An elementary mathematical model for MMT (Modern Monetary Theory)

Tanaka, Yasuhito

2 August 2021

An elementary mathematical model for MMT (Modern Monetary Theory)

Yasuhito Tanaka

Faculty of Economics, Doshisha University, Kamigyo-ku, Kyoto, 602-8580, Japan
E-mai: yatanaka@mail.doshisha.ac.jp

Abstract

In recent years, a school of economics called MMT (Modern Monetary Theory) has been attracting attention, but it has not been analyzed theoretically or mathematically. This study aims to provide a theoretical basis for the skeleton of the MMT argument, while maintaining the basics of the neoclassical microeconomic framework, such as utility maximization of consumers by means of utility functions and budget constraint, profit maximization of firms in monopolistic competition, and equilibrium of supply and demand of goods. Using a simple static model that includes economic growth due to technological progress, we will argue that: 1) a continuous budget deficit is necessary to maintain full employment when the economy is growing, and that this deficit does not have to be covered by future surpluses; 2) Inflation is caused when the actual budget deficit exceeds the level necessary and sufficient to maintain full employment. In order to avoid further inflation, it is necessary to maintain a certain level of budget deficit; 3) A shortfall in the budget deficit leads to recession and involuntary unemployment. To recover from this, a budget deficit that exceeds the level necessary to maintain full employment is required. However, since a continuous budget deficit is necessary after full employment is restored, the deficit created to overcome the recession does not need to be covered by future budget surpluses, nor should it be.

Keywords: MMT, Economic growth, Budget deficit, Inflation

1. Introduction

The outstanding amount of government bonds in Japan is over 900 trillion yen, which is said to be in a critical situation. On the other hand, there is a view that there is no problem in accumulating budget deficits if the debt is not owed to foreign countries, and that fiscal policy should be evaluated only in terms of its effects, such as preventing inflation and achieving full employment and stable economic growth. The so-called Functional Finance Theory by Lerner (1943, 1944), a famous economist, is one such theory. MMT (Modern Monetary Theory, Wray (2015), Mitchell, Wray and Watts (2019) and Kelton(2020)), which has spread in the U.S. in recent years and has become a hot topic in Japan, is another representative example, and the proponents of MMT themselves admit that Lerner's theory is the source of their thinking. However, it is often pointed out that MMT lacks theoretical analysis based on mathematical models compared to mainstream economics, which is based on a neoclassical framework.

In this paper, we try to argue positively for the idea of the effect of fiscal policy, which is the backbone of functional finance theory and MMT's argument, using a simple mathematical model, while maintaining the basics of the neoclassical microeconomic framework, such as utility maximization of consumers by utility function and budget constraint, profit maximization of firms in monopolistic competition, and equilibrium of supply and demand of goods.

Specifically, we will argue the following points.

1) In order to maintain full employment when the economy is growing, it is necessary to run a continuous budget deficit, which does not have to be covered by future surpluses.

2) Inflation is caused when the actual budget deficit exceeds the level necessary and sufficient to maintain full employment. In order to prevent further inflation, it is necessary to maintain a stable and constant budget deficit.

3) Insufficient budget deficit causes recession and involuntary unemployment. To recover from this, a budget deficit that exceeds the level necessary to maintain full employment is required. However, since a continuous budget deficit is necessary after full employment is restored, the deficit created to overcome the recession does not need to be covered by future budget surpluses, nor should it be.

As will be discussed in Section 2.2, fiscal expenditures create demand for goods, while taxes play a role in reducing consumer income (disposable income) and suppressing consumption, and are not a source of revenue for fiscal expenditures. The size of the budget deficit should be considered from the perspective of adjusting and managing aggregate demand.

2. The model

A static model is used, but economic growth due to technological progress is included. Taxes will be proportional to income rather than a lump-sum tax. First, we analyze the behavior of consumers, the government, and firms in a given period. We assume that consumers live, work, consume, and save for only one period, and that they inherit the savings left by the previous generation. The productivity of the economy grows at a constant rate due to technological progress.

2.1 Consumers

Consumers employed by firms and unemployed consumers consume and save based on the income they earn. The first step is to find the solution to utility maximization by a consumption basket of the goods and savings, and the second step is to maximize the consumption basket of the goods given expenditures[1]. We denote the consumption basket of an employed consumers by C^e, his savings by S^e, disutility of labor by $\ln\beta$, consumption basket of an unemployed consumer by C^u, and his savings by S^u. Labor supply is inslastic, that is, it is 1 or 0. The unemployed consumer also wishes to work if the utility of being employed is greater, but if there is not enough demand for labor, this wish will not be realized and the consumer will become involuntarily unemployed.

Let P be the price of the consumption basket, w be the nominal wage rate, L be the employment level. Denote the total labor suppy, or the employent level at the full employment state by L_f. Let Π be the profit distributed from firms to a consumer. The profit is received by both employed and unemployed consumers.

The consumption baskets for employed and unemployed consumer consist of consumtion of each good, c_i^e, c_i^u, which exists on the continuum $[0,1]$. They are written as

$$C^e=\left(\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}},\ C^u=\left(\int_0^1 (c_i^u)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}.$$

σ is the elasticity of substitution among consumption goods. $\sigma>1$. The price of the consumption basket, P, is represented as

$$P=\left(\int_0^1 p_i^{1-\sigma}di\right)^{\frac{1}{1-\sigma}},$$

where p_i is the price of each good.

The utility of an employed consumer is reprsented by the following function.

$$U^e=\alpha\ln C^e+(1-\alpha)\ln\frac{S^e}{P}-\ln\beta.$$

$\beta>1$. His budget constraint is

$$PC^e+S^e=w+\Pi-t(w+\Pi)+M=(1-t)(w+\Pi)+M.$$

t>0 is the tax rate. M represents the per capita savings inherited from the previous generation of consumers. Taxes are assumed to be paid at the same rate for both employed and unemployed consumers.

Similarly, the utility of an unemployed consumer is represented by the following function.

$$U^u=\alpha\ln C^u+(1-\alpha)\ln\frac{S^u}{P}.$$

His budget constraint is

$$PC^u+S^u=(1-t)\Pi+M.$$

From the first order conditions and the budget constraint for employed and unemployed consumers, we obtain the following demand functions for the consumption baskets and the savings function.

$$C^e=\frac{\alpha[(1-t)(w+\Pi)+M]}{P},\ S^e=(1-\alpha)[(1-t)(w+\Pi)+M],$$

$$C^u=\frac{\alpha[(1-t)\Pi+M]}{P},\quad S^u=(1-\alpha)[(1-t)\Pi+M].$$

It may seem impossible that the unemployed, whose only income is the distribution of profits, would save, but if utility is determined by consumption and savings, then they will certainly save. It is

[1] The calculation can also be done in one step instead of two steps. The appendix contains a rather detailed calculation process.

possible to incorporate unemployment insurance and other mechanisms to support the unemployed into the model, but the essence of the argument remains the same.

Given the expenditure in the second step the Lagrange functions for maximization of consumption baskets are

$$\mathcal{L}^e = \left(\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} - \lambda^e \left[\int_0^1 p_i c_i^e di - \alpha[(1-t)(w+\Pi)+M]\right],$$

and

$$\mathcal{L}^u = \left(\int_0^1 (c_i^u)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} - \lambda^u \left[\int_0^1 p_i c_i^u di - \alpha[(1-t)\Pi+M]\right].$$

λ^e and λ^u are Lagrange multipliers. Solving these maximization problems, we get the following demand functions for each good of employed and unemployed consumers.

$$c_i^e = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{\alpha[(1-t)(w+\Pi)+M]}{P},$$

and

$$c_i^u = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{\alpha[(1-t)\Pi+M]}{P}.$$

2.2 The government

The government purchases each good so as to maximize the consumption basket through fiscal expenditure. Let g_i be the amount of the good purchased by the government. The consumption basket of the government is

$$\left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}.$$

Let *G* be the fiscal expenditure. Then,

$$\int_0^1 p_i g_i di = G,$$

and, the Lagrange function for the government is

$$\mathcal{L}_G = \left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} - \lambda_G \left(\int_0^1 p_i g_i di - G\right).$$

λ_G is the Lagrange multiplier. Similarly to the case of consumers, solving this maximization problem[2], the demand for each good by the government is obtained as follows;

$$g_i = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{G}{P}.$$

It should be noted that it is the total amount of fiscal expenditure, G, that constrains the purchase of goods by the government, not the tax revenue. As mentioned earlier, fiscal spending creates demand for goods, while taxes have, or only have, the role of reducing demand by reducing consumer income. Taxes are not a source of financing for fiscal spending.

2.3 Firms

The number of firms is set to 1, assuming a short run of time with no entry or exit of firm (this does not mean that there is only one firm, but that there are countless small firms, the sum of which is 1).

Each good i exists on the continuum [0, 1] and each firm produces one consumption good among them. Each firm is a monopoly about its good, but there are countless alternative. It decides the price of the good it produces given the price *P* of the consumption basket of the goods. Production is done

[2] The last part of the appendix explains this calculation in some detail.

solely by labor, and each unit of output has an additional cost of one. This is due to productivity in the reference period, and productivity in the next period will increase with technological progress. Let y_i be the output of Firm i. Then, the labor demand of this firm is

$$l_i = y_i.$$

This is common to all firms.

Government spending also creates a demand for goods. Let d_i be the demand for the good of Firm $i. d_i$ is the sum of the demand of the consumers and demand of the government. Thus, it is

$$d_i = Lc_i^e + (L_f - L)c_i^u + g_i = \left(\frac{p_i}{P}\right)^{-\sigma}\left(\frac{\alpha[(1-t)(wL+L_f\Pi)+L_fM]+G}{P}\right).$$

Since c_i^e and c_i^u are, respectively, the demand of an employed consumer and an unemployed consumer, they are multiplied by L or $L_f - L$. The proft of Firm i is

$$\pi_i = p_i y_i - w y_i = (p_i - w)\left(\frac{p_i}{P}\right)^{-\sigma}\left(\frac{\alpha[(1-t)(wL+L_f\Pi)+L_fM]+G}{P}\right).$$

At the equilibrium we have $d_i = y_i$. The first order condition for profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = \left(\frac{p_i}{P}\right)^{-\sigma}\left(\frac{\alpha[(1-t)(wL+L_f\Pi)+L_fM]+\mathrm{G}}{P}\right) - \sigma(p_i - w)\left(\frac{p_i^{-\sigma-1}}{P^{-\sigma}}\right)\left(\frac{\alpha[(1-t)(wL+L_f\Pi)+L_fM]+\mathrm{G}}{P}\right) = 0.$$

From this we get

$$p_i = \frac{\sigma}{\sigma-1}w.$$

Let $\mu = \frac{1}{\sigma}$. Then,

$$p_i = \frac{1}{1-\mu}w.$$

Since all firms are symmetric,

$$P = p_i = \frac{1}{1-\mu}w. \tag{1}$$

The outputs y_i's of all firms are equal. Let denote them by Y.

Since the revenue of a good is equal to the sum of the wages for labor and the distribution of profits , we obtain

$$PY = wY + L_f\Pi.$$

From this,

$$L_f\Pi = (P - w)Y.$$

By (1),

$$L_f\Pi = [P - (1-\mu)P]Y = \mu PY.$$

We have $Y = L$, and $Y = L_f$ at the full employment state.

When (1) holds, the total nominal demand is

$$\alpha[(1-t)\big(wL + L_f\Pi\big) + L_fM] + G = \alpha[(1-t)PL + L_fM] + G.$$

2.4 Firm behavior after economic growth

After the economy grows at the rate $\gamma - 1$, the relation between the output and the labor demand is

$$\gamma l_i = y_i.$$

The nominal wage rate increases to γw, and the profit of Frim i is

$$\pi_i = p_i y_i - \gamma w l_i = p_i y_i - w y_i.$$

This is the same shape as before growth. The price is

$$P = p_i = \frac{1}{1-\mu}w.$$

w is not the nominal wage rate after growth, but it is the nominal wage rate in the reference period. If inflation occurs, it will also increase the nominal wage rate, which will be reflected in prices, but increase in the nominal wage rate due to productivity growth will not increase the cost per unit of output and therefore will not affect prices. The output Y is γL. At the full employment state, it is γL_f. If the nominal wage rate, the profit of a firm, the savings and the fiscal expenditure increase at the rate γ, the total nominal demand after growth is equal to

$$\alpha[(1-t)\gamma(wL+L_f\Pi)+\gamma L_f M]+G=\alpha[(1-t)P\gamma L+\gamma L_f M]+\gamma G.$$

3. Budget deficit for full employment under economic growth

In this section we consider budget deficit to maintain full employment under economic growth with constant prices. If full employment is achieved, the employment and the output of a firm is equal to L_f, and the demand and supply of the goods satisfy the following equation.

$$\alpha[(1-t)PL_f+L_f M]+G=PL_f. \qquad (2)$$

Then, the savings of a consumer is

$$(1-\alpha)[(1-t)PL_f+L_f M].$$

Since the economy is growing at a rate of $\gamma-1>0$ due to technological progress while maintaining full employment under constant prices, this savings must satisfy the following condition.

$$(1-\alpha)[(1-t)PL_f+L_f M]=\gamma L_f M \qquad (3)$$

Adding (2) and (3) together, we get

$$(1-t)PL_f+L_f M+G=PL_f+\gamma L_f M.$$

From this we obtain

$$G-tPL_f=(\gamma-1)L_f M. \qquad (4)$$

Since tPL_f is tax revenue, the left-hand side of (4) is budget deficit. (4) means that if $\gamma>1$, that is, the economy grows at a positive rate, the budget deficit should be positive. We get the following propositin.

Proposition 1 In an economy that grows at a positive rate through technological progress, a continuous budget deficit is necessary to maintain full employment at constant prices.

The budget deficit is equal to the increase in savings due to growth. Since this budget deficit is necessary as long as the economy is growing, there is no need to make up for it later with a surplus, and this should not be done.

Although (2) does not explicitly include the rate of economic growth, we can assume that the output in the previous period was $\frac{1}{\gamma}$ times the current period's output (and thus production costs were γ times the current period's costs). If the economy is growing at constant prices, the nominal wage rate will increase at the same rate as the growth rate, reflecting the increase in productivity.

4. Excessive budget deficits and inflation

Suppose that the budget deficit has been satisfying (4) until the previous period, and full employment has been achieved under constant prices, and in a certain period the budget deficit is G' which is larger than the budget deficit satisfying (4), and the price is P'. They satisfy

$$G' - tP'L_f = (\zeta - 1)L_f M > (\gamma - 1)L_f M. \quad (5)$$

We will show that in this case inflation will occur. (2) is rewritten as

$$\alpha[(1-t)P'L_f + L_f M] + G' = P'L_f. \quad (6)$$

Let M' be the savings of consumers. Since

$$L_f M' = (1-\alpha)[(1-t)P'L_f + L_f M], \quad (7)$$

(6) means

$$L_f M' = G' - tP'L_f + L_f M.$$

Further, since from (5)

$$L_f M' - L_f M = (\zeta - 1)L_f M,$$

we have

$$L_f M' = \zeta L_f M.$$

From (3)

$$\gamma L_f M = (1-\alpha)[(1-t)PL_f + L_f M], \quad (8)$$

and

$$M = \frac{(1-\alpha)(1-t)P}{\gamma - 1 + \alpha}.$$

With the inflation rate $\rho - 1$, (7) and (8) mean

$$(1-\alpha)(1-t)(\rho - 1)P = (\zeta - \gamma)M.$$

Since $\zeta > \gamma$ implies $\rho > 1$, inflation occurs. Also,

$$\rho = \frac{\zeta - 1 + \alpha}{\gamma - 1 + \alpha}.$$

We obtain the following proposition.

Proposition 2 In an economy that grows at a constant rate through technological progress while maintaining full employment, inflation will occur if the budget deficit is larger than the level necessary and sufficient to sustain full employment under growth with constant prices.

If $0 < \alpha < 1$, ρ is smaller than $\frac{\zeta}{\gamma}$. The reason for this is that the savings inherited from the previous generation are not affected by price increases. In addition, when inflation occurs, the nominal wage rate and the prices increase at the same rate compared to the case without inflation.

If the budget deficit becomes excessive, inflation will occur, but if it is restored, it will be possible to maintain full employment under constant prices. "it is restored" means that the fiscal expenditure in the next period, G'', should be so that the following equation is satisfied (the output is γL_f).

$$G'' - tP'\gamma L_f = (\gamma - 1)L_f M' = (\gamma - 1)\zeta L_f M.$$

Let $\tilde{G}$ be the fiscal expenditure without inflation. Then, since

$$\tilde{G} - tP\gamma L_f = (\gamma - 1)\gamma L_f M,$$

we have

$$G'' - tP'\gamma L_f - [\tilde{G} - tP\gamma L_f] = (\gamma - 1)(\zeta - \gamma)L_f M > 0.$$

Also we get

$$\frac{G'' - tP'\gamma L_f}{\tilde{G} - tP\gamma L_f} = \frac{\zeta}{\gamma}.$$

If we try to keep the price level constant after inflation, the budget deficit will be correspondingly larger in nominal terms than it would have been if prices had remained low, but there is not much difference in real terms since $\frac{\zeta}{\gamma}$ and ρ have close values. We should not use surpluses to make up for excessive budget deficits caused by inflation. If we do so, it will cause a recession and unemployment, as we will see in the next section.

5. Recession and unemployment caused by insufficient budget deficit and recovery from it

Suppose that the budget deficit has been satisfying (4) until the previous period, full employment has been achieved under constant prices, and in a certain period the budget deficit is G'' which is smaller than the budget deficit satisfying (4). It satisfies

$$G' - tPL < (\gamma - 1)L_f M.$$

The employment L is not necessarily equal to L_f. Then, the demand and supply of the goods satisfy

$$\alpha[(1-t)PL + L_f M] + G' = PL$$

Ley G be the fiscal expenditure when full employment is achieved, and comparing this equation with (2), then, by (4)

$$G - tPL_f - (G' - tPL) = (1-\alpha)P(L_f - L) > 0.$$

This means that with constant prices insufficient budget deficit creates involuntary unemployment. We have proved the following proposition.

Proposition 3 If the actual budget deficit is smaller than the level necessary and sufficient to sustain full employment and growth at constant prices, there will be a recession and involuntary unemployment.

When the budget deficit is small and involuntary unemployment occurs in a given period, we consider policies to restore full employment. In the next period after the period analyzed above, we assume that fiscal spending is set to G'' and full employment is restored. The following equation will then hold (the output is γL_f).

$$\alpha[(1-t)P\gamma L_f + L_f M'] + G'' = P\gamma L_f.$$

$L_f M'$ is the savings by consumers in the period of involuntary unemployment. It is expressed as follows.

$$L_f M' = (1-\alpha)[(1-t)PL + L_f M].$$

Then, we have

$$\alpha\{(1-t)P\gamma L_f + (1-\alpha)[(1-t)PL + L_f M]\} + G'' = P\gamma L_f,$$

and we get

$$G'' - tP\gamma L_f = (1-t)(1-\alpha)P\gamma L_f - \alpha(1-\alpha)[(1-t)PL + L_f M]. \qquad (9)$$

Let $\tilde{G}$ be the budget deficit in the same period in the steady-state case where full employment is continuously maintained. From (2)

$$\alpha[(1-t)P\gamma L_f + \gamma L_f M] + \tilde{G} = P\gamma L_f.$$

Since by (3)

$$\gamma L_f M = (1-\alpha)[(1-t)PL_f + L_f M],$$

we obtain,

$$\tilde{G} - tP\gamma L_f = (1-t)(1-\alpha)P\gamma L_f - \alpha(1-\alpha)[(1-t)PL_f + L_f M]. \quad (10)$$

Comparing (9) with (10), we get

$$G'' - tP\gamma L_f - (\tilde{G} - tP\gamma L_f) = \alpha(1-\alpha)(1-t)P(L_f - L) > 0. \quad (11)$$

This equation represents the additional budget deficit required to restore full employment from a situation of involuntary unemployment. Therefore, we obtain

Proposition 4 In order to restore full employment from a recession that includes involuntary unemployment caused by a shortfall in the budget deficit, a larger budget deficit is required than would be the case if full employment were maintained continuously.

After full employment is restored, a continuous budget deficit is necessary to maintain it, so the additional budget deficit created to overcome the recession should not be offset by a subsequent budget surplus.

About the period following the period in which full employment is restored

Denote the period in which involuntary unemployment occurs by *t,* the period in which full employment is restored by *t*+1. What will happen to the budget deficit in period *t*+2? Denote the savings in period $t+1$ by M^{t+1}, the fiscal spending in period $t+2$ by G^{t+2}. Then, the following equation holds (the output is $\gamma^2 L_f$).

$$\alpha[(1-t)P\gamma^2 L_f + L_f M^{t+1}] + G^{t+2} = P\gamma^2 L_f.$$

Let M' be the savings in peiod t in which involuntary unemployment occurs. Then,

$$L_f M^{t+1} = (1-\alpha)[(1-t)P\gamma L_f + L_f M'].$$

Since

$$L_f M' = (1-\alpha)\big[(1-t)PL + L_f M\big],$$

we have

$$L_f M^{t+1} = (1-\alpha)(1-t)P\gamma L_f + (1-\alpha)^2[(1-t)PL + L_f M].$$

Thus, the budget deficit in period $t+2$ is equal to

$$G^{t+2} - tP\gamma^2 L_f = (1-\alpha)(1-t)P\gamma^2 L_f - \alpha(1-\alpha)(1-t)P\gamma L_f - \alpha(1-\alpha)^2[(1-t)PL + L_f M]. \quad (12)$$

Let $\tilde{G}^{t+2}$ be the budget deficit in the same period of the steady-state case where full employment is continuously maintained. Then, we get

$$\alpha[(1-t)P\gamma^2 L_f + \gamma^2 L_f M] + \tilde{G}^{t+2} = P\gamma^2 L_f.$$

From

$$\gamma L_f M = (1-\alpha)[(1-t)PL_f + L_f M],$$

and

$$\gamma^2 L_f M = (1-\alpha)[(1-t)P\gamma L_f + \gamma L_f M] = (1-\alpha)(1-t)P\gamma L_f + (1-\alpha)^2[(1-t)PL_f + L_f M],$$

we get

$$\tilde{G}^{t+2} - tP\gamma^2 L_f = (1-\alpha)(1-t)P\gamma^2 L_f - \alpha(1-\alpha)(1-t)P\gamma L_f - \alpha(1-\alpha)^2[(1-t)PL_f + L_f M]. \quad (13)$$

Note that *M* is the savings in the period before involuntary unemployment occurred. *M* in (12) and that in (13) represent the same thing. Comparing (12) with (13) yields

$$G^{t+2} - tP\gamma^2 L_f - (\tilde{G}^{t+2} - tP\gamma^2 L_f) = \alpha(1-\alpha)^2(1-t)P(L_f - L) > 0.$$

It can be seen that the budget deficit is larger than that in the steady-state case where full employment is maintained continuously. This is because the shortage of savings during the period of involuntary unemployment will continue to have an impact on the future, but its value is smaller than that of (11) because it is $1-\alpha$ times. Inductively, for $n \geq 3$, we will have

$$G^{t+n} - tP\gamma^n L_f - (\tilde{G}^{t+n} - tP\gamma^n L_f) = \alpha(1-\alpha)^n(1-t)P(L_f - L)>0.$$

This value becomes smaller and converges to zero with each passing period.

6 Receipt and payment of money

Let's organize the receipt and payment of money. Let's list them.

Receit of money

R1: Receipt of wages and profits by consumers
R2: Recipt by firms from the sale of goods
R3: Receipt of taxes by the government
R4: Receipt by consumers of savings from previous generation of consumers

Payment of money

P1: Payment for the purchase of goods by consumers
P2: Payment of wages and profits by firms
P3: Payment by the government for the purchase of goods (fiscal expenditure)
P4: Payment of taxes by consumers

R1=P2, R2=P1+P3, R3=P4 and R2=R1 are satisfied. Let S=savings of consumers. Then, S=R1+R4−P1−P4 = R2+R4−P1−P4 holds. Thus,

S =P1+P3+R4−P1−P4= P3+R4−R3,

and we have

S−R4=P3−R3. (14)

This means that the savings of consumers is equal to the budget deficit.

A case where savings are not passed on and disappear

If savings are not passed on to the next generation or to the government and disappear, R4 is zero, so we have

S=P3-R3,

and the savings is equal to the budget deficit. Since $L_f M$ in the left-hand sides of (2) and (3) disappear, (4) is rewritten as

$$G - tPL_f = \gamma L_f M.$$

A case where there is a tax on savings

If some (or all) of the savings inherited from the previous generation are taxed, the tax on savings is added to R3 and P4 above, but the rest remains the same. (14) also holds, but the tax on savings is included in R3 on the right-hand side.

If all savings are confiscated by the government (as in the case of a 100% inheritance tax), then all R4 is included in R3, and from

S=P3−(R3−R4)

the budget deficit excluding the confiscated savings will be equal to the consumer's savings. As in the above case, since $L_f M$ in the left-hand sides of (2) and (3) disappear, we have.

$$G - tPL_f = \gamma L_f M.$$

This can be written as

$$G - tPL_f - L_f M = (\gamma - 1) L_f M.$$

This equation implies that the budget deficit when confiscated savings are considered as a tax is equal to the increase in savings. The fiscal expenditure required to maintain full employment is equal in the two cases (when savings are extinguished and when the tax is 100%).

A case where consumers save in government bonds

Up to this point, we have assumed that consumers save in money, but the same is true if we assume that they save in government bonds, which are redeemed, with interest, to the next generation of consumers. In this case the meaning of R4 is as follows.

R4: Receipt of government bonds by consumers from the previous generation consumers

About government bonds,

B1: Handover of government bonds (inherited from previous generations) by consumers to the government
B2: Receipt of government bonds by the government

Further,

P5: Payment for purchase of government bonds by consumers (savings in government bonds)
R6: Receipt by the current generation of consumers of the redemption of government bonds left by the previous generation of consumers
R5: Receipt by the government of the sale of government bonds from the current generation of consumers (issuance of government bonds to be saved)
P6: Payment by the government to the current generation of consumers of the redemption (plus interest) of government bonds left by the previous generation of consumers

We have R5=P5, R6=P6, B1=B2. R6 is equal to B1 plus interest. Since the current generation of consumers earn interest on the bonds left by the previous generation of consumers, we have

$$S = P3 + R6 - R3 = R5,$$

and so

$$S - B1 = R5 - B1 = P3 + R6 - B1 - R3. \qquad (15)$$

S is the savings by the government bonds. S−B1=R5−B1 represents increase in the savings by the government bonds, and R6−B1 is equal to the payment of interest to the government bonds. Therefore, (15) means that budget deficit including interest payments on government bonds equals increase in savings.

7 Concluding Remark

In this paper, we have examined MMT's claims about budget deficits using a simple static model that includes consumers' utility maximization and firms' profit maximization, and have found that they are generally correct.

In particular, it is important to note that taxes are not a source of revenue for fiscal spending, but that fiscal spending increases the demand for goods, on the other hand taxes reduce the demand for consumption goods by reducing people's income, and the budget deficit is merely the difference between the resulting fiscal spending and taxes.

The content of fiscal expenditures, the balance between public and private goods, and the fairness of taxes are important issues, but they should be discussed from the standpoint of public economics and public finance, not macroeconomics.

In this paper, for simplicity, we have used a static model and assumed that consumers live for only one period. It is possible to generalize and make dynamic the analysis in this paper using the overlapping generations model, for example, by Otaki (2007, 2009, 2015), which is currently under study.

Appendix: Utility maximization of consumers and the demand for the goods of the government

We consider the demand for the goods of an employed consumer. The same is true for the calculation for unemployed consumers. The Lagrange function with the Lagrange multiplier λ is

$$\mathcal{L}=\alpha\ln\left(\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}+(1-\alpha)\ln\frac{S^e}{P}-\ln\beta-\lambda\left(\int_0^1 p_ic_i^e di+S^e-(1-t)(w+\Pi)-M\right).$$

The condition sfor utility maximization for each consumer i are

$$\frac{\partial\mathcal{L}}{\partial c_i^e}=\frac{\alpha}{\left(\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}}\left(\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{1}{\sigma-1}}(c_i^e)^{-\frac{1}{\sigma}}-\lambda p_i \quad \text{(A-1)}$$

$$=\frac{\alpha}{\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di}(c_i^e)^{-\frac{1}{\sigma}}-\lambda p_i=0,$$

and

$$\frac{\partial\mathcal{L}}{\partial S^e}=\frac{1-\alpha}{\frac{S^e}{P}}\frac{1}{P}-\lambda=\frac{1-\alpha}{S^e}-\lambda=0. \quad \text{(A-2)}$$

From (A-1)

$$\frac{\alpha}{\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di}(c_i^e)^{\frac{\sigma-1}{\sigma}}-\lambda p_ic_i^e=0.$$

Further

$$\frac{\alpha}{\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di}\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di-\lambda\int_0^1 p_ic_i^e di=\alpha-\lambda\int_0^1 p_ic_i^e di=0. \quad \text{(A-3)}$$

Again from (A-1),

$$\frac{\alpha}{\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di}\left(\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{1}{1-\sigma}}-\lambda\left(\int_0^1 p_i^{1-\sigma}di\right)^{\frac{1}{1-\sigma}}=\frac{\alpha}{\left(\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}}-\lambda\left(\int_0^1 p_i^{1-\sigma}di\right)^{\frac{1}{1-\sigma}}=0.$$

Therefore,

$$\alpha=\lambda PC^e.$$

By (A-3)

$$PC^e = \int_0^1 p_i c_i^e di.$$

By (A-2)

$$1-\alpha = \lambda S^e.$$

Hence, we obtain

$$PC^e = \int_0^1 p_i c_i^e di = \alpha[(1-t)(w+\Pi)+M],$$

and

$$S^e = (1-\alpha)[(1-t)(w+\Pi)+M].$$

By (A-1)

$$\frac{\alpha}{\int_0^1 (c_i^e)^{\frac{\sigma-1}{\sigma}} di}(c_i^e)^{-\frac{1}{\sigma}} = \frac{\alpha}{(C^e)^{\frac{\sigma-1}{\sigma}}}(c_i^e)^{-\frac{1}{\sigma}} = \frac{\alpha}{PC^e} p_i.$$

Thus,

$$(C^e)^{\frac{1}{\sigma}}(c_i^e)^{-\frac{1}{\sigma}} = \frac{p_i}{P}.$$

From this

$$c_i^e = \left(\frac{p_i}{P}\right)^{-\sigma} C^e,$$

and

$$PC^e = \alpha[(1-t)(w+\Pi)+M].$$

Therefore, we get

$$c_i^e = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{\alpha[(1-t)(w+\Pi)+M]}{P}.$$

Finally, we consider the demand for the goods of the government. The Lagrange function is

$$\mathcal{L}_G = \left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} - \lambda_G\left(\int_0^1 p_i g_i di - G\right).$$

The first order condition is

$$\left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{1}{\sigma-1}} g_i^{-\frac{1}{\sigma}} - \lambda_G p_i = 0. \qquad \text{(A-4)}$$

From this

$$\left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{1}{\sigma-1}} \int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di - \lambda_G \int_0^1 p_i g_i di = 0.$$

Further,

$$\left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} - \lambda_G \int_0^1 p_i g_i di = 0.$$

By (A-4)

$$\left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{-1} \int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di - \lambda_G^{1-\sigma} \int_0^1 p_i^{1-\sigma} di = 1 - \lambda_G^{1-\sigma} \int_0^1 p_i^{1-\sigma} di = 0.$$

Thus,

$$\lambda_G\left(\int_0^1 p_i^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} = \lambda_G P = 1.$$

Hence,

$$\lambda_G = \frac{1}{P}, \left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} = \frac{\int_0^1 p_i g_i di}{P} = \frac{G}{P}.$$

By (A-4)

$$\left(\int_0^1 g_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} g_i^{-1} = \frac{G}{P} g_i^{-1} = \left(\frac{p_i}{P}\right)^{\sigma},$$

and so we get

$$g_i = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{G}{P}.$$

References

S. Kelton. *The Deficit Myth: Modern Monetary Theory and the Birth of the People's Economy*. Public Affairs, 2020.

A. P. Lerner. Functional finance and the federal debt. *Social Research*, 10:38–51, 1943.

A .P. Lerner. *The Economics of Control: Principles of Welfare Economics*. Macmillan, 1944.

W. Mitchell, L. R. Wray, and M. Watts. *Macroeconomics*. Red Gbole Press, 2019.

M. Otaki. The dynamically extended Keynesian cross and the welfare-improving fiscal policy. *Economics Letters*, 96, 23–29, 2007.

M. Otaki. A welfare economics foundation for the full-employment policy. *Economics Letters*, 102, 1–3, 2009.

M. Otaki. *Keynsian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Springer, 2015.

L. Randall Wray, *Modern Money Theory: A Primer on Macroeconomics for Sovereign Monetary Systems*. Palgrave Macmillan; 2nd ed. 2015.