A theory of optimal paid parental leave policies

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Abstract

This study examines paid parental leave policies in which a pregnant worker can choose not to take a temporary leave and a worker on leave can choose not to return to work after the leave period ends. An optimal paid parental policy is defined as a policy that maximizes social welfare. In the optimal paid parental leave policy, where a pregnant worker voluntarily chooses to take a leave and a worker on leave voluntarily chooses to return to work after the leave period ends, the income risk caused by the leave is not necessarily perfectly shared among workers and workers on leave. In addition, by lengthening the leave period, another feasible parental leave policy that does not satisfy the incentive constraint improves social welfare, implying that the length of the leave under the optimal paid parental leave policy is "short." A numerical example of the model justifies that both a short-leave-with-generous-leave-benefits policy and the long-leave-with-less-generous-leave-benefits policy observed across most OECD countries can be optimal.

Keywords: Paid parental leave policy, lack of commitment, incentive constraint, optimal policy

JEL Classification: E24, J38, D61

Declarations

1 Introduction

Most OECD countries have adopted mandatory paid parental leave policies. For pregnant workers (and their spouses and partners), these countries provide compensation during the leave and the right to return to their workplace after the leave period ends. There is a large amount of research that examines the effects of leave policies on leave-taking rates, job continuity of women, children's ability and health, and

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so on.[1] According to empirical studies such as Waldfogel (1999), Han et al. (2009), Baum and Ruhm (2016), Yamaguchi (2019) and so on, leave polices increase leave-taking rates. Additionally, empirical studies such as Baum (2003), Baker and Milligan (2008), Kluve and Tamm (2013), and so on show that job-protected leave policies raise the likelihood that women will return to the job. Since taking leave and returning to the job are not mandatory, these findings suggest that the parental leave policies are designed so that a pregnant worker has an incentive to take leave and a worker on leave has incentive to return to the job after the leave period ends. Taking these facts into account, I investigate how the optimal paid parental leave policies look theoretically.

This study analyzes the following model. In each period, a risk-averse worker produces output and derives utility from consumption. At the end of each period, a worker gets pregnant with a certain probability, and she then has to decide whether to take a temporary leave or not. If such a shock does not hit a worker, she continues to work. A worker on leave derives utility from consumption and enjoying time with her child. At the end of each period, another shock hits a worker on leave with a certain probability. If this shock hits a worker on leave, then she has to decide whether to return to work. If a worker on leave decides to return to work, then she will work in the next period. If not, she stays home forever. A worker who stays home derives utility from enjoying time with her child. In such a model, I characterize an optimal paid parental leave policy, in which a worker hit by a shock voluntarily chooses to take leave and a worker on leave voluntarily chooses to return to work after the leave period ends.

The structure of optimal paid parental leave policies depends on whether the incentive constraint that a worker on leave faces binds. Since a worker on leave voluntarily chooses to return to the job at the optimum, the value of that worker has to be higher than or equal to the value of a worker who stays home forever. When the constraint does not bind, the consumption of a worker and that of a worker on leave should be equal. Since workers are risk averse, consumption should be unchanged based on the worker's status. As for the length of the leave, the optimal length should equalize the marginal welfare gain and marginal welfare loss caused by the additional extension of the leave period. An increase in the leave period decreases the number of workers in the steady state, which lowers aggregate output and consumption. Thus, social welfare marginally decreases. At the same time, an increase in the leave period gives a worker on leave additional utility from leisure, which increases social welfare. The optimal length of the leave thus equalizes the marginal welfare loss and marginal welfare gain.

When this incentive constraint binds, the above results no longer hold. The consumption of a worker is greater than that of a worker on leave. Hence, to give a worker on leave an incentive to return to work after the leave period ends, an optimal policy has to guarantee higher (or at least equal) values when a worker on leave chooses to return to work than when she does not. The efficient way to achieve this goal is to increase the consumption of a worker and decrease the consumption of a worker on leave. This makes the consumption of a worker and that of a worker on leave different, which implies that the optimal policy offers imperfect income insurance. In addition, the present study shows that a worker's consumption when the incentive constraint does not bind is larger than that when the incentive constraint binds.

As for the length of parental leave, the leave period in the optimal paid parental leave policy is "short" in the sense that we can find another policy whose leave period is slightly longer than the optimal one and it raises social welfare. The intuition here is as follows. To give a worker on leave an incentive to return to work, the consumption of a worker should be large. Social welfare is measured by the sum of

[1] Rossin-Slater (2017) is a good survey article for maternity and family leave policy.

all the statuses of workers' utilities in a steady state. Since the number of workers in the steady state depends on the length of the leave, a short leave period increases the number of workers, which can raise social welfare. Another paid parental leave policy may raise social welfare above that achieved by the optimal paid parental leave policy even though the incentive constraint is not satisfied.

By a numerical example based on the model, I claim that the optimal paid parental leave policy exhibits a negative relationship between leave benefits and the duration of the leave. This numerical study shows that a country whose probability that a worker gets pregnant is low tends to provide imperfect income insurance and a longer leave period, because there are more workers in the country with low probability of a worker getting pregnant. Thus, both short-leave-with-generous-benefits paid parental policy and a long-leave-with-less-generous-benefits paid parental leave policy can be optimal ones. In Figure 1, each point shows a combination of the ratio of cash benefits to previous earnings, called "Average payment rate" for females, and the length of the leave, called "Length in weeks," of each OECD country.[2] As is shown, the cross-country data shows the negative relationship between the leave duration and replacement ratio. Assuming OECD countries are similar to each other, the numerical example conducted in this paper suggests that the paid parental policies adopted by OECD countries can be optimal ones.

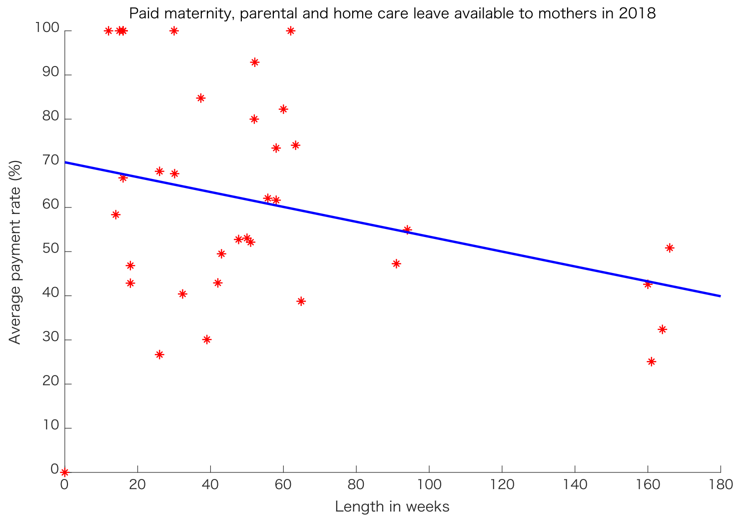

Figure 1: Paid leave entitlements available to mothers in April 2018 (Data source: OECD Family Database)

Additionally, the study compares consumption and the length of the leave at the optimum in two economies, say A and B. In economy A, the discount factor is sufficiently high that the incentive constraint does not bind at the optimum. In economy B, the discount factor is so low that the incentive constraint binds at the optimum. It is shown that the consumption of a worker in economy B is larger

[2]More precisely, "Average payment rate" refers to the proportion of previous earnings replaced by the benefit over the length of the paid leave entitlement for a person earning 100% of the average national (2018) earnings. "Length in weeks" in the figure is the sum of weeks of paternity and maternity leaves available to mothers.

than that in economy A and that the *total* consumption of a worker on leave during the leave period in economy B is smaller than that in economy A. Note that the *periodic* consumption of a worker on leave in economy B is not necessarily smaller than that in economy A. This result contrasts with that in literature of providing insurance with limited commitment (see, for instance, Chapter 20 in Ljungqvist and Sargent (2012).) This difference comes from the model setting under which the length of the leave is an endogenous variable in this study.

Most analyses of parental leave policies have been conducted by empirical work, whereas theoretical analyses are scarce except Erosa et al. (2010) and Del-Rey et al. (2017). One of the closest works to this study is Erosa et al. (2010), where the researcher investigated the effect of paid maternity leave policies on fertility, leave-taking behavior, employment, and welfare using a labor search model. Although the model is rich, it is hard to solve analytically; thus, the study calibrated the model consistent with U.S. data, finding that parental leave policies lead to an aggregate welfare loss even though female welfare actually increases. However, they focused on the redistribution role of parental leave policies, whereas this study sheds light on their insurance role. Del-Rey et al. (2017) considered a similar model to the one in Erosa et al. (2010) and showed that the effects of length of leave on wages and unemployment are ambiguous. Again, however, Del-Rey et al. (2017) did not focus on the insurance role of parental leave policies, as this study does. Furthermore, to the best of my knowledge, no research has investigated paid parental leave policies from an optimality point of view.

A paid parental leave policy can be considered as a type of unemployment insurance, since payment provides the main source of income for the period when a worker cannot work. Indeed, unemployment insurance does not fully cover previous earnings, although the policy can be constrained optimal. In optimal unemployment insurance literature, for instance, Hopenhayn and Nicolini (1997) showed that since an unemployed worker's effort in searching for a job is unobservable, to prevent the moral hazard problem, unemployment insurance partially covers previous earnings. Thus, they claimed that observed unemployment insurance can be constrained optimal. However, this study's driving force is not a worker's hidden action as in Hopenhayn and Nicolini (1997), but rather her incentive constraint, under which a worker on leave voluntarily returns to work after the leave period ends.[3]

In this study, a worker's lack of commitment prevents workers from sharing the income risk perfectly. The mechanism is similar to that in Thomas and Worrall (1988) and Kocherlakota (1996). Thomas and Worrall (1988) considered long-term wage contracts between a risk-neutral firm and a risk-averse worker. Both agents cannot commit to future wage contracts, so the optimal contracts should be self-enforcing. Kocherlakota (1996) considered a pure exchange economy in which two infinitely lived agents face an income risk in every period. From an ex-ante welfare point of view, it is socially optimal that both agents consume the same amount in all periods. That is, in every period, a rich agent transfers some amount to a poor agent. However, since agents cannot commit to future cooperative behavior, an agent may defect if they want. To avoid such defection by agents, an allocation has to give a rich agent an incentive not to defect. Then, the resulting allocation does not perfectly share the income risk between two agents even though the allocation is constrained-efficient. This study applies a similar logic to a different setting where the length of the leave is part of a policy.

The remainder of the paper is organized as follows. Section 2 describes the economic model and Section 3 characterizes a stationary equilibrium. Section 4 characterizes optimal paid parental leave

[3] In a model extending Hopenhayn and Nicolini (1997) such as Mitchell and Zhang (2010), the same result on optimal consumption holds.

policy, and Section 5 discusses the model. Section 6 concludes.

2 Model

2.1 Environment

The economy is populated by a continuum of agents, distributed uniformly on the unit interval, and a social planner. Time is discrete and continues forever, $t = 0, 1, 2, \ldots$.

At the beginning of the economy, the social planner sets a lump-sum tax, $\tau \in [0, y]$, levied on a worker, and the probability with which the leave period ends, $\gamma \in \Gamma := [\underline{\gamma}, 1]$, where y is the amount of output a worker produces and $\underline{\gamma} > 0$ can take any small number. The assumption of a lower bound of γ is only a technical assumption that guarantees the existence of a solution to the optimization problem defined below. A pair, (τ, γ), is called a *paid parental leave policy*.

In every period, an agent, called a *worker*, produces $y > 0$, which is constant over time and given exogenously. In each period, the worker pays a lump-sum tax and consumes $c_t = c := y - \tau$. She derives utility from consumption. If the worker consumes c units of consumption, then her periodic utility is $u(c)$, where $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave, and continuously differentiable. Assume that the good is perishable and there is no storage technology. At the end of each period, a shock arrives with probability $\sigma \in (0, 1)$. This shock can be considered to represent a chance to have a child and take a temporary leave. If the shock hits a worker, then the worker has a child and has to decide whether to leave the job temporarily or not.[4] Let $a_t \in \{0, 1\}$ be the *action* taken by a worker in period t; $a_t = 1$ implies that a worker decides to leave the job temporarily and $a_t = 0$ means that she does not. If the worker chooses to have a child and leave the job temporarily, then she will be a *worker on leave* in the next period; otherwise, she will continue to work in the next period.

A worker on leave produces nothing and derives utility from consumption and time with her child. If a worker on leave consumes d_t units of consumption, then her periodic utility is $u(d_t) + \nu$, where $\nu > 0$ is the utility from time spent with her child. At the end of the period, with probability γ, the leave period ends. With probability $1 - \gamma$, a worker on leave continues to be a worker on leave. The inverse of γ, $\frac{1}{\gamma}$, is the expected duration of leave in this model. If the leave period ends, then a worker on leave has to decide whether to return to work. If she chooses to return to work, she starts working in the next period. If not, she stays home with her child. Hence, she is *out of the labor force*. Let $s_t \in \{0, 1\}$ be the *action* taken by a worker on leave in period t; $s_t = 1$ implies that a worker on leave decides to return to her job and $s_t = 0$ means that she does not.

An agent out of the labor force consumes nothing and enjoys time with her child in each period. Thus, her periodic utility is $u(0) + \nu$. I assume that an agent stays out of the labor force once she leaves.

I assume that a worker chooses to leave the job temporarily if and only if the value of a worker on leave is greater than or equal to that of a worker, and that a worker on leave chooses to return to her job if and only if the value of a worker is greater than or equal to that of an agent out of the labor force. I assume that every shock in this economy is i.i.d. across workers and over time.

Let W_t, L_t, and Q_t be the values of a worker, a worker on leave, and an agent out of the labor force in

[4] If a worker has a child but does not take a temporary leave, it means that she returns to work right after the mandatory leave period ends.

period t, respectively. Then, the values of each status of an agent are given by

$$W_t = u(c) + \beta \left[\sigma \max_{a_t \in \{0,1\}} \{(1-a_t)W_{t+1} + a_t L_{t+1}\} + (1-\sigma)W_{t+1} \right], \tag{1}$$

$$L_t = u(d_t) + \nu + \beta \left[\gamma \max_{s_t \in \{0,1\}} \{s_t W_{t+1} + (1-s_t)Q_{t+1}\} + (1-\gamma)L_{t+1} \right], \tag{2}$$

and

$$Q_t = u(0) + \nu + \beta Q_{t+1}, \tag{3}$$

where $\beta \in (0,1)$ is a common discount factor. Let $\mu_t := (\mu_t^w, \mu_t^l, \mu_t^q)$ be the distribution of agents in period t, where μ_t^w is the ratio of workers in the economy in t, μ_t^l is the ratio of workers on leave in the economy in t, and μ_t^q is the ratio of agents out of the labor force in the economy in t. The distribution μ_t satisfies $\mu_t^w + \mu_t^l + \mu_t^q = 1$ and $\mu_t^x \geq 0$ for all $x \in \{w, l, q\}$. The social planner finances the consumption for workers on leave, d_t, through tax revenue. I assume the social planner balances the budget in every period, that is, in every period t,

$$\mu_t^w \times \tau = \mu_t^l \times d_t \tag{4}$$

is satisfied. Given an agent's actions in period t, (a_t, s_t), the distribution of agents evolves in the following manner:

$$\mu_{t+1}^w - \mu_t^w = -\mu_t^w \sigma a_t + \mu_t^l \gamma s_t, \tag{5}$$

$$\mu_{t+1}^l - \mu_t^l = -\mu_t^l \gamma + \mu_t^w \sigma a_t, \tag{6}$$

and

$$\mu_{t+1}^q - \mu_t^q = \mu_t^l \gamma (1-s_t). \tag{7}$$

The definition of an equilibrium in this model is as follows:

Definition 2.1. *Given (τ, γ) and the initial distribution of agents, μ_0, an* **equilibrium** *consists of a sequence of values, $(W_t^*, L_t^*, Q_t^*)_{t=0}^\infty$, a sequence of actions, $(a_t^*, s_t^*)_{t=0}^\infty$, a sequence of consumption for workers on leave and workers out of labor force, $(d_t^*, b_t^*)_{t=0}^\infty$, and a sequence of distributions of agents, $(\mu_t^*)_{t=1}^\infty$ such that:*

1. *Given $(a_t^*, s_t^*)_{t=0}^\infty$ and $(d_t^*, b_t^*)_{t=0}^\infty$, $(W_t^*, L_t^*, Q_t^*)_{t=0}^\infty$ satisfies Equations (1), (2), and (3).*
2. *Given $(d_t^*, b_t^*)_{t=1}^\infty$, for all $t \geq 1$, $a_t^* = 1$ if and only if $L_{t+1}^* \geq W_{t+1}^*$ and $s_t^* = 1$ if and only if $W_{t+1}^* \geq Q_{t+1}^*$.*
3. *For all $t \geq 0$, d_t^* and μ_t^* satisfy Equation (4).*
4. *Given μ_0, for all $t \geq 1$, $(\mu_t^*)_{t=1}^\infty$ satisfies Equation (5), (6), and (7).*

An equilibrium is a **stationary** *equilibrium if an equilibrium is time-invariant.*

Hereafter, I focus on a stationary equilibrium. Additionally, since paid parental leave policies are designed to let a female worker take a temporary leave voluntarily and most female workers return to work after the leave period ends, I also focus on a stationary equilibrium in which $a^* = 1$ and $s^* = 1$.

3 Characterizing a stationary equilibrium where $a^* = 1$ and $s^* = 1$

From Equation (3), in a stationary equilibrium, the value of an agent out of the labor force is

$$Q^* := \frac{u(0)+\nu}{1-\beta} \tag{8}$$

for all t.

$a^* = 1$ and $s^* = 1$ imply

$$W^* = \frac{u(y-\tau)+\beta\sigma L^*}{1-\beta(1-\sigma)} \tag{9}$$

and

$$L^* = \frac{u(d^*)+\nu+\beta\gamma W^*}{1-\beta(1-\gamma)}. \tag{10}$$

Combining these equations, I have

$$W^* = \frac{[1-\beta(1-\gamma)]u(y-\tau)+\beta\sigma[u(d^*)+\nu]}{(1-\beta)(1-\beta+\sigma\beta+\gamma\beta)}. \tag{11}$$

In a stationary equilibrium, from Equation (5), $\mu^{w*}\sigma = \mu^{l*}\gamma$ holds. Since $\sigma > 0$ and $\gamma > 0$, $\mu^{w*} > 0$ and $\mu^{l*} > 0$. Hence, Equation (4) implies

$$d^* = \frac{\gamma}{\sigma}\tau. \tag{12}$$

In this stationary equilibrium, the incentive constraints, $L^* \geq W^*$ and $W^* \geq Q^*$, need to hold, which induce

$$u(\frac{\gamma}{\sigma}\tau)+\nu \geq u(y-\tau) \tag{13}$$

and

$$u(y-\tau)+\frac{\beta\sigma}{1-\beta(1-\gamma)}u(\frac{\gamma}{\sigma}\tau) \geq \frac{1-\beta+\sigma\beta+\gamma\beta}{1-\beta(1-\gamma)}u(0)+\nu. \tag{14}$$

Combining Equation (5) with $\mu^{w*}+\mu^{l*}+\mu^{q*} = 1$, $\mu^{q*} = 0$,

$$\mu^{w*} = \frac{\gamma}{\sigma+\gamma} \tag{15}$$

and

$$\mu^{l*} = \frac{\sigma}{\sigma+\gamma} \tag{16}$$

Lemma 3.1. *There is a stationary equilibrium where a worker hit by a shock takes a leave voluntarily and a worker on leave voluntarily returns to the job if and only if Equations (13) and (14) hold. This stationary equilibrium is characterized by W^*, L^*, and Q^* given by Equations (9), (10), and (8), the consumption of a worker on leave is $d^* = \frac{\gamma}{\sigma}\tau$, and the distribution of workers is given by Equations (15) and (16).*

4 Characterizing optimal paid parental leave policy

The social planner's problem is to maximize the sum of all workers' values in a stationary equilibrium by choosing (τ,γ). Given that a worker chooses $a^* = 1$ and a worker on leave chooses $s^* = 1$ in a stationary equilibrium, from Lemma 3.1, the social planner's objective function is:

$$\mu^{w*}W^* + \mu^{l*}L^* + \mu^{q*}Q^* = \frac{\gamma}{\sigma+\gamma}\frac{u(y-\tau)}{1-\beta} + \frac{\sigma}{\sigma+\gamma}\frac{u(\frac{\gamma}{\sigma}\tau)+\nu}{1-\beta}. \tag{17}$$

The constraints that the social planner faces are Equations (13) and (14). Therefore, the social planner's problem (SP) is:

$$\begin{aligned}
(\mathrm{SP}) \quad \max_{(\tau,\gamma)\in[0,y]\times\Gamma} \quad & \frac{\gamma}{\sigma+\gamma}\frac{u(y-\tau)}{1-\beta} + \frac{\sigma}{\sigma+\gamma}\frac{u(d)+\nu}{1-\beta} \\
\text{s.t.} \quad & u(\frac{\gamma}{\sigma}\tau)+\nu \geq u(y-\tau), && (18)\\
& u(y-\tau) + \frac{\beta\sigma}{1-\beta(1-\gamma)}u(\frac{\gamma}{\sigma}\tau) \geq \left[1+\frac{\beta\sigma}{1-\beta(1-\gamma)}\right]u(0)+\nu. && (19)
\end{aligned}$$

Definition 4.1. *A paid parental leave policy, (τ,γ), is* **optimal** *if (τ,γ) is a solution to the problem (SP).*

If the utility from enjoying time with a worker's child, ν, is too large, then Equation (19) will not hold for any $(\tau,\gamma)\in[0,y]\times\Gamma$. Hence, to avoid the non-existence of the solution to the problem (SP), define $\overline{\nu}$ by

$$\overline{\nu} := \max_{(\tau,\gamma)\in[0,y]\times\Gamma}\left\{u(y-\tau) + \frac{\beta\sigma}{1-\beta(1-\gamma)}u(\frac{\gamma}{\sigma}\tau) - \frac{1-\beta+\beta\sigma+\beta\gamma}{1-\beta(1-\gamma)}u(0)\right\}.$$

Since the objective function is continuous in (τ,γ) and the set, $[0,y]\times\Gamma$, is non-empty and compact, from the Weierstrass theorem, $\overline{\nu}$ is well defined. Hereafter, I assume that ν is not so large in the sense that $\nu\in(0,\overline{\nu}]$.

Lemma 4.1. *The problem (SP) has a solution.*

Proof. Since the objective function is continuous in (τ,γ) and the constraint set is non-empty and compact, from the Weierstrass theorem, a solution exists. *Q.E.D.*

To characterize the constrained-optimal allocations, consider the following problem:

$$\begin{aligned}
(\mathrm{RSP}) \quad \max_{(\tau,\gamma)\in[0,y]\times\Gamma} \quad & \frac{\gamma}{\sigma+\gamma}\frac{u(y-\tau)}{1-\beta} + \frac{\sigma}{\sigma+\gamma}\frac{u(\frac{\gamma}{\sigma}\tau)+\nu}{1-\beta} \\
\text{s.t.} \quad & u(y-\tau) + \frac{\beta\sigma}{1-\beta(1-\gamma)}u(\frac{\gamma}{\sigma}\tau) \geq \left[1+\frac{\beta\sigma}{1-\beta(1-\gamma)}\right]u(0)+\nu. && (20)
\end{aligned}$$

This is a relaxed problem of problem (SP). The constraint (18) is dropped. Later, I will show that the solution to this problem actually satisfies Equation (18).

Letting λ be Lagrange multiplier to Equation (20), respectively, set the Lagrangean as

$$\mathscr{L} := \frac{\gamma}{\sigma+\gamma}\frac{u(y-\tau)}{1-\beta} + \frac{\sigma}{\sigma+\gamma}\frac{u(\frac{\gamma}{\sigma}\tau)+\nu}{1-\beta} + \lambda\left\{u(y-\tau) + \frac{\beta\sigma}{1-\beta(1-\gamma)}u(\frac{\gamma}{\sigma}\tau) - \left[1+\frac{\beta\sigma}{1-\beta(1-\gamma)}\right]u(0) - \nu\right\}.$$

The first-order necessary conditions for an interior solution are

$$-\frac{\gamma}{\sigma+\gamma}\frac{u'(y-\tau)}{1-\beta} + \frac{\sigma}{\sigma+\gamma}\frac{u'(\frac{\gamma}{\sigma}\tau)}{1-\beta}\frac{\gamma}{\sigma} + \lambda\left[-u'(y-\tau) + \frac{\beta\sigma}{1-\beta(1-\gamma)}u'(\frac{\gamma}{\sigma}\tau)\frac{\gamma}{\sigma}\right] = 0, \tag{21}$$

$$\begin{aligned}&\frac{\sigma}{(\sigma+\gamma)^2}\frac{u(y-\tau)}{1-\beta} - \frac{\sigma}{(\sigma+\gamma)^2}\frac{u(\frac{\gamma}{\sigma}\tau)+\nu}{1-\beta} + \frac{\sigma}{\sigma+\gamma}\frac{u'(\frac{\gamma}{\sigma}\tau)}{1-\beta}\frac{\tau}{\sigma}\\ &+\lambda\frac{\beta\sigma}{1-\beta(1-\gamma)}\left[-\frac{\beta}{1-\beta(1-\gamma)}u(\frac{\gamma}{\sigma}\tau) + u'(\frac{\gamma}{\sigma}\tau)\frac{\tau}{\sigma} + \frac{\beta}{1-\beta(1-\gamma)}u(0)\right] = 0,\end{aligned} \tag{22}$$

$$\lambda\left\{u(y-\tau) + \frac{\beta\sigma}{1-\beta(1-\gamma)}u(\frac{\gamma}{\sigma}\tau) - \left[1+\frac{\beta\sigma}{1-\beta(1-\gamma)}\right]u(0) - \nu\right\} = 0.$$

4.1 When the constraint in Problem (RSP) does not bind

First, consider the case in which an interior solution satisfies Equation (20) with strict inequality.

Since Equation (20) holds with strict inequality, in the first-order conditions, $\lambda^\star = 0$. Let $(\tau^\star, \gamma^\star)$ be a solution in this case. Then, Equation (21) implies

$$\frac{u'(y-\tau^\star)}{1-\beta} = \frac{u'(\frac{\gamma^\star}{\sigma}\tau^\star)}{1-\beta}.$$

Since u is strictly concave, $y-\tau^\star = \frac{\gamma^\star}{\sigma}\tau^\star$, or $c^\star = d^\star$. Thus,

$$\tau^\star = \frac{\sigma}{\sigma+\gamma^\star}y \text{ and } c^\star = d^\star = \frac{\gamma^\star}{\sigma+\gamma^\star}y.$$

This implies that when Equation (20) does not bind, at the optimum, an income risk caused by a shock should be fully insured and that consumption should be smoothed across the different statuses of a worker.

Plugging $c^\star = d^\star$ into Equation (22), $\gamma^\star$ satisfies

$$u'(\frac{\gamma^\star}{\sigma+\gamma^\star}y)y = \nu. \tag{23}$$

Since u is strictly concave, $\gamma^\star$ is uniquely determined by this equation. An increase in γ raises the number of workers in the economy, which increases output and consumption. At the same time, an increase in γ shortens the leave period, which loses the opportunity of enjoying time with a child. At the optimum, the marginal welfare gain from an increase in γ and the marginal welfare loss from that are equalized, as implied by Equation (23).

Proposition 4.1. *Suppose that $(\tau^\star, \gamma^\star)$ is an interior, optimal paid parental leave policy that satisfies the constraint in Problem (RSP) with strict inequality. Then, $(\tau^\star, \gamma^\star)$ satisfies*

$$\tau^\star = \frac{\sigma}{\sigma+\gamma^\star}y \text{ and } c^\star = d^\star = \frac{\gamma^\star}{\sigma+\gamma^\star}y,$$

and

$$u'(\frac{\gamma^\star}{\sigma+\gamma^\star}y)y=\nu.$$

Under $(\tau^\star,\gamma^\star)$, the self-enforcing constraint, Equation (20), is

$$\begin{aligned}&\left[1+\frac{\sigma\beta}{1-\beta(1-\gamma^\star)}\right]u(c^\star)\geq\left[1+\frac{\beta\sigma}{1-\beta(1-\gamma^\star)}\right]u(0)+\nu\\ \Rightarrow\quad&\left[1+\frac{\sigma\beta}{1-\beta(1-\gamma^\star)}\right][u(c^\star)-u(0)]\geq\nu.\end{aligned}\tag{24}$$

Note that $c^\star$ and $\gamma^\star$ are independent of β. Since $\frac{\sigma\beta}{1-\beta(1-\gamma^\star)}$ is increasing in β, one case in which Equation (20) does not bind is when β is sufficiently high, that is, a worker is sufficiently patient.

Since $c^\star=d^\star$ and $\nu>0$, $u(c^\star)+\nu>u(d^\star)$ holds, which implies that Equation (18) holds with strict inequality. At the optimal paid parental leave policy in this case, a worker voluntarily chooses to take temporary leave when a shock hits a worker. Hence, $(\tau^\star,\gamma^\star)$ is also a solution to Problem (SP).

4.2 When the constraint in Problem (RSP) binds

Next, I characterize an optimal policy when Equation (20) binds. Let $(\tau^{\star\star},\gamma^{\star\star},\lambda^{\star\star})$ denote an interior, optimal policy in this case. Since the constraint binds, $\lambda^{\star\star}>0$. From Equation (21),

$$\frac{\gamma^{\star\star}}{\sigma+\gamma^{\star\star}}\frac{1}{1-\beta}\left[u'(y-\tau^{\star\star})-u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\right]=-\lambda^{\star\star}\left[u'(y-\tau^{\star\star})-\frac{\beta\gamma^{\star\star}}{1-\beta(1-\gamma^{\star\star})}u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\right].\tag{25}$$

Since $\frac{\gamma^{\star\star}}{\sigma+\gamma^{\star\star}}\frac{1}{1-\beta}>0$ and $\lambda^{\star\star}>0$, the sign of $u'(y-\tau^{\star\star})-u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})$ and that of $u'(y-\tau^{\star\star})-\frac{\beta\gamma^{\star\star}}{1-\beta(1-\gamma^{\star\star})}u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})$ are opposite. Since $\frac{\beta\gamma^{\star\star}}{1-\beta(1-\gamma^{\star\star})}<1$ and $u'>0$,

$$u'(y-\tau^{\star\star})-u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})<u'(y-\tau^{\star\star})-\frac{\beta\gamma^{\star\star}}{1-\beta(1-\gamma^{\star\star})}u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}).$$

This implies that in order to satisfy Equation (21),

$$u'(y-\tau^{\star\star})-u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})<0<u'(y-\tau^{\star\star})-\frac{\beta\gamma^{\star\star}}{1-\beta(1-\gamma^{\star\star})}u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})$$

must be satisfied. Thus,

$$u'(y-\tau^{\star\star})<u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\tag{26}$$

holds. Since u is strictly concave,

$$c^{\star\star}=y-\tau^{\star\star}>\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}=d^{\star\star}$$

holds.

Next, to characterize $\gamma^{\star\star}$, consider the first three terms in the left-hand side of Equation (22), which is $\frac{\sigma}{(\sigma+\gamma)^2}\frac{u(y-\tau)}{1-\beta}-\frac{\sigma}{(\sigma+\gamma)^2}\frac{u(\frac{\gamma}{\sigma}\tau)+\nu}{1-\beta}+\frac{\sigma}{\sigma+\gamma}\frac{u'(\frac{\gamma}{\sigma}\tau)}{1-\beta}\frac{\tau}{\sigma}$. Since this term is 0 when the constraint does not bind, the sign of this term is informative to characterize $\gamma^{\star\star}$.

Lemma 4.2.

$$\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(y-\tau^{\star\star})}{1-\beta} < \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})+\nu}{1-\beta} - \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})}{1-\beta}\tau^{\star\star}\left(1+\frac{\gamma^{\star\star}}{\sigma}\right) \tag{27}$$

holds.

Proof. To claim $\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(y-\tau^{\star\star})}{1-\beta} < \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})+\nu}{1-\beta} - \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})}{1-\beta}\tau^{\star\star}\left(1+\frac{\gamma^{\star\star}}{\sigma}\right)$, suppose, by way of a contradiction, that

$$\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(y-\tau^{\star\star})}{1-\beta} - \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})+\nu}{1-\beta} + \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})}{1-\beta}\tau^{\star\star}\left(1+\frac{\gamma^{\star\star}}{\sigma}\right) \geq 0.$$

Since $\lambda^{\star\star} > 0$,

$$-\frac{\beta}{1-\beta(1-\gamma^{\star\star})}u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}) + u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\frac{\tau^{\star\star}}{\sigma} + \frac{\beta}{1-\beta(1-\gamma^{\star\star})}u(0) \leq 0. \tag{28}$$

Since the constraint binds,

$$u(y-\tau^{\star\star}) + \frac{\beta\sigma}{1-\beta(1-\gamma^{\star\star})}u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}) - \left[1+\frac{\beta\sigma}{1-\beta(1-\gamma^{\star\star})}\right]u(0) = \nu \geq u(y-\tau^{\star\star}) + u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\tau^{\star\star} - u(0),$$

where Equation (28) is applied to the second inequality. From this equation

$$\begin{aligned}
& \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(y-\tau^{\star\star})}{1-\beta} - \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})+\nu}{1-\beta} + \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})}{1-\beta}\tau^{\star\star}\left(1+\frac{\gamma^{\star\star}}{\sigma}\right) \\
\leq\ & \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(y-\tau^{\star\star})}{1-\beta} - \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})+u(y-\tau^{\star\star})+u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\tau^{\star\star}-u(0)}{1-\beta} \\
& + \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})}{1-\beta}\tau^{\star\star}\left(1+\frac{\gamma^{\star\star}}{\sigma}\right) \\
=\ & -\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{1}{1-\beta}\left[u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}) + u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\tau^{\star\star} - u(0) - u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\frac{\sigma+\gamma^{\star\star}}{\sigma}\tau^{\star\star}\right] \\
=\ & -\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{1}{1-\beta}\left[\underbrace{u(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}) - u(0) - u'(\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma})\frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}}_{>0}\right] < 0,
\end{aligned}$$

where the strict concavity of u implies the last inequality. This is a contradiction. Thus, Equation (27) holds. *Q.E.D.*

An implication of Equation (27) is that by lengthening the leave period, another policy improves social welfare. In this sense, $\gamma^{\star\star}$ is too large, or the length of the parental leave is too short.[5] The explanation is as follows. Suppose γ slightly decreases from $\gamma^{\star\star}$. This decreases the number of workers

[5] Of course, this other policy breaks the self-enforcing constraint.

and increases the number of workers on leave. The first term on the left-hand side of Equation (27) is the marginal welfare loss when the number of workers decreases, while the term on the right-hand side of Equation (27) is the marginal welfare gain when the number of workers on leave increases. A change in γ also changes output and consumption. A decrease in the number of workers lowers total output by y and the total consumption of workers by $c^{\star\star} = y - \tau^{\star\star}$, whereas an increase in the number of workers on leave increases the total consumption of workers on leave, $d^{\star\star} = \frac{\gamma^{\star\star}\tau^{\star\star}}{\sigma}$. Overall, a decrease in γ reduces total resources by $\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}(y - c^{\star\star} + d^{\star\star}) = \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\tau^{\star\star}\left(1 + \frac{\gamma^{\star\star}}{\sigma}\right)$. To satisfy the government budget constraint, some workers have to consume less than before. The second term on the right-hand side of Equation (27) represents the marginal welfare loss if the consumption of workers on leave uniformly decreases by $\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}(y - c^{\star\star} + d^{\star\star})$. This implies that $(\tau^{\star\star}, \gamma)$, where γ is slightly smaller than $\gamma^{\star\star}$, raises social welfare above that achieved by $(\tau^{\star\star}, \gamma^{\star\star})$.

A summary of the findings so far is as follows.

Proposition 4.2. *Suppose that $(\tau^{\star\star}, \gamma^{\star\star})$ is an interior, optimal paid parental leave policy that satisfies the constraint in Problem (RSP) with equality. Then,*

1. *$c^{\star\star} > d^{\star\star}$.*
2. *There is a $\gamma < \gamma^{\star\star}$, such that a policy, $(\tau^{\star\star}, \gamma)$, raises social welfare above that achieved by $(\tau^{\star\star}, \gamma^{\star\star})$.*

In this case, since Equation (20) binds,

$$u(c^{**}) + \frac{\beta\sigma}{1-\beta(1-\gamma^{**})}u(d^{**}) = \left[1 + \frac{\beta\sigma}{1-\beta(1-\gamma^{**})}\right]u(0) + v$$

holds. Then,

$$\begin{aligned} u(d^{**}) + v &= u(d^{**}) + u(c^{**}) - u(0) + \frac{\beta\sigma}{1-\beta(1-\gamma^{**})}[u(d^{**}) - u(0)] \\ &= u(c^{**}) + \left[1 + \frac{\beta\sigma}{1-\beta(1-\gamma^{**})}\right][u(d^{**}) - u(0)] > u(c^{**}), \end{aligned}$$

because $d^{**} > 0$ and u is strictly increasing. This implies that Equation (18) holds at the optimal policy. Hence, when a worker is hit by a shock, she will take temporary leave voluntarily at the optimal paid parental leave policy.

5 Discussion

5.1 A numerical example

As shown in Figure 1 in the Introduction, the cross-country data on paid parental leave policies show the negative relationship between the average payment rate and the length of the leave. How can we explain this negative relationship?

By showing numerical examples, I claim that this observation may result from the implementation of optimal paid parental leave policies, and the key parameter is σ. To show this, consider the following numerical example. The periodic utility function is $u(x) = \frac{x^{0.9}}{0.9}$, $\beta = 0.99$, $y = 2$, and $\nu = 2$. σ can take a value between 0.025 and 0.05. The values of σ and β are taken from Erosa et al. (2010),[6], while the other values are chosen arbitrarily. The replacement rate is measured by $\frac{d}{c}$ and the length of the leave is measured by $\frac{1}{\gamma}$ in the model. Figures 2 and 3 plot the simulation result.

Figure 2 shows the extent to which c and d vary as σ changes. In this case, $c^\star = d^\star = 1$, and $d^{\star\star} > d^\star$ holds when the incentive constraint binds. In Figure 3, as σ rises, a point moves from the bottom right to the top left. As σ increases, the incentive constraint does not bind. Thus, once the replacement rate reaches one, only the length of the leave shortens.

Assuming OECD countries typically share similar characteristics, this simulation result suggests that current paid parental leave policies can be optimal policies, and that the key parameter across countries is the probability that a worker leaves her job temporarily, σ, as discussed next.

For countries whose average payment rate is one, the length of the leave also varies. To derive this result, from Equation (23), different σ values lead to different $\gamma^\star$ values, while $d^\star/c^\star$ is constant at one. More precisely, as σ rises, so does $\gamma^\star$. When σ is too low, the constraint, that is, Equation (19), is more likely to bind. In the social welfare function, more weight is placed on workers' welfare than the welfare for workers on leave. Therefore, as σ falls, the gap between $c^{\star\star}$ and $d^{\star\star}$ increases and thus $d^{\star\star}/c^{\star\star}$ is lowered. In Equation (19), a smaller γ raises the left-hand side of Equation (19). Thus, $\gamma^{\star\star}$ becomes smaller, or $\frac{1}{\gamma^{\star\star}}$ becomes larger as σ becomes smaller. This explains why σ is my focus here.

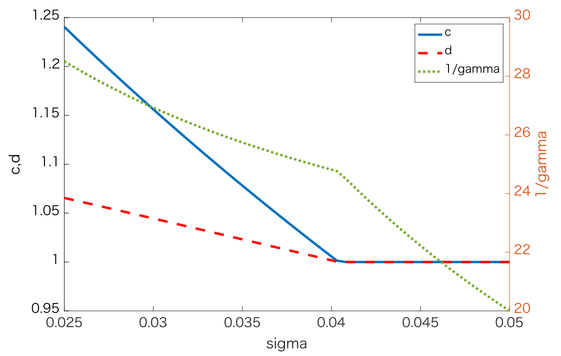

Figure 2: Results of c, d, and $\frac{1}{\gamma}$ for different σ

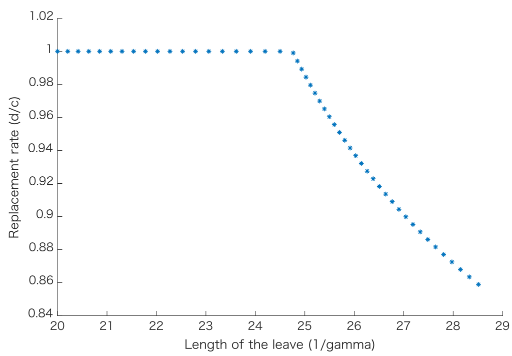

Figure 3: Results of the length of the leave and replacement rate

5.2 Comparison of $(c^\star, d^\star, \gamma^\star)$ with $(c^{\star\star}, d^{\star\star}, \gamma^{\star\star})$

In this subsection, I compare $(c^\star, d^\star, \gamma^\star)$ with $(c^{\star\star}, d^{\star\star}, \gamma^{\star\star})$. To do this, consider two economies in which all the exogenous variables besides β are the same. The discount factor in one economy, say economy A, is so high that Equation (24) holds, while the discount factor in the other economy, say economy B, is too low to satisfy Equation (24). Thus, the optimal policy in economy A is characterized by $(\tau^\star, \gamma^\star)$ and that in economy B is characterized by $(\tau^{\star\star}, \gamma^{\star\star})$. Some might expect $c^{\star\star} > c^\star = d^\star > d^{\star\star}$, but this inequality does not necessarily hold here.

[6]To be precise, the lower value of σ is $\sigma(4)$ and the higher value of σ is $\sigma(1)$.

First, consider $c^\star$ and $c^{\star\star}$. Since $u'(c^{\star\star}) < u'(d^{\star\star})$, Equation (27) implies

$$\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(c^{\star\star})}{1-\beta}+\frac{u'(c^{\star\star})}{1-\beta}\frac{\sigma}{(\sigma+\gamma^{\star\star})^2}(y-c^{\star\star}+d^{\star\star}) < \frac{\sigma}{(\sigma+\gamma^{\star\star})^2}\frac{u(d^{\star\star})+\nu}{1-\beta}, \tag{29}$$

where $\tau^{\star\star}\left(1+\frac{\gamma^{\star\star}}{\sigma}\right) = y-c^{\star\star}-d^{\star\star}$ is applied here. The strict concavity of u and $c^{\star\star} > d^{\star\star}$ implies

$$u(c^{\star\star})-u(d^{\star\star})-u'(c^{\star\star})(c^{\star\star}-d^{\star\star}) > 0. \tag{30}$$

Applying Equation (30) to Equation (29), I obtain

$$u'(c^{\star\star})y < \nu.$$

Since $u'(c^\star)y = \nu$ from Equation (23),

$$u'(c^{\star\star}) < u'(c^\star)$$

holds. The strict concavity of u implies $c^{\star\star} > c^\star$.

Next, consider $(d^\star,\gamma^\star)$ and $(d^{\star\star},\gamma^{\star\star})$. From Equations (12), c and d satisfy

$$y-c = \frac{\sigma}{\gamma}d.$$

Thus, $c^{\star\star} > c^\star$ implies

$$\frac{\sigma}{\gamma^\star}d^\star = y-c^\star > y-c^{\star\star} = \frac{\sigma}{\gamma^{\star\star}}d^{\star\star}. \tag{31}$$

Since $\sigma > 0$, Equation (31) is equivalent to

$$\frac{d^\star}{\gamma^\star} > \frac{d^{\star\star}}{\gamma^{\star\star}}.$$

What does this equation mean? Since $\frac{1}{\gamma}$ is the expected duration of the leave and d is the consumption of workers on leave in each period, $\frac{d}{\gamma}$ is interpreted as the total consumption of workers on leave during the leave period. Which of $d^\star$ and $d^{\star\star}$ is larger is inconclusive. However, if total consumption during the leave period is considered, consumption when the constraint does not bind is larger than consumption when the constraint binds.

A summary of the findings in this subsection is as follows.

Proposition 5.1. *Suppose there are two identical economies except β and that the discount factor in one economy is so high that Equation (24) holds, whereas that in the other economy is too low to satisfy Equation (24). Then,*

1. $c^{\star\star} > c^\star$.
2. $\frac{d^\star}{\gamma^\star} > \frac{d^{\star\star}}{\gamma^{\star\star}}$.

This result does not necessarily imply that $d^\star > d^{\star\star}$. Consider the following numerical example. The periodic utility function is $u(x) = \frac{x^{0.9}}{0.9}$, $\sigma = 0.1$, $y = 2$, and $\nu = 2$. β can take a value between 0.95 and 0.99. Figure 4 illustrates c and d with different β values, showing that $d^{\star\star}$ for low β is larger than $d^\star = 1$. From Proposition 3.3 (2), this implies $\gamma^\star < \gamma^{\star\star}$.

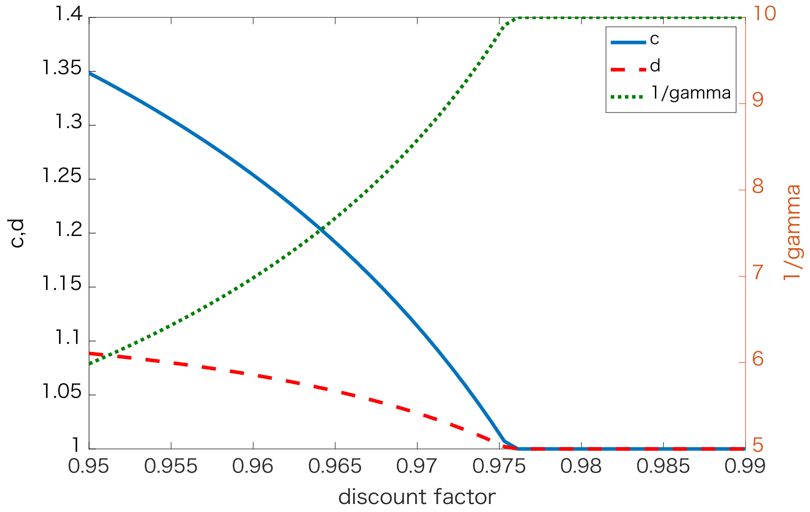

Figure 4: Simulation results of c, d, and $\frac{1}{\gamma}$ for different β

6 Conclusion

This study examines a paid parental leave policy using a dynamic model in which a key friction is that a worker on leave cannot commit to returning to work after the leave period ends. Therefore, the optimal policy has to give a worker on leave an incentive to return to work. Optimal paid parental leave policies are characterized by whether the incentive constraint binds.

When the incentive constraint does not bind, the consumption of a worker and that of a worker on leave should be equal; that is, the income risk caused by the leave should be shared among workers. The length of the leave is set to equalize the marginal welfare gain and marginal welfare loss from a marginal increase in the length of the leave. When the incentive constraint binds, the consumption of a worker is greater than that of a worker on leave to satisfy the incentive constraint. Thus, the imperfect risk sharing caused by the leave might be optimal. As for the length of the leave, in the optimal policy, it is too short in the sense that social welfare can be improved by lengthening the leave period, although the incentive constraint is not satisfied.

Based on the theoretical analysis, I examined whether the paid parental leave policies across OECD countries are optimal. Using a numerical simulation, I showed that a negative relationship between the length of the leave and replacement rate could result from optimal paid parental leave policies and that the key parameter is the probability that a worker gets pregnant.

Furthermore, the present study compares optimal policies when the discount factor is sufficiently high to satisfy the incentive constraint strictly with those when the discount factor is too low to satisfy the incentive constraint. I show that the consumption of a worker when the incentive constraint does not bind is larger than that when the incentive constraint binds. Regarding the consumption of workers on leave and length of the leave, it is shown that total consumption during the leave period when the incentive constraint does not bind is larger than that when the incentive constraint binds. This, however, does not necessarily imply that the periodic consumption of a worker on leave when the incentive constraint does not bind is larger than that when the incentive constraint binds.

Since the model used in this study is simple, it can be extended in many directions. One interesting extension of the model could be to add a worker's savings as Ábrahám and Laczó (2018) extended Kocherlakota (1996) and Mitchell and Zhang (2010) extended Hopenhayn and Nicolini (1997).

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