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To the memory of Ralph Braid

Abstract

We consider a mixed ownership duopoly delegation model with spatial price discrimination and constant, albeit different, marginal production costs. In contrast to what holds true for a private duopoly, the Nash equilibrium, absent delegation, for a mixed duopoly with discriminatory pricing according to location is both consistent and socially optimal. We find that under Nash conjectures, in most cases, firm owners have a strong incentive to delegate location decisions to managers. In such cases, firms locate closer to each other. The intensity of the competition leads to lower prices, lower profits, for both firms, and increased surplus for the consumer.

Keywords: mixed duopoly; delegation; spatial competition; consistent conjectures; Nash equilibrium

JEL Classification: L13, L21, L22, D43, R32

1 Introduction

In recent years private and public firms are forced to coexist in many sectors of the economic activity. Privatisation occupies a prominent place in practically every political discourse vying for effective economic reform. Understanding aspects of the competition between private and public firms has gained in popularity for both scientists and practitioners. Besides its natural theoretical interest, such an understanding has the potential to dictate policy.

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On the other hand, the separation between ownership and decision taking has been hailed as the breakthrough of modern industrial organisation. This is modelled by means of an incentive contract offered to the manager by the owner designed to promote profit, sales, etc. A representative sample of papers along this line would certainly include [Vickers (1985)], [Fershtman and Judd (1987)],[Sklivas (1987)]. All these papers deal with private firms. It is important, however, to be able to determine whether the separation between ownership and control is equally effective when a public firm competes against a private one. Despite the numerous papers examining strategic delegation, few consider the canonical spatial price discrimination model. A notable exception is [Heywood and Wang (2014)].

Spatial price discrimination introduced by Hoover, [Hoover (1937)], and Lerner and Singer, [Lerner and Singer (1937)], is a different form of price competition in which companies bear transportation costs in setting delivery price schedules. Discriminatory pricing due to geographical location is, for example, often applied when we purchase goods online. In generalised spatial models the actual location as the price discriminating parameter can be replaced by any - product - characteristic that may be ordered. Other illustrative examples include - without being limited to - models attempting to describe newspaper editorial content differentiation along the ideological spectrum from left to right, departure times for itineraries of transportation means between fixed destinations fron morning till night, etc.. The social optimality of the Nash equilibrium of the standard model has been established, first for a private duopoly, by Hurter and Lederer, [Lederer and Hurter (1986)]. More recently, Eleftheriou and Michelacakis, [Eleftheriou and Michelacakis (2016)], have extended this result for a mixed oligopoly in a multi-varieties setting.

In spite of the long history, the canonical model of spatial price discrimination has not been studied for conjectural consistency - as defined by Bresnahan in [Bresnahan (1981)] - until very recently. Conjectural variations are often viewed as reflecting the belief formation of each competitor, a feature absent in Nash where rivals stubbornly refuse to take each other's behaviour into account. One may hope that they present advantages in unveiling the intrinsic rivalry of competition in a more faithful way. Consistency of beliefs is often viewed as a static shortcut to full dynamic modelling. As a solution concept, it often receives criticism - from a strict game-theoretic perspective - for attempting to discuss the dynamics of what is essentially a static game. Despite its drawbacks, many authors believe in its use as a tool to disclose inner aspects of competition hidden by Nash. Theoretical foundation for such beliefs has been provided by works, such as Cabral's, [Cabral (1995)]. Heywood and Wang, [Heywood and Wang (2016)], are the first to look at the equilibrium generated by consistent location conjectures for a private duopoly. They prove that, under equal (constant) marginal production costs and linear travel cost, the two firms collocate at the middle, away from their socially optimal locations at the quartiles. The tendency of closing the separating gap between the firm locations is verified even when travel cost is convex.

We are first to examine delegation in the spatial context when a private firm competes against a public one. To the end of understanding, at the same time, the much less studied solution concept of conjectural consistency we compare conjectural variations to classic Nash for the location sub-game. On the one hand conjectural consistency, albeit one-shot, is considerd as reflecting, to a certain extent, players' belief formation in response to other players' choices. On the other hand Nash is taking each player's reaction as fixed and independent of the reaction of the other player.

We consider a duopoly of one private and one public firm. Marginal production costs are different, while being constant. Demand is inelastic and delivery cost due to location is increased by td, proportional to, d, the distance shipped. We maintain the assumption of linear travel cost throughout. We consider a four or five-stage game according to whether the two owners consider delegation simultaneously or sequentially. Managers are offered the final form of their contracts at the end of the second stage - if delegation is simultaneous - or else at the end of the third stage. At the next stage, the two managers decide on locations and at the last stage they set delivery price schedules, price discriminating acording to customer location, and compete for quantity or market share. The location sub-game is solved by means of two different solution concepts namely, conjectural consistency and Nash conjectures and the results are being compared.

In sharp contrast to findings regarding a private duopoly, [Heywood and Wang (2016)], we show that, absent delegation, the Nash equilibrium of the mixed duopoly model is consistent. The deeper reason why this is true is because the private firm conjectures that the reaction of the public firm is independent of its own reaction. We prove that when the location sub-game is decided via consistent conjectures the delegation order is immaterial to final results as both firms do not delegate. The situation changes when Nash conjectures is the solution concept adopted for the location sub-game. With the exception of the case where the private firm delegates first, in the other two cases both owners delegate the location decision to managers. The result of this behaviour is fewer profits for both firms but higher consumer surplus. Simultaneous delegation, in this context, leads to lower delivery price schedules for all customers as well.

In section 2 we present the model and carry out the analysis. Section 2 is further divided into sub-sections according to the type of delegation and the solution concept adopted for the location sub-game. All results are in section 2. Section 3 concludes with a synthesis and a discussion of the deeper reasons behind the results emerged through this comparative study. All proofs are collected in the Appendix I.
Appendix II is ad-hoc and is provided for completeness as we were unable to find a specific reference answering the question: "when Nash is consistent?".

2 The model and results

Two firms, $R_\imath$, $\imath = 1, 2$, compete in a one-dimensional (linear) market, uniformly populated, by consumers represented by the compact interval $[0, 1]$. R_1 is privately owned whereas R_2 is a public firm. The locations of R_1 and R_2 are denoted by x and y, respectively, with $x < y$ in $[0, 1]$. Inelastic demand is assumed. The two firms produce a common good at constant marginal costs w_1 and w_2 respectively. We make the assumption that $w_2 \geq w_1$, i.e. the private firm being more flexible and adaptive uses a more efficient technology to produce, in general, at a lower cost than the public one. Transportation costs are equal to td, where t is a positive scalar and d is the distance shipped. The maximum reservation price a consumer is willing to pay for the produced commodity, k, is assumed to be sufficiently large(1). In setting their respective price schedules the two competitors bear transportation costs. Their marginal delivery cost for selling at location z is equal to the marginal production cost w_i for $i = 1, 2$, plus the transportation cost for shipping the good to the consumer's location z. The produced good is the object of Bertrand competition à la Hoover, [Hoover (1937)], and Lerner and Singer (1937). Specifically, the price charged for the good by the firm the consumer chooses to buy from, is equal to (or infinitesimally less than) the delivery cost of the firm that is further away. Managers are offered a contract which is a convex combination of profit and sales.

When simultaneous delegation is taking place the game is played in four stages. In the first stage, owners decide whether or not to delegate to managers the choice of location by writing out an incentive contract. If they decide to do so, the terms of the incentive contract are simultaneously chosen in the second stage. Both contracts are convex combinations of profit and output. In the third stage, managers choose locations and in the fourth stage they compete engaging in spatial price discrimination in a context of conjectural consistency. The sub-game perfect equilibrium is sought and backward induction is applied.

To care for sequential delegation we retain the basic structure of the game adding an extra stage following the decision of both owners to delegate the location choice to managers. A five-stage game is then played. In the first stage, owners decide whether or not to delegate to managers the choice of location by writing out incentive contracts that are convex combinations of profit and output. If they decide to do so, the terms of the first contract are chosen in the second stage. The terms of the contract offered to the competing firm are determined in the third stage. In the fourth stage, managers choose locations and in the fifth and final stage the two firms compete à la Cournot engaging in price discrimination according to buyers' location. In reality, the above game skeleton encompasses two games differing in the order the two contracts are written out. When considering Nash equilibria, the above games are replicated, the sole exception being the solution concept adopted by the two managers in determining their location choices.

(1) In fact, it is enough to assume $k \geq t + \max\{w_1, w_2\} = t + w_2$.

In order to write down the profit functions of the downstream firms, we first need to determine the location, s, of the indifferent consumer. To this end, we equate the respective delivered schedules to get $t\,(y-s)+w_2 = t\,(s-x)+w_1 \Longrightarrow s = \frac{x+y}{2}+\frac{w_2-w_1}{2t}$. It should also be observed that if $\frac{|w_2-w_1|}{2t} > \frac{y-x}{2}$, both firms are reduced to spatial-price discriminating monopolists, where the common good is now provided only by either R_1 or R_2. We consider this case trivial and focus only on the case

$$\frac{|w_2-w_1|}{2t} \leq \frac{y-x}{2}\,. \tag{2.0.1}$$

The profit functions of R_1 and R_2, are respectively:

$$\Pi_{R_1}(x,y) = \int_0^x [t(y-x)+w_2-w_1]dz + \int_x^s [t(x+y-2z)+w_2-w_1]dz\,, \tag{2.0.2}$$

$$\begin{aligned} \Pi_{R_2}(x,y) &= \int_s^y [t(2z-x-y)+w_1-w_2]dz + \int_y^1 [t(y-x)+w_1-w_2]dz \\ &\quad +g(x,y)\,, \end{aligned} \tag{2.0.3}$$

with

$$\begin{aligned} g(x,y) &= \Pi_{R_1}(x,y) \\ &\quad + \left(\int_0^s [k-t(y-z)-w_2]dz + \int_s^1 [k-t(z-x)-w_1]dz\right)\,. \end{aligned} \tag{2.0.4}$$

The term inside the parentheses in (2.0.4) corresponds to the total consumer surplus $\mathcal{CS}$. Accordingly, contractual incentives offered to managers are given by:

$$I_1(x,y) = a_1\Pi_{R_1}(x,y) + (1-a_1)q_1(x,y)\,, \tag{2.0.5}$$

$$I_2(x,y) = a_2\Pi_{R_2}(x,y) + (1-a_2)q_2(x,y)\,, \tag{2.0.6}$$

where $q_i, i = 1,2$ is the quantity sold and $a_1, a_2 \in (0,1]$. Equivalently, (2.0.5) and (2.0.6) become:

$$I_1(x,y) = a_1 2t\left(\frac{s^2}{2}-\frac{x^2}{2}\right) + (1-a_1)s\,, \tag{2.0.7}$$

$$I_2(x,y) = a_2\left[-2t\left(\frac{y^2}{2}+\frac{x^2}{2}-\frac{s^2}{2}\right)+ty-w_2+k-\frac{t}{2}\right] + (1-a_2)(1-s)\,. \tag{2.0.8}$$

2.1 Consistent location conjectures

Each manager maximises her own profit with respect to her choice of location generating best response functions. Differentiating (2.0.7) with respect to x, and setting equal to 0 we get

$$2a_1t\left[s\left(1+r_{21}\right)-2x\right]+(1-a_1)(1+r_{21})=0\,. \tag{2.0.9}$$

Similarly differentiating (2.0.8) with respect to y, and setting equal to 0 we get

$$a_2\left[-2t\left(2y+2xr_{12}-s(r_{12}+1)\right)+2t\right]-(1-a_2)(r_{12}+1)=0\,, \tag{2.0.10}$$

with $W:=w_2-w_1$, $r_{12}=\frac{dx}{dy}$ and $r_{21}=\frac{dy}{dx}$ the location conjectures of firm 2 about firm 1 and firm 1 about firm 2 respectively.

The consistent conjectures equilibrium requires that each firm correctly anticipates the rival's location choice in response to own choice, [Bresnahan (1981)]. Implicitly differentiating (2.0.9) with respect to y and (2.0.10) with respect to x, we get respectively

$$2a_1t\left[\left(1+r_{21}\right)\left(r_{12}+1\right)-4r_{12}\right]=0 \tag{2.0.11}$$

and

$$2a_2t\left[4r_{12}+4r_{21}-\left(1+r_{12}\right)\left(1+r_{21}\right)\right]=0\,, \tag{2.0.12}$$

which solve to

$$r_{21}=0 \quad \text{and} \quad r_{12}=\frac{1}{3}\,. \tag{2.0.13}$$

The asymmetry of the conjectures is, more or less, expected and differs from what is found in [Heywood and Wang (2016)] relative to a private duopoly. What is far less expected is the first part of (2.0.13). It means that the private firm conjectures that the public firm will pick the same spot to locate regardless of own location choice.

To find the equilibrium locations we return the above values into (2.0.9) and (2.0.10) to obtain

$$x^C(W,t,a_1,a_2)=\frac{6a_1a_2W+3a_1a_2t-2a_1a_2-2a_1+4a_2}{12a_1a_2t} \tag{2.0.14}$$

and

$$y^C(W,t,a_1,a_2)=\frac{-2a_2W-3a_2t-2a_2+2}{-4a_2t}\,. \tag{2.0.15}$$

Equation (2.0.15), in contrast to equation (2.0.14), reveals that the public firm remains focused on its own priorities regardless of the degree of aggressiveness of its private competitor, i.e. the location choice of the manager of the public firm does not depend on the incentive parameter, a_1, offered to her counterpart by the owner of the private firm.

2.1.1 Simultaneous delegation

We return (2.0.14) and (2.0.15) into (2.0.2) and (2.0.3), differentiate with respect to a_1 and a_2 respectively and solve the resulting best response functions

$$\frac{\vartheta \Pi_{R_1}(a_1, a_2)}{\vartheta a_1} = -\frac{1}{6}\frac{-1+a_1}{t a_1^3}, \tag{2.0.16}$$

$$\frac{\vartheta \Pi_{R_2}(a_1, a_2)}{\vartheta a_2} = -\frac{1}{3}\frac{a_2-1}{t a_2^3}, \tag{2.0.17}$$

simultaneously for a_1 and a_2. The common solution yields $a_1 = 1 = a_2$[(2)].

Proposition 2.1 *In a mixed duopoly, location game with simultaneous delegation and constant but unequal production costs, both firms do not delegate and spatial discrimination in prices leads to a consistent equilibrium in locations*

$$x^{C_{\text{sim}}}(W,t) = x^C(W,t) = \frac{2W+t}{4t},$$

$$y^{C_{\text{sim}}}(W,t) = y^C(W,t) = \frac{2W+3t}{4t}$$

that are socially optimal.

Proof. Referred to the Appendix.

Both $\frac{\vartheta x^C}{\vartheta W} > 0$ and $\frac{\vartheta y^C}{\vartheta W} > 0$ proving that the private firm converts its cost advantage into market gain by locating closer to the public firm. For higher t the public firm locates to the left towards the private firm which recedes giving up market share as both $\frac{\vartheta x^C}{\vartheta t}$ and $\frac{\vartheta y^C}{\vartheta t}$ are negative. The equilibrium locations are determined by two opposing effects: the marginal production cost difference - that benefits the private firm (under the assumption that the public firm produces at a higher marginal cost) - and the transportation cost whose high values work in favour of the public firm.

2.1.2 Sequential delegation

The basic structure of the game is preserved but for the addition of an extra stage to care for the sequential delegation. Thus, in stages two and three the owners of the two firms take turns in deciding the contractual incentives they will offer their respective managers.

First, consider the case where, at stage two, the private firm decides first on the details of the contract it will offer its manager and then, at stage three, the public firm follows with its respective offer. To the end of finding the best response of the

(2) Note that $\frac{\vartheta^2 \Pi_{R_\imath}(a_1,a_2)}{\vartheta a_\imath^2} < 0$ provided $a_\imath < \frac{3}{2}$ for $\imath = 1, 2$.

public firm to the choice of the private owner, a_1, we return equations (2.0.14) and (2.0.15) into (2.0.3). Differentiating with respect to a_2, we get

$$\frac{\vartheta \Pi_{R_2}(a_1, a_2)}{\vartheta a_2} = \frac{1 - a_2}{3ta_2^3}\,. \tag{2.1.1}$$

Setting it equal to 0 yields the best response of the owner of the welfare maximising firm which is 1 regardless of the value of a_1, the incentive offered to the private manager, i.e. $a_2(a_1) = 1$. Filling this value into equation (2.0.2), differentiating with respect to a_1,

$$\frac{\vartheta \Pi_{R_1}(a_1, a_2 = 1)}{\vartheta a_1} = \frac{1 - a_1}{6ta_1^3} \tag{2.1.2}$$

and solving for a_1, we find the value of a_1 maximising the profit of the private firm to be 1.

If the delegation order is reversed, the public firm decides first and the private firm follows. Again, we substitute the locations from equations (2.0.14) and (2.0.15) into (2.0.2), differentiate with respect to a_1,

$$\frac{\vartheta \Pi_{R_1}(a_1, a_2)}{\vartheta a_1} = \frac{1 - a_1}{6ta_1^3} \tag{2.1.3}$$

and solve with respect to a_1, to get the best response of the owner of the private firm which is, once more, constant, equal to 1, i.e. $a_1(a_2) = 1$. Letting $a_1 = 1$ in (2.0.3) and differentiating with respect to a_2 we have

$$\frac{\vartheta \Pi_{R_2}(a_1 = 1, a_2)}{\vartheta a_2} = \frac{1 - a_2}{3ta_2^3}\,. \tag{2.1.4}$$

We see that in either situation both firms do not delegate. This does not come as a surprise because

$$\frac{\vartheta \Pi_{R_1}(a_1, a_2)}{\vartheta a_2} = 0 = \frac{\vartheta \Pi_{R_2}(a_1, a_2)}{\vartheta a_1}\,,$$

which means that the system is uncoupled and therefore, at the delegation stages, the two owners decide independently of each other.

Proposition 2.2 *In a mixed duopoly, location game with sequential delegation and constant but unequal production costs, both firms do not delegate and spatial discrimination in prices leads to a consistent equilibrium in locations*

$$x^{C_{\mathrm{seq}}}(W, t) = x^C(W, t) = \frac{2W + t}{4t}\,,$$

$$y^{C_{\mathrm{seq}}}(W, t) = y^C(W, t) = \frac{2W + 3t}{4t}\,,$$

that is socially optimum, regardless of delegation leader or follower.

The deeper explanation of both Propositions 2.1 and 2.2 lies with equation (2.0.15). Given this particular solution concept for the location sub-game, the welfare maximising firm locates independently of the contractual incentive offered to the manager of the private firm. Thus, the owner of the public firm has no reason why to use the contractual incentive to counter her counterpart's choice and sticks to a zero-weight policy on sales in all cases. The owner of the private firm anticipating correctly this behaviour opts for a full-weight on profits policy and does not delegate as well. As a result they both locate at their socially optimal spots.

We move on to evaluate firm profits. To this end we return the equilibrium locations found, in either Proposition 2.1 or Proposition 2.2, into equations (2.0.2) and (2.0.3) to get

$$(2.2.1)\quad \Pi^C_{R_1}(W,t) = \frac{3t^2+12tW+12W^2}{16t} = \frac{3(2W+t)^2}{16t} = \Pi^{C_{\mathrm{sim}}}_{R_1}(W,t) = \Pi^{C_{\mathrm{seq}}}_{R_1}(W,t)$$

and

$$(2.2.2)\qquad \Pi^C_{R_2}(W,t) = \frac{4Wt+4W^2-t^2+8kt-8w_2t}{8t} = \Pi^{C_{\mathrm{sim}}}_{R_2}(W,t) = \Pi^{C_{\mathrm{seq}}}_{R_2}(W,t)\,.$$

The proof of the non-negativity of $\Pi^C_{R_2}$ is referred to the Appendix I.

Consumer surplus, $\mathcal{CS}$, emerges as the difference between the maximum price consumers are willing to pay, k, and the total cost required for the whole market to be served; equivalently:

$$(2.2.3)\qquad \mathcal{CS}(W,t) = \int_0^s [k-t(y-z)-w_2]dz + \int_s^1 [k-t(z-x)-w_1]dz\,.$$

After integration, (2.2.3) becomes

$$(2.2.4)\qquad -sW+k-w_1-\frac{t}{2}+ts^2-ts(x+y)+tx\,.$$

Returning the equilibrium locations of either Proposition 2.1 or Proposition 2.2 into (2.2.4), we get the consumer surplus:

$$(2.2.5)\qquad \begin{aligned} \mathcal{CS}^C(W,t) &= \tfrac{-2W^2-tW+2kt-2w_1t-t^2}{2t} \\ &= \mathcal{CS}^{C_{\mathrm{sim}}}(W,t) = \mathcal{CS}^{C_{\mathrm{seq}}}(W,t)\,. \end{aligned}$$

We have made the assumption that the reservation price k is sufficiently large for the game to unroll. In fact $k > t + w_2$ suffices to ensure the positivity of the consumer surplus. For a proof we refer to the Appendix I.

2.2 Nash equilibrium in locations

In this sub-section, we drop the requirement that beliefs be consistent and search for location equilibria under strategic delegation in a mixed duopoly setting. We solve for the associated traditional Nash equilibria in locations. The comparison will shed light and enable us to understand the properties of the consistent behaviour and why it differs sharply from the associated Nash behaviour. Differentiating (2.0.7) with respect to x, and (2.0.8) with respect to y, and setting both derivatives equal to 0 we get the following system:

$$\begin{aligned} \frac{\vartheta I_1}{\vartheta x} &= 2a_1 t(s-2x)+1-a_1=0 \\ \frac{\vartheta I_2}{\vartheta x} &= -2a_2 t(2y-s)+2a_2 t-1+a_2=0\,. \end{aligned} \tag{2.2.6}$$

Solving the system yields for the two locations the following formulae:

$$\begin{aligned} x^N(W,t;a_1,a_2) &= \frac{4a_1a_2W+2a_1a_2t-2a_1a_2-a_1+3a_2}{8a_1a_2t}\,, \\ y^N(W,t;a_1,a_2) &= \frac{4a_1a_2W+6a_1a_2t+2a_1a_2-3a_1+a_2}{8a_1a_2t}\,. \end{aligned} \tag{2.2.7}$$

Absent delegation, $a_1=1, a_2=1$, the firms locate at

$$x^N(W,t;1,1)=\frac{2W+t}{4t}\,,\quad y^N(W,t;1,1)=\frac{2W+3t}{4t}\,, \tag{2.2.8}$$

their respective socially optimal locations.
In contrast to what happens in the consistent case, the choices of both owners, in terms of delegations incentives, now affect both managers' location decisions. The signal of aggressive marketing sent out by hiring a manager is answered by the competition in a similar way because Nash is all about intentions. For any pair of the incentive parameters a_1, a_2 both $\vartheta x^N/\vartheta W > 0$ and $\vartheta y^N/\vartheta W > 0$. Thus, the effect of the marginal production cost disadvantage of the public firm is so strong that cannot be countered by the choice of a suitable incentive parameter offered to its manager.

2.2.1 Simultaneous delegation

To evaluate the optimum contracts, we return (2.2.7) into (2.0.2) and (2.0.3). This gives the owners' best response functions in terms of managerial incentives a_1, a_2 which solved simultaneously give

$$a_1^{N_{\mathrm{sim}}}=\frac{8}{6W+3t+8}\,,\quad a_2^{N_{\mathrm{sim}}}=\frac{8}{2W+t+8}\,. \tag{2.2.9}$$

Again, in contrast to what we found when the quantity game was solved through conjectural consistency, when solving for Nash equilibrium both firms always delegate as both owners know that they need to perk up their managers. The asymmetry in their respective choices of incentives reflects the different objectives the two firms have set themselves. The owner of the welfare maximising firm makes, naturally, more restrained use of the contractual tool to incentivise her manager. Both owners understand that the larger t (or W) is the less weight on profit is required to spur their managers on.

Returning equations (2.2.9) to (2.2.7) yields the equilibrium locations in this case.

Proposition 2.3 *In a mixed duopoly, location game with simultaneous delegation and constant but unequal production costs, both firms delegate and spatial discrimination in prices leads to the following Nash equilibrium in locations:*

$$x^{N\text{sim}}(W,t) = \frac{3}{8}\frac{2W+t}{t} \quad \textit{and} \quad y^{N\text{sim}}(W,t) = \frac{1}{4}\frac{2W+3t}{t}.$$

The private firm increases its market share by moving to the right of its socially optimal location, while the public firm manages to contain the attack holding on its socially optimal location(3)*.*

When transportation cost is high the comparative production cost advantage of the private firm is minimised and $y^{N\text{sim}} \to 3/4$ as $t \to \infty$. Not only is the Nash location choice of the public firm consistent but, regardless of transportation costs or marginal production cost handicap, it is socially optimal. The hiring of a manager on behalf of the private owner prompts a similar reaction on the part of the public firm to counter the attack. This is done successfully as the public firm manages to maintain its socially optimal location against the advancement of its private competitor. Should the public firm have opted to remain passive, without hiring a manager to prop its interests, the corresponding location choices would be $\frac{50W+25t}{64t}$, for the private, and $\frac{38W+51t}{64t}$, for the public firm respectively, i.e. to the right of their Nash locations, and it would amount to the welfare maximising firm giving away even more market share. The result in Proposition 2.3 is in contrast and differs sharply to findings in related literature concerning private duopolists. Notably, in [Heywood and Wang (2016)] the authors prove that private duopolists locate closer to the center under conjectural consistency than under Nash conjectures minimising their respective profits.

To the end of evaluating firm profits when simultaneous delegation is taking place we fill into equations (2.0.2) and (2.0.3) the locations found in Proposition 2.3. Let $\Pi_{R_\jmath}^{N\text{sim}}$ denote the profit of firm $\jmath = 1, 2$, then

$$\Pi_{R_1}^{N\text{sim}}(W,t) = \frac{45}{256}\frac{(2W+t)^2}{t} \tag{2.3.1}$$

and

$$\Pi_{R_2}^{N\text{sim}}(W,t) = \frac{1}{256}\frac{116W^2 + 116Wt - 35t^2 - 256w_2t + 256kt}{t}. \tag{2.3.2}$$

To calculate the corresponding consumer surplus, $\mathcal{CS}^{N\text{sim}}$, we return the equilibrium locations of Proposition 2.3 into (2.2.4):

$$\mathcal{CS}^{N\text{sim}}(W,t) = \frac{-324W^2 - 132tW + 256kt - 256w_1t - 113t^2}{256t}. \tag{2.3.3}$$

The non-negativity of both expressions (2.3.2) and (2.3.3) is guaranteed by the fact that k is assumed to be sufficiently large. For an additional argument, we refer to the proofs of the non-negativity of (2.2.2) and (2.2.5) found in the Appendix I.

It follows,

(3) Condition (2.0.1) implies that $3t \geq 10W$ which suffices to guarantee $0 \leq x^{N\text{sim}}(W,t) \leq y^{N\text{sim}}(W,t) \leq 1$.

Proposition 2.4 *In a mixed duopoly, location game with simultaneous delegation and constant but unequal production costs*

i) the consistent-conjectures equilibrium is socially optimal,

ii) under Nash conjectures, only the private firm locates closer to the center

$$x^{C\mathrm{sim}}(W,t) < x^{N\mathrm{sim}}(W,t) < y^{N\mathrm{sim}}(W,t) = y^{C\mathrm{sim}}(W,t)\,,$$

iii) profits of both firms are lower under Nash than under conjectural consistency, and

iv) consumer surplus is higher under Nash.

The details of the proof of Proposition 2.4 may be found in the Appendix I.

Proposition 2.4 sheds light into the difference between the two solution concepts. When firms locate, trying to correctly guess the opponent's reaction to own choice there is no need for delegation. If, however, the location sub-game is solved according to Nash, each firm conjectures that the reaction of the other firm will be fixed, regardless of own choice. Both owners, now, know that they have to incentivise their managers by offering them contracts with weight on sales. The welfare maximising firm manages to defend its previous location, when delegation is simultaneous. At the same time the private firm closes the gap pushing the indifferent consumer closer to where the public firm is based at, thus, gaining market share. This behaviour results in lower or - at best - equal selling prices for all customers, lower profits for both firms and increased customer surplus.

To better understand what happens, divide the market into three adjacent segments, $[0, s^{C\mathrm{sim}}(W,t)]$, $[s^{C\mathrm{sim}}(W,t), s^{N\mathrm{sim}}(W,t)]$ and $[s^{N\mathrm{sim}}(W,t), 1]$, where $s^{C\mathrm{sim}}(W,t)$ and $s^{N\mathrm{sim}}(W,t)$ denote the locations of the indifferent consumer under consistent conjectures and under Nash, respectively. Selling prices to customers in $[0, s^{C\mathrm{sim}}(W,t)]$ remain the same. Delivery price schedules to customers in $[s^{C\mathrm{sim}}(W,t), s^{N\mathrm{sim}}(W,t)]$ are strictly lower than before, because these customers, now, buy from the private firm at a - decreased - price determined by their distance from the public firm. The same holds true for those customers in $[s^{N\mathrm{sim}}(W,t), 1]$, because although they still buy from the public firm, the corresponding delivery price schedules are lower than before since they are determined by their respective distance from the private firm, which has moved closer to them. As a result customer surplus is on the rise. The public firm suffers a profit loss due to the dual effect of shrinkage of market share on the one hand, and lower revenue due to reduced price schedules, on the other. The private firm suffers a loss, as well, because the expansion of its market share is not sufficient to overcome the rise in cost of servicing its customers due to its new location. Total welfare lags behind, despite the increase in consumer surplus.

2.2.2 The private firm leads the delegation sub-game

To care for the situation when owners decide in succession, an extra stage is added to the structure of the original game as described above, in Section 2. During stages two and three, the two owners decide, in turns, on the contractual incentives they are going to offer to their respective managers.

Assume, first, that the delegation rounds are led by the private firm at stage two, followed by the public firm at stage three. To find the best response of the owner of the public firm to the choice, a_1, of the private owner we return (2.2.7) into (2.0.3) and differentiate with respect to a_2. This gives

$$\frac{\vartheta \Pi_{R_2}(a_1, a_2)}{\vartheta a_2} = -\frac{1}{16}\frac{-3a_1 + 2a_2a_1 + a_2}{a_1 t a_2^3}.$$

Setting the above equal to 0 gives for a_2

$$a_2(a_1) = \frac{3a_1}{2a_1 + 1}. \tag{2.4.1}$$

Filling (2.2.7) and (2.4.1) into (2.0.2) gives the function the owner of the private firm needs to maximise

$$\frac{1}{48}\frac{36W^2a_1^2 + 36Wa_1^2t - 4 + 8a_1 - 4a_1^2 + 9a_1^2t^2}{a_1^2 t}.$$

Differentiating first and solving for a_1 yields $a_1 = 1$. Substituting this value in equation (2.4.1), we get $a_2 = 1$ as well.

Proposition 2.5 *If in a mixed duopoly, location game with sequential delegation and constant but unequal production costs, the private firm leads the delegation sub-game, both firms do not delegate and spatial discrimination in prices leads, under Nash, to an equilibrium in locations*

$$x^{N_{\mathrm{seq}}1}(W, t) = \frac{2W + t}{4t},$$

$$y^{N_{\mathrm{seq}}1}(W, t) = \frac{2W + 3t}{4t},$$

that is consistent and socially optimum.

Consistency follows from Proposition 2.2. The social optimality is explained in the proof of Proposition 2.1.

The reaction function of the owner of the public firm, given by equation (2.4.1), is increasing in a_1. The owner of the private firm anticipates that the owner of the welfare maximising firm, acting second, would try to top her choice of weight on profit. She chooses the maximum possible weight, i.e. $a_1 = 1$, leaving no room to her counterpart to better her choice, practically forcing her to choose $a_2 = 1$, as well. Thus, no one delegates and Nash recovers the socially optimal location equilibrium.

2.2.3 The public firm leads the delegation sub-game

Now, the owner of the public firm moves first in the delegation stages of the game. In a fashion similar to the one in subsection 2.2.2, to find the best reaction of the following private firm, we return equations (2.2.7) into (2.0.2) and differentiate with respect to a_1 to obtain

$$\frac{\vartheta \Pi_{R_1}(a_1, a_2)}{\vartheta a_1} = -\frac{1}{32} \frac{4a_2 W a_1 - 5a_2 + 6a_2 a_1 - a_1 + 2a_1 a_2 t}{t a_2 a_1^3},$$

which, solved for a_1, gives

$$a_1(a_2) = \frac{5a_2}{-1 + 4a_2 W + 6a_2 + 2ta_2}, \tag{2.5.1}$$

the best response of the private owner to the choice, a_2, the public owner has already made. The objective function of the public owner is the result of substitution of equations (2.2.7) and (2.5.1) into equation (2.0.3), i.e.

$$\frac{44a_2^2W^2+8a_2W-8a_2^2W+44a_2^2Wt-11+22a_2+4ta_2}{100a_2^2t}$$
$$+\quad \frac{-11a_2^2-54a_2^2t+36t^2a_2^2-100w_2a_2^2t+100ka_2^2t}{100a_2^2t}.$$

Differentiating and solving the above for a_2, gives

$$a_2^{N_{\text{seq}2}} = \frac{11}{11 + 4W + 2t}. \tag{2.5.2}$$

Substituting equation (2.5.2) into equation (2.5.1) gives the equilibrium incentive in the private contract

$$a_1^{N_{\text{seq}2}} = \frac{11}{11 + 8W + 4t}. \tag{2.5.3}$$

The public firm, forseeing that the manager of the private firm will put more weight on sales to spur her manager on, uses its strategic advantage to counter the offensive by taking pre-emptive action.

Returning equations (2.5.2) and (2.5.3) to (2.2.7) yields the equilibrium locations.

Proposition 2.6 *If in a mixed duopoly, location game with sequential delegation and constant but unequal production costs, the welfare maximising firm leads the delegation sub-game, both firms delegate and spatial discrimination in prices leads, under Nash, to a socially suboptimal equilibrium in locations*

$$x^{N_{\text{seq}2}}(W,t) = \frac{4}{11}\frac{2W + t}{t},$$

$$y^{N_{\text{seq}2}}(W,t) = \frac{1}{11}\frac{5W + 8t}{t}.$$

In fact both firms abandon their socially optimal locations and move closer to the center intensifying competition(4) .

The owner of the public firm knows that acting first has the drawback of revealing her delegation choice to her counterpart. This will allow the owner of the private firm, acting in her wake, to properly "motivate" her manager to put up a successful defence. Thus, the owner of the public firm uses her "leader's" advantage to contain the counter-attack by committing to a managerial contract with larger weight on sales, compared to the situation where delegation for both firms is concurrent. This is manifested in

$$\frac{11}{11+4W+2t} = a_2^{N_{\mathrm{seq}2}} < a_2^{N_{\mathrm{sim}}} = \frac{8}{2W+t+8}\,.$$

Both owners know that for large t or W more weight on sales will be required. The result of the increased aggressiveness is the intensification of competition as both firms locate, away from their socially optimal locations,

$$\frac{2W+t}{4t} < x^{N_{\mathrm{seq}2}}(W,t) < y^{N_{\mathrm{seq}2}}(W,t) < \frac{2W+3t}{4t}\,, \tag{2.6.1}$$

closer to the center and to each other.

To evaluate firm profits in this situation, we return the locations of Proposition 2.6 into equations (2.0.2) and (2.0.3). Let $\Pi_{R_\jmath}^{N_{\mathrm{seq}2}}$ denote the profit of firm $\jmath = 1, 2$ then

$$\Pi_{R_1}^{N_{\mathrm{seq}2}}(W,t) = \frac{20}{121}\frac{(2W+t)^2}{t} \tag{2.6.2}$$

and

$$\Pi_{R_2}^{N_{\mathrm{seq}2}}(W,t) = \frac{1}{22}\frac{10W^2+10Wt-3t^2-22w_2t+22kt}{t}\,. \tag{2.6.3}$$

To calculate the corresponding consumer surplus, $\mathcal{CS}^{N_{\mathrm{seq}2}}$, we fill the equilibrium locations of Proposition 2.6 into (2.2.4):

$$\mathcal{CS}^{N_{\mathrm{seq}2}}(W,t) = \frac{-288W^2-112tW+242kt-242w_1t-105t^2}{242t}\,. \tag{2.6.4}$$

The non-negativity of expressions (2.6.3) and (2.6.4) follows, again, from the assumption that k is assumed to be sufficiently large. An additional argument may be crafted along the lines of the proofs of the non-negativity of (2.2.2) and (2.2.5), found in the Appendix I.

Proposition 2.7 *When comparing two mixed duopoly location games of constant but unequal marginal production costs, the Nash location equilibria with simultaneous delegation differ sharply from the associated equilibria with sequential delegation. In particular, if the public firm leads the delegation rounds, we have that*

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(4) Condition (2.0.1) implies that $3t \geq 5W$ which suffices to guarantee $0 \leq x^{N_{\mathrm{seq}2}}(W,t) \leq x^{N_{\mathrm{seq}2}}(W,t) \leq 1$

i) under sequential delegation both firms locate to the left of their equilibrium locations under simultaneous delegation

$$x^{N_{\text{seq2}}}(W,t) < x^{N_{\text{sim}}}(W,t) < y^{N_{\text{seq2}}}(W,t) < y^{N_{\text{sim}}}(W,t) \, .$$

Competition is more intense under sequential delegation and the public firm capitalises on its strategic advantage to increase its market share,

ii) the private firm has a higher profit under simultaneous delegation while the public firm, capitalising on its advantage, earns more under sequential delegation[(6)] *,*

iii) consumer surplus is higher under sequential delegation.

The detail of the proof of Proposition 2.7 is in the Appendix I.

In sequential delegation with the owner of the public firm, acting first, the owner of the private firm has full knowledge of her counterpart's choice and adjusts her reaction accordingly. To overcome this handicap, the owner of the public firm offers her manager a contract with increased weight on sales, compared to the contract she offers her manager when the delegation decision is taken simultaneously. The new locations are to the left of the old ones and closer to each other. Mimicking the analysis following Proposition 2.4, the result of this new location geography is reduced profits for the private firm - due: i) to access to a smaller market segment and ii) to lower delivery price schedules, because the competitor has also moved to the left and closer to private firm location. Profits of the the public firm due to direct sales lag behind, as well, despite the increase in number of customers. All changes taken into account, when the delegation is carried out sequentially, the price distribution favours a higher consumer surplus and a higher total welfare.

3 Conclusion

The present paper is the first to examine, in a comparative way, strategic delegation effects on spatial competition between a private and a public firm under consistent location and Nash conjectures.

Collecting all results in the previous section in a single Proposition, we have proved that:

Proposition 3.1 *In a mixed duopoly, location game with delegation of constant but unequal marginal production costs, the location equilibria depend on the type of delegation and solution concept adopted for the location sub-game. In particular,*

[(6)] We specify that total welfare is on the rise, under sequential delegation, while profits, due to direct sales of the public firm, still lag behind the simultaneous delegation case.

i)

$$\begin{aligned} x^{C}(W,t) &= x^{N_{\mathrm{seq}}1}(W,t) \\ &< x^{N_{\mathrm{seq}}2}(W,t) < x^{N_{\mathrm{sim}}}(W,t) < y^{N_{\mathrm{seq}}2}(W,t) < y^{N_{\mathrm{sim}}}(W,t) \\ &= y^{N_{\mathrm{seq}}1}(W,t) = y^{C}(W,t)\,. \end{aligned}$$

Competition is more fierce when the public firm delegates first and managers decide on location under Nash conjectures,

ii) both firms maximise profit under conjectural consistency,

$$\Pi_{R_1}^{N_{\mathrm{seq}}2}(W,t) < \Pi_{R_1}^{N_{\mathrm{sim}}}(W,t) < \Pi_{R_1}^{N_{\mathrm{seq}}1}(W,t) = \Pi_{R_1}^{C}(W,t)\,,$$

$$\Pi_{R_2}^{N_{\mathrm{sim}}}(W,t) < \Pi_{R_2}^{N_{\mathrm{seq}}2}(W,t) < \Pi_{R_2}^{N_{\mathrm{seq}}1}(W,t) = \Pi_{R_2}^{C}(W,t)\,,$$

and

iii) consumer surplus tops under Nash, when the public firm delegates first

$$\mathcal{CS}^{C}(W,t) = \mathcal{CS}^{N_{\mathrm{seq}}1}(W,t) < \mathcal{CS}^{N_{\mathrm{sim}}}(W,t) < \mathcal{CS}^{N_{\mathrm{seq}}2}(W,t)\,^{(6)}\,.$$

The proof of the Proposition 3.1 is in the Appendix I.

Implicit in Proposition 3.1 are several key results. Most importantly, absent delegation, not only is the Nash equilibrium socially optimal but it is also consistent. The social optimality is known and holds true in a far more general context, [Eleftheriou and Michelacakis (2016)]. The consistency, though, is novel. It comes in sharp contrast to related findings regarding a private duopoly. Notably, Heywood and Wang, [Heywood and Wang (2016)], find that, with linear transportation costs, the two private firms collocate in the middle of the market if consistency of conjectures is applied. The key difference here is that the welfare maximising nature of the public firm makes the private firm conjecture that the public firm will locate independently of own location choice, equation (2.0.13). This asymetric result is manifested again in equations (2.0.14) and (2.0.15); for fixed W and t, the location choice of the public firm depends only on a_2 - the strategic choice of its owner - a result that is not true for the location of the private firm.

Another consequence of equation (2.0.15) is that the private owner - realising that her delegation choice will not affect the location choice of her public competitor - appears to have no reason why to settle for anything less than full weight on profit. This is in fact true because, no matter what the delegation order is, marginal profits depend only on own delegation choices - equations (2.0.16) and (2.0.17), equations (2.1.1) and (2.1.2) and equations (2.1.3) and (2.1.4). As a result, whether the two owners decide simultaneously on delegation or take turns is immaterial to the consistent equilibrium

(6) We remind the reader that the exponents N_{sim}, $N_{\mathrm{seq}}1$, $N_{\mathrm{seq}}2$ denote Nash conjectures, simulataneous delegation, private firm delegates first, public firm delegates first, respectively. When conjectural consistency is applied the corresponding notations are C_{sim}, $C_{\mathrm{seq}}1$, $C_{\mathrm{seq}}2$.

locations.
The situation is replicated under Nash, when the private firm decides on delegation first. In this situation, the reaction function of the public owner, equation (2.4.1), is an increasing function of a_1, the incentive choice of the private owner. Anticipating this reaction, the private owner, acting first, can prevent her counterpart from topping her choice by chosing the maximum possible value for a_1, i.e. $a_1 = 1$. They are both led to a no-delegation decision which produces, under Nash now, a socially optimum and consistent equilibrium in locations.

For the remaining two cases, the concept solution of the location sub-game is Nash conjectures. Delegation, in the first case, is carried out simultaneously, in the second, sequentially with the public firm taking the lead. The reaction of each one of the two competitors is taken as fixed without any requirement for correctly matching each other's "guess". The owner of the public firm, when she has the lead, commits herself to an increased weight on sales - as compared to the situation where the delegation decisions are taken simultaneously - in an effort to pre-empt her counterpart's reaction. This behaviour results in stiffer competition favouring both total welfare and the consumer surplus.

It is safe to say that, in a mixed duopoly, consistency of conjectures in locations brings in the minimisation of the total cost of servicing the market and higher profits for both firms, at the expence of the consumer. Nash, on the other hand stiffens competition, lowers profits (of both firms) but benefits the consumer.

In spite of the criticism that has been associated with the use of the conjectural variations model as a shortcut to fully dynamic modelling, we believe the contribution of this paper is important, to both theorists and practioners, in capturing aspects of the oligopolistic competition not known until now. It is fair to mention that, albeit theoretically appealing, consistent location conjectures remain, essentially, untested. Perhaps, experimental evidence could provide a convincing argument in favour or against this particular behavioural assumption.

4 Appendix I

Proof of Proposition 2.1.

Filling $a_1 = 1 = a_2$ into (2.0.14) and (2.0.15), respectively, we get the consistent conjectures equilibrium in locations at

$$x^C(W,t) = x(W,t) = \frac{2W+t}{4t}, \qquad y^C(W,t) = y(W,t) = \frac{2W+3t}{4t}.$$

Condition (2.0.1) amounts to $2W \leq t$ which suffices to prove that both $x^C(W,t)$ and $y^C(W,t)$ are internal points of the market. The fact that these locations are socially optimal stems directly from Proposition 4 of [Eleftheriou and Michelacakis (2016)]. For completeness, we provide a short proof here. The socially optimal locations can be found by minimizing the total production cost to satisfy aggregate demand, $T(x,y)$, where

$$T(x,y) = \int_0^x [t(x-z)+w_1]dz + \int_x^s [t(z-x)+w_1]dz + \int_s^y [t(y-z)+w_2]dz + \int_y^1 [t(z-y)+w_2]dz\,.$$

The FOC for T give the equations

$$\frac{\vartheta T(x,y)}{\vartheta x} = -\frac{\vartheta \Pi_{R_1}(x,y)}{\vartheta x} = 0 \quad \text{and} \quad \frac{\vartheta T(x,y)}{\vartheta y} = -\frac{\vartheta \Pi_{R_2}(x,y)}{\vartheta y} = 0\,,$$

completing the proof.

Proof of the non-negativity of $\Pi^C_{R_2}$.

For $W, t \geq 0$

$$\begin{aligned}
(2W-t)^2 + 6t^2 \geq 0 \quad &\Leftrightarrow \\
4W^2 - 4tW + t^2 + 6t^2 \geq 0 \quad &\Leftrightarrow \\
8t^2 - 8tW + 4W^2 + 4Wt - t^2 \geq 0 \quad &\Leftrightarrow \\
8t(t-W) + 4W^2 + 4Wt - t^2 \geq 0 &
\end{aligned}$$

By assumption

$$k \geq t + w_1 \;\Leftrightarrow\; k - w_2 \geq t - W\,.$$

Combining the above two inequalities we see that

$$8t(k-w_2) + 4W^2 + 4Wt - t^2 \geq 0$$

completing the proof.

Proof of the non-negativity of $\mathcal{CS}^C$.

We claim that

$$2t(t+W) \geq 2W^2 + tW + t^2\,. \tag{4.0.1}$$

$$\begin{aligned}
2t(t+W) \geq 2W^2 + tW + t^2 \quad &\Leftrightarrow \\
2t(t+W) \geq W(W-t) + (W+t)^2 \quad &\Leftrightarrow \\
(t-W)(2W+t) \geq 0 &
\end{aligned}$$

which is true because of (2.0.1). On the other hand, by assumption,

$$k \geq t + w_2 \;\Leftrightarrow\; k - w_1 \geq t + W\,.$$

Combining this with (4.0.1) we get

$$2t(k-w_1) \geq 2t(t+W) \geq 2W^2 + tW + t^2$$

completing the proof.

Proof of Proposition 2.4.

i) The first part follows directly from Proposition 2.1.

ii) The second part is a straightforward combination of Proposition 2.1 with Proposition 2.3.

iii) For the third part, from equations (2.3.1) and (2.2.1) we have

$$\Pi_{R_1}^{N\mathrm{sim}}(W,t) = \frac{45}{256}\frac{(2W+t)^2}{t} < \frac{3(2W+t)^2}{16t} = \Pi_{R_1}^{C\mathrm{sim}}(W,t)\,.$$

Similarly, equations (2.3.2) and (2.2.2) give

$$\Pi_{R_2}^{C\mathrm{sim}}(W,t) - \Pi_{R_2}^{N\mathrm{sim}}(W,t) = \frac{3}{256}\frac{4W^2+4Wt+t^2}{t} \geq 0\,.$$

iv) Regarding the consumer surplus, subtracting equation (2.3.3) from equation (2.2.5) we get

$$\mathcal{CS}^{C\mathrm{sim}}(W,t) - \mathcal{CS}^{N\mathrm{sim}}(W,t) = \frac{1}{256}\frac{68W^2+4Wt-15t^2}{t}\,.$$

The positive root of the numerator of the above difference is $W = 60/136t$. Because we want both Proposition 2.1 and Proposition 2.3 to hold true $W \leq \min\{3/10t, 1/2t\} = 3/10t$ and $3/10 < 60/136$ thus, the sign of the above difference is negative proving the result.

Proof of Proposition 2.7.

i) The triple inequality

$$x^{N\mathrm{seq2}}(W,t) < x^{N\mathrm{sim}}(W,t) < y^{N\mathrm{seq2}}(W,t) < y^{N\mathrm{sim}}(W,t)$$

follows directly from Propositions 2.3, Proposition 2.6 and $3t \geq 10W$. While the inequality

$$y^{N\mathrm{seq2}}(W,t) - x^{N\mathrm{seq2}}(W,t) = \frac{-3W+4t}{11t} < \frac{-2W+3t}{8t} = y^{N\mathrm{sim}}(W,t) - x^{N\mathrm{sim}}(W,t)$$

proves that the two firms locate closer to each other under sequential delegation leading to fiercer competition, the location of the indifferent consumer is to the left of the location of the indifferent consumer under simultaneous delegation because

$$x^{N\mathrm{seq2}}(W,t) + y^{N\mathrm{seq2}}(W,t) = \frac{13W+12t}{11t} < \frac{10W+9t}{8t} = x^{N\mathrm{sim}}(W,t) + y^{N\mathrm{sim}}(W,t)$$

proving that the welfare maximising firm extends its market share when its owner decides first on the delegation terms of her manager.

ii) To the end of proving part ii) of Proposition 2.7, it suffices to compare equality (2.3.1) with equality (2.6.2) and equality (2.3.2) with equality (2.6.3), respectively. Thus, regarding the private firm, we see

$$\Pi_{R_1}^{N_{\text{seq}2}}(W,t) = \frac{20}{121}\frac{(2W+t)^2}{t} < \frac{45}{256}\frac{(2W+t)^2}{t} = \Pi_{R_1}^{N_{\text{sim}}}(W,t)\,.$$

The situation is reversed as proved by

$$\begin{aligned}
\Pi_{R_2}^{N_{\text{seq}2}}(W,t) - \Pi_{R_2}^{N_{\text{sim}}}(W,t) &= \frac{1}{22}\frac{10W^2+10Wt-3t^2-22w_2t+22kt}{t} \\
&\quad -\frac{1}{256}\frac{116W^2+116Wt-35t^2-256w_2t+256kt}{t} \\
&= \frac{4W^2+4Wt+t^2}{(256)(11)t}\,.
\end{aligned}$$

iii) Consider the difference

$$\begin{aligned}
\mathcal{CS}^{N_{\text{seq}2}}(W,t) - \mathcal{CS}^{N_{\text{sim}}}(W,t) &= \frac{-288W^2-112tW+242kt-242w_1t-105t^2}{242t} \\
&\quad -\frac{-324W^2-132tW+256kt-256w_1t-113t^2}{256t} \\
&= \frac{4680W^2+4680Wt+466t^2}{(242)(256)t}\,.
\end{aligned}$$

It is always positive, proving the claim.

Proof of Proposition 3.1

i) Part i) is the combined result of Propositions 2.1, 2.2, 2.4, 2.5, 2.7 and inequality

$$x^{N_{\text{seq}2}}(W,t) = \frac{2W+t}{4t} < \frac{4}{11}\frac{2W+t}{t}\,.$$

ii) Similarly part ii) follows, again, from Propositions 2.1, 2.2, 2.4, 2.5, 2.7 and inequality

$$\Pi_{R_2}^{C}(W,t) - \Pi_{R_2}^{N_{\text{seq}2}}(W,t) = \frac{8W^2+8Wt+2t^2}{(220(8)t} > 0\,.$$

iii) Finally, part iii) is the direct outcome of the combination of the same set of Propositions, namely Proposition 2.1, Proposition 2.2, Proposition 2.4, Proposition 2.5 and Proposition 2.7.

5 Appendix II

The aim of this ad-hoc appendix is to give a precise answer to a question arising, in a natural way, from the above analysis: "when is Nash consistent?". Scattered evidence throughout the relative literature manifests the implicit knowledge of the answer, at least to specialists. We decided to icorporate it here for completeness and because we could not find a reference containing it to cite. To desist from being pedantic, we consider a two-player game and prove:

Proposition 5.1 *Let $\mathcal{G} = \{\mathcal{I} = \{1,2\}, (I_\imath, \pi_\imath)_{\imath\in\mathcal{I}}\}$ be a two-player game with $I_\imath \subset \mathbf{R}$, closed intervals, and $\pi_\imath|_{I_\imath}$ differentiable real functions. A Nash equilibrium in pure strategies, (x_1^*, x_2^*), is consistent if and only if either $x_{\jmath\imath}(x_1^*, x_2^*) := \frac{dx_\jmath}{dx_\imath}|_{x_\imath^*} = 0$ or (x_1^*, x_2^*) is a critical point of $\pi_\imath$.*

Proof. The reverse being trivial, we prove the direct statement. If (x_1^*, x_2^*) is a Nash equilibrium for the above game, it satisfies the system

$$\frac{\vartheta\pi_1}{\vartheta x_1} = 0 \quad \text{and} \quad \frac{\vartheta\pi_2}{\vartheta x_2} = 0\,.$$

For (x_1^*, x_2^*) to be a consistent equilibrium, it must satisfy the system

$$\frac{\vartheta\pi_1}{\vartheta x_1} + \frac{dx_2}{dx_1}\frac{\vartheta\pi_1}{\vartheta x_2} = 0 \quad \text{and} \quad \frac{\vartheta\pi_2}{\vartheta x_2} + \frac{dx_1}{dx_2}\frac{\vartheta\pi_2}{\vartheta x_1} = 0\,,$$

where $x_{21} := dx_2/dx_1$ and $x_{12} := dx_1/dx_2$ are the respective conjectures. It follows that (x_1^*, x_2^*) satisfies the system

$$\frac{\vartheta\pi_1}{\vartheta x_2} = 0 \quad \text{and} \quad \frac{\vartheta\pi_2}{\vartheta x_1} = 0\,,$$

as well, completing the proof.

Alternatively, player $\imath$, in order to best respond to the choice of player $\jmath$, in a consistent way, not only does she look at own objective function - solving $\vartheta\pi_\imath/\vartheta x_\imath = 0$ - but she puts herself into the shoes of her opponent solving $\vartheta\pi_\jmath/\vartheta x_\imath = 0$, as well.

Evidently, Proposition 5.1 generalises to any number of players.

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