Convergence across castes

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Abstract

India has witnessed a remarkable catch-up by the historically disadvantaged scheduled castes and tribes (SC/STs) towards non-SC/ST levels in their education attainment levels, occupation choices as well as wages during the period 1983-2012. Using a heterogenous agent, multi-sector model we show that sectoral productivity growth during this period can explain 75 percent of the observed wage convergence between the castes. Inter-sectoral net flows of workers are key as they account for 3/4 of the predicted convergence. Absent these net flows, the caste wage gaps would have marginally widened. Selection effects, while present in these net flows, account for just a quarter of the predicted wage convergence. We also find that affirmative action policies that reduced skilling costs for SC/STs may have reduced the levels of the caste wage gaps at all times but played a limited role in accounting for the dynamics of the wage gap. Growth was key for the dynamic wage convergence.

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1 Introduction

A perennial challenge of managing the development process is to balance the macroeconomic goals of growth and development with the microeconomic goals of equity and distributional fairness. These

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challenges often come to the fore during periods of rapid economic changes in growing economies. An example of this phenomenon is India over the past 30 years. This period has witnessed a rapid takeoff of the Indian economy with average annual growth rates doubling relative to the pre-reform phase. What have been the distributional consequences of the growth take-off? Did growth lift all groups or were there tradeoffs? What are the mechanisms that linked the two?

We approach the issue by focussing on the experience in India of Scheduled Castes and Scheduled Tribes (SC/STs) – an historically underprivileged section of Indian society.[1] These groups experienced a rapid catch-up towards non-SC/ST levels with the mean wage gap shrinking by 15 percentage points and the median wage gap declining by an even more striking 20 percentage points between 1983 and 2012. This wage convergence was accompanied by convergence in education attainment levels, occupation choices, and consumption levels (see Hnatkovska et al. (2012)).[2]

The goal of the paper is to assess the roles of growth and affirmative action programs in driving the declining economic gaps between non-SC/STs and SC/STs. Constitutionally mandated affirmative action programs that carved out reserved seats in higher education and legislative houses for SC/STs have been a key feature of public policy in India since the 1950s. How important have these protections been in accounting for the changes in the observed caste disparities since 1983 relative to the role of the faster productivity and economic growth in India after 1991? Did faster growth succeed in breaking down historical barriers to entry of SC/STs into higher-skill occupations?

To answer these questions, we first examine data on the evolution of caste gaps between SC/STs and non-SC/STs in education attainment rates broken down by age and birth cohorts. A key finding from this exercise is that caste education gaps within birth cohorts remained relatively unchanged till 1993-94 after which there was a decrease. This pattern characterizes most of the economically active cohorts, including the older ones. We interpret this as evidence that is suggestive of people perceiving educational attainment as a key determinant of changing employment.

As a next step, we conduct a statistical decomposition of the change in the overall wage gap between non-SC/STs and SC/STs during 1983-2012. We do so by noting that in a multi-sector economy, caste wage gaps could change either due to changes in the employment distribution of castes across sectors or due to changes in sectoral caste wage gaps or some combination of the two.

[1] SC/STs comprise a list of castes that have been listed in a schedule of the Indian constitution as being historically disadvantaged and consequently eligible for affirmative action programs in higher education, public sector jobs and political representation. Crucially, caste identities are inherited by birth and hence immutable over time. Moreover, the extent of reservations for SC/STs have also remained constant over time.

[2] There has also been sharp convergence in the intergenerational mobility rates in these three indicators (see Hnatkovska et al. (2013)).

Our decomposition exercise reveals that the relatively faster entry of SC/STs into the service sector during this period was the most important driver of the caste wage convergence.

Based on this evidence, we develop a three-sector model (agriculture, manufacturing and services) of an economy with two types of agents (non-SC/STs and SC/STs) and non-homothetic preferences. Employment in the non-agricultural sectors requires additional skills beyond innate ability. Agents differ in their abilities and their costs of acquiring skills to work in the non-agricultural sectors. Based on their abilities, the skill-acquisition costs and the sectoral returns to labor, agents sort into employment in the three sectors. This endogenously generates caste gaps in employment and wages in each sector as well as an overall caste wage gap.

After calibrating the model to match the sectoral caste gaps in employment and wages in 1983, we conduct a sequence of quantitative experiments to examine the importance of productivity growth and skill acquisition costs in inducing the observed changes in the caste gaps during 1983-2012. Our experiments yield three main results. First, sectoral productivity growth during 1983-2012 accounts for 75 percent of the percentage decline in the caste wage gap during this period.

Second, the observed sectoral labor productivity shocks induce dynamics of sectoral labor shares and sectoral prices in the model that reproduce their patterns in the data. Specifically, the model induces contractions in the labor shares of agriculture and manufacturing along with an expansion of the labor share in services. Crucially, the model predicts a decline in the prices of manufacturing and services relative to agriculture, a feature that also matches the data. We view these aggregate features of the model, including its structural transformations predictions, as indicative of the model being a good fit to the data.[3]

Using counterfactual experiments we show that the key contributor to the caste convergence is the endogenous re-sorting of workers into sectors induced by the differential sectoral productivity growths. Absent this re-sorting, the overall caste wage gap would likely have marginally increased as productivity growth tended to be higher in sectors that were historically over-represented by non-SC/STs. Consistent with the statistical decomposition exercise, we find that the wage convergence in the model is accounted for mostly by the decrease in the caste employment gap in services.

There are two aspects of the worker re-sorting in the model that are potentially important. The first is just the net movement of workers across sectors. The second is the selection by ability of the workers that choose to move. We find that 3/4 of the wage convergence in the baseline model is due

[3]Interestingly, while the model allows for non-homothetic preferences, our quantitative exercises reveal that it is differential sectoral productivity growth that accounts for the sectoral dynamics of prices and the structural transformation in the model. Non-homothetic preferences, by themselves, imply counterfactual sectoral price movements.

to net flows of worker across sectors. Selection effects quantitatively account for just a quarter of the wage convergence in the model. We interpret this result as being consistent with the statistical decomposition exercise which attributed most of the convergence to just inter-sectoral labor flows.

Third, we assess the role of affirmative action programs by varying the costs of acquiring skills for the two groups. In the baseline calibration, the skilling costs of SC/STs are lower than that for non-SC/STs. We view this to be the consequence of reservations for SC/STs. When we equalize the cost of acquiring skills across castes while leaving all other parameters in the baseline model unchanged, the caste wage gap rises by 21 percent in 1983 and by 11 percent in 2012 relative to the gap under the baseline calibration. This suggests to us that affirmative action was likely important in lowering the ***levels*** of the caste wage gaps.

The counterfactual experiments on skilling costs also suggest that affirmative action likely did not materially affect the ***dynamics*** of caste wage gaps between 1983 and 2012. The predicted decrease in the wage gap during 1983-2012 in the baseline case where reservations exist throughout, is 8.1 percent. When reservations are removed in the model by equalizing skilling costs across castes at all times, the same productivity growth causes the caste wage gap to decline by an even greater 15 percent during 1983-2012. The higher rate of decrease without reservations is due to low "base" effects: without reservations, SC/STs localize in the low wage agriculture sector which reduces their 1983 wages. This base effect raises the implied growth rate of SC/ST wages in 2012 when higher non-agricultural sectoral productivities induce some higher ability SC/STs to switch to the higher wage manufacturing sector. Most importantly however, growth appears to drive a reduction in caste wage gaps, with or without reservations.

Overall, our results suggest that the rapid growth take-off in India over the past three decades has been the key factor driving the dramatic narrowing of the historical economic disparities faced by SC/STs. Growth appears to have impacted the wage gaps by inducing a re-sorting of workers across sectors. Pre-existing reservations for SC/STs in higher education that were enshrined into the Indian constitution, probably played a role in reducing the level of the caste wage gaps. However, these reservations appeared to have had limited effects on the dynamics of the caste wage gaps between 1983 and 2012.[4] Rather, the results suggest that growth succeeded in breaking down historical barriers to entry into some occupations for SC/STs.

The paper is related to three distinct bodies of work. The first is the work on castes in India

[4]We should note that the focus of the model is on the reservation policies for higher education. There also exist reservations for SC/STs in public sector employment. However, given the limited number and scope of public sector jobs, we have chosen to focus on the education side of affirmative action programs in this paper.

and their impact on economic outcomes. Aside from the contributions of Hnatkovska et al. (2012). and Hnatkovska et al. (2013) cited above, notable other contributors to this literature are Banerjee and Knight (1985), Madheswaran and Attewell (2007) and Borooah (2005) who examined the discrimination against SC/STs in labor markets in urban India. On a related theme, Ito (2009) studied labor market discrimination in two Indian states – Bihar and Uttar Pradesh. Exploring the theme of castes as networks, Munshi and Rosenzweig (2006) and Munshi and Rosenzweig (2016) show how castes induce network effects that impact labor mobility, education choices and employment.[5]

A second literature that is related to our work is the extensive work on structural transformation of countries along the development path wherein countries gradually switch their economic focus from agriculture to non-agricultural sectors. This is a voluminous literature that spans both empirical and theoretical work. Key contributions in this are are Matsuyama (1992), Kongsamut et al. (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). An excellent overview can be found in Herrendorf et al. (2014) who describe both the empirical regularities as well as the theoretical explanations for this phenomenon. Our paper shares the focus on structural transformation with these papers but differs in its focus on the distributional effects of this transformation.

Our work also relates to the literature that has examined the sources of productivity differences between countries. Two lines of research in this broad tent are closely connected with this paper. The first is the work on misallocation of talent by Hsieh et al. (2019) who analyze the consequences of misallocating talent by gender and race on productivity and growth in the USA. We share their interest in the implications of misallocating labor across sectors due to discrimination or other factors. A second branch of work in this area has focused on the role of occupation selection in accounting for income differences between rural and urban workers (see Young (2013)) and between agricultural and non-agricultural workers (see Lagakos and Waugh (2013)). This list is illustrative rather than being exhaustive.

The next section describes the key facts on caste economic gaps and structural transformation in India. Section 3 conducts a decomposition exercise to determine the statistical drivers of the caste wage convergence while Section 4 presents the model and some analytical results. Section 6 presents the calibration and quantitative results as well as the counterfactual experiments. The

[5] Another paper that is related to our work is Banerjee and Munshi (2004). They examined the differences between entrants belonging to the incumbent traditional community of *Gounders* in the garment industry in Tirupur in India in the early 1990s relative to entrants from other communities. They found evidence of sharp catch-up of capital and output of outsider firms to the levels of entrants from the Gounder community.

last section concludes.

2 Empirical regularities

Our data comes from different sources. The primary data source is the National Sample Survey (NSS) rounds 38 (1983), 43 (1987-88), 50 (1993-94), 55 (1999-2000), 61 (2003-04), 66 (2009-10) and 68 (2011-12).[6] The NSS provides household-level data on approximately 600,000 individuals on education, employment, consumption, and wages as well as other social characteristics. We consider individuals between the ages 16-65 belonging to male-headed households who were not enrolled full time in any educational degree or diploma. The sample is restricted to those individuals who provided their 4-digit industry of employment information as well as their education information.[7] Our focus is on full-time working individuals who are defined as those that worked at least 2.5 days per week. This selection leaves us with a working sample of around 165,000-182,000 individuals, depending on the survey round. The wage data is more limited. This is primarily due to the prevalence of self-employed individuals in rural India who do not report wage income. As a result, the sub-sample with wage data is limited to about 48,000 individuals on average across rounds. Details on the data are contained in the Data Appendix to this paper.

We start by reporting some aggregate facts regarding the education and wage gaps between SC/STs and non-SC/STs since 1983. These facts are extensions of the results reported in Hnatkovska et al. (2012). Figure 1 reports the wage gaps between the castes. Panel (a) shows the mean wage gaps between the groups across the NSS rounds while panel (b) shows the corresponding median gaps. The solid lines depict the unconditional wage gaps while the dashed lines show the wage gaps after controlling for the age characteristics of workers.[8] Both plots reveal an unambiguous pattern of wage convergence between the two groups since 1983, with the mean wage gap declining by 10.5% and the median gap falling by 14%.

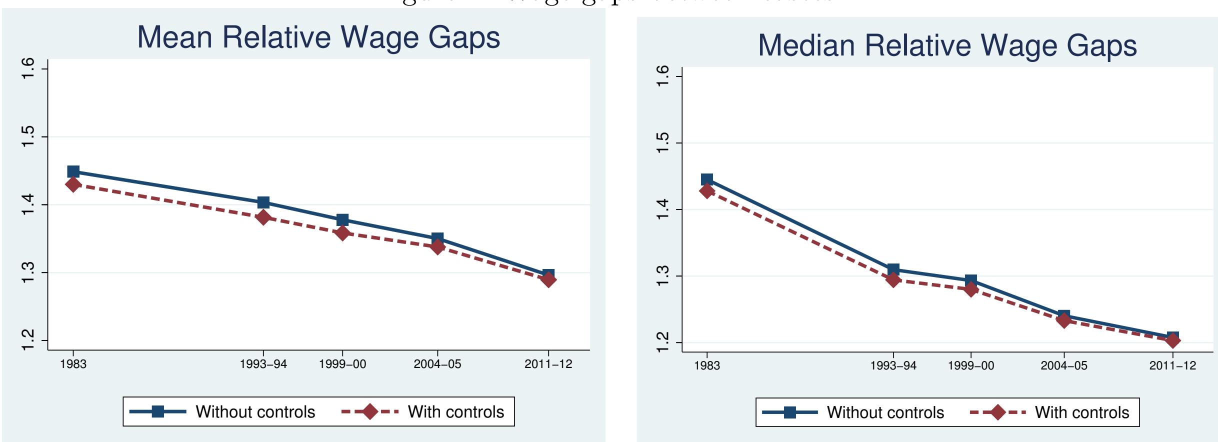
Next we examine the education patterns of the two groups during this period. Figure 2 shows the relative gaps in the years of education between non-SC/STs and SC/STs. Panel (a) of the Figure shows the gaps for different age cohorts while panel (b) shows the corresponding gaps in

[6] The 68th NSS round is the latest available and released by the Indian government.

[7] We also consider a narrower sample in which we restrict the sample to only males and find that our results remain robust.

[8] Specifically, to obtain unconditional wage gaps we estimated an OLS regression (for mean) and a RIF regression (for median) of log wages on a constant and an SC/ST dummy. The conditional gaps are computed from the same regression with age and age squared controls.

Figure 1: Wage gaps between castes

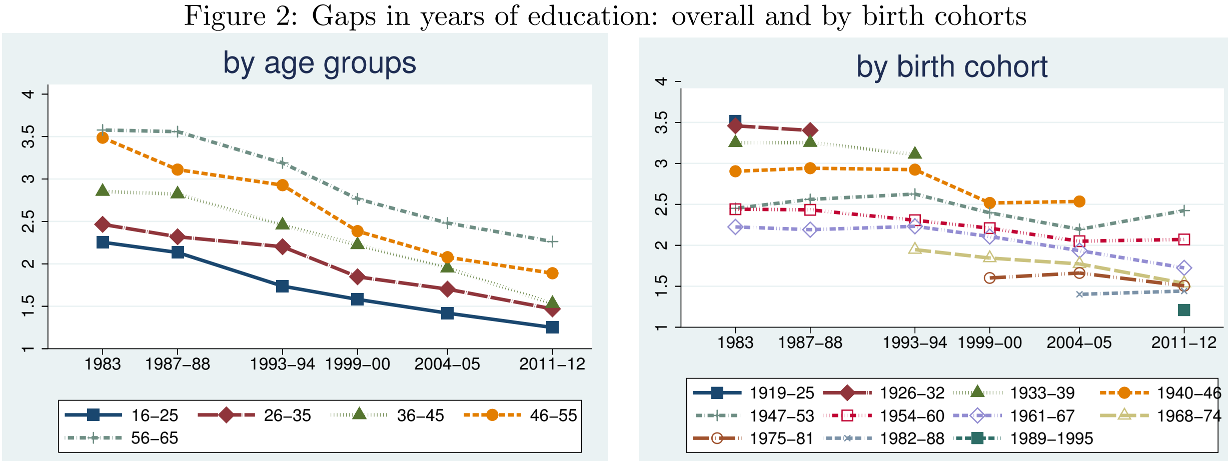


(a) (b)

Notes: Panel (a) of this Figure presents the mean wage gaps between SC/STs and non-SC/STs (expressed as non-SCST/SCST) from the 1983 to the 2011-12 NSS rounds. Panel (b) shows the corresponding median wage gaps (non-SCST/SCST). The dashed lines in the two panels show the computed wage gaps after controlling for the age characteristics of workers while the solid lines are the gaps without such controls.

the average years of schooling by birth cohorts. Both panels reveal a pattern of convergence in education attainment rates between the two groups. Remarkably, caste education gaps even within older birth cohorts appear to have shrunk after 1991-92. In fact, the education convergence trends are even sharper than the trends in wage convergence.

Figure 2: Gaps in years of education: overall and by birth cohorts



(a) (b)

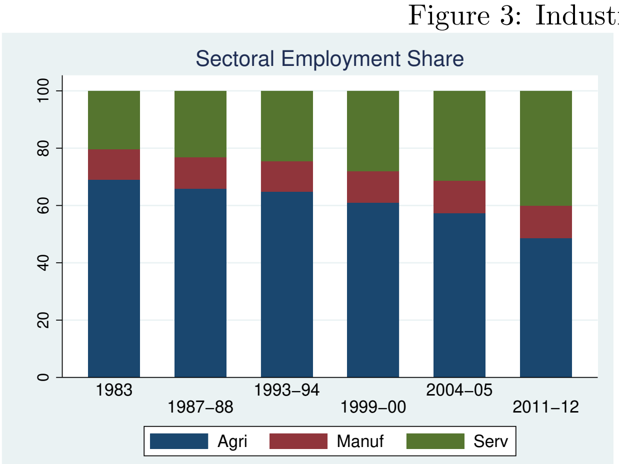
Notes: Panel (a) of this Figure shows the relative gap in average years of education (non-SCST/SCST) across the NSS rounds for different age cohorts while Panel (b) shows the gaps by birth cohorts.

Given the trends in Figures 1 and 2, the natural question to ask is how much of the wage convergence between the two groups is due to convergence in education attainment. Hnatkovska et al. (2012) examined precisely this question and found that most of the wage convergence is, in fact, due to education convergence.

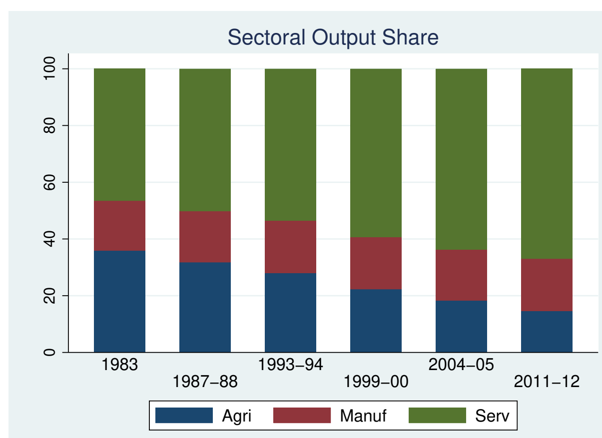
These trends, while interesting by themselves, raise the logical question about the deeper reasons behind the observed convergence between the groups during this period. While there may have been multiple factors operating simultaneously, in this paper we focus on the two biggest changes that occurred in the Indian economy during this period. As is well known, this period – 1983 to 2012 – was a period of major changes in economic policy which was accompanied by a sharp economic take-off in India. There were large scale trade and industrial reforms carried out in the mid-1980s and in the 1990s. Economic growth in India took off from an average of around 3 percent in the period between 1950 and 1985 to consistently being above 6 percent by the end of the 1990s. Second, this period was also marked by a very sharp structural transformation of the economy.

Before proceeding, it is useful to document some of the key data facts related to the structural transformation of the economy since the early 1980s. In order to present the structural transformation facts, we combine 4-digit industry categories in the data into three broad categories: Agriculture, Manufacturing, and Services. See Appendix 9.1 for more details on the industry grouping.

Figure 3: Industry distribution



(a)



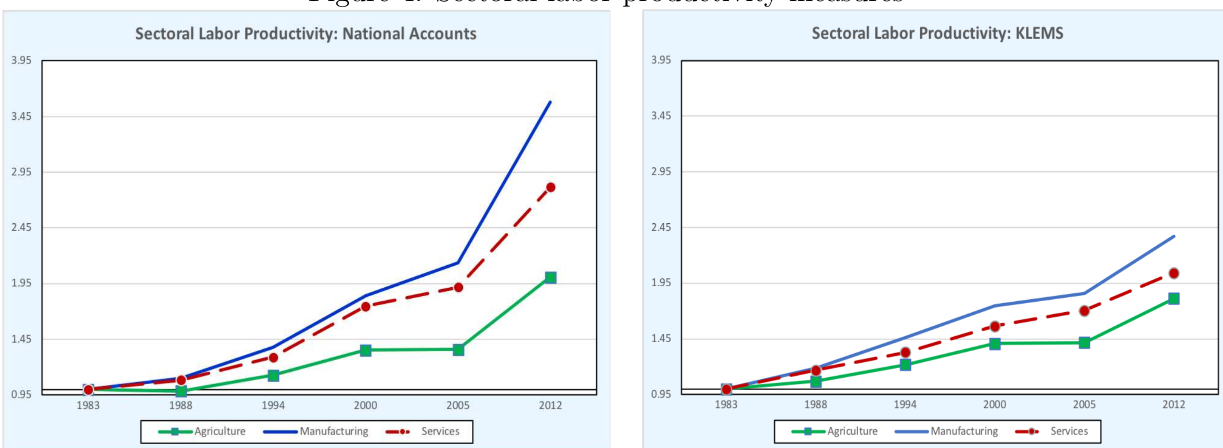
(b)

Notes: Panel (a) of this Figure presents the distribution of workforce across three industry categories for different NSS rounds. Panel (b) presents distribution of output (measured in constant 1980-81 prices) across three industry categories.

Figure 3 shows that the period 1983-2012 was marked by a gradual contraction in the traditional agricultural sector while the service sector expanded both in terms of its share of output as well as employment (there was an expansion in the manufacturing sector too but much more tepid relative to that of the service sector).

This process of structural transformation coincided with rapid growth in productivity at the aggregate and sectoral levels. Figure 4 reports labor productivity in each sector. Panel (a) is measured as output per worker computed from the national accounts data, while panel (b) reports the sectoral labor productivity numbers that are reported in the KLEMS dataset for India. All the series are normalized by their values in 1983, so they all start at unity.

Figure 4: Sectoral labor productivity measures



(a) (b)

Notes: Panel (a) of this Figure presents labor productivity, measured as GDP (in constant 1980-81 prices) divided by number of workers in each sector. Panel (b) shows the sectoral labor productivity computed from the KLEMS database for India. All series are normalized by their 1983 values.

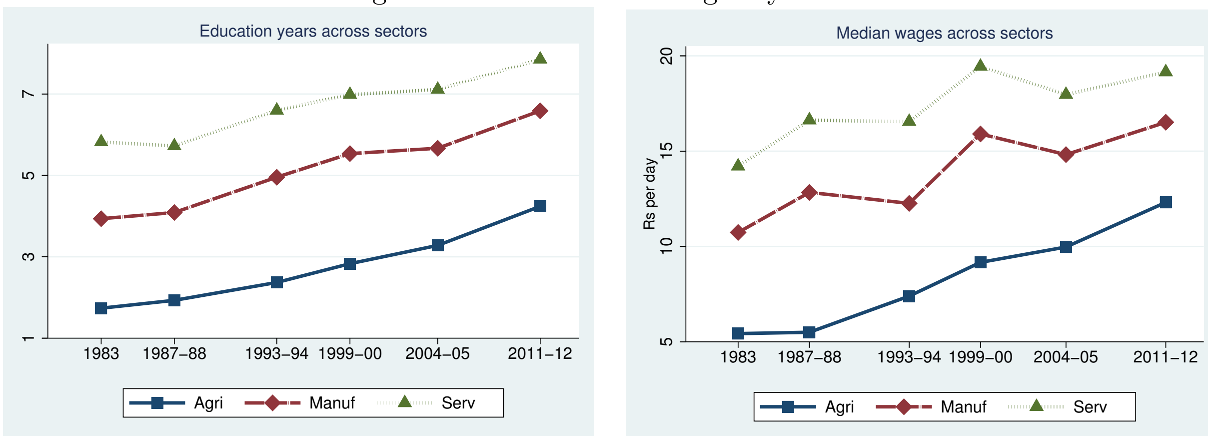
Figure 4 shows that productivity growth across the three sectors, especially in the non-agricultural sectors, is a feature of both the national income accounts and KLEMS data. Both datasets also reveal a common rank-ordering of sectoral labor productivity growth during 1983-2012: manufacturing grew the fastest followed by services with agriculture being the slowest growing sector.[9]

Figure 5 reports mean years of education and median wages in the three sectors for various survey rounds. The figures reveal a dramatic increase in both education attainments and median wages in India during 1983-2012 period. Importantly, the figures show that the rank order of educational attainment and wages across the sectors in descending order were services, manufacturing and agriculture. This will be important for the calibration of our model later.

So, how did this overall transformation of the economy affect the two groups? Figure 6 reports

[9]Quantitatively, the KLEMS data reports lower productivity growth for all three sectors relative to the numbers computed from the national income accounts. In the calibration section below, where we examine the effect of productivity growth on caste gaps, we shall examine the robustness of our quantitative results to these alternative measures of productivity growth.

Figure 5: Education and wages by sector



(a) (b)

Notes: Panel (a) of this Figure presents average years of education of workers employed in each of the three sectors. Panel (b) reports median wages in the three sectors.

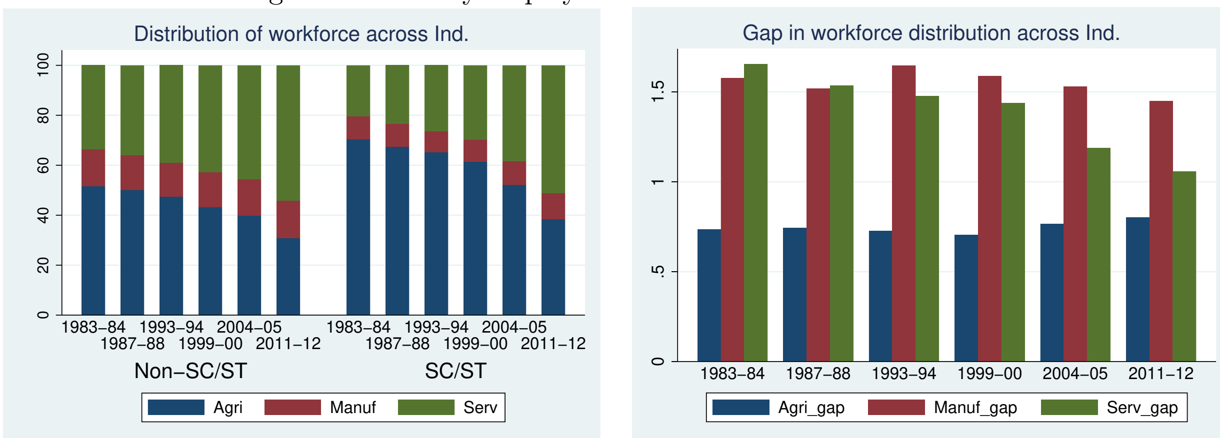
the industry distribution of working individuals among SC/STs and non-SC/STs, and the relative gaps in this distribution. Clearly, SC/STs were and remain more likely to be employed in agriculture and other farming activities than non-SC/STs. However the gap narrowed somewhat in the last ten years of our sample. The second largest industry of employment for both social groups is services, whose share has risen steadily over time. Interestingly, services also exhibits the sharpest convergence pattern between non-SC/STs and SC/STs. The relative gap between non-SC STs and SC/STs in employment shares in services has shrunk from 60 percent in 1983 to 21 percent in 2012. Manufacturing shows relatively little changes in the employment shares of the two groups over time.

Figures 7 reports the relative gaps in education attainments and median wages between non-SC/STs and SC/STs employed in each sector. The education gaps have narrowed significantly over time between the two caste groups. Median wage gaps on the other hand declined in Services, stayed unchanged in Manufacturing, but widened somewhat in Agriculture.

To summarize the data features documented above, the period 1983-2012 was characterized by high aggregate growth in the economy, rising output per worker in all three sectors and productivity growth across the sectors. Concurrently, there was a gradual transformation of the economy that was underway as well with services becoming a larger share of the economy both in terms of output and employment while the corresponding agriculture shares shrank.

In terms of the caste distributions, both SC/STs and non-SC/STs exited from agriculture and moved into service sector employment during this period. The education gap between the castes

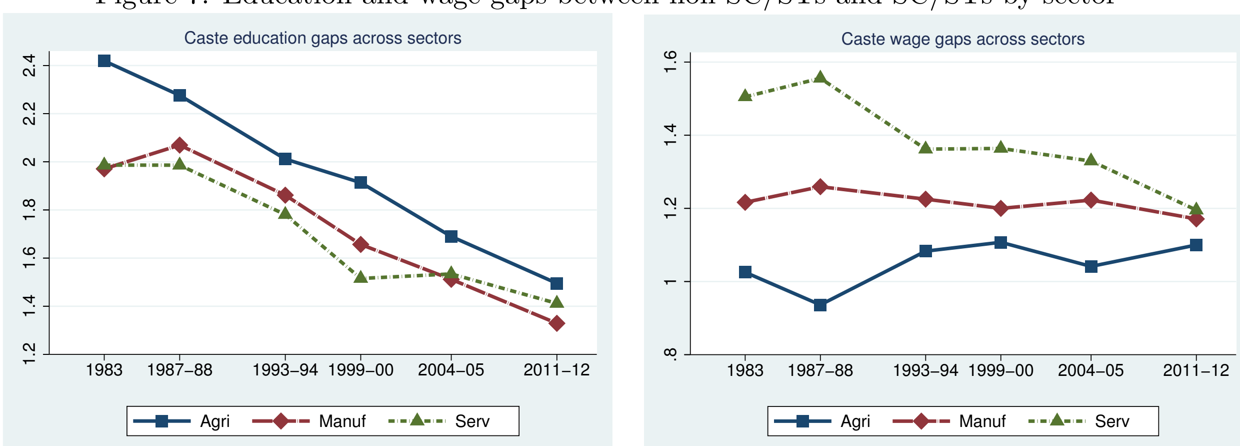
Figure 6: Industry employment distribution across castes



(a) (b)

Notes: Panel (a) of this Figure presents the distribution of workforce across the three industry categories for different NSS rounds. The left set of bars on each Figure refers to non-SC/STs, while the right set is for SC/STs. Panel (b) presents relative gaps in the distribution of non-SC/STs relative to SC/STs across three industry categories.

Figure 7: Education and wage gaps between non-SC/STs and SC/STs by sector



(a) (b)

Notes: Panel (a) of this Figure presents relative gap in years of education between non-SC/STs and SC/STs. Panel (b) presents the ratio of non-SC/STs median wages to SC/STs median wages.

declined in all three sectors. Moreover, while wages were converging *overall* between the castes, there were interesting contrasts in the patterns across the sectors. The wage convergence was strong in the service sector. The agricultural sector however saw a divergence in wages between the castes. Interestingly, the wage gaps in the manufacturing sector remained relatively stable over this period.

3 Decomposing the Wage Convergence

We saw above that the period 1983-2012 witnessed changes in the sectoral employment distribution of SC/STs which occurred at different rates than that for non-SC/STs. The period also saw changes in the sectoral wages of SC/STs relative to non-SC/STs.

How much of the wage convergence was due to convergence in the sectoral employment distributions? How much was due to differential movements in the sectoral wages of the castes? This would provide guidance for the margins to focus on in building a structural explanation. In this section we conduct precisely this kind of decomposition using a three-sector structure.

3.1 Notation

We use the following notation for our sectoral decomposition exercise:

Let s denote SC/STs and n denote non-SC/STs individuals. Also let a, m, h denote, respectively, agriculture, manufacturing and services.

- $W^{j,k}$: Average wage of caste $j = s, n$ in sector $k = a, m, h$
- W^{j}: Average wage of caste $j = s, n$
- W^{k}: Average wage of sector $k = a, m, h$
- $\ell^{j,k}$: Employment of caste j in sector k
- $S^{j,k} = \frac{\ell^{j,k}}{\sum_k \ell^{j,k}}$: Share of sector k in total employment of caste j
- $\Delta W = \frac{W^n}{W^s}$: Overall caste wage gap
- $\Delta W^k = \frac{W^{n,k}}{W^{s,k}}$: Caste wage gap in sector k
- $\Delta S^k = \frac{S^{n,k}}{S^{s,k}}$: Sectoral caste labor gap

3.2 Decomposition of the wage gap

The overall caste wage gap at any date t can be written as:

$$\Delta W_t \equiv \frac{W_t^n}{W_t^s} = \frac{\sum_{k=a,m,h} W_t^{n,k} S_t^{n,k}}{\sum_{k=a,m,h} W_t^{s,k} S_t^{s,k}} = \sum_{k=a,m,h} \Delta W_t^k \Delta S_t^k \underbrace{\frac{W_t^{s,k} S_t^{s,k}}{W_t^s}}_{\equiv \omega_t^{s,k}}$$

Using the above, we can write the change of the overall wage gap between dates $t-1$ to t as

$$\frac{\Delta W_t}{\Delta W_{t-1}} = \frac{\sum_{k=a,m,h} \Delta W_t^k \Delta S_t^k \omega_t^{s,k}}{\Delta W_{t-1}} = \sum_{k=a,m,h} \left(\frac{\Delta W_t^k \Delta S_t^k}{\Delta W_{t-1}^k \Delta S_{t-1}^k} \right) \left(\omega_t^{s,k} \frac{\Delta W_{t-1}^k \Delta S_{t-1}^k}{\Delta W_{t-1}} \right)$$

The expression above shows that the change in the overall wage gap between dates $t-1$ and t can be decomposed into changes in (a) the three sectoral wage gaps between castes; and (b) the three sectoral labor share gaps between the castes. These are the terms in the first bracket on the right hand side of the equation above. The terms in the second bracket on the right hand side of this equation are weights on these sectoral changes.

The decomposition above allows us to quantitatively assess the individual contribution of each of the six margins identified above to the overall wage convergence. Table 1 reports these results.

Table 1: Overall wage convergence: Contributions of sectoral gaps

Caste Gap	$\frac{Gap_{2012}}{Gap_{1983}}$	$\frac{\Delta W_{2012}}{\Delta W_{1983}}$ Predicted	$\frac{\Delta W_{2012}}{\Delta W_{1983}}$ Data
ΔS^A	0.98	0.98	0.895
ΔS^M	1.09	1.00	0.895
ΔS^H	0.76	0.88	0.895
ΔW^A	1.04	1.00	0.895
ΔW^M	0.95	0.98	0.895
ΔW^H	0.92	0.95	0.895

Notes: 1. Each row of this table shows the change in a specific sectoral caste gap between 1983 and 2012 as well as the predicted change in the overall caste *wage* gap if the other five sectoral caste gaps had remained unchanged during this period.

Column 1 of Table 1 lists the caste gaps in sectoral labor shares and sectoral mean wages. The second column reports the actual changes in these caste gaps between 1983 and 2012. The third column reports the predicted counterfactual change in the overall caste *wage* gap if the caste gap for that row was the only gap to have changed between 1983 and 2012. The last column compares this counterfactual change with the actual change in the overall caste mean wage gap equal to 10.5%.

As an example, the 0.98 entry in column 3 next to ΔS^A indicates that relative caste labor share gap in Agriculture declined by 2 percent between 1983 and 2012. The other two entries for the row labelled ΔS^A show that while the overall relative caste wage gap declined by 10.5 percent in the data between 1983 and 2012 (last column), the wage gap would have declined by just 2 percent had the caste labor gap in agriculture been the only gap to have changed during this period.

The main takeaway from the Table is that the role of convergence in the caste labor gap in services was key for the overall wage convergence. The measured decline in the services labor share gap alone reduces the overall wage gap by 12 percent between 1983 and 2012. In comparison, the other sectoral caste gaps had relatively minor effects on the overall wage gap. Consequently, any structural explanation for the observed wage convergence has to generate a big convergence in the caste labor gap in services.

4 Model

We now ask whether an aggregate productivity shock can have a differential impact on the two groups and cause the education and wage gaps between the castes to fall? If so, what are the conditions under which that can happen? Would such an environment also induce sectoral outcomes that are consistent with the facts that we just outlined above, in particular the large changes in the caste labor gap in services?

Consider a one-period lived closed economy that is inhabited by a continuum of agents of measure L. A measure S of these agents belong to caste s (for scheduled castes and tribes or SC/STs) while a measure $N = L - S$ belong to caste n for non-SC/ST. Each agent i maximizes utility from

$$u(c_i) = \frac{c_i^{1-\rho}}{1-\rho}$$

where

$$c_i = (c_i^a - \bar{c})^{\theta} (c_i^m)^{\eta} \left(c_i^h\right)^{1-\theta-\eta}$$

$\bar{c}$ is the minimum level of consumption of the a good (which we think of as the agricultural good). As before, we shall refer to the a good as the agricultural good, the m good as the manufacturing good and the h good as the high skill good.

Each agent i is born with one unit of labor time that is supplied inelastically to the market and an endowment of ability e_i. The ability distribution is caste specific. So, for agents belonging to caste s the ability e_i is drawn from an $i.i.d.$ process that follows the cumulative distribution function $G_s(e)$, $\quad e \in [\underline{e}_s, \bar{e}_s]$. Similarly, for agents belonging to caste n the ability type is drawn from an *i.i.d.* process summarized by the distribution function $G_n(e)$, $e \in [\underline{e}_n, \bar{e}_n]$. [10]

Ability is a productive input in both production and in skill acquisition. An agent can work in either of the three sectors. Sector a does not require any training or special skills, hence agents who choose to work in this sector can supply their labor endowment to this sector as is. Working in sector m requires some special, sector-specific skill. Agent i can acquire this skill by spending f_{ji}^m units of the sector m good where $j = s, n$ denotes the caste to which agent i belongs. This specification allows the skill acquisition costs to be caste specific. Similarly, to work in sector h the worker i needs to acquire a different skill level which can be acquired by expending f_{ji}^h units of the m good. Throughout the paper we shall use the sectoral identifiers a, m, h to refer to agriculture, manufacturing and services, respectively.

In the following we shall make the following assumptions regarding the costs of acquiring skills:

Assumption 1: $f_{ji}^k = f_j^k(e_i) = \bar{f}_j^k + f(e_i)$, $\quad \frac{\partial f}{\partial e_i} \leq 0$, $\quad j = s, n$, $\quad k = m, h$
Assumption 2: $\bar{f}_j^h > \bar{f}_j^m$, $\quad j = n, s$

Assumption 1 says that skilling costs have two components. The first, $\bar{f}_j^k$, is a fixed cost that is specific to sector and caste. The second component, $f(e_i)$, is decreasing in the ability level of the individual. Assumption 2 says that the fixed component of the skilling cost for sector-h is greater than that for sector-m for both castes. This assumption reflects the data pattern in Figure 5 where we showed that education attainment rates were the highest for workers employed in the services sector. Since the second component of the sectoral skilling cost, $f(e_i)$, is independent of sector, Assumption 2 also implies that $f_j^h(e_i) > f_j^m(e_i)$ for all e and $j = n, s$.

There are thus two sources of differences across castes: the distribution of abilities, $G_j(e), j = n, s$; and the sectoral skilling costs f_j^k, $j = n, s$, and $k = m, h$. We shall expand on the implications of these differences below.

The technologies for producing the three goods are all linear in the labor input. In particular, an unskilled worker with ability e_i supplying one unit of labor time to sector a produces

$$y_i^a = Ae_i$$

[10] The evolution of these supports of the distributions is clearly a dynamic issue and endogenous to time and investment decisions regarding education that are made by families. We intend to address these issues more fully in future work.

An m-sector worker with ability e_i produces the manufacturing good m according to

$$y_i^m = Me_i$$

Lastly, an $h-$sector worker with ability e_i produces the high skill good according to

$$y_i^h = He_i$$

Note that labor supply is inelastic and indivisible. So each worker supplies one unit of labor time to whichever sector he/she works in.

The budget constraints of worker i is given by

$$c_i^a + p_m c_i^m + p_h c_i^h = \hat{y}_i$$

where

$$\hat{y}_i = \max\left\{y_i^a, p_m\left(y_i^m - f^m\left(e_i\right)\right), p_h y_i^h - p_m f^h\left(e_i\right)\right\}$$

The subscript i refers to household i with ability e_i. Recall that each worker will be working in only one sector.

Household optimality conditions and optimal consumption of th three goods are derived in subsection 9.2 of Section 9.

4.1 Occupation and Skill Choice

The decisions about which occupation to choose and what skill level to acquire are joint in this model since skills are matched uniquely to sectors. Thus, an agent of caste j with ability e_i will choose to remain unskilled and work in sector a if and only if

$$Ae_i \geq p_m\left(Me_i - f_j^m\left(e_i\right)\right)$$
$$Ae_i \geq p_h He_i - p_m f_j^h\left(e_i\right)$$

where p_m is the relative price of good m and p_h is the relative price of good h. Throughout we shall use good a as the numeraire. In addition, an agent who chooses to get skilled will choose to work in sector h rather than sector m if and only if

$$p_h H e_i - p_m f_j^h(e_i) \geq p_m \left(M e_i - f_j^m(e_i)\right)$$

The three conditions can be rewritten as

$$\begin{aligned} \frac{f_j^m(e_i)}{e_i} &\geq M - \frac{A}{p_m} \\ \frac{f_j^h(e_i)}{e_i} &\geq \frac{p_h}{p_m} H - \frac{A}{p_m} \\ \frac{f_j^h(e_i)}{e_i} - \frac{f_j^m(e_i)}{e_i} &\geq \frac{p_h}{p_m} H - M \end{aligned}$$

The right hand sides of these conditions are the relative gains from working in sector m or h while the left hand sides are the relative costs. Crucially, the right hand side variables are aggregate variables that private agents take as given. The left hand sides, on the other hand, are individual specific and are clearly decreasing functions of e_i. These conditions define the cutoff thresholds:

$$z_j^m\left(\hat{e}_j^m\right) = M - \frac{A}{p_m}, \quad j = s, n \tag{4.1}$$

$$z_j^h\left(\hat{e}_j^h\right) = \frac{p_h}{p_m} H - \frac{A}{p_m}, \quad j = s, n \tag{4.2}$$

$$z_j^h\left(\tilde{e}_j^h\right) - z_j^m\left(\tilde{e}_j^h\right) = \frac{p_h}{p_m} H - M, \quad j = s, n \tag{4.3}$$

where we have used the definitions $z_j^m(e) \equiv \frac{f_j^m(e)}{e}$ and $z_j^h(e) \equiv \frac{f_j^h(e)}{e}$.

Lemma 4.1. *All individuals $i \in$ caste $j = n, s$ with ability e_i prefer employment in sector-m to employment in sector-a if $e_i \geq \hat{e}_j^m$; employment in sector-h to sector-a if $e_i \geq \hat{e}_j^h$; and employment in sector-h to sector-m if $e_i \geq \tilde{e}_j^h$.*

Proof. The statements follow directly from Assumptions 1 and 2 which imply that $z_j^k(e)$ and $z_j^h(e) - z_j^m(e)$ are both decreasing in e for $k = m, h$ and $j = s, n$.[11] ∎

Note that since the right hand sides of equations (4.1) and (4.2) are not caste specific (they are

[11] Since $z_j^h(e) - z_j^m(e) > 0$ for all $e \in [\underline{e}_j, \bar{e}_j]$, if $\frac{p_h}{p_m} H < M$ then employment in sector-m would dominate sector-h employment for all ability types. In our analysis below we will focus on the case $\frac{p_h}{p_m} H > M$ to ensure a positive measure of employment in sector-h.

aggregate variables), the threshold conditions also imply that

$$z_s^m(\hat{e}_s^m) = z_n^m(\hat{e}_n^m)$$
$$z_s^h\left(\hat{e}_s^h\right) = z_n^h\left(\hat{e}_n^h\right)$$
$$z_s^h\left(\tilde{e}_s^h\right) = z_n^h\left(\tilde{e}_n^h\right)$$

4.1.1 Mapping Abilities to Sectors

How do agents get distributed across sectors in this economy? This depends on the relative rank ordering of the three thresholds $\hat{e}_j^m, \hat{e}_j^m$, and $\tilde{e}^h$. In order to characterize the sectoral distribution of abilities, there are two possible configurations of sectoral productivities and skilling costs to consider: (i) $\frac{p_h H}{p_m} \geq M$ and $z_j^h(e) \geq z_j^m(e)$; and (ii) $\frac{p_h H}{p_m} < M$ and $z_j^h(e) \geq z_j^m(e)$. Assumption 2 eliminates the other two possibilities associated with $z_j^h(e) < z_j^m(e)$. Case (ii) is inconsistent with an interior solution for $\tilde{h}$ in equation 4.3 above. This reduces the set of cases to (i) above.

It is easy to check that one can have $\hat{e}^h \gtreqless \hat{e}^m$ when $\frac{p_h H}{p_m} \geq M$ and $z_j^h(e) \geq z_j^m(e)$. Given this ambiguity, the following lemma is useful for characterizing the different possibilities:

Lemma 4.2. *The rank order of the three ability thresholds are*

$$\tilde{e}_j^h < \hat{e}_j^h < \hat{e}_j^m \quad \textit{if} \quad \hat{e}_j^h = min[\hat{e}_j^m, \hat{e}_j^h]$$
$$\tilde{e}_j^h > \hat{e}_j^h > \hat{e}_j^m \quad \textit{if} \quad \hat{e}_j^h = max[\hat{e}_j^m, \hat{e}_j^h]$$

Proof. With no loss of generality, suppose $\hat{e}_j^m < \hat{e}_j^h$. We first show that $\tilde{e}^h \notin (\hat{e}_j^m, \hat{e}_j^h)$. Suppose $\hat{e}_j^m < \tilde{e}_j^h < \hat{e}_j^h$. From equation 4.3

$$\frac{p_h}{p_m}H - M = z_j^h\left(\tilde{e}_j^h\right) - z_j^m\left(\tilde{e}_j^h\right) > z_j^h\left(\hat{e}_j^h\right) - z_j^m\left(\hat{e}_j^m\right)$$

where the last inequality follows from the fact that $z_j^k(e)$, $k = m, h$, is decreasing in e. But $z_j^h\left(\hat{e}_j^h\right) - z_j^m\left(\hat{e}_j^m\right) = \frac{p_h}{p_m}H - M$ from equations 4.1 and equation 4.2, which is a contradiction. The other case $\hat{e}_j^h < \tilde{e}_j^h < \hat{e}_j^m$ leads to a contradiction by a similar logic.

Now, suppose $\tilde{e}_j^h < \hat{e}_j^m < \hat{e}_j^h$. Consider $e \in (\hat{e}_j^m, \hat{e}_j^h)$. Since $e < \hat{e}_j^h$ we must have $\frac{A}{p_m} > \frac{p_h}{p_m}H - z_j^h(e)$. Moreover, $e > \hat{e}_j^m$ implies that $M - z_j^m(e) > \frac{A}{p_m}$. But, $e > \tilde{e}_j^h$ implies that $\frac{p_h}{p_m}H - z_j^h(e) > M - z_j^m(e)$, which is a contradiction.

Next, suppose $\tilde{e}_j^h > \hat{e}_j^m > \hat{e}_j^h$. Consider $e \in (\hat{e}_j^m, \hat{e}_j^h)$. Here, $\hat{e}_j^m > e > \hat{e}_j^h$ implies that $\frac{p_h}{p_m}H -$

$z_j^h(e) > \frac{A}{p_m}$ and $M - z_j^m(e) < \frac{A}{p_m}$. But $e < \tilde{e}_j^h$ implies that $\frac{p_h}{p_m}H - z_j^h(e) < M - z_j^m(e)$, which is a contradiction. ■

Lemma 4.2 describes the relationship between the three thresholds in the model. Specifically, it says that $\tilde{e}_j^h$ cannot lie in between $\hat{e}_j^m$ and $\hat{e}_j^h$. Rather, it lies on the same side of $\hat{e}_j^m$ as $\hat{e}_j^h$.

Since the model structure can give rise to $\hat{e}^h \gtrless \hat{e}^m$, the following Proposition characterizes the mapping of the abilities to sectoral employment under both these cases:

Proposition 4.1. *(a) When $\hat{e}_j^h > \hat{e}_j^m$, $j = n, s$, the sectoral distribution of abilities is*

$$e_i \in \begin{cases} [\underline{e}_j, \hat{e}_j^m) & : \ i \in A \\ [\hat{e}_j^m, \tilde{e}_j^h) & : \ i \in M \\ [\tilde{e}_j^h, \bar{e}_j] & : \ i \in H \end{cases}$$

b) When $\hat{e}_j^h < \hat{e}_j^m$, $j = n, s$, the sectoral distribution of abilities is

$$e_i \in \begin{cases} [\underline{e}_j, \hat{e}_j^h) & : \ i \in A \\ [\hat{e}_j^h, \tilde{e}_j^m) & : \ i \in H \\ [\tilde{e}_j^m, \bar{e}_j] & : \ i \in H \end{cases}$$

Proof. (a) When $\hat{e}_j^m < \hat{e}_j^h$, Lemma 4.2 says that we must have $\hat{e}_j^m < \hat{e}_j^h < \tilde{e}_j^h$. The distribution of ability types across the three sectors in this case follows directly from equations 4.1, 4.2, 4.3, and Lemma 4.1. Ability types below $\hat{e}_j^m$ remain unskilled and work in sector-a while those in between $\hat{e}_j^m$ and $\hat{e}_j^h$ choose sector-m. For ability types between $\hat{e}_j^h$ and $\tilde{e}_j^h$, equation 4.3 implies that employment in sector-m is strictly preferred to sector-h. Those with ability above $\tilde{e}_j^h$ choose to work in sector-h, which follows directly from equation 4.3.

(b) When $\hat{e}_j^h < \hat{e}_j^m$, from Lemma 4.2 we have $\tilde{e}_j^h < \hat{e}_j^h < \hat{e}_j^m$. In this case, the distribution of ability types across sectors follows directly from equations 4.1-4.2 and Lemma 4.1. Ability types below $\hat{e}_j^h$ strictly prefer employment in sector-a to both sectors h and m. For all ability types in caste $j = n, s$ with $e \in [\hat{e}_j^h, \hat{e}_j^m)$, employment in sector-$h$ dominates both sectors a and m. For $e \geq \hat{e}_j^m > \tilde{e}_j^h$, equation 4.1 says that sector-m dominates sector-a while equation 4.3 says that working in sector-h is strictly preferred by these types over sector-m employment. ■

While the message of Proposition 4.1 is self-explanatory, a comment on part (b), which describes allocations when $\hat{e}_j^h < \hat{e}_j^m$, is useful. The ability distribution described in Proposition 4.1 for this case implies that labor from both castes choose employment in either sector-a or sector-h, thereby rendering sector-m empty. This is clearly counterfactual since our data analysis revealed that both castes were employed in all three sectors. In the remainder of the paper we ignore this case and focus exclusively on parameter configurations such that $\hat{e}_j^h > \hat{e}_j^m$ for $j = n, s$.[12]

4.2 Market clearing and Equilibrium

Markets for each good must clear individually. Hence, we must have

$$c^a = y^a \tag{4.4}$$

$$c^m = y^m - F \tag{4.5}$$

$$c^h = y^h \tag{4.6}$$

where F denotes the total skill acquisition costs incurred by workers employed in sector m and sector h, while y^a, y^m and y^h denote aggregate output of goods a, m and h, respectively. The market clearing condition for the m good recognizes that part of the use of the good is for acquiring skills.

Definition: *The Walrasian equilibrium for this economy is a vector of prices $\{p_m, p_h\}$ and quantities $\left\{c^a, c^m, c^h, y^a, y^m, y^h, F^m, F^h,\ \hat{e}_s^m, \hat{e}_s^h, \hat{e}_n^m, \hat{e}_n^h\right\}$ such that all worker-households satisfy their optimality conditions, budget constraints are satisfied and all markets clear.*

In subsection 9.2 of Section 9 we describe the derivation of the aggregate variables of the model and its equilibrium construction.

5 A two-sector illustration

A key motivation for this paper is to understand the effects of productivity growth on the macroeconomy and caste gaps. In order to gain some analytical insights on this issue, we now specialize the three-sector model developed above to the two-sector case. In particular, we assume that there are only two sectors – a and m, i.e., we eliminate the h sector. We should note that in this two-sector example there is only one cut-off ability threshold for each group. Hence, in the following we drop the sectoral superscripts from the notation for the thresholds and use $\hat{e}_j$ to denote the threshold

[12]The case $\hat{e}_j^h = \hat{e}_j^m = \tilde{e}_j^j$ is possible but clearly non-generic. Consequently, we ignore this pathological possibility.

ability of caste $j = s, n$ so that all individuals with ability levels greater than $\hat{e}_j$ will get skilled in order to work in the $m-$sector.

We also impose two additional assumptions:

Assumption 1': The skill acquisition cost is given by $f_j(e) = \phi(\gamma_j - \alpha e)$ for $j = s, n$ with $\gamma_j > \alpha \bar{e}_j$.

Assumption 3: $G_j(e)$ is uniform on the support $\left[\underline{e}_j, \bar{e}_j\right]$ for $j = s, n$.

Assumption 1' imposes linearity on the skill cost function with the marginal effect of ability on the cost assumed to be identical for both castes and sectors. This is clearly a very restrictive assumption which we shall relax in the quantitative exercises below. We impose it here to facilitate some analytical insights. Crucially though, this specification allows the intercept term on the cost function to vary by sector and caste. The condition $\gamma_j > \alpha \bar{e}^j$ ensures that getting skilled involves a positive cost for even the highest ability type. Assumption 3 imposes the uniform distribution for ability which just makes the analytics simple.[13]

Under this formulation, it is easy to check that

$$\frac{\hat{e}_n}{\hat{e}_s} = \frac{\gamma_n}{\gamma_s}$$

In other words, the relative sectoral ability thresholds of the two groups are proportional to their relative fixed costs of acquiring skills to work in that sector. Crucially, the conditions say that the ability cutoff of caste n for working in sector m will be greater than the corresponding cutoff for caste s if and only if their fixed skill costs exceed the corresponding cost for caste s.

Under these assumptions, the equilibrium of the economy is determined by the system of equations:

$$p_m = \frac{\left(\frac{1-\theta}{\theta}\right)\left[y^A - \bar{c}L\right]}{y^M - F}$$
$$p_m = \frac{A\hat{e}_s}{M\hat{e}_s - \phi(\gamma_s - \alpha\hat{e}_s)}$$
$$\frac{\hat{e}_n}{\hat{e}_s} = \frac{\gamma_n}{\gamma_s}$$

These variables are given by equations (9.20), (9.21) and (9.23) in the Appendix (without the

[13]It is important to recognize that the assumption that ability is uniformly distributed does not imply that the distribution of skills or the distribution of sectoral employment will be uniform as well. Since skills are acquired based on both costs and ability, the equilibrium skill distribution will not, in general, inherit the distribution of abilities.

h-sector terms).

The first equation above is the market clearing condition for the agricultural good while the second equation gives the ability threshold for group s members to get skilled in order to work in sector m. The third equation comes from the relation $z_s(\hat{e}_s) = z_n(\hat{e}_n)$. Under the assumed linear skill cost technology, this condition implies that the relative ability thresholds of the two castes are proportional to the relative fixed costs of getting skilled.

Note that in this case sectoral outputs and total skill expenditure are

$$y^A - \bar{c}L = \frac{A}{2}\left[S\left(\frac{\left(\frac{\gamma_s}{\gamma_n}\hat{e}_n\right)^2 - \underline{e}_s^2}{\bar{e}_s - \underline{e}_s}\right) + N\left(\frac{\hat{e}_n^2 - \underline{e}_n^2}{\bar{e}_n - \underline{e}_n}\right) - \frac{2\bar{c}L}{A}\right]$$

$$y^M = \frac{M}{2}\left[S\left(\frac{\bar{e}_s^2 - \left(\frac{\gamma_s}{\gamma_n}\hat{e}_n\right)^2}{\bar{e}_s - \underline{e}_s}\right) + N\left(\frac{\bar{e}_n^2 - \hat{e}_n^2}{\bar{e}_n - \underline{e}_n}\right)\right]$$

$$F = \phi S\left(\frac{\bar{e}_s - \hat{e}_s}{\bar{e}_s - \underline{e}_s}\right)\left[\gamma_s - \frac{\alpha}{2}\left(\bar{e}_s + \frac{\gamma_s}{\gamma_n}\hat{e}_n\right)\right] + \phi N\left(\frac{\bar{e}_n - \hat{e}_n}{\bar{e}_n - \underline{e}_n}\right)\left[\gamma_n - \frac{\alpha}{2}\left(\bar{e}_n + \hat{e}_n\right)\right]$$

A key question of interest to us is whether the aggregate changes in the Indian economy over the past two decades can explain, at least qualitatively, the wage and education convergence across the castes. Toward that end, we define

$$A = \mu\bar{A} \tag{5.7}$$

$$M = \mu\bar{M} \tag{5.8}$$

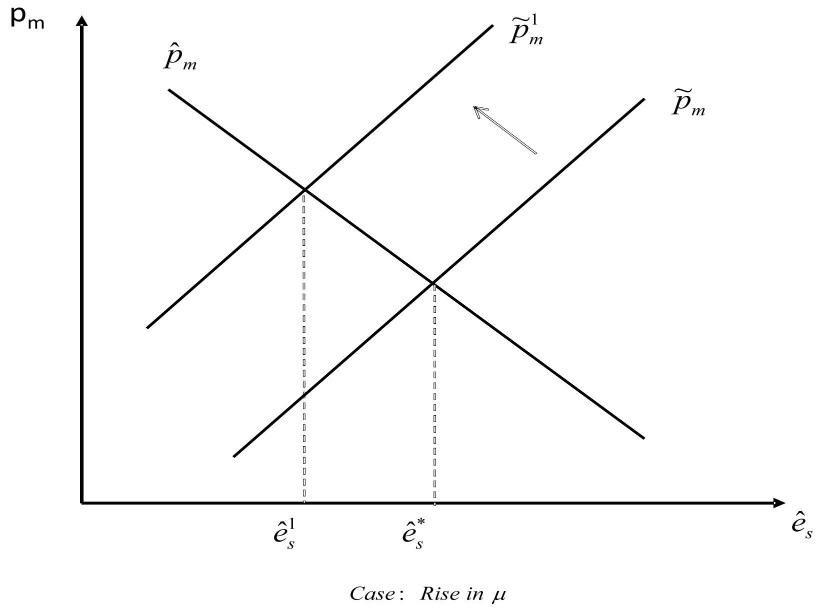
This specification nests sectoral and aggregate productivity changes with changes in μ being aggregate shocks while changes in $\bar{A}$ and $\bar{M}$ are sector-specific productivity shocks. Moreover, to avoid scale effects we also set

$$\phi = \frac{\mu}{\bar{\phi}} \tag{5.9}$$

If the skill acquisition cost were not indexed to the aggregate productivity parameter μ, the cost of acquiring skills would become progressively smaller as a share of total output of the economy simply in response to aggregate productivity growth. The specification avoids this scale effect of growth.

In order to determine the effect of aggregate productivity growth on the economy, it is useful

Figure 8: An aggregate productivity shock



to define the following:

$$\hat{p}_m = \frac{A\hat{e}_s}{M\hat{e}_s - \phi(\gamma_s - \alpha\hat{e}_s)} \tag{5.10}$$

$$\tilde{p}_m = \frac{\left(\frac{1-\theta}{\theta}\right)\left(y^A - \bar{c}L\right)}{y^M - F} \tag{5.11}$$

Figure 8 plots the two equations. Clearly, $\hat{p}_m$ is a downward sloping function of $\hat{e}_s$ while $\tilde{p}_m$ is an upward sloping function. The equilibrium threshold ability is $\hat{e}_s^*$. All ability types above this critical level get skilled and work in the $m-$sector while the rest work in the $a-$sector. Recall that $\hat{e}_n = \frac{\gamma_n}{\gamma_s}\hat{e}_s$, so this solves for the ability threshold for type-n individuals as well.

Our primary interest is in determining the effect of an aggregate productivity shock on this economy. The following Lemma summarize the effects of a common productivity shock on the ability threshold $\hat{e}_s$:

Lemma 5.3. *An increase in aggregate labor productivity μ decreases the ability threshold $\hat{e}_s$.*

Proof. The lemma follows by equating equations 5.10 and 5.11 and totally differentiating the resultant expression with respect to $\hat{e}_s$ and μ. ∎

The logic behind Lemma 5.3 is easiest to describe using Figure 8 which shows the effect of an aggregate TFP shock on the equilibrium system of equations 5.10-5.11. A rise in μ leaves the $\hat{p}_m$

locus unchanged but shifts the $\tilde{p}_m$ locus up and to the left. As a result the equilibrium threshold ability level $\hat{e}_s$ declines. Intuitively, the aggregate TFP shock leaves unchanged the relative gains and losses from getting skilled since the productivity of all sectors (including the education sector) are affected symmetrically. However, a higher μ raises the aggregate supply of the agricultural good *net* of the subsistence amount $\bar{c}L$ while leaving the aggregate supply of the manufacturing good net of the training cost unchanged. The resultant excess supply of the agricultural good induces a terms of trade worsening of the agricultural good. As agents increase their demand for good m its relative price p_m rises. All else equal, this increases the attractiveness of working in sector-m. Consequently, the threshold ability falls and agents with lower ability now begin to get trained. This effect is stronger for caste-s as there are more of them in sector-a in the initial equilibrium.

The experiment above focused on aggregate productivity shocks that are balanced or proportional across all sectors. What would happen instead if productivity changes are biased against the agricultural sector, a feature that characterizes India during this period? Specifically, suppose A remains unchanged while M rises. The direct effect of this shock would be to shift the $\hat{p}_m$ schedule down and to the left while simultaneously shifting the $\tilde{p}_m$ schedule down and to the right. The effect of these changes would be an unambiguous decline in the relative price of the manufacturing good. The effect on $\hat{e}_s$ is ambiguous and depends on the net strength of the two shifts. The key feature to note though is that the structure is perfectly consistent with both an improvement in the agricultural terms of trade as well as an increase in the share of each group getting skilled.

Before closing this section, we analyze a special case of the model to highlight the power of growth in reducing historical caste gaps. We consider the case where the ability distribution of the two castes being identical. Specifically, assume that

$$\bar{e}_n = \bar{e}_s = \bar{e};\ \underline{e}_n = \underline{e}_s = \underline{e}$$

Under the uniform distribution for ability given by Assumption 3, the relative caste wage gap, $\Delta W \equiv \frac{\bar{w}_n}{\bar{w}_s}$ is

$$\Delta W = \left(\frac{\bar{e}_s - \underline{e}_s}{\bar{e}_n - \underline{e}_n}\right)\left[\frac{\left(\hat{e}_n^2 - \underline{e}_n^2\right)A + \left(\bar{e}_n^2 - \hat{e}_n^2\right)p_m M}{\left(\hat{e}_s^2 - \underline{e}_s^2\right)A + \left(\bar{e}_s^2 - \hat{e}_s^2\right)p_m M}\right]$$

Substituting equation 5.10 in the above gives the relative wage gap as

$$\Delta W = \left(\frac{\bar{e}_s - \underline{e}_s}{\bar{e}_n - \underline{e}_n}\right)\left[\frac{\left(\bar{e}_n^2 - \underline{e}_n^2\right)M\hat{e}_s + \phi\left(\gamma_s - \alpha\hat{e}_s\right)\left(\underline{e}_n^2 - \hat{e}_n^2\right)}{\left(\bar{e}_s^2 - \underline{e}_s^2\right)M\hat{e}_s + \phi\left(\gamma_s - \alpha\hat{e}_s\right)\left(\underline{e}_s^2 - \hat{e}_s^2\right)}\right] \tag{5.12}$$

Noting that $\frac{\hat{e}_n}{\hat{e}_s} = \frac{\gamma_n}{\gamma_s}$, it is easy to check from equation (5.12) that when the ability distributions are identical, $\Delta W \gtreqless 1$ as $\gamma_n \lesseqgtr \gamma_s$. This is intuitive. If the ability distributions are identical for the two castes then the only reason for there to be any difference in their average wages would be differences in skilling costs. If the fixed costs of getting skilled are greater for caste s then a relatively smaller fraction of caste s acquires the skills to work in the relatively high wage non-agriculture sector. As a result, their average wages are lower.

Proposition 5.2. *When $\bar{e}_n = \bar{e}_s = \bar{e}$; $\underline{e}_n = \underline{e}_s = \underline{e}$, the conditions $\gamma_n \leq \gamma_s$ and $\gamma_s \geq 2\alpha\bar{e}$ are jointly sufficient for an increase in aggregate labor productivity μ to weakly reduce the wage gap ΔW.*

Proof. The proposition follows from differentiating equation (5.12) with respect $\hat{e}_s$ which gives that ΔW is a weakly decreasing function of $\hat{e}_s$ under the two jointly sufficient conditions. The rest of the proposition follows directly from Lemma 5.3. ∎

Proposition 5.2 is striking in that it shows that even when castes have equal ability and SC/STs have a higher relative fixed cost of acquiring skills, a fall in the threshold ability cutoff $\hat{e}$ induced by higher productivity growth would reduce the relative caste wage gap as long as the fixed cost of skilling is large enough. Intuitively, higher productivity reduces the fixed costs of skilling. These gains are proportionately greater for the group with the higher fixed cost. Hence, a relatively larger fraction of caste s workers get skilled and transit to sector m which reduces the overall wage gap.

6 A Quantitative Evaluation

We now turn to a quantitative implementation of the full version of the three-sector model. Specifically, we examine whether a calibrated version of the three sector model can explain the observed caste gap dynamics through the observed macroeconomic growth; and whether the caste education subsidization in India through reservations were crucial for the observed convergence.

The quantitative strategy of this section is to calibrate the model to the mimic the 1983 distribution of skills and sectoral output. Next, we estimate the sectoral productivity changes from the National Income and Product Accounts data and feed those estimated paths into the calibrated model. The resulting distributional implications of the model at each date are then compared to the data in order to evaluate the explanatory power of aggregate productivity shocks for the caste wage gap dynamics.

While we stick to the functional forms outlined in the previous sections, we make one change here in that we change Assumption 1' to

Assumption $1''$: The skill acquisition cost is given by $f_j^k(e) = \phi\left(\gamma_j^k - \alpha e^2\right)$ with $\gamma_j^k > \alpha\left(\bar{e}^j\right)^2$ for $j = s, n$ and $k = m, h$.

Two comments on this change in the specification of the skill acquisition costs are in order. First, this specification switches the cost function for skill acquisition from linear to quadratic. This is a more flexible specification that allows a better fit of the model to the data. Second, the cost function isn't scaled to productivity levels in the other sectors of the economy. This is in contrast to the specification in equation 5.9 above. Without scaling, productivity growth in the other sectors will reduce the relative education costs. This will potentially create an independent transmission mechanism of productivity shocks to aggregate output. We explore the quantitative effects of scaling ϕ in Section 6.3.4 below.

Table 2: Calibration of Key Variables

Variable	Value	Variable	Value
$\underline{c}$	0.5	θ	0.46
η	0.15	α	1
$\underline{e}_s$	1	$\bar{e}_s$	2
$\frac{M}{A}_{1983}$	7.33	$\frac{H}{A}_{1983}$	5.49
S	0.25	L	1
Calibrated variables for 1983			
$\gamma^m_{s,1983}$	6.9271	$\gamma^h_{s,1983}$	21.7544
$\frac{\gamma^m_{n,1983}}{\gamma^m_{s,1983}} = \frac{\gamma^h_{n,1983}}{\gamma^h_{s,1983}}$	1.2966	ϕ	0.8086
$\frac{\underline{e}_n}{\underline{e}_s}$	0.7672	$\frac{\bar{e}_n}{\bar{e}_s}$	1.3503

Notes: The table gives the parameters used for calibrating the model. The top panel lists the parameter values that were taken from other studies. The parameters in the bottom panel of the table were picked to match data moments from 1983.

Our focus is on six key data moments: the three sectoral occupation distribution gaps and the three sectoral wage gaps between non-SCSTs and SCSTs. Our calibration strategy is to choose the skilling cost parameters $\left(\gamma_s^m, \gamma_s^h, \frac{\gamma_n^m}{\gamma_s^m}\right)$, the two relative ability ratios $\left(\frac{\bar{e}_n}{\bar{e}_s}, \frac{\underline{e}_n}{\underline{e}_s}\right)$ and the scaling parameter ϕ for the skilling cost functions to match the six sectoral occupation and wage gaps between the castes in 1983. We then freeze these parameters at the 1983 values and recompute the equilibrium by feeding in the observed change in sectoral labor productivity between 1983 and 2012. Note that since the model has no intrinsic dynamics, each new level for productivity generates a new equilibrium.

Table 2 reports the key parameters. The upper panel of the table gives the parameters that were either normalizations or values that were taken from other studies. The numbers in the lower panel are the ones that were calibrated to match the moments of the 1983 caste distribution. A couple of features of the parameter values reported above are worth highlighting. First, the absolute levels of the ability supports for SCSTs are unimportant. The crucial objects instead are the relative ability mark-ups of non-SCSTs at the lower and upper supports of the ability distribution.

Table 3 below shows the match between the targeted variables under the calibration in Table 2 and their corresponding data values in 1983. Two features in the Table are worth noting. First, the model fits the rank order and magnitudes of the sectoral caste gaps in both labor shares and wages gaps quite well. Thus, both in the model and in the data the gaps are the lowest in agriculture and the highest in services. Second, the model is able to get quite close to the overall caste wage gap in 1983 in the data despite it not being a targeted moment of the calibration.

Table 3: Data and Model Moments

	1983		2012	
Variable	Data	Model	Data	Model
Δs^a	0.80	0.89	0.79	0.94
Δs^m	1.43	1.40	1.57	1.35
Δs^h	1.61	1.68	1.21	0.98
Δw^a	1.04	1.04	1.08	1.04
Δw^m	1.20	1.25	1.14	1.25
Δw^h	1.45	1.32	1.33	1.32
Δw	1.45	1.27	1.30	1.17

Notes: The table reports the sectoral caste gaps in employment and wages. $\Delta s^j,\ j = a, m, h$ is the ratio of the fraction of all of all non-SC/STs working in sector j to the fraction of all SC/STs working in sector j. $\Delta w^j,\ j = a, m, h$ is the ratio of the mean non-SC/ST to mean SC/ST wage in sector j. Δw is the ratio of the mean non-SC/ST to mean SC/ST wage.

Having described the fit of the model to the targeted data moments in 1983, we now examine its dynamic predictions for caste gaps. We are specifically interested in examining the predicted paths of the caste gaps in sectoral labor shares and wages over time in response to the actual sectoral productivity changes since 1983.

Table 3 also gives the labor and wage gaps across castes in the model and the data in 2012. The main takeaway from the Table is in the last row. The wage gap between non-SC/STs and SC/STs declined by 0.15/1.45 or 10.5 percent between 1983 and 2012 in the data. The corresponding reduction generated by the model is 7.9 percent. Thus, the baseline model with quadratic skilling costs can explain 75 percent of the observed decline in the percentage wage gap.

Underneath the success in reproducing the overall caste wage gap dynamics, the model has mixed success in matching the sectoral dynamics in wages and employment shares. While the sectoral caste wage gaps did exhibit some changes in the data, the model predicts relatively unchanged sectoral wage gaps. However, the model has both qualitative and quantitative success in generating the observed dynamics of the caste gaps in sectoral employment shares.

As we showed through the decomposition exercise, the size of the change in the caste labor share gap in services was the largest amongst all the sectoral gaps, and the key driver of the overall wage convergence in the data. The model delivers on this margin by generating a 42 percent reduction in the labor share gap in service between 1983 and 2012. While this overshoots the actual 25 percent decline in the data by, it allows the model to reproduce the overall wage gap dynamics.

The labor share gap in agriculture remained relatively stable both in the data and the model. Where the model misses is in matching the change in the labor share gap in manufacturing. In the data this gap actually increased by 9.8 percent while the model generates a 3.6 percent reduction. A key feature of the data is that there was a switch between the relative rank orders of the labor share gaps between manufacturing and services. While services had the largest caste gap in labor shares in 1983, by 2012 it was manufacturing that had the largest caste labor share gap. The model successfully reproduces this switch.

How well does the model reproduce the aggregate dynamics of the Indian economy during this period? Table 4 describes the dynamic behavior of the sectoral shares of labor in agriculture, manufacturing and services (L^a/L, L^m/L and L^h/L, respectively) along with the relative sectoral prices of manufacturing and services (p_M and p_H, respectively). The results for the baseline model are shown in the last column of Table 4 under the $\underline{c} = 0.5$. Both the model and the data predict decreases in the labor shares of agriculture and manufacturing and a decline in the prices of manufacturing and services relative to agriculture.

It is important to note that these aggregate results are not sensitive to the presence of non-

Table 4: Sectoral Price and Labor Shares

Growth rate 1983-2012			
Variable	Data	$\underline{c} = 0$	$\underline{c} = 0.5$
$\frac{L^a}{L}$	-0.153	-0.062	-0.108
$\frac{L^m}{L}$	-0.143	-0.227	-0.111
$\frac{L^h}{L}$	0.407	0.692	1.250
p_m	-0.323	-0.509	-0.529
p_h	-0.264	-0.578	-0.593

Notes: The table reports the rate of change in sectoral labor shares and sectoral relative prices. L^j denotes labor employed in sector j and L is total labor employed. $p_j,\ j = m, h$ denotes the sector j price relative to sector a.

homotheticities since the same pattern emerges when $\underline{c} = 0$. It is also worth noting that that standard structural transformation in models with non-homotheticities predict deteriorating agricultural terms of trade. That is clearly not consistent with the Indian data but a feature that our model can match. We shall show below that the key for our model to match these sectoral dynamics is differential sectoral growth.

6.1 Deconstructing the wage convergence

In order to understand the source of wage convergence across the castes in the model, note that rising sectoral productivities induces a fall in the thresholds $\hat{e}_k^m, \tilde{e}_k^h, k = n, s$. A decrease in the threshold causes workers to flow from one sector to another. The size of these flows depends on (a) the number of people to the left of the threshold; (b) the elasticity of the distribution function of abilities around the threshold; and (c) how much the threshold changes. Castes can differ on all or any of (a), (b) and (c) above. The effect on the caste wage gap ultimately depends on the relative magnitudes of these effects and the differences in sectoral productivities.

To see these mechanisms at play, it is useful to write the wage gap in general terms. Recall that the caste wage gap is $\Delta w = \frac{w_n}{w_s}$. Using $G_k(e)$ to denote the cumulative distribution function of ability for caste $k = n, s$ with the associated probability density function $g_k(e)$, the mean wage of each caste can be written as the share weighted sum of its sectoral average wages:

$$w_n = G_n(\hat{e}_n^m)w_n^a + [G_n(\tilde{e}_n^h) - G_n(\hat{e}_n^m)]w_n^m + [1 - G_n(\tilde{e}_n^h)]w_n^h$$

$$w_s = G_s(\hat{e}_s^m)w_s^a + [G_s(\tilde{e}_s^h) - G_s(\hat{e}_s^m)]w_s^m + [1 - G_s(\tilde{e}_s^h)]w_s^h$$

where the sectoral wages for caste $k = n, s$ are given by

$$w_k^a = A\int_{\underline{e}_k}^{\hat{e}_k^m} e\frac{dG_k(e)}{G_k(\hat{e}_k^m)},\ w_k^m = p_m M\int_{\hat{e}_k^m}^{\tilde{e}_k^h} e\frac{dG_k(e)}{G_k(\tilde{e}_k^h) - G_k(\hat{e}_k^m)},\ w_k^h = p_h H\int_{\tilde{e}_k^h}^{\bar{e}_k} e\frac{dG_k(e)}{1 - G_k(\tilde{e}_k^h)}$$

For given sectoral relative prices and productivities, the caste specific wages can be differentiated and rearranged to get the percent change in the caste wage gap in response to percent changes in the ability thresholds as

$$\begin{aligned}\frac{d\Delta w}{\Delta w} &= (A - p_m M)\left[\left\{\frac{\hat{e}_n^m}{w_n}G_n(\hat{e}_n^m)\eta_n^m(\hat{e}_n^m) - \frac{\hat{e}_s^m}{w_s}G_s(\hat{e}_s^m)\eta_s^m(\hat{e}_s^m)\right\}\frac{d\hat{e}_n^m}{\hat{e}_n^m} + \frac{\hat{e}_s^m}{w_s}G_s(\hat{e}_s^m)\eta_s^m(\hat{e}_s^m)\left(\frac{d\hat{e}_n^m}{\hat{e}_n^m} - \frac{d\hat{e}_s^m}{\hat{e}_s^m}\right)\right] \\ &+ (p_m M - p_h H)\left[\left\{\frac{\tilde{e}_n^h}{w_n}G_n(\tilde{e}_n^h)\eta_n^h(\tilde{e}_n^h) - \frac{\tilde{e}_s^h}{w_s}G_s(\tilde{e}_s^h)\eta_s^h(\tilde{e}_s^h)\right\}\frac{d\tilde{e}_n^h}{\tilde{e}_n^h} + \frac{\tilde{e}_s^h}{w_s}G_s(\tilde{e}_s^h)\eta_s^h(\tilde{e}_s^h)\left(\frac{d\tilde{e}_n^h}{\tilde{e}_n^h} - \frac{d\tilde{e}_s^h}{\tilde{e}_s^h}\right)\right]\end{aligned} \tag{6.13}$$

where

$$\eta_k^m(\hat{e}_k^m) \equiv \frac{\hat{e}_k^m g_k(\hat{e}_k^m)}{G_k(\hat{e}_k^m)},\ \eta_k^h(\tilde{e}_k^h) \equiv \frac{\tilde{e}_k^h g_k(\tilde{e}_k^h)}{G_k(\tilde{e}_k^h)},\ k = n, s$$

denote the elasticities of the CDF of the ability of caste k at the thresholds $\hat{e}_k^m$ and $\tilde{e}_k^h$.

To see the different forces at work, recall that changes in sectoral labor productivities alter the ability thresholds $\hat{e}_k^m$ and $\tilde{e}_j^h$ for $k = n, s$. Changes in these thresholds change the output produced by the two types of workers. The magnitude of this effect for each caste depends on the product of the marginal effect of moving a worker at the threshold from one sector to another and the total movement of workers of that caste across sectors due to the change in the threshold.

For example, an increase in the threshold $\hat{e}_k^m$ changes the proportion of caste $k = n, s$ working in sector m by the elasticity η_k^m of the CDF G_k at the threshold weighted by the fraction of workers below the threshold $G_k(\hat{e}_k^m)$. $(A - p_m M)\frac{\hat{e}_k^m}{w_k}$, in turn, is the marginal net effect on output of moving a worker of caste k with ability $\hat{e}_k^m$ from sector m to sector a, expressed relative to the average output of caste k in sector-m. Hence, $(A - p_m M)\frac{\hat{e}_k^m}{w_k}G_k(\hat{e}_k^m)\eta_k^m(\hat{e}_k^m)$ is the total effect on output produced by caste $k = n, s$ due to a change in the ability threshold $\hat{e}_k^m$ by one percent.

Since $A - p_m M$ is common to both castes, the difference in the output effect of changing $\hat{e}_k^m$ for the two castes depends on the relative magnitudes of $\frac{\hat{e}_k^m}{w_k}G_k(\hat{e}_k^m)\eta_k^m(\hat{e}_k^m)$. The effects of changes in $\tilde{e}_k^h$ are similarly described by the expression $(p_m M - p_h H)\frac{\tilde{e}_k^h}{w_k}G_k(\tilde{e}_k^h)\eta_k^h(\tilde{e}_k^h)$.

Under our baseline calibration $\frac{d\hat{e}_n^m}{\hat{e}_n^m} \approx \frac{d\hat{e}_s^m}{\hat{e}_s^m}$ and $\frac{d\tilde{e}_n^h}{\tilde{e}_n^h} \approx \frac{d\tilde{e}_s^h}{\tilde{e}_s^h}$. Hence, the sign of $\frac{d\Delta w}{\Delta w}$ depends on the terms in the curly brackets in both lines of equation (6.13). Under the assumed uniform distributions for ability, the elasticities $\eta_k(e)$, $k = n, s$ are decreasing in e. The calibration also implies that $G_n(\hat{e}_n^m) < G_s(\hat{e}_s^m)$, $G_n(\tilde{e}_n^h) < G_s(\tilde{e}_s^h)$, $\hat{e}_n^m > \hat{e}_s^m$ and $\tilde{e}_n^h > \tilde{e}_s^h$, $j = m, h$. Since, $\frac{\hat{e}_n^m}{w_n} < \frac{\hat{e}_s^m}{w_s}$ while $\frac{\tilde{e}_n^h}{w_n} \approx \frac{\tilde{e}_s^h}{w_s}$, the terms in the curly brackets in equation (6.13) are negative.

Under our calibration $A < p_m M < p_h H$ in 1983. This, along with the fact that the terms in the curly brackets are negative, imply that the caste wage gap moves in the same direction of the ability thresholds. In the quantitative model, rising sectoral productivities reduce the ability thresholds for both castes. Hence, the overall caste wage gap falls.

Intuitively, the higher ability thresholds for caste-n implies a lower elasticity of their CDF around the thresholds. When the thresholds fall proportionately for both groups, the smaller numbers of type-n workers below the thresholds along with the smaller increase of their probability weight on higher productivity sectors due to the lower elasticity mean that the shares of type-n workers in sectors m and h increase at slower rates than the corresponding expansion of employment shares of type-s workers in these sectors. This is the primary driver of the overall wage convergence.

Put differently, the overall caste wage convergence in the model is driven primarily by SC/STs switching to higher wage sectors faster than non-SC/ST workers. Within-sector wage convergence plays a very minor role since the ability thresholds of the castes move similarly.

6.2 Robustness to alternative productivity growth measures

The exogenous driver of dynamic changes in our model are the sectoral labor productivity growth rates. As we had showed in the empirics above, there are variations in the estimated labor productivity numbers produced by different data sources. Specifically, we had seen in Figure 4 that the sectoral productivity growth rates estimated in the KLEMS dataset were lower than the productivity numbers estimated from the national accounts statistics. How robust are our convergence results to using the lower productivity growth numbers from KLEMS?

Table 5 reports the caste labor and wage gaps by sector for 2012. The second column reports the data moments for 2012, the third column gives the moments from the baseline model while the last column gives the predicted numbers for 2012 under the alternative productivity growth estimates from the KLEMS dataset. The lower productivity growth estimates in KLEMS induces marginal changes in the predicted sectoral caste labor gaps but leaves the predicted sectoral caste wage gaps unchanged. The joint effect of these changes is that the predicted decline in the overall

Table 5: Alternative Productivity Growth Estimates

	2012		
Variable	Data	Baseline	KLEMS
Δs^a	0.79	0.94	0.93
Δs^m	1.57	1.35	1.27
Δs^h	1.21	0.98	1.05
Δw^a	1.08	1.04	1.04
Δw^m	1.14	1.25	1.25
Δw^h	1.33	1.33	1.33
Δw	1.30	1.17	1.19

The table reports the moments from the model computed under sectoral labor productivity growth for 1983-2012 computed from the national accounts of India and from the KLEMS dataset release 2020.

caste wage gap is very slightly lower under the KLEMS estimates relative to the baseline. This suggests to us that the results are robust to alternative estimates of productivity growth.

We view these results as being broadly supportive of the baseline model in explaining the static and dynamic patterns in both the overall caste wage gap and sectoral labor gaps. While the model has difficulty in reproducing the observed dynamics of the sectoral wage gaps, the key to the overall wage convergence is the dynamic behavior of the sectoral labor gaps across castes, especially in services, a feature that the model generates successfully.

6.3 Mechanisms at Play

The model developed and quantified above has a few important features built into it. These include selection effects in sectoral choices by workers, non-homothetic preferences, education costs as well as differential sectoral productivity growth. While the education costs are more specific to our model, selection effects, non-homothetic preferences and unbalanced sectoral growth have been emphasized by a number of authors in models of structural transformation.

Which of these features is key for our baseline results? We try to uncover the answer by re-solving the quantitative model under combinations of four special cases: (a) random sorting of workers across sectors; (b) homothetic preferences with $\underline{c} = 0$; (c) balanced sectoral labor productivity growth with productivity growth rates equalized across sectors at the rate of the observed agricultural labor productivity growth;[14] and (d) education costs that are scaled to the growth rate labor productivity in agriculture. Recall that not scaling skilling costs implies that education costs decline in relative terms as productivity grows in the other sectors. This induces an endogenous expansion of output as more people get skilled.

6.3.1 Random sorting: No selection effects

In the first special case of the model, we shut down all selection effects in the sectoral re-sorting of workers in 2012 in response to productivity shocks. Specifically, we feed the same sectoral productivity growth as in the baseline model but instead of allowing workers to endogenously make their skilling and sectoral employment choices in response, we assign random selections of workers from agriculture to manufacturing and from manufacturing to services. The random selections are done such that the overall shares of workers of each caste changing sectors in 2012 relative to 1983 matches the corresponding shares in the baseline model.

This experiment effectively keeps the number of workers changing sectors the same as in the baseline model but does not permit any selection effects to dictate the identity of those changing sectors. The results are shown in Table 6.

The main takeaway can be gleaned from the row corresponding to ΔW in the top panel which gives the overall caste wage gaps in the different cases. While in the baseline model with selection the caste wage gap falls by 8 percent (0.102/1.274), the corresponding decline under random sorting is 6 percent (0.076/1.274). Clearly, labor flows across sectors are the key driver of wage convergence. Selection effects, while present, have relatively smaller effects. This is consistent with our statistical decomposition exercise where we had found that sectoral caste labor convergence was the main contributor to the overall wage convergence.

The effects of selection are illustrated by the last two columns of the other panels of Table 6. When labor is reassigned across sectors randomly rather than through selection the average ability of both castes rises in agriculture while falling in the other two sectors. This is intuitive. Under selection, only the most able types move from agriculture to manufacturing. When the re-sorting is random, the average ability of the group that moves from agriculture is lower. This raises the average ability of the pool that remains in agriculture while simultaneously reducing the average ability of the new pool of workers in manufacturing. A similar force explains the decline of the

[14] The results presented below are not sensitive to choosing the rate of labor productivity of one of the other two sectors as the common growth rate.

Table 6: Wage Gaps with Random Resorting

Variable	Baseline 1983	Baseline 2012		Random Sorting 2012
Wage Gaps and Growth				
ΔW^a	1.0394	1.0382	$<$	1.0403
ΔW^m	1.2452	1.2492	$<$	1.3920
ΔW^h	1.3238	1.3247	$>$	1.3012
ΔW	1.2740	1.1720	$<$	1.1974
Sectoral average ability				
Ee^a_{nst}	1.4601	1.3992	$<$	1.4615
Ee^m_{nst}	2.3395	2.1934	$>$	2.0066
Ee^h_{nst}	2.6126	2.5302	$>$	2.4804
Ee^a_{st}	1.4048	1.3465	$<$	1.4049
Ee^m_{st}	1.8779	1.7563	$>$	1.4415
Ee^h_{st}	1.9729	1.9099	$>$	1.9062
Output and Price				
Y^a	1.0710	1.8466	$<$	1.9283
Y^m	2.9324	8.8260	$>$	7.9371
Y^h	1.1226	6.5094	$>$	6.4048
p_m	0.1755	0.0827	$<$	0.1080
p_h	0.4312	0.1754	$<$	0.1891

average ability of service sector workers of both castes under random sorting.

The altered sectoral composition of workers under random sorting has macroeconomic implications. The higher average ability of agricultural workers raises agricultural output in 2012 relative to the baseline. The corresponding output levels of manufacturing and services declines due to the declining ability pool of their workers. This raises the sectoral relative prices of manufacturing and services relative to the baseline case.

The relatively muted selection effects in the model is due to our assumption that ability is uniformly distributed. Assuming a Pareto distribution for ability would likely raise the contribution of selection effects. However, it is worth recalling that the presence of skilling costs in the model induces Pareto distributions for education and sectoral employment despite the uniform distribution for abilities. We view this feature to be a strength of the model. Since ability is private information, heaping a large part of the explanation on a Pareto distribution of an unobservable would potentially be problematic.

6.3.2 Non-homothetic Preferences

Next, we examine the importance of non-homothetic preferences in generating the results. The last column of Table 7 gives the relevant changes in the caste labor and wage gaps between 1983 and 2012 predicted by the model under our baseline calibration where we have non-homothetic preferences ($\underline{c} = 0.5$), differential sectoral productivity growth and no scaling of the skilling costs.

The main result to note is that non-homotheticity is quantitatively not important in generating the caste wage convergence. Comparing the changes in the overall caste wage gap for $\underline{c} = 0$ and $\underline{c} = 0.5$ for common sectoral growth under scaled skilling costs shows that the predicted changes in the wage gaps are quite similar (0 and -1.41 percent, respectively). Similarly, the predicted changes under $\underline{c} = 0$ and $\underline{c} = 0.5$ for common sectoral growth when skilling costs are not scaled are also very similar (-6.87% and -6.44%).

Table 7: Assessment of Different Mechanisms

		Percentage Growth Rate 1983-2012							
		Common Growth				Differential Growth			
		Scaled skill costs		Unscaled skill costs		Scaled skill costs		Unscaled skill costs	
Variable	Data	$\underline{c} = 0$	$\underline{c} = 0.5$	$\underline{c} = 0$	$\underline{c} = 0.5$	$\underline{c} = 0$	$\underline{c} = 0.5$	$\underline{c} = 0$	$\underline{c} = 0.5$
Δs^a	-1.52%	0%	4.72%	2.51%	5.73%	2.09%	5.66%	3.84%	6.21%
Δs^m	9.37%	0%	-21.41%	6.18%	-14.04%	4.77%	-15.87%	17.46%	-3.16%
Δs^h	-24.37%	0%	-15.92%	-27.43%	-35.91%	-24.77%	-34.03%	-35.50%	-42.09%
Δw^a	4.12%	0%	-0.08%	0%	-0.11%	0.01%	-0.10%	0.01%	-0.12%
Δw^m	-5.00%	0%	0.34%	0.12%	0.34%	0.11%	0.35%	0.16%	0.33%
Δw^h	-8.19%	0%	0.01%	0%	0%	0.05%	0%	0.11%	0%
Δw	-10.51%	0%	-1.41%	-6.87%	-6.44%	-6.09%	-5.91%	-9.28%	-8.01%

Notes: The table reports the percent changes in sectoral caste gaps in employment and wages between 1983 and 2012. $\Delta s^j,\ j = a, m, h$ is the ratio of the fraction of all of all non-SC/STs working in sector j to the fraction of all SC/STs working in sector j. $\Delta w^j,\ j = a, m, h$ is the ratio of the mean non-SC/ST to mean SC/ST wage in sector j. Δw is the ratio of the mean non-SC/ST to mean SC/ST wage.

These results do not imply that homothetic preferences are irrelevant in the model. As can be seen from the column for $\underline{c} = 0$ in the common sectoral growth rate panel, both labor and wage caste gaps remain unaffected by growth when preferences are homothetic. Since all sectors expand proportionately under balanced growth, and demand grows proportionately as well due to homotheticity, relative sectoral prices remain unchanged in this case. This leaves the caste-specific ability cut-offs for switching sectors unaffected by growth. Consequently, caste gaps are invariant to growth. Introducing non-homotheticity changes this as growth induces unbalanced demand increases across sectors. The resultant changes in sectoral relative prices affect the ability thresholds but it only induces a small reduction in the overall caste wage gap. Hence, the quantitative effect of non-homotheticity is small.

6.3.3 Differential Sectoral Labor Productivity Growth

Table 7 also illustrates the key role played by differential sectoral productivity growth in inducing the wage convergence. Even with homothetic preferences and scaled skilling costs, the observed sectoral productivity growth rates imply a 6.09% reduction in the overall caste wage gap. In addition to the differential sectoral productivity growth, if we also remove the scaling of skilling costs and introduce non-homothetic preferences, the caste wage gap declines by an additional 1.9 percentage points, taking the overall decrease of the wage gap to 8.01%. Clearly, differential sectoral productivity growth seems to be key for the caste convergence.

6.3.4 Scaling of Skilling Costs

In the quantitative implementation of the model, we did not scale up the costs of skilling with productivity growth in the agriculture, manufacturing and services (see the discussion following Assumption 1" above). The non-scaling effectively reduces relative skilling costs when productivity grows in the other three sectors. This mechanism acts as a form of endogenous propagation of exogenous productivity shocks.

Table 7 shows that unscaled skilling costs do have quantitatively strong effects on the overall caste convergence. Comparing the results for scaled and unscaled skilling costs for otherwise identical growth and homotheticity specifications of the model shows that there is a sharp increase in the wage convergence when skilling costs remain unscaled. Clearly, this endogenous propagation mechanism is quantitatively strong.

7 Counterfactual experiments

Having described the fit of the model to the 1983 sectoral labor and wage gaps across castes, we now turn to the issue of identifying the key drivers of the observed dynamics in these gaps. Two questions are of particular interest to us. We examine them below.

7.1 Re-sorting of workers

A key narrative surrounding the issue of castes in India is that it often acts as a barrier to entry into different occupations and sectors. In our model, productivity changes induce a sectoral re-sorting of workers based on their cost of skill acquisition and sectoral wages. How important was this re-sorting for the declining caste gaps?

We assess the importance of sectoral sorting by workers by conducting a counterfactual experiment wherein we subject the model to the same observed sectoral productivity shocks as in the baseline case but keep the sectoral labor choice of workers of all castes fixed at the 1983 values. Hence, the only changes over time in worker wages under this experiment will be due to changes in sectoral productivity since the sectoral worker composition remains unchanged.

The last column of Table 8 shows the the sectoral labor and wage gaps along with the overall caste wage gap in 2012 when sectoral allocations are frozen at the 1983 levels. The last column and row of the table reveals that without any labor re-sorting across sectors, sectoral productivity growth alone would have induced a *widening* of the relative caste wage gap from 1.27 to 1.30. The reason for this is that productivity growth was faster in the non-agricultural sectors where SC/STs were under-represented in 1983. Without the possibility of reallocation of SC/ST labor towards the faster growing sectors, the overall wage gap increases even though each sectoral wage gap remains unchanged in this case since the relative worker abilities of the two types in each sector remains constant.

This result shows the importance of re-sorting of workers across sectors for the caste wage convergence. The importance of growth for narrowing inter-group inequality arises due to the fact that it encourages workers to re-optimize their sector of employment in accordance with comparative advantage based on their ability.

7.2 Importance of Affirmative Action Policies

As we noted in the introduction to the paper, India has had affirmative action protection enshrined in the constitution with reservations extended to SC/STs in seats in public institutions of tertiary education, in public sector employment and in political representation. In terms of the model developed in this paper, our proxy for reservations for SC/STs is the lower cost of getting skilled in order to access manufacturing and service sector employment. How important were these reservation policies for the observed caste convergence between 1983 and 2012?

We examine the importance of reservations by making one change to the baseline model. Specifically, we set $\frac{\gamma_n^m}{\gamma_s^m} = \frac{\gamma_n^h}{\gamma_s^h} = 1$ while leaving γ_n^m and γ_n^h at their baseline levels. In other words, we

Table 8: Counterfactual Experiments

	1983			2012			
Variable	Data	Baseline	Equal costs	Data	Baseline	Equal costs	No re-sorting
Δs^a	0.80	0.89	0.67	0.79	0.94	0.65	0.89
Δs^m	1.43	1.40	∞	1.57	1.35	2.05	1.40
Δs^h	1.61	1.68	∞	1.21	0.98	∞	1.68
Δw^a	1.04	1.04	0.94	1.08	1.04	0.92	1.04
Δw^m	1.20	1.25	∞	1.14	1.25	1.07	1.25
Δw^h	1.45	1.32	∞	1.33	1.32	∞	1.32
Δw	1.45	1.27	1.54	1.30	1.17	1.31	1.30

Notes: The table reports the sectoral caste gaps in employment and wages in 1983 and 2012 in the data and under different special cases of the model. $\Delta s^j,\ j = a, m, h$, is the ratio of the fraction of all of all non-SC/STs working in sector j to the fraction of all SC/STs working in sector j. $\Delta w^j,\ j = a, m, h$ is the ratio of the mean non-SC/ST to mean SC/ST wage in sector j. Δw is the ratio of the mean non-SC/ST to mean SC/ST wage.

raise the fixed cost component of skilling costs for SC/STs to non-SC/STs levels in each sector thereby eliminating the advantage of reservations for SC/STs. All the other baseline calibration parameters are left unchanged.

The column "Equal cost" in the left panel of Table 8 shows the effect of this re-calibration on the computed moments for 1983. SC/ST share of employment in the non-agricultural sectors goes to zero as they all switch to the agricultural sector. Compared to the baseline, the caste wage gap in agriculture declines as a result of SC/STs with *relatively* higher ability than the incumbent SC/STs switching to agriculture. The net effect of this re-sorting of workers is that the overall caste wage gap widens to 1.54. This is a 21 percent increase in the caste wage gap relative to the baseline case.

The effects of equalizing skilling costs on the caste gaps in 2012 are shown in the right panel of Table 8 under the column "Equal Costs". The sectoral productivity growths during 1983-2012 does make manufacturing employment attractive for some higher ability SC/STs in 2012. However, the services sector continues to remain unpopulated by SC/STs. The overall caste wage gap in 2012 is 1.31, which is 11 percent higher than the wage gap of 1.17 in the baseline case.

The higher caste wage gaps in 1983 and 2012 predicted by the model due to equalizing skilling costs across castes suggest that reservations in higher education did succeed in reducing caste wage gaps. The policy likely worked by inducing some relatively higher ability SC/STs to take advantage of the lower education costs and move out of agriculture to take up employment in the non-agricultural sectors with higher wages. Without these reservations in higher education, the caste wage gaps would have been higher at all points.

We should note that it is slightly tricky to compare the reductions in the caste wage gaps between 1983 and 2012 implied by the model with and without reservations. In the absence of reservations, the SC/STs completely exit the manufacturing and services sectors in 1983. This lowers their mean wage a lot in 1983 and thus raises the implied percentage decrease in the caste wage gap during 1983-2012 due to this base effect.

The impact of base effects on assessing the dynamic effects of reservations can be seen through three different comparisons. If one compares caste wage gaps in 1983 under equal skilling costs ($\Delta w = 1.54$) with the corresponding gap in 2012 with equal skilling costs ($\Delta w = 1.31$), the implied reduction in the caste wage gap between 1983 and 2012 is 14.9 percent.

On the other hand, if we compare the caste wage gap in 1983 in the baseline case with reservations ($\Delta w = 1.27$) with the caste gap in 2012 with equal skilling costs ($\Delta w = 1.31$), the implied reduction in the caste wage gap is -3.1 percent. The latter experiment corresponds to a counterfactual where 2012 differs from 1983 along two dimensions: sectoral labor productivity and skilling costs. It is thus a joint shock which results in a widening of the overall caste wage gap. Effectively, removing reservations in education more than offset the wage gains of SC/STs due to higher sectoral productivities.

The main takeaway from these "equal costs" counterfactual experiments is that for given productivity levels, removal of reservations for SC/STs induce an increase in the caste wage gap. In this sense, the educational reservations policy in India that have been in effect since the 1950s have indeed raised the average wages of SC/STs by facilitating their access to higher wage employment.

8 Conclusion

The past three decades have seen a significant convergence in the education attainments, occupation choices and wages of scheduled castes and tribes (SC/STs) in India to toward the corresponding levels of non-SC/STs. In this paper we have examined the possibility that the large aggregate changes that were occurring in the Indian economy at this time may have jointly contributed to both the caste convergence as well as the large scale structural transformation in the economy observed in the data.

Using a multi-sector, heterogenous agent model we find that sectoral labor productivity increases can account for 75 percent of the observed wage convergence between SC/STs and non-SC/STs. Importantly, the model generates sectoral labor shares and price dynamics that are consistent with the data for this period. Our statistical decomposition of the overall caste wage gap which identified the decline in the caste labor gap in services as a key component of the overall wage convergence. The quantitative model mimics this feature.

While the model incorporates both non-homothetic preferences and differential sectoral productivity growth as potential drivers of structural transformation, we find differential sectoral productivity growth to be key for both the aggregate and caste gap dynamics.

Our counterfactual experiments on the model suggest that the main mechanism driving the caste convergence was worker re-sorting across sectors. SC/STs, who were severely under-represented in the manufacturing and service sectors, responded to faster productivity growth by switching employment disproportionately into the higher paying service sector. Absent this re-sorting, the caste wage gap would have actually widened despite the productivity growth. We also find that the net flows of workers across sectors account for 75 percent of the wage convergence generated by the model while selection effects account for just a quarter of the convergence.

We also find that affirmative action policies that lowered the relative costs of acquiring skills for SC/STs may have played an important role in raising SC/ST wages by facilitating the training and entry of relatively higher ability SC/STs into non-agricultural sectors. Absent these policies, SC/ST representation in the manufacturing and services sectors would have been much lower. Consequently, the overall caste wage gap would have been much larger in both 1983 and 2012.

It bears stressing however that our baseline results show that even *without any change* in affirmative action policies, growth not only raises the wages of all groups but also benefits SC/STs marginally more as the caste wage gap falls. In a world with no education reservations at any point, productivity growth reduces the caste wage gap by 14.9 percent. On the other hand, our baseline model, which has reservations in place throughout, predicts a 8.1 percent decrease in the caste wage gap in response to sectoral productivity growth. In other words, growth appears to lift all boats, with or without reservations.

A key primitive in our model is the ability distribution. In a fully dynamic intergenerational model, the ability distribution for the young would endogenously evolve in response to human capital bequests from parents which, in turn, would depend on parents' human capital. Policies that facilitate access to education for disadvantaged groups would consequently affect the ability distribution of future generations both directly and indirectly. We intend to return to this issue in future work.

The model that we have explored here focused specifically on the costs of acquiring specialized skills in order to acquire non-agricultural employment. This focus is intended to ascertain the effects of reservations in higher education for SC/STs which have been provided by the Indian Constitution. The constitution of India also reserves a fraction of public sector jobs for SC/STS. We have not focused on that margin here since public sector employment comprises a small share of overall employment in India.

An important mechanism that we have ignored here is changes in upper caste discrimination against SC/STs. As is well known from the work of Becker (1957), rising competition can make discrimination more expensive for business owners. This could reduce barriers for SC/STs and the caste gaps. To examine this hypothesis we would need to conduct a detailed analysis of more disaggregated sectoral data to both document changes in the degree of competition and changes in employment patterns. We hope to examine this in future work.

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9 Appendix

9.1 Data

Table 9 summarizes one-digit industry codes in our dataset. In the presentation in the text we group these codes further into three broad industry categories: Ind 1 refers to Agriculture, Hunting, Forestry and Fishing; Ind 2 collects Manufacturing and Mining and Quarrying; while Ind 3 refers to all Service industries. These groupings are detailed in Table 9.

Table 9: Industry categories

Industry code	Industry description	Group
A	Agriculture, Hunting and Forestry	Ind 1
B	Fishing	Ind 1
C	Mining and Quarrying	Ind 2
D	Manufacturing	Ind 2
E	Electricity, Gas and Water Supply	Ind 3
F	Construction	Ind 3
G	Wholesale and Retail Trade; Repair of Motor Vehicles, motorcycles and personal and household goods	Ind 3
H	Hotels and Restaurants	Ind 3
I	Transport, Storage and Communications	Ind 3
J	Financial Intermediation	Ind 3
K	Real Estate, Renting and Business Activities	Ind 3
L	Public Administration and Defence; Compulsory Social Security	Ind 3
M	Education	Ind 3
N	Health and Social Work	Ind 3
O	Other Community, Social and Personal Service Activities	Ind 3
P	Private Households with Employed Persons	Ind 3
Q	Extra Territorial Organizations and Bodies	Ind 3

9.2 Model: Optimality Conditions and Aggregation

9.2.1 Household optimality

The optimality conditions governing consumption of the three goods for household i are

$$\frac{\eta}{\theta}\left(\frac{c_i^a-\bar{c}}{c_i^m}\right)=p_m$$
$$\frac{(1-\eta-\theta)}{\theta}\left(\frac{c_i^a-\bar{c}}{c_i^h}\right)=p_h$$

Using these solutions along with the household's budget constraint gives

$$c_i^a=\bar{c}+\theta\left(\hat{y}_i-\bar{c}\right)$$
$$p_m c_i^m=\eta\left(\hat{y}_i-\bar{c}\right)$$
$$p_h c_i^h=(1-\eta-\theta)\left(\hat{y}_i-\bar{c}\right)$$

9.2.2 Aggregation

There are six variables to aggregate – aggregate consumption of the three types of goods as well as their aggregate productions. We start with the consumption side. Aggregate consumption of each good is the sum of the consumptions of the good by each type of household. Recall that there are three types of households in this economy and that each household opts into only one of the three available occupations. The solution for c_i^a derived above says that

$$c_i^a = (1-\theta)\,\bar{c} + \theta \hat{y}_i$$

Using the net income of each type of worker, $\hat{y}_i$, aggregate consumption of sector a goods are given by

$$c_a = \theta \left[\sum_{j=s,n} s_j \left[\int_{\underline{e}_j}^{\hat{e}_j^m} Ae_i dG_j(e) + p_m \int_{\hat{e}_j^m}^{\hat{e}_j^h} \left\{ Me_i - f_j^m(e_i) \right\} dG(e) + \int_{\hat{e}_j^h}^{\bar{e}_j} \left\{ p_h He_i - p_m f_j^h(e_i) \right\} dG_j(e) \right] \right] \tag{9.14}$$

$$+ L(1-\theta)\,\bar{c}$$

where $s_s = S$, and $s_n = N$ represent the population sizes of the two two castes. Similarly, we have $p_m c_{mi} = \eta(\hat{y}_i - \bar{c})$ which when aggregated across all households gives

$$p_m c_m = \eta \left[\sum_{j=s,n} s_j \left[\int_{\underline{e}_j}^{\hat{e}_j^m} Ae_i dG_j(e) + p_m \int_{\hat{e}_j^m}^{\hat{e}_j^h} \left\{ Me_i - f_j^m(e_i) \right\} dG(e) + \int_{\hat{e}_j^h}^{\bar{e}_j} \left\{ p_h He_i - p_m f_j^h(e_i) \right\} dG_j(e) \right] \right] \tag{9.15}$$

$$- L\eta\bar{c}$$

Lastly, since $p_h c_{hi} = (1-\eta-\theta)(\hat{y}_i - \bar{c})$, aggregate consumption of the h good is

$$p_h c_h = (1-\eta-\theta) \left[\sum_{j=s,n} s_j \left[\int_{\underline{e}_j}^{\hat{e}_j^m} Ae_i dG_j(e) + p_m \int_{\hat{e}_j^m}^{\hat{e}_j^h} \left\{ Me_i - f_j^m(e_i) \right\} dG(e) + \int_{\hat{e}_j^h}^{\bar{e}_j} \left\{ p_h He_i - p_m f_j^h(e_i) \right\} dG_j(e \right. \right.$$

$$- L(1-\eta-\theta)\,\bar{c} \tag{9.16}$$

Next, the expected output of a worker i of caste j in each sector is given by

$$\bar{w}_j^a = Ey_{ji}^a = \int_{\underline{e_j}}^{\hat{e}_m^j} Ae_i \frac{dG_j(e)}{G_j\left(\hat{e}_j^m\right)} \tag{9.17}$$

$$\bar{w}_j^m = Ey_{ji}^m = \int_{\hat{e}_m^j}^{\hat{e}_h^j} Me_i \frac{dG_j(e)}{G_j\left(\hat{e}_j^h\right) - G_j\left(\hat{e}_j^m\right)} \tag{9.18}$$

$$\bar{w}_j^h = Ey_{ji}^h = \int_{\hat{e}_h^j}^{\bar{e}_j} He_i \frac{dG_j(e)}{1 - G_j\left(\hat{e}_j^h\right)} \tag{9.19}$$

Given the linearity of the production technologies in ability, these are just the conditional means of the relevant distributions of ability in each sector.

Using these expected outputs, the aggregate output of each sector is

$$y^k = \sum_{j=s,n} y_j^k \quad , \quad k = a, m, h$$

where k indexes the sector. Clearly, $y_j^a = s_j G_j\left(\hat{e}_j^m\right)\bar{w}_j^a$, $y_j^m = s_j\left[G_j\left(\hat{e}_j^h\right) - G_j\left(\hat{e}_j^m\right)\right]\bar{w}_j^m$ and $y_j^h = s_j\left[1 - G_j\left(\hat{e}_j^h\right)\right]\bar{w}_j^h$, where $j = s, n$ and $s_j = S, N$. Substituting in the relevant expressions gives

$$y^a = S\int_{\underline{e}_s}^{\hat{e}_s^m} Ae_i dG_s(e) + N\int_{\underline{e}_n}^{\hat{e}_n^m} Ae_i dG_n(e) \tag{9.20}$$

$$y^m = S\int_{\hat{e}_s^m}^{\hat{e}_s^h} Me_i dG_s(e) + N\int_{\hat{e}_n^m}^{\hat{e}_n^h} Me_i dG_n(e) \tag{9.21}$$

$$y^h = S\int_{\hat{e}_s^h}^{\bar{e}_s} He_i dG_s(e) + N\int_{\hat{e}_n^h}^{\bar{e}_n} He_i dG_n(e) \tag{9.22}$$

Note that the aggregation above represents case (b) of Figure 1 above where $\hat{e}_j^h > \hat{e}_j^m$. The limits of integration would have to be altered appropriately for the opposite configuration.

To derive the skill acquisition costs, note that the average costs of skill acquisition by caste j conditional on the sector of employment is

$$F_j^m = \int_{\hat{e}_j^m}^{\hat{e}_j^h} f_j^m(e_i)\frac{dG_j(e)}{G_j\left(\hat{e}_j^h\right) - G_j\left(\hat{e}_j^m\right)}$$

$$F_j^h = \int_{\hat{e}_j^h}^{\bar{e}_s} f_j^h(e_i)\frac{dG_j(e)}{1 - G_j\left(\hat{e}_j^h\right)}$$

Hence, total skill acquisition cost of caste j is

$$F_j = s_j \left[\left\{ G_j\left(\hat{e}_j^h\right) - G_j\left(\hat{e}_j^m\right) \right\} F_j^m + \left\{ 1 - G_j\left(\hat{e}_j^h\right) \right\} F_j^h \right], \quad j = s, n$$

where $s_s = S$ and $s_n = N$. Summing the costs across the castes then gives the total cost of acquiring skills by the different groups as

$$F = S\left[\int_{\hat{e}_s^m}^{\hat{e}_s^h} f_s^m\left(e_i\right) dG_s\left(e\right) + \int_{\hat{e}_s^h}^{\bar{e}_s} f_s^h\left(e_i\right) dG_s\left(e\right)\right] + N\left[\int_{\hat{e}_n^m}^{\hat{e}_n^h} f_n\left(e_i\right) dG_n\left(e\right) + \int_{\hat{e}_n^h}^{\bar{e}_n} f_n^h\left(e_i\right) dG_s\left(e\right)\right] \tag{9.23}$$

These relationships can then be used in equations (4.4-4.6) to derive the specific market clearing conditions for the a, m, and h goods. Note that only two of the three market clearing conditions are free - if two markets clear then the third must clear as well.

The equilibrium for the economy can be reduced to a system of four equations in four unknowns, $\hat{e}_s^m, \hat{e}_s^h, \hat{e}_n^m, \hat{e}_n^h$. The four equations that jointly determine these variables are two out of the three market clearing conditions along with the conditions $z_s^m\left(\hat{e}_s^m\right) = z_n^m\left(\hat{e}_n^m\right)$ and $z_s^h\left(\hat{e}_s^h\right) = z_n^h\left(\hat{e}_n^h\right)$. Note that the prices p_m and p_h can be eliminated from this system of equations by using any two of the four threshold conditions $z_j^m\left(\hat{e}_j^m\right) = M - \frac{A}{p_m}$ and $z_j^h\left(\hat{e}_j^h\right) = \frac{p_h}{p_m}H - \frac{A}{p_m}, \quad j = s, n.$