following proposition.

**Proposition 3** Under regime two, equilibrium profits are monotonically increasing in \( a \) on \([0, a_{ff}^*]\) and decreasing thereafter. When \( a = a_{ff}^* \), banks leave a positive surplus to the indifferent consumer.

As under regime one, banks can collude by setting the interchange fee jointly. Nevertheless, their ability to extract consumers surplus by raising the interchange fee is reduced because of the existence of foreign fees: contrary to regime one, cardholders react to an increase of the interchange fee (and hence of the foreign fee) by adjusting their demand for foreign withdrawals downward in order to pay less to their bank.

The size of the network for the profit-maximizing interchange fee is

\[
n_{ff}^*(a_{ff}^*) = \left( \frac{\alpha^2(3 + \gamma)}{64\beta c} \right)^{-\frac{1}{2\gamma}}
\]  

(25)

Note that assumption 2 guarantees that \( n_{ff}^*(a_{ff}^*) \) is lower than one. The associated individual profit is

\[
\pi_{ff}^*(a_{ff}^*) = \frac{t}{2} + 4^{\frac{2\gamma - 4}{2\gamma}} (1 - 2\gamma)(3 + \gamma)^{\frac{2\gamma - 4}{2\gamma}} \alpha^{\frac{2\gamma}{2\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{2\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{2\gamma}}
\]  

(26)
We will compare the profits for the profit-maximizing interchange fees across the three regimes subsequently.

4 Regime three: the case with foreign fees and surcharges

Under regime three, each bank $i$ charges its own cardholders the two-part tariff $p_i + f_i q_i^f$ and charges non-customers the linear tariff $s_i q_j^f$.

4.1 The equilibrium for a given interchange fee

As in regime two, banks maximize their cardholders surplus by setting the foreign fee equal to the marginal cost of a foreign withdrawal ($f_i^* = a$) and extract it back through the account fee $p_i$. This is proved in appendix 3. Using expression (8)(ii) and the fact that $D_i = 1/2$ at the symmetric equilibrium, we obtain the equilibrium account fee of bank $i$:

$$p^*_{i,sur}(a) = t + c_b + (a + s^*_i(a))q^f_{j,sur}(a)$$

(27)

The opportunity cost of accepting an additional cardholder must now take into account the surcharges $s_i q_j^f$ that this cardholder would pay to bank $i$ if he had chosen bank $j$. As under regime two, the interchange outflows per cardholder, $aq_i^f$, are recovered through the foreign fees revenues $f_i q_i^f$ and not through the account fee $p_i$.

At the symmetric equilibrium, expression (8)(i) can be written as

$$t \frac{\delta D_i}{\delta n_i} + (a + s_i) \frac{\partial q_i^f}{\partial n_i} (1 - D_i) + (f_i - a) \frac{\partial q_i^f}{\partial n_i} D_i = c$$

(A3)  (B3)  (C3)

(28)
This expression shows that the qualitative reasons to deploy ATMs are the same as under regime 2. First each bank deploys ATMs in order to increase its deposit market share: term (A3) is positive. We show in appendix 8 that at the symmetric equilibrium $\delta D_i/\delta n_i = (\alpha/2t)(a + s_i)n^{\gamma-1}$. Second, each bank wants to increase its revenues from the interchange inflows it perceives: term (B3) is positive. As under regime two, the third term is null at equilibrium. In fact expression (28) is the same as expression (18), except that $a$ is replaced by $a + s_i$: for a given level of interchange fee and a positive surcharge $s_i$, bank $i$ has more incentives to open ATMs than under regime two. The surcharge adds to the interchange fee and permits double marginalization: the revenue per foreign withdrawal is higher. Consequently, surcharging boosts ATM deployment.

Let us now examine the factors determining the level of the surcharge. For that, we have to interpret expression (8) (iv). Using (27), expression (8) (iv) can be written as

$$t \frac{\partial D_i}{\partial s_i} + \left( (a + s_i) \frac{\partial q_j^f}{\partial s_i} + q_j^f \right) (1 - D_i) = 0$$

(29)

The LHS of expression (29) measures the effect of a marginal increase of $s_i$ on bank $i$’s profit.

- The first term of the LHS is positive and equal to $\frac{1}{2}q_j^f$ (see appendix 9): a higher surcharge $s_i$ increases the price per foreign withdrawal for the cardholders of bank $j$. This has a negative effect on the surplus derived from the affiliation to bank $j$ and bank $i$ becomes more attractive for depositors. Hence, a higher surcharge permits bank $i$ to enlarge its deposit market share and increase its profit. This effect appears in the literature as the “depositor-stealing motive” for surcharges (see McAndrews (1998)).

- Bank $i$ has also to consider the effect of rising the surcharge on the revenue from non customers measured by the second term of expression (29). A higher surcharge yields higher revenues per foreign withdrawal but non customers use less frequently bank $i$’s machines.
Using the fact that $f_i^* = a$, expressions (28), (29), and symmetry, we obtain

$$
\begin{cases}
    s_1^*(a) = s_2^*(a) = s^*(a) = \alpha n^* \gamma(a)/3 \beta - a \\
    n^*(a) = \left( \frac{\alpha (3 + \gamma)(a + s^*(a))}{4c} \right)^{\frac{1}{\gamma}}
\end{cases}
$$

The two previous equations show that there is a reinforcement effect between the surcharge level and the network size: for a given level of $a$, double marginalization induces a bigger ATM network than under regime two. Demands for foreign withdrawals shift upward and banks can set higher surcharges. In turn the higher surcharges increase the double margin, and so on. Solving the previous system we have

$$
n^*_\text{sur} = \left( \frac{\alpha^2 (3 + \gamma)}{12 \beta c} \right)^{\frac{1}{1-2\gamma}}
$$

We verify the second order conditions of maximization in appendix 10.

### 4.2 The neutrality of the interchange fee

When foreign fees and surcharges are permitted the interchange fee affects neither the equilibrium number of ATMs, $n^*_\text{sur}$, nor the total price paid by cardholders for foreign withdrawal, $f^* + s^* = \alpha n^*_\gamma / 3 \beta$. Banks’ profits are therefore independent from $a$:

$$
\pi^*_\text{sur} = \frac{t}{2} - 2^{\frac{3-\gamma}{2\gamma}} \frac{2\gamma - 2}{2+2\gamma} (5 + 3\gamma) \frac{2}{1-2\gamma} \alpha^{\frac{1}{2-2\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{1-2\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{1-2\gamma}}
$$

To understand the neutrality of the interchange fee, consider the situation where bank $i$ obtains interchange revenues equal to $a + s^*_i$ for each withdrawal made by a cardholder of bank $j$, and bank $j$ receives $f^*_j - a$ for each foreign withdrawals made by its own cardholders. Cardholders of bank $j$ pay $f^*_j + s^*_i$ per foreign withdrawal. Suppose now that the interchange fee is increased by $\Delta a$. Banks can preserve the equilibrium payoffs and cardholders’ demands are unchanged if bank $i$ reduces its surcharge by $\Delta a$ while bank $j$ increases its foreign fee by $\Delta a$. The total usage fee paid by bank $j$’s cardholder is still equal to $f^*_j + s^*_i$, the number
of foreign withdrawals and banks’ revenues are unchanged. Since the equilibrium network size and the total usage fee are unaffected by the interchange fee, consumers’ welfare is also independent from \( a \).

We sum up the results in the following proposition:

**Proposition 4** When foreign fees and surcharges are allowed, the interchange fee is neutral in the following sense: (i) its level does not affect the equilibrium deployment of ATM nor banks’ equilibrium profits. (ii) its level does not affect consumers’ welfare.

The main consequence of this result is that under regime three the interchange fee cannot be a collusive tool for banks anymore. Interestingly, bank \( i \)’s accounting net revenue per ATM is equal to

\[
\frac{(a + s_i^*)q_{i,\text{sur}}^* (a) D_j^*}{n_{i,\text{sur}}^* (a)} - c = \frac{7 + 3\gamma c}{9 + 3\gamma c}
\]

which is negative. As noted before, this prediction is consistent with empirical observations that in the USA, ATMs operated by banks lose money on average (see footnote 6).

5 **Comparison of the three pricing regimes**

In this section, we compare banks’ profits and consumers’ surplus across the three regimes when banks choose the interchange fee to maximize their joint profits. We also study the consequences of replacing the interchange system by a direct charging scheme.

5.1 **Comparison of profits and consumers’ surplus**

Under regime one, individual profits \( \pi^*(a) \) are monotonically increasing in the interchange fee up to \( a^* \), where \( a^* \) is defined by condition (14) verified with equality. Consequently,

\[12\] This intuitive argument explaining neutrality was first developed by Salop (1990).
banks never choose an interchange fee lower than \(a^*\) and the individual profits cannot be lower than \(\pi^*(a^*)\). There could exist equilibria for interchange fees above \(a^*\) yielding higher equilibrium profits. However it will be sufficient to work with the profits associated to the type of equilibrium we have described to compare the three regimes. Under regime two, banks choose \(a_{ff}^*\) defined by (24) to maximize their joint profits. Each bank obtain \(\pi_{ff}^*(a_{ff}^*)\) defined by (25). Under regime three the choice of the interchange fee is irrelevant and each bank obtains \(\pi_{sur}^*\) defined by (31).

**Proposition 5** Suppose that under each regime banks jointly set the interchange fee at the level that maximizes their joint profits, then

(i) the network is larger under regime three than under regime two: \(n_{ff}^*(a_{ff}^*) < n_{sur}^*\);

(ii) banks prefer regime one to regime two and regime two to regime three:

\[
\pi^*(a^*) > \pi_{ff}^*(a_{ff}^*) > \pi_{sur}^*;
\]

(iii) there exists a value \(\gamma^*(\beta'/\beta)\) above which consumers prefer regime three to regime two:

\[
\forall \gamma < \gamma^*(\beta'/\beta), \quad \tilde{w}_{ff}^*(a_{ff}^*) > \tilde{w}_{sur}^* > \tilde{w}^*(a^*) = 0;
\]

\[
\forall \gamma > \gamma^*(\beta'/\beta), \quad \tilde{w}_{sur}^* > \tilde{w}_{ff}^*(a_{ff}^*) > \tilde{w}^*(a^*) = 0.
\]

where \(\gamma^*(\beta'/\beta) = \frac{1}{2} \left(1 - \ln \frac{16}{3} / \ln \left(\frac{99 + 29 \frac{\beta'}{\beta}}{8 + 16 \frac{\beta'}{\beta}}\right)\right)\)

Proof: appendix 11.

\(\gamma^*(\beta'/\beta)\) is represented in figure 2.

When banks choose the profit-maximizing interchange fee, fewer ATMs are deployed under regime two than under regime three. This is because the revenue per foreign withdrawal \(a_{ff}^*\) under regime two is lower than the revenue with surcharges, \(a_{ff}^* + s^*(a_{ff}^*)\). Regime three is the worst for banks because they incur very high deployment cost compared to regime two and they cannot use the interchange fee to relax price competition on the deposit market.
For a given substitution rate $\beta'/\beta$, consumers prefer regime three to regime two for sufficiently high values of the dispersion parameter $\gamma$. In this case, travel costs to withdraw cash are higher ceteris paribus and consumers appreciate the large network of regime three even if they have to pay higher usage fees per foreign withdrawal. When the shopping space is more concentrated consumers prefer the smaller and less expensive network of regime two. Interestingly, this prediction is consistent with the empirical work of Knittel and Stango (2004) who find that after the introduction of surcharging in 1996 in the USA, consumers’ welfare increased in high travel cost counties while it decreased in low travel cost counties.

Potentially, regime one is the most profitable for banks and the worst for consumers because the collusive power of the interchange fee is only limited by the participation constraint of the marginal consumer. By banning usage fees banks can maximize consumers’ gross surplus and extract it back through the account fee. Under regime two, banks’ individual objectives do not coincide with the industry objectives: at equilibrium banks independently set the foreign fee equal to the interchange fee to maximize their individual profits while it would be better for them to set it equal to zero and increase the account fee.

5.2 Direct charging

The ATM markets in Australia and South Africa work under a regime close to our regime two, with interchange fees and foreign fees but no surcharges. Recently, the Australian and the South African regulation authorities have proposed to use a “direct charging model” whereby the interchange fee applicable to each foreign transaction would be removed and ATM owners would be free to set their own fee for foreign ATM transactions (see Reserve Bank of Australia & the Australian Competition and Consumer Commission (2000) ; Competition Commission of South Africa (2007)). According to its proponents, there are two main objectives of the reform. First, removing the interchange system limits banks’ possibility to collude. The
second objective is to favor price competition on ATM fees between ATM deployers. The resulting price flexibility is to be opposed to the stickiness of interchange fees. One can study the consequences of such a regulation scheme in our model.

**Proposition 6** The “direct charging model” is equivalent to regime three.

Proof: setting \( a = f_1 = f_2 = 0 \) in the system of equations (8) yields the solution characterized by (30).

Hence, the model predicts that switching from regime two to direct charging increases consumers’ welfare if travel costs are sufficiently high. However direct charging hurts banks: because of the deposit stealing effect, surcharges are set above the level that would maximize the revenue per ATM, which in turn favors the ATM deployment, higher surcharges and so on. The advocates of the direct charging scheme forget this interaction between the withdrawal market and the deposit market.

### 6 Conclusion

We have proposed a tractable model to study the effect of ATM pricing on welfare in which the relationships between the deposit market and the withdrawal market are highlighted. We have shown that increasing the number of usage fees make ATM networks more differentiated which provide banks with more incentives to deploy ATMs. The potential increase in revenues from adding usage fees is not sufficient to cover the additional deployment costs and the model predicts that banks’ profits diminish when one switches from regime one to two and from regime two to three. Regime three is specially bad for banks since the neutrality of the interchange fee is further added to the large ATM deployment.

From the regulator’s perspective, our analysis shows the importance of the travel costs to reach cash within the shopping space when deciding to ban surcharges or not: consumers
prefer regime three to regime two only when they incur high travel costs as it is the case when the shopping space is spread out. The model also predicts that direct charging induces an intense competition to attract both depositors and withdrawals. This leads banks to deploy a large number of ATMs and their profits are adversely affected.
Appendix

Appendix 1. Surplus from consuming withdrawals

We assume that the variable gross surplus from consuming withdrawals takes the following quadratic shape:

\[ u_i = \frac{1}{\beta^2 - \beta'^2} \left[ (\alpha\beta \frac{n_i}{n} \gamma + \alpha\beta' \frac{n_j}{n} \gamma^2) q^d_i - \frac{\beta}{2} (q^d_i)^2 + (\alpha\beta' \frac{n_j}{n} \gamma + \alpha\beta' \frac{n_i}{n} \gamma^2) q^f_i - \frac{\beta}{2} (q^f_i)^2 - \beta' q^d_i q^f_i \right] \]

Writing \( \frac{\partial v_i}{\partial q^d_i} = 0 \) and \( \frac{\partial v_i}{\partial q^f_i} = 0 \) and inverting the system yields the individual demands for withdrawals.

Using expressions (2), (32), (3) and (4) we get the optimized expression of the net surplus from consuming withdrawals,

\[ v_i = u_i - (f_i + s_j) q^f_i \]

\[ = \frac{\alpha^2}{2(\beta - \beta') \left[ n_i^2 n^2 \gamma^2 - 2n_i^2 n^2 \gamma - n_j^2 n^2 \gamma - 2 \right] + \beta (f_i + s_j)^2 - \alpha (f_i + s_j) n_j n^\gamma - 1} \]

Appendix 2. Characterization of \( a^* \) under regime 1

Under regime one, the two last terms of expression (33) are equal to zero. By setting \( n_i^*(a) = n_j^*(a) = n^*(a)/2 \) we obtain

\[ v_1 = v_2 = \frac{\alpha^2 n^2 \gamma(a)}{4(\beta - \beta')} \]

Condition (14) can be rewritten with equality under the following shape:

\[ v_b + \frac{\alpha^2 n^2 \gamma(a)}{4(\beta - \beta')} - \frac{t}{2} - p_i^*(a) = 0 \]

or using expressions (10) and (12),

\[ v_b - c_b - \frac{3t}{2} + \frac{\alpha a^{2 \gamma}}{(2c/\alpha)^{1/\gamma}} - \frac{a^{\frac{2}{1+2\gamma}}}{4(\beta - \beta')(2c/\alpha)^{1/\gamma}} = 0 \]
The LHS of expression (35) is the function represented in figure 1 and the equation has a unique positive solution, \( a^* \).

It must be the case that the interchange fee \( a^* \) does not induce a network \( n^*(a^*) \) larger than one. In other words, we must have

\[
v_b + \frac{\alpha^2}{4(\beta - \beta')} - \frac{t}{2} - p_i^*(a) < 0 \quad \text{for} \quad a = 2c/\alpha
\]

which can be rewritten

\[
v_b - c_b < \frac{3t}{2} + 2c - \frac{\alpha^2}{4(\beta - \beta')}
\]

The expression is not very demanding since under assumption 2, we can verify that

\[2c - \alpha^2/4(\beta - \beta') > c.\]

Generally, it is not possible to determine \( a^* \) explicitly. However, by setting \( v_b - c_b - 3t/2 = 0 \), we obtain the minimum interchange fee that we denote by \( a^*_\text{min} \).

We have

\[
a^*_\text{min} = \left( \frac{\alpha}{2^{2-\gamma}(\beta - \beta')^{1-\gamma}c^\gamma} \right)^{\frac{1}{1-2\gamma}}
\]

The associated network size is

\[
n^*(a^*_\text{min}) = \left( \frac{\alpha^2}{8(\beta - \beta')c^\gamma} \right)^{\frac{1}{1-2\gamma}}
\]

**Appendix 3.** The Hessian matrix of the profit function under regime 1

The Hessian matrix of second derivatives of the profit function must be negative definite.

The matrix is

\[
H = \begin{pmatrix}
\partial^2 \pi_i / \partial n_i^2 & \partial^2 \pi_i / \partial n_i \partial p_i \\
\partial^2 \pi_i / \partial n_i \partial p_i & \partial^2 \pi_i / \partial p_i^2
\end{pmatrix} = \begin{pmatrix}
\alpha a(\gamma - 1) & \alpha \frac{a}{2t} \gamma n^{\gamma - 1} \\
\alpha \frac{a}{2t} \gamma n^{\gamma - 1} & -1/t
\end{pmatrix}
\]

We obtain \( \text{Det}(H_{11}) = \alpha a(\gamma - 1) < 0 \). Furthermore \( \text{Det}(H) = \alpha a(1 - \gamma)n^{\gamma - 2}/t - \alpha^2 a^2 \gamma^2 n^{2\gamma - 2}/4t^2 \). Using the fact that \( n^*(a) = (\alpha a/2c)^{\frac{1}{1-\gamma}} \) we get \( \text{Det}(H) = 2c(1 - \gamma)/tn - \)
Clearly $\text{Det}(H) > 0$ if $\gamma = 0$. Suppose $\gamma > 0$, $\text{Det}(H) > 0$ is equivalent to $n^*(a) < 2(1 - \gamma)t/c\gamma^2$ or equivalently,
\[
a < \frac{2c}{\alpha} \frac{(2(1 - \gamma)t)^{1-\gamma}}{c\gamma^2} \equiv a_{\text{max}}
\]
The latter condition is verified if $a^* < a_{\text{max}}$, where $a^*$ is the solution of (35). This last inequality can be rewritten
\[
v_b - c_b < \frac{3t}{2} + \frac{2(1 - \gamma)t}{\gamma^2} - \frac{\alpha^2}{4(\beta - \beta')} \frac{(2(1 - \gamma)t)^{2\gamma}}{c\gamma^2}
\]
(40)
One can verify that the RHS of expression (40) is decreasing with $\gamma$. When $\gamma \to 0$, the RHS of expression (40) is infinite. When $\gamma = 1/2$, the RHS is close to $11t/2$ because assumption 2 guarantees that $\frac{\alpha^2}{4(\beta - \beta')}$ is small. Conditions (36) and (40) must hold together with assumption 1.

**Appendix 4.** Proof that $f_i = a$ under regimes 2 and 3

Under regimes 2 and 3, bank $i$’s demand for deposits is
\[
D_i = \frac{1}{2} + \frac{1}{2t}(v_i - v_j - p_i + p_j)
\]
Hence
\[
\frac{\partial D_i}{\partial p_i} = -\frac{1}{2t}
\]
Let us calculate the effect of $f_i$ on bank $i$’s deposit market share, $D_i$. We have
\[
\frac{\partial D_i}{\partial f_i} = \frac{1}{2t} \frac{\partial v_i}{\partial f_i}
\]
Using (33), we obtain
\[
\frac{\partial v_i}{\partial f_i} = \frac{\partial u_i}{\partial q_i} \frac{\partial q_i^f}{\partial f_i} - q_i^f - (f_i + s_j) \frac{\partial q_i^f}{\partial f_i}
\]
As $\partial v_i/\partial q_i^f = 0$, we have $\partial u_i/\partial q_i^f = f_i + s_j$ so that $\partial v_i/\partial f_i = -q_i^f$. Finally
\[
\frac{\partial D_i}{\partial f_i} = -\frac{1}{2t} q_i^f
\]
(41)
Condition (8)\((ii)\) can be rewritten:

\[
p_i - c_b - (a - f_i)q_i^f - (a + s_i)q_j^f = -D_i/\partial D_i/\partial p_i = 2tD_i
\]

Plugging this result in condition (8)\((iii)\), we obtain

\[
2tD_i \partial D_i/\partial f_i + (f_i - a) \partial q_i^f /\partial f_i D_i + q_i^f D_i = 0
\]

Using (41), we have

\[
(f_i - a) \partial q_i^f /\partial f_i D_i = 0
\]

hence \(f_i = a\) for any \(D_i\).

**Appendix 5.** Determination of the equilibrium network size under regime 2.

Using expressions (6) and (33), we have

\[
\frac{\partial D_i}{\partial n_i} = \frac{1}{2t} \frac{\partial (v_i - v_j)}{\partial n_i} = \frac{\alpha}{2t} (f_j \left(n_i^{\gamma - 1}\right) + (\gamma - 1)n_i n_i^{\gamma - 2}) - f_i (\gamma - 1)n_j n_j^{\gamma - 2}
\]

(42)

At the symmetric equilibrium we have \(f_i = f_j = a\) and \(n_i = n_j\). Hence,

\[
\frac{\partial D_i}{\partial n_i} = \frac{\alpha}{2t} a n_i^{\gamma - 1}
\]

(43)

Using (16) and (43), we can rewrite expression (18) as

\[
\frac{\alpha}{2} a n_i^{\gamma - 1} + \frac{\alpha (1 + \gamma)}{4} a n_i^{\gamma - 1} - c = 0
\]

which yields expression (19).

**Appendix 6.** The Hessian matrix of the profit function under regime 2

Let us calculate the Hessian matrix at \((n^*_i(a), p^*_f(a), f^* = a)\). We have

\[
H = \begin{pmatrix}
\partial^2 \pi_i /\partial n_i^2 & \partial^2 \pi_i /\partial n_i \partial p_i & \partial^2 \pi_i /\partial n_i \partial f_i \\
\partial^2 \pi_i /\partial n_i \partial p_i & \partial^2 \pi_i /\partial p_i^2 & \partial^2 \pi_i /\partial p_i \partial f_i \\
\partial^2 \pi_i /\partial n_i \partial f_i & \partial^2 \pi_i /\partial f_i \partial p_i & \partial^2 \pi_i /\partial f_i^2
\end{pmatrix}
\]
or

\[
H = \begin{pmatrix}
-A - B/t & c/t & cq_i^*(a)/t \\
c/t & -1/t & -q_i^*(a)/t \\
\lambda_i(a)/t & -q_i^*(a)/t & -\beta/2 - q_i^*(a)^2/t
\end{pmatrix}
\]

with \( A = \frac{c}{\pi_j(a)} \frac{(1-\gamma)(\gamma+6)}{(3+\gamma)} > 0 \) and \( B = 8c^2 \frac{1+\gamma}{(3+\gamma)} > 0 \). \( A \) and \( B \) do not depend on \( t \).

\( \text{Det}(H_{11}) = -A - B/t \) is negative.

\( \text{Det}(H_{22}) = (tA + B - c^2)/t^2 \) is positive for \( t \) sufficiently large.

\( \text{Det}(H) = -0.5\beta(tA + B - c^2)/t^2 \) is negative for \( t \) sufficiently large.

**Appendix 7.** Comparison of \( \pi^*(a) \) and \( \pi^*_f(a) \). Comparison of \( w^*_f(a) \) and \( w^*(a) \)

We compare \( \pi^*(a) \) and \( \pi^*_f(a) \). We express everything in \( n^*(a) \). We have \( n^*_f(a) = \lambda n^*(a) \) with

\[
\lambda = \left( \frac{3 + \gamma}{2} \right)^{\frac{1}{1-\gamma}} > 1
\]

and \( a = 2cn^{*1-\gamma}(a)/\alpha \). Hence,

\[
\pi^*(a) = \alpha \frac{a}{2} n^{*\gamma}(a) - \frac{c}{2} n^*(a) = \frac{c}{2} n^*(a)
\]

Furthermore,

\[
\pi^*_f(a) = \alpha \frac{a}{2} n^{*\gamma}_f(a) - \beta a^2 - \frac{c}{2} n^*_f(a) = \alpha \frac{a}{2} \lambda n^{*\gamma}(a) - \beta a^2 - \frac{c}{2} \lambda n^*(a)
\]

\[
= (2\lambda\gamma - \lambda) \frac{a}{2} n^*(a) - \beta a^2 = \left( \frac{3 + \gamma}{2} \right)^{\frac{1}{1-\gamma}} \left( \frac{1 - \gamma}{2} \right) \frac{c}{2} n^*(a) - \beta a^2
\]

As \( \left( \frac{3 + \gamma}{2} \right)^{\frac{1}{1-\gamma}} \left( \frac{1 - \gamma}{2} \right) < 2 \cdot \frac{1}{2} = 1 \), we have \( \pi^*(a) > \pi^*_f(a) \).

We calculate the sign of \( w^*_f(a) - w^*(a) \):

\[
w^*_f(a) - w^*(a) = \left( \frac{\alpha^2 n^{*2\gamma}_f(a)}{4(\beta - \beta')} + \frac{3\beta}{2} a^2 - \alpha n^{*\gamma}_f(a) \right) - \left( \frac{\alpha^2 n^{*2\gamma}(a)}{4(\beta - \beta')} - \alpha n^{*\gamma}(a) \right) \quad (44)
\]

Note first that \( w^*_f(a) - w^*(a) > 0 \) for \( \gamma = 0 \). Let us take \( \gamma > 0 \). We express everything in \( n^*(a) \). Using (12) one can write \( a = 2cn^{*1-\gamma}(a)/\alpha \). Furthermore \( n^*_f(a) = \lambda n^*(a) \) with
\[ \lambda = \left( \frac{3 + \gamma}{2} \right)^{\frac{1}{1 - \gamma}} > 1. \] Hence dropping \((a)\), (44) becomes

\[
w^*_f(a) - w^*(a) = \frac{\alpha^2 n^{2\gamma}}{4(\beta - \beta')} (\lambda^{2\gamma} - 1) + \frac{6\beta c^2 n^{2 - 2\gamma}}{\alpha^2} - 2(\lambda^\gamma - 1)cn^*
\]

\[
= n^* \left( \frac{\alpha^2 n^{2\gamma - 1}}{4(\beta - \beta')} (\lambda^{2\gamma} - 1) + \frac{6\beta c^2 n^{1 - 2\gamma}}{\alpha^2} - 2(\lambda^\gamma - 1) \right)
\]

However

\[
\argmin_n \left( \frac{\alpha^2 n^{2\gamma - 1}}{4(\beta - \beta')} (\lambda^{2\gamma} - 1) + \frac{6\beta c^2 n^{1 - 2\gamma}}{\alpha^2} - 2(\lambda^\gamma - 1) \right) = \left( \sqrt{\frac{\beta}{\beta' - \beta'}} (\lambda^{2\gamma} - 1) - \sqrt{4(\lambda^\gamma - 1)^2} \right) c
\]

which is positive because

\[
6\frac{\beta}{\beta' - \beta'} (\lambda^{2\gamma} - 1) > 6(\lambda^{2\gamma} - 1) > 4(\lambda^\gamma - 1)^2
\]

for any \(\lambda > 1\). Hence \(w^*_f - w^* > 0\) for any \(n > 0\), that is for any \(a > 0\).

Under regime 2, the surplus of the indifferent consumer is

\[
w^*_f(a^*_f) = v_b - c_b - \frac{3t}{2} + \left( \frac{\alpha^2}{4(\beta - \beta')} - \frac{29c^2}{512\beta} \right) (n^*_f)^{2\gamma} > 0
\]

**Appendix 8.** Effect of \(n_i\) on bank \(i\)'s deposit market share, under regime 3.

Using expressions (6) and (33), we have

\[
\frac{\partial D_i}{\partial n_i} = \frac{1}{2t} \frac{\partial (v_i - v_j)}{\partial n_i} = \frac{\alpha}{2t} ((s_i + f_j) \left( n^{\gamma - 1} + (\gamma - 1)n_i n^{\gamma - 2} \right) - (s_j + f_i)(\gamma - 1)n_j n^{\gamma - 2} \right) (45)
\]

For \(f_i = f_j = a, n_i = n_j and s_i = s_j = s\) we obtain

\[
\frac{\partial D_i}{\partial n_i} = \frac{\alpha}{2t} (a + s)n^{\gamma - 1}
\]

**Appendix 9.** Effect of \(s_i\) on bank \(i\)'s deposit market share, under regime three

Under regime three, we have

\[
\frac{\partial D_i}{\partial s_i} = \frac{1}{2t} q_i^f
\]

Indeed using (6) and (2) one can write

\[
\frac{\partial D_i}{\partial s_i} = -1 \frac{\partial v_i}{2t \partial s_i} = -1 \left( \frac{\partial u_i}{\partial q_i^f} + \frac{\partial q_i^f}{\partial s_i} - q_i^f - (f_j + s_i) \frac{\partial q_i^f}{\partial s_i} \right)
\]

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However \( \partial u_j/\partial q_j^i = f_j + s_i \) so that \( \partial v_j/\partial s_i = -q_j^i \).

**Appendix 10.** Hessian matrix under regime 4

We show that the Hessian matrix at \( (n_{\text{sur}}^*, P_{\text{sur}}^*, f^*, s^*) \) is negative definite when \( t \) is large enough. We have

\[
H = \begin{pmatrix}
\frac{\partial^2 \pi_i}{\partial n_i^2} & \frac{\partial^2 \pi_i}{\partial n_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial n_i \partial f_i} & \frac{\partial^2 \pi_i}{\partial n_i \partial s_i} \\
\frac{\partial^2 \pi_i}{\partial n_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial f_i} & \frac{\partial^2 \pi_i}{\partial p_i \partial s_i} \\
\frac{\partial^2 \pi_i}{\partial n_i \partial f_i} & \frac{\partial^2 \pi_i}{\partial f_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial f_i \partial f_i} & \frac{\partial^2 \pi_i}{\partial f_i \partial s_i} \\
\frac{\partial^2 \pi_i}{\partial n_i \partial s_i} & \frac{\partial^2 \pi_i}{\partial p_i \partial s_i} & \frac{\partial^2 \pi_i}{\partial f_i \partial s_i} & \frac{\partial^2 \pi_i}{\partial s_i^2}
\end{pmatrix}
\]

or

\[
H = \begin{pmatrix}
-A - B/t & c/t & cD/t & E/t + F \\
c/t & -1/t & -D/t & 0 \\
cD/t & -D/t & -\beta/2 - D^2/t & 0 \\
E/t + F & 0 & 0 & -3\beta/2 + D^2/t
\end{pmatrix}
\]

with \( A = \frac{c}{n_{\text{sur}}^* (1-\gamma) (\gamma + 0)} > 0 \), \( B = 8c^2 \frac{1+\gamma}{(3+\gamma)^2} > 0 \), \( D = \alpha (n_{\text{sur}}^*)^\gamma / 6 > 0 \), \( E = \alpha^2 \frac{1-\gamma}{72\beta} (n_{\text{sur}}^*)^{3\gamma - 1} > 0 \) and \( F = \alpha^{\frac{7+1}{2}} (n_{\text{sur}}^*)^{\gamma - 1} > 0 \)

\[\text{Det}(H_{11}) = -A - B/t < 0\]

\[\text{Det}(H_{22}) = (tA + B - c^2)/t^2 \] is positive for \( t \) sufficiently large.

\[\text{Det}(H_{33}) = -0.5\beta(tA + B - c^2)/t^2 \] is negative for \( t \) sufficiently large.

\[\text{Det}(H_{44}) = 0.25\beta [t^2(3A\beta - 2F^2) - 2AtD^2 + 3B\beta t - 2BD^2 - 3c^2\beta t + 2c^2D^2 - 2E^2 - 4EFt] / t^3\]

is positive when \( t \) is large enough. Indeed \( \gamma < \frac{\sqrt{25 - 9}}{6} \approx 0.393 \) guarantees that \( 3A\beta - 2F^2 > 0 \).

**Appendix 11:** Comparison of \( \pi^*(a^*) \) and \( \pi_{ff}^*(a_{ff}^*) \). Comparison of \( \tilde{w}_{ff}^*(a_{ff}^*) \) and \( \tilde{w}_{sur}^* \).

Let us consider particular values of the parameters: \( u_0^0, c_0^0 \) and \( t_0 \) that satisfy \( u_0^0 - c_0^0 - 3t_0^0/2 = 0 \). They yield the minimum interchange fee \( a_{\text{min}}^* \) that we obtained in appendix 2
As \( n \) under regime 2, according to expression (26), we have

\[
\pi^*(a^*_{\text{min}}) = \frac{t^0}{2} + \left( \frac{\alpha a^*_{\text{min}}}{2^{2-\gamma}c^7} \right)^{\frac{1}{1-\gamma}} = \frac{t^0}{2} + 4^{\frac{1-\gamma}{2-\gamma}} \alpha^{\frac{1}{2-\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{2-\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{2-\gamma}}
\]

Let us consider parameters \( v_{b}, c_{b} \) and \( t \) such that \( v_{b} - c_{b} - 3t/2 \geq 0 \). The corresponding interchange fee \( a^* \) is higher than \( a^*_{\text{min}} \). The associated profit is

\[
\pi^*(a^*) = \frac{t}{2} + \left( \frac{\alpha a^*}{2^{2-\gamma}c^7} \right)^{\frac{1}{1-\gamma}}
\]

Under regime 2, according to expression (26), we have

\[
\pi^*_f(a^*_f) = \frac{t}{2} + 4^{\frac{1-\gamma}{2-\gamma}} (1 - 2\gamma) (3 + \gamma) \frac{\alpha^2}{2^{2-\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{2-\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{2-\gamma}}
\]

One can verify that

\[
4^{\frac{1-\gamma}{2-\gamma}} \alpha^{\frac{1}{2-\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{2-\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{2-\gamma}} > 4^{\frac{1-\gamma}{2-\gamma}} (1 - 2\gamma) (3 + \gamma) \frac{\alpha^2}{2^{2-\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{2-\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{2-\gamma}} = \pi^*_f(a^*_f)
\]

Hence,

\[
\pi^*(a^*) = \frac{t}{2} + \left( \frac{\alpha a^*}{2^{2-\gamma}c^7} \right)^{\frac{1}{1-\gamma}} > \frac{t}{2} + \left( \frac{\alpha a^*_{\text{min}}}{2^{2-\gamma}c^7} \right)^{\frac{1}{1-\gamma}} = \frac{t}{2} + 4^{\frac{1-\gamma}{2-\gamma}} \alpha^{\frac{1}{2-\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{2-\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{2-\gamma}}
\]

\[
> \frac{t}{2} + 4^{\frac{1-\gamma}{2-\gamma}} (1 - 2\gamma) (3 + \gamma) \frac{\alpha^2}{2^{2-\gamma}} \left( \frac{1}{\beta} \right)^{\frac{1}{2-\gamma}} \left( \frac{1}{c} \right)^{\frac{2\gamma}{2-\gamma}} = \pi^*_f(a^*_f)
\]

QED

We compare the surplus of the indifferent consumer under regimes 2 et 3. We have

\[
\bar{w}^*_f(a^*_f) = v_{b} - c_{b} - \frac{3t}{2} + \left( \frac{\alpha^2}{4(\beta - \beta')} - \frac{29\alpha^2}{512\beta} \right) (n^*_f)^{2\gamma}
\]

and

\[
\bar{w}^*_s(v) = v_{b} - c_{b} - \frac{3t}{2} + \left( \frac{\alpha^2}{4(\beta - \beta')} - \frac{\alpha^2}{6\beta} \right) (n^*_s)^{2\gamma}
\]

As \( n^*_s = (\frac{16}{3})^{\frac{1}{2-\gamma}} n^*_f(a^*_f) \), writing \( w^*_f(a^*_f) > w^*_s(v) \) yields \( \gamma < \frac{3}{4} (\beta' / \beta) \).
References


Figure 1: determination of $a^*$

Figure 2: comparison of consumers’ welfare under regime two and regime three