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Abstract

This paper considers the problem of a water management authority which is faced with the threat of a drought that will take place at an uncertain date. Three management policies are investigated: i) the *laissez-faire* policy of automatic regulation through Open Access mechanisms, ii) the policy of keeping a rationed level of water usage until the water table is restored, and iii) an economically optimal policy taking account of the probability of a drought and the fact that water is a capital. In particular, it is shown that the optimal pre-drought steady-state stock of water is smaller than the stock that would be optimal in the no-drought case. Hence, no precautionary stock is built up. However, a more extensive drought period will lead to a larger pre-drought steady-state stock.

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1. Introduction

Each year deficit rainfall and drought around the world takes a heavy toll in terms of destroyed crops, famine, and losses of human life. The drought periods may be single event phenomena or recurring events, possibly linked to the ENSO (El Nino/Southern Oscillation)-phenomenon (Diaz and Markgraf, 1992). One of the most disastrous droughts took place in the Sahel region on the southern edge of the Sahara in 1968-1973. In its last year this long and persistent drought alone took 100 000 human lives and reduced the livestock to a half. Erratic swings in climatic conditions are typical in this region and periods of drought tend to occur every 25 to 30 years, always claiming their share of victims (Sinn, 1988). A less dramatic but more frequently occurring drought is the monsoon phenomenon on the Dirre-plateau in Ethiopia. Each year during the deficient winter rainfall period from November to March the Boran people have to water theirs large herds of sheep, camels and horses from a small number of all-year wells (the *Taulas*), organized around an intricate system of distribution rights based upon bequest and labor input in water collection. Occasionally the drought extends for a longer period of time implying increased water shortage, sinking water table and lost livestock.

The major characteristics of aquifers that affect the costs of water supply are summarized in Young and Haveman (1985). These characteristics include: the depth of the water table, the thickness of the saturated zone (the water-bearing formation), the transmissivity (the rate at which the water is transmitted through the aquifer) and the characteristics of the geological formations through which the well must be drilled. (See also Burt, 1964, 1965 and Carruthers and Clark, 1981). When groundwater is abstracted from an aquifer by artificial means, a lowering of the water table occurs. This leads to increasing recharge and decreasing discharge (Huisman, 1972). The recharge increases by a shift of the water divide which enlarges the catchment area, as well as by increased infiltration of surface water from influent streams. The discharge decreases by a reduction of evapo-transpiration, by a smaller return of groundwater to the surface and by reduced percolation to bounding aquifers. As long as the amount of artificial recovery is limited, the resulting increase in recharge, together with the decrease in discharge, is able to balance this abstraction and a new equilibrium with a lower water table is reached. However, a further increase of artificial abstraction may lead to depletion. The deficit will then be taken from storage, continuously lowering the groundwater table. Such abstractions will only be allowed for a limited (though often long) period (e.g. London basin, some coastal aquifers in southern California). Hence, the maximum possible groundwater abstraction reduces the outflow to zero and brings the amount of water entering the aquifer up to the highest value obtainable under the geo-hydrologic conditions present. Depletion of an aquifer may be an irreversible event, where the tiny channels otherwise supported by the water within them, may collapse through subsidence and be unable to carry water in the future (Neher, 1990). Problems of serious water table reduction is a matter of concern in places such as northeast India and parts of sub-Saharan Africa.

The level of the water table fluctuates with precipitation, possibly with considerable lags. This depends upon the category of the aquifer involved, notably the permeability of the material and the size of the aquifer. The time of movement of infiltrating water is a function of the thickness of the unsaturated zone and the vertical unsaturated hydrolic conductivity. Small and shallow aquifers are able to adapt within hours after a rainfall, while larger and more complicated aquifers like in the Hualapai Plateau area of northwestern Arizona, water may take years to pass through the unsaturated zone. The response time of a water table aquifer is more rapid the thinner is the unsaturated zone above the water table (Fetter, 1980).

In the following we consider a stylized problem of groundwater management, where there is a probability of a shortfall of precipitation followed be normal precipitation conditions. It is assumed that the cost of water extraction depends upon the size of the aquifer (the level of the water table). Hence, water may be extracted during the drought period (when local rainfall fails and surface water courses dries up) but only at increasing costs. We consider three management policies of increasing sophistication; i) a laissez faire policy, ii) a rationing policy and ii) an economically optimal policy. According to the first policy nothing is done to alleviate the burden of the drought. In the second some effort is made to ration water once the drought sets in, while the third is an optimal policy which take account of the fact that water is a capital and that the size of the probability of a drought also may influence the management decision.

The first policy seems to be a fair description of water management in many countries. Around the world groundwater basins are typically exploited by a large number of independent pumpers withdrawing from a common groundwater supply. As groundwater ordinarily can move in response to withdrawals, the action of any pumper affects the conditions experienced by other users; thus they are interdependent and external costs (or benefits) are imposed (Young and Haveman, 1985). In this setting nothing is normally done when the drought strikes and the aquifer is left to the Open Access solution. Hence, in the laissez-faire policy of Open Access, the stock of water will automatically be restored to its pre-drought level by way of the zero-marginal-benefit-condition.

The rationing policy amounts to keeping a constrained level of water supply from the date the drought sets in until the aquifer has regained its normal size. Policies like this are in use around the world, in particular in countries with an organized system of public water provision but without unit water pricing. It may take various forms, for instance by cutting water supply during specific hours of the day or by prohibiting certain usage of water e.g. the watering of private gardens. The third policy is an optimal policy whose implementation may call for pricing according to marginal scarcity cost. Water markets and pricing of water are now emerging and are in fact common in many developed countries and regions around the world e.g. Denmark, United States (California, Colorado), Chile, the Canary Islands (Ringskog, 1997)

2. The basic model and assumptions

We consider a management authority or a «sole owner» with perpetual tenure managing the aquifer. The stock of groundwater at date t is denoted x_t and the recharge rate (emanating from precipitation) g is assumed constant and stock independent. The flow rate of extraction from the aquifer at date t is endogenously determined and denoted h_t. The objective of the management authority is to maximize the present value of net social benefit, which is defined as the difference between (gross) benefits U and costs C. The benefits are assumed to increase with extraction at a decreasing rate i.e. $U(h_t)$; with $U'(h_t) > 0$ and $U''(h_t) < 0$. The costs are assumed linearly dependent such that $C = c(x_t)h_t$ with $c'(x_t) < 0$ and $c''(x_t) \geq 0$, i.e. the higher is the water table the lower is the average cost of extraction. The social discount rate is denoted δ.

Hence, the objective of the management authority is

$$\max_{h_t, x_t} \int_0^{\infty} (U(h_t) - c(x_t)h_t)e^{-\delta t}dt$$

subject to

1) $\dot{x}_t = g - h_t$

2) $x_0 = x(0)$

The corresponding Hamiltonian to this problem reads

$$\mathrm{H} = \left[U(h_t) - c(x_t)h_t\right]e^{-\delta t} + \gamma_t(g - h_t)$$

where γ_t denotes the adjoint variable

The first order necessary conditions are

3) $$\frac{\partial \mathrm{H}}{\partial h_t} = \left[U'(h_t) - c(x_t)\right]e^{-\delta t} - \gamma_t = 0$$

4) $$\frac{\partial \mathrm{H}}{\partial x_t} = -c'(x_t)h_t e^{-\delta t} = -\dot{\gamma}_t$$

Taking the total differential of 3), substituting into 4), using 1) and rearranging terms we arrive at the following relationship

5) $$\frac{U''\dot{h}_t - c'(x_t)h_t}{U' - c(x_t)} = \delta$$

The steady-state solution, h^*, x^* (where $\dot{h}_t = \dot{x}_t = 0$ and $h^* = g$) is characterized by

6) $$-\frac{c'(x^*)g}{U'-c(x^*)}=\delta$$

The solution to this problem is well known and described by Neher, 1990, (i.e. for the case of U'' equal to a negative constant). Assuming that the initial value $x(0) < x^*$, the optimal solution is to bring the stock of water asymptotically along a stable arm to the steady-state stock, x^*. The optimal strategy is to set the initial harvest rate at a value less than $h^* = g$ and let it grow over time at a decreasing rate to approach the steady-state value h^*. Since the harvest rate is below the recharge rate, the aquifer will also grow over time and approach the optimal steady-state stock x^*. Hence, the stable arm for an initial stock of water less than the steady-state stock is characterized by $\dot{h}_t > 0, \dot{x}_t > 0$. During the steady-state the adjoint variable, γ_t decreases at a rate equal to $-\delta$, (i.e. $(\dot{\gamma}_t/\gamma_t) = -\delta$) which implies a constant (current) implicit price of water.

Into this basic model we introduce a drought period of length ε that will start at an uncertain date τ. To represent the uncertainty of τ we apply an exponential function which is frequently used in uncertainty problems dealing with the arrival of some event (see Barlow and Proschan, 1975). The essential assumption here is that the conditional probability of the event happening, provided that it has not already happened, remains the same as time proceeds. Hence, the length of the time period prior to the event does not matter. Thus, the distribution function is given by $F(t) = \Pr(\tau \le t) = (1 - e^{-\lambda t})$ and the density function by $f(t) = \Pr(\tau) = \lambda e^{-\lambda t}$. During the drought period we assume that the recharge to the aquifer vanishes. Hence, during this period $g = 0$. After the drought period normal precipitation and recharge rates are restored. It is essential to note that we assume the drought will happen sooner or later. The expected waiting time until it happens is equal to $1/\lambda$.

3. Open Access restoration

The Open Access solution is characterized by a marginal benefit equal to zero, i.e.

7) $$U'(h) - c(x) = 0$$

The pre-drought steady-state stock, x_∞ is determined from 7) by recognizing that $h_\infty = g$. When the drought sets in, the stock of water starts to diminish since there is a net use of water. During the drought period, ε, the stock of water will be drawn down at the rate h_t (determined by 7)) such that it becomes equal to $x_{\tau+\varepsilon}$ at date $\tau+\varepsilon$, i.e.

$$x_{\tau+\varepsilon} = x_\infty - \int_\tau^{\tau+\varepsilon} h_t dt$$

During this period the extraction of water will be falling, since the marginal extraction cost is increasing. This can be seen by utilizing 7) to obtain

$$\dot{h} = -\frac{c'(x_t)h_t}{U''(h_t)} < 0$$

However, at date $\tau+\varepsilon$, the recharge is again positive (equal to g) and the stock starts to increase until the Open Access pre-drought steady-state stock, x_∞ is reached. During this period the extraction is increasing as determined by

$$\dot{h} = \frac{c'(x_t)(g-h_t)}{U''(h_t)} > 0$$

Thus, under Open Access, water extraction is sensitive to the drought and there is a built in restoration mechanism as determined by the zero-marginal-benefit-condition. However, this solution is not sensitive to the size of the probability of a drought since the solution is not planned but simply sets in at the date the drought hits. The solution is, however, sensitive to the length of the drought period. A longer drought period will lead to a further reduced extraction, a smaller water stock and an increased period of stock restoration.

4. Rationing

This is a somewhat more active policy than the previous one, even though it does not take fully account of the fact that the aquifer is a capital. We assume that the size of the aquifer at the outset is equal to some steady-state stock, $\bar{x}$. The problem is in an optimal way to determine a constant rate of water extraction, $\bar{h}$, which is to be kept from the date the drought sets in until

$$U'(\bar{h}) - c(x_t) > 0$$

This implies that the rationed extraction is less than the extraction under Open Access ($\bar{h} < h_{\infty,t}$). From this it follows that the stock will not be reduced by as much under rationing as under the automatic adjustment of Open Access.

5. Optimal policy

The policy discussed above is not optimal since it does not fully take into account that the aquifer is a capital. In order to determine the optimal management strategy we start by solving the optimization problem at date $\tau + \varepsilon$, i.e. after the drought period is over. At this date we are faced with the basic problem discussed in the previous paragraph. Denoting the size of the aquifer at date $\tau + \varepsilon$ by $x_{\tau+\varepsilon}$, the net present value of the future benefits (evaluated at date $\tau + \varepsilon$) is denoted $S(h_t^*, x_t^*; x_{\tau+\varepsilon})$. Hence, the aquifer is managed according to condition 5) and 6) with an initial value equal to $x_{\tau+\varepsilon}$.

Next, we move one step forward and consider the optimization problem at date τ (shortly after the drought sets in). The optimization problem at this date may be formulated as

$$\max_{h_t, x_t} \left\{ \int_{\tau}^{\tau+\varepsilon} [U(h_t) - c(x_t)h_t] e^{-\delta t} dt + S(h_t^*, x_t^*; x_{\tau+\varepsilon}) e^{-\delta(\tau+\varepsilon)} \right\}$$

subject to

8) $\quad \dot{x}_t = -h_t$

9) $\quad x_\tau = x(\tau)$

Condition 9) gives the restriction on the starting value of the stock of water at date τ.
The Hamiltonian of the above problem is

$$\text{H} = [U(h_t) - c(x_t)h_t] e^{-\delta t} - \varphi_t h_t$$

where φ_t denotes the adjoint variable

The first order necessary conditions are

10) $$\frac{\partial H}{\partial h_t} = \left[U(h_t) - c(x_t)h_t\right]e^{-\delta t} - \varphi_t = 0$$

11) $$\frac{\partial H}{\partial x_t} = -c'(x_t)h_t e^{-\delta t} = -\dot{\varphi}_t$$

12) $$\varphi_{\tau+\varepsilon} = \frac{\partial S(h_t^*, x_t^*; x_{\tau+\varepsilon})e^{-\delta(\tau+\varepsilon)}}{\partial x_{\tau+\varepsilon}}$$

Condition 12) is the transversality condition for an optimal control problem with fixed te time and a scrap value function (e.g. see Seierstad and Sydsæter, 1987). The adjoint vari φ_t may be interpreted as the net present value of the marginal scarcity rent of water.

Taking the total differential of 10), substituting into 11) and rearranging terms we arrive

13) $$\frac{U''\dot{h}_t}{U'(h_t) - c(x_t)} = \delta$$

Condition 13) is seen to be identical to 5) with $g = 0$. It represents the Hotelling rule fo non-renewable resource. Hence, from date τ on, water in the aquifer is considered a non renewable resource which is to be extracted in an optimal way bringing the stock of wat down to $x_{\tau+\varepsilon}$ while satisfying 12).

To simplify notation we next define the following maximum function

$$F(x_\tau) = \max_{h_t, x_t}\left\{\int_0^\varepsilon \left[U(h_t) - c(x_t)h_t\right]e^{-\delta t}dt + S(h_t^*, x_t^*, x_{\tau+\varepsilon})e^{-\delta\varepsilon}\right\},$$ s.t. 8), and 9).

The derivative of this maximum function with respect to the stock available at date τ, is equal to the current value of the adjoint variable at this date, i.e. $F'(x_\tau) = \varphi_\tau e^{\delta\tau} > 0$. Furthermore, we have $F''(x_\tau) = (d\varphi_\tau / dx_\tau) < 0$, i.e. the scarcity rent decreases as the size of the stock increases.

The full (*ex ante*) problem to be considered is as follows

$$\max_{h_t, x_t} \int_0^\infty \lambda e^{-\lambda t} \left\{ \int_0^\tau \left[U(h_t) - c(x_t)h_t \right] e^{-\delta t} dt + F(x_\tau) e^{-\delta\tau} \right\} d\tau$$

subject to

14) $\quad \dot{x}_t = g - h_t$

Integrating the objective functional by parts , the problem can be reformulated as

$$\max_{h_t, x_t} \int_0^\infty \left[U(h_t) - c(x_t)h_t + \lambda F(x_t) \right] e^{-(\delta+\lambda)t} dt$$

s.t. 14).

The corresponding Hamiltonian to this problem is

$$\mathrm{H} = \left[U(h_t) - c(x_t)h_t + \lambda F(x_t) \right] e^{-(\delta+\lambda)t} + \mu_t (g - h_t)$$

where μ_t denotes the adjoint variable

The necessary first order conditions are

15) $$\frac{\partial H}{\partial h_t} = \left[U'(h_t) - c(x_t)\right]e^{-(\delta+\lambda)t} - \mu_t = 0$$

16) $$\frac{\partial H}{\partial x_t} = \left[-c'(x_t)h_t + \lambda F'(x_t)\right]e^{-(\delta+\lambda)} = -\dot{\mu}_t$$

17) $$\mu_\tau = F'(x_\tau)e^{-\delta\tau}$$

Taking the total differential of 15), substituting into 16), using 15) again and rearranging te we obtain the following condition for the optimal steady-state solution $\tilde{h}$, $\tilde{x}$ (with $\dot{h}_t = \dot{x}_t = 0$)

18) $$-\frac{c'(\tilde{x})g}{U'(\tilde{h}) - c(\tilde{x})} + \frac{\lambda\left[F'(\tilde{x}) - (U'(\tilde{h}) - c(\tilde{x}))\right]}{U'(\tilde{h}) - c(\tilde{x})} = \delta$$

Condition 18) represents a «modified golden rule» of stock management. It essentially says that the rate of return from keeping a marginally larger stock in optimum must be equal to social discount rate. The rate of return consists of two elements. Firstly, there is the rate of return known as the «stock- cost effect» which gives the rate of return in terms of reduced extraction cost by keeping a marginally larger stock. This is the first expression on the left side of 18). Secondly, there is the expected marginal rate of return in terms of future benei i.e. the expected increase in future benefits following from keeping a marginally larger sto the date the drought sets in. This is the second element on the left hand side of 18). The numerator gives the (instantaneous) expected net benefit from investment while the denominator is the investment. The gross benefit is equal to $F'(\tilde{x})$ and the investment is e to $(U'(\tilde{h}) - c(\tilde{x}))$ (the marginal water rent foregone).

It turns out that the expected net benefit of keeping a marginally larger stock (the numera the second element) is negative i.e. the return is not sufficient to cover the investment. Th be seen by utilizing 15) and 17)

$$\left[U'(\widetilde{h})-c(\widetilde{x})\right]e^{-(\delta+\lambda)\tau}=F'(\widetilde{x})e^{-\delta\tau}$$

Hence, $\left[U'(\widetilde{h})-c(\widetilde{x})\right]>F'(\widetilde{x})$ and the marginal rate of return in this element is negative.

Next, the problem is to determine how $\widetilde{x}$ is related to x^{*} and how it reacts to changes in λ and ε. In order to do this we take the implicit derivatives of condition 18) and obtain

19) $$\frac{d\widetilde{x}}{d\lambda}=\frac{\left[1-\dfrac{F'}{U'-c}\right]}{\dfrac{\lambda\left[F''(U'-c)+F'c'\right]-g\left[c''(U'-c)+c'^{2}\right]}{\left[U'-c\right]^{2}}}\leq 0$$

20) $$\frac{d\widetilde{x}}{d\varepsilon}=\frac{-\dfrac{dF'}{d\varepsilon}}{\dfrac{\lambda\left[F''(U'-c)+F'c'\right]-g\left[c''(U'-c)+c'^{2}\right]}{\left[U'-c\right]^{2}}}>0$$

Inspection of signs show that the denominators of both expressions are negative. Furthermore, the numerator of 19) is strictly positive (for $\varepsilon>0,\lambda>0$) since the expected net benefit is strictly negative. Hence, an increase of the «failure rate», λ leads to a reduction of the optimal pre-drought steady-state stock. The implication of this is that it is always optimal under uncertainty to keep a steady-state stock, $\widetilde{x}$, which is smaller than the certainty steady-state stock , x^*. It is, thus, not optimal under this kind of uncertainty to build up a «reserve stock» of water to be used when the drought sets in.

To determine the sign of the numerator of 20) we first recall that $F'(\widetilde{x})=\varphi_{\tau}e^{\delta t}$ i.e. the future value of a marginal increase of the water stock is equal to the value of the costate variable (the scarcity rent of water) at the date the drought starts. Furthermore, we recognize that an increase of the drought period will lead to an increased scarcity value of the existing stock of water $\widetilde{x}$. Hence, $(d\varphi_{\tau}/d\varepsilon)>0$ and 20) is, thus, positive. This is to say that an increase of the drought period will lead to a larger pre-drought steady-state stock $\tilde{x}$,but it will still be less than x^*.

7. Concluding remarks

Three management policies for an aquifer confronted with the threat of a limited drought period have been considered. It is shown that Open Access water usage automatically restores the water table to its pre-drought level through the zero-marginal-benefit-condition. Furthermore, it is shown that rationing speeds up the restoration process as compared to the Open Access policy but is otherwise non-economical since it does not take into account that the water stock is a capital that should be managed according to the size of the social discount rate as well as to the size of the probability of the occurrence of a drought.

With respect to an optimal management of the water resource, reasoning could lead one to believe that the threat of a drought would call for an extra precautionary stock of water (as compared to the no-drought case) to be drawn upon during the drought period (i.e. a smooth-pasting argument). However, the contrary is the case. Economically optimal stock management implies that the pre-drought stock should be kept smaller than the optimal no-drought stock, and the larger is the probability of a drought the smaller should the pre-drought stock be.

This result implies that the consumers should take the advantage of using the water stock during the period of normal precipitation before the drought starts. Technically, the failure rate λ functions as an increment to the social discount rate and leads to heavier discounting and therefore a smaller steady-state stock. This result holds true even if the drought period extends into infinity, such that the aquifer change from being a renewable resource to becoming a non-renewable resource at the date the drought sets in. Also, even if the drought trigger a salination process which makes all or a part of the water stock unusable for consumption (and thus increases water scarcity) this result will hold. The optimal water stock prior to this event will still be smaller than the optimal non-drought steady-state stock.

Literature

Burt, O.R. (1964), «Optimal Resource Use over Time with an Application to Ground Water», . Burt, O.R. (*Management Science* 11, 80-93 .

1975), «Ground Water Management and Surface Water Development for Irrigation» in: R.M. Thrall (ed.) *Economic Modelling for Water Policy Evaluation* (North Holland , Amsterdam)

Caldwell, J.C. (1975) «The Sahelian Drought and its Demographic Implications», American Council on Education, OLC Paper No.8, 1975.

Carruthers, I. and C. Clark (1981), *«The Economics of Irrigation»,* Liverpool University Press, Liverpool.

Diaz, H.F. and Markgraf, V. (eds.) (1992) «*El Nino*», Cambridge University Press.

Fetter, C.W. (1980) «*Applied hydrology*», Charles Merill Publishing Company, Columbus, Ohio.

Huisman, L. (1972) «*Groundwater Recovery*», Macmillan, London, 1972.

Neher, P. (1990) «*Natural resource economics: Conservation and exploitation*», Cambridge University Press, Cambridge.

Ringskog, K. (1997) «*Water Markets in the Americas*», World Bank Report.

Sinn, H.-W. (1988) « The Sahel Problem», *Kyklos*, Vol. 41, Fasc. 2, 187-213.

Young, R.A. and R.H. Haveman (1985) «Economics of Water Resources» in Kneese, A.V. and J.L. Sweeney (eds.) «*Handbook of Natural Resource and Energy Economics*», Vol III, North Holland.