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6. October 2008

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Abstract : *The aim of this paper is to analyze the relationship between competition and growth in a model of human capital accumulation and research by disentangling the monopolistic mark-up in the intermediate goods sector and the returns to specialization in order to have a better measure of competition. We find that the steady-state output growth rate depends on the parameters describing preferences, human capital accumulation technology and R&D activity. We also show that the relationship between competition and growth is inverse U shaped. This result that seems to be in line this empirical results (Aghion and Griffith (2005)) is explained by the resource allocation effect.*

Keywords : Endogenous growth, Horizontal differentiation, Technological change, Imperfect competition, Human capital

JEL Classification : D43, J24, L16, 031, 041

*I would like to thank, without implicating J-L. Gaffard, C. Le Van for useful suggestions and comments on an earlier draft of this paper. Moreover, the author has benefited from discussions at the Vth Conference of the ACDD at the university of Strasbourg. Opinions expressed in this paper are of author only and do not necessarily reflect the views of CRP Henri Tudor or STATEC. The usual disclaimer applies.

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1 Introduction

Economists have long been interested in the relationship between competition and growth, but economic theory seems to be contradicted by the evidence. Indeed, the most important growth models in which there exists an imperfect competition (Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992)) show a decreasing or increasing relationship between competition and growth. However, recent empirical work (Aghion and Griffith (2005)) which disputes the form of this relationship, finds an inverted-U relationship between competition and growth which is robust to many alternative specifications and remains true in the data for many individual industries. In order to reconcile theory with evidence, Aghion and Griffith (2005) and Bucci (2005a) extend the basic Schumpeterian endogenous growth model. The first one introduces an escape competition effect in the Aghion and Howitt (1992) model whereas the second one introduces an resource allocation effect in the Romer (1990) model.

On the other hand, recent theoretical works (Ziesemer (1991), Eicher (1996), Redding (1996), Arnold (1998), Blackburn, Hung, and Pozzolo (2000), Sjögren (1998) and Lloyd-Ellis and Roberts (2002)) analyze the relationship between R&D investment and human capital accumulation and their impact on growth. However, all these studies don't focus on the effect of competition on growth. More recently, Bucci (2003) and Bucci (2005b) focus explicitly on the relationship between competition and growth.

The purpose of this paper is to extend these works. Indeed, contrary to all these models, first, by following Benassy (1998) and Bianco (2006), we introduce a distinction between the returns to specialization and the market power parameter which allows us to have a better measure of the competition. Indeed, in our model the market power parameter is not strongly related to the returns to specialization but it is completely independent. Secondly, we introduce the degree of R&D difficulty in the sense that higher values of skilled labor force require more skilled labor allocated in the research sector to achieve the same level of the growth rate of knowledge.

Our paper is structured as follows. Section 2 presents our model. Section 3 analyses the market equilibrium. Section 4 discuss the relationship between competition and growth in a general case. Finally, section 5 concludes.

2 The model

The model developed is based on Bucci (2003)[4] model in which we disentangle the returns to specialization from the market power. The economy is structured by three sectors : final good sector, intermediate goods sector and R&D sector.

2.1 The final good sector

In this sector atomistic producers engage in perfect competition. The final goods sector produces a composite good Y by using all the jth type of intermediate goods x_j and skilled labor H_Y.[5] Production is given by :

$$Y = AH_Y^{1-\alpha} n^{\gamma-1+\alpha} \int_0^n x_j^\alpha dj, \tag{1}$$

[4]We use the notations of Bucci (2003) in order to have a direct comparison with his model.

[5]Time subscripts are omitted whenever there is no risk of ambiguity.

where α and $\gamma \in]0,1[$ and A are technological parameters. This production function allows us to disentangle the degree of market power of monopolistic competitors in the intermediate sector ($\frac{1}{\alpha}$) and the degree of returns from specialization (γ).[6] In this sense, this model is a generalization of the Bucci (2003) and the Arnold (1998) models.[7] Under perfect competition in the final output market and the factor inputs markets, the representative firm chooses intermediate goods and skilled labor in order to maximize its profit taking prices as given and subject to its technological constraint. The first order conditions are the followings :

$$\frac{\partial \pi_Y}{\partial x_j} = \alpha A H_Y^{1-\alpha} n^{\gamma-1+\alpha} x_j^{\alpha-1} - p_j = 0, \tag{2}$$

$$\frac{\partial \pi_Y}{\partial H_Y} = (1-\alpha) A H_Y^{-\alpha} n^{\gamma-1+\alpha} - w_Y = 0, \tag{3}$$

where w_Y is the wage rate in the final good sector and p_j is the price of the jth intermediate good. Equation (2) is the inverse demand function for the firm that produces the jth intermediate good whereas equation (3) characterizes the demand function of skilled labor.

2.2 The intermediate goods sector

In the intermediate goods sector, producers engage in monopolistic competition. Each firm produces one horizontally differentiated intermediate good and have to buy a patented design before producing it. Following Grossman and Helpman (1991), Bucci (2003), Bucci (2005b) and Bucci (2005c), we assume that each local intermediate monopolist has access to the same technology employing only skilled labor h_j :

$$x_j = B h_j. \tag{4}$$

Using the first order condition, we obtain the price of the jth intermediate good :

$$p_j = \frac{w_j}{B\alpha}, \tag{5}$$

where w_j is wage rate in the intermediate goods. At the symmetric equilibrium, all the firms produce the same quantity of the intermediate good, face the same wage rate and by consequence fix the same price for their production. The price is equal to a constant mark up $\frac{1}{\alpha}$ over the marginal cost $\frac{w}{B}$. Defining by $H_j = \int_0^n h_j dj$, the total amount of labor employed in the intermediate goods sector, we can rewrite the equation (4) :

$$x_j = \frac{BH_j}{n}, \tag{6}$$

Finally, the profit function of the firm[8] which produces the jth intermediate good is

$$\pi_j = A\alpha(1-\alpha)B^{\alpha} n^{\gamma-1} H_j^{\alpha} H_Y^{1-\alpha}. \tag{7}$$

[6] Benassy (1996) made a simple modification of the Dixit and Stiglitz (1977) model which clearly disentangles taste for variety and market power. At the same time, Benassy (1998) and de Groot and Nahuis (1998) show that the introduction of this modification in an endogenous growth model with expanding product variety *à la* Grossman and Helpman (1991) affects the welfare analysis.

[7] Indeed, we obtain the Bucci (2003) model by introducing the following constraint $\gamma = 1-\alpha$ in our model. In the same way, we obtain the Arnold (1998) model by introducing the following constraints $\gamma = \frac{1}{\alpha} - 1$ and $A = 1$ in our model.

[8] In order to have a negative relationship between competition and profit, we assume that $0 < \gamma < 1$.

2.3 The R&D sector

There are competitive research firms undertaking R&D. Following Dinopoulos and Thompson (1999), we assume that new blueprints are produced using old blueprint n, an amount of R&D skilled labor H_n and the skilled labor force H :

$$\dot{n} = \frac{nCH_n}{H}, \tag{8}$$

where C is a productivity parameter. Unlike Bucci (2003), we explicitly assume that positive spillover effect is attached to the available stock of disembodied knowledge (approximated by the existing number of designs, n) in discovering a new product variety. Another and more important difference is the existence of a dilution effect[9] in our model. Indeed, the equation (8) captures in a very simple way the idea that R&D difficulty grows with the labor force.[10] According to this effect, the invention of a new intermediate requires a share of R&D skilled labor in the skilled labor force $H_n = \frac{H}{Cn}$, which changes over time because of both innovation and skilled labor growth. While innovation generates a positive inter-temporal externality, the skilled labor growth tends to reduce innovation via a fall in the R&D productivity. Because of the perfect competition in the R&D sector, we can obtain the real wage by using the zero profit condition :

$$w_n H_n = \dot{n} V_n, \tag{9}$$

where w_n represents the real wage earned by R&D skilled labor and V_n is the real value of such a blueprint which is equal to :

$$V_n = \int_t^{\infty} \pi_j e^{-r(\tau - t)} d\tau, \tau > t, \tag{10}$$

since the research sector is competitive, the price of the jth design at time t will be equal to the discounted value of the flow of instantaneous profits that is possible to make in the intermediate goods sector by the jth firm from t onwards. Given V_n, the free entry condition leads to :

$$w_n = \frac{nV_n}{H}, \tag{11}$$

2.4 The consumer behavior

The demand side is characterized by the representative household who holds asset in the form of ownership claims on firm and chooses plans for consumption (c), asset holdings (a) and human capital (h).[11] Following Lucas (1988), we assume that the household is endowed with one unit of time and optimally allocates a fraction u of this time endowment to productive activities (final good, intermediate goods and research production) and the remaining fraction $(1 - u)$ to non productive activities (education). Following Romer (1990), we assume that the utility function of this consumer is [12] :

$$U = \int_0^{\infty} e^{-\rho t} log(c) dt, \tag{12}$$

[9]Dinopoulos and Segestrom (1999) have introduced this kind of effect in an endogenous growth model. In our model, contrary to the Dinopoulos and Segestrom (1999) model, this effect is also linked to the human capital accumulation.

[10]For more details about this issue, see Dinopoulos and Thompson (1999), Dinopoulos and Sener (2006) and Jones (2005).

[11]Like Bucci (2003), Bucci (2005b) and Bucci (2005c), for the sake of simplicity, we assume that there is no population growth.

[12]This specification of the utility function does not alter our results.

where c is private consumption, $\rho > 0$ is the rate of pure time preference. The flow budget constraint for the household is :

$$\dot{a} = wuh + ra - c, \tag{13}$$

where w is the wage rate per unit of skilled labor services. The human capital supply function is given by :

$$\dot{h} = \delta(1-u)h, \tag{14}$$

where $\delta > 0$ is a parameter reflecting the productivity of the education technology. From the maximization program of the consumer,[13] the first order conditions are :

$$\lambda_1 = \frac{1}{c}e^{-\rho t}, \tag{15}$$

$$-\dot{\lambda}_1 = \lambda_1 r, \tag{16}$$

$$-\dot{\lambda}_2 = \lambda_1 wu + \lambda_2 \delta(1-u), \tag{17}$$

$$\lambda_1 = \lambda_2 \frac{\delta}{w}. \tag{18}$$

Equation (15) gives the discounted marginal utility of consumption which satisfies the dynamic optimality condition in equation (16). Equation (18) gives the static optimality condition for the allocation of time. The marginal cost of an additional unit of skills devoted to working evolves optimally as in equation (17). Conditions (15) through (18) must satisfy the constraints (13) and (14), together with the transversality conditions :

$$\lim_{t\to\infty} \lambda_{1t} a_t = 0, \tag{19}$$

$$\lim_{t\to\infty} \lambda_{2t} h_t = 0. \tag{20}$$

3 The equilibrium and the steady state

In this section, we characterize the equilibrium and give some analytical characterization of a balanced growth path.

3.1 The equilibrium

It is now possible to characterize the skilled labor market equilibrium in the economy considered. On this market, because of the homogeneity and the perfect mobility across sectors, the arbitrage ensures that the wage rate that is earned by employees who work in the final good sector, intermediate goods sector or R&D sector is equal. As a result, the following three conditions must simultaneously be checked :

$$u^* = s_Y + s_j + s_n, \tag{21}$$

$$w_j = w_Y, \tag{22}$$

$$w_j = w_n. \tag{23}$$

Equation (21) is a resource constraint, saying that at any point in the time the sum of the skilled labor demands coming from each activity must be equal to the total available

[13]The control variables of this problem are c and u whereas a and h are the state variables. λ_1 and λ_2 denote the shadow price of the household's asset holdings and human capital stock.

supply. Equation (22) and equation (23) state that the wage earned by one unit of skilled labor is to be the same irrespective of the sector where that unit of skilled labor is actually employed.

We can characterize the product market equilibrium in the economy considered. Indeed, on this market, the firms produce a final good that can be consumed. Consequently, the following condition must be checked :

$$Y = C. \tag{24}$$

Equation (24) is a resource constraint on the final good sector.

We can describe the capital market equilibrium in our economy. Because the total value of the household's assets must be equal to the total value of firms, the following condition must be checked :

$$a = nV_n, \tag{25}$$

where V_n is given by the equation (10) and satisfies the following asset pricing equation :

$$\dot{V}_n = rV_n - \pi_j, \tag{26}$$

3.2 The steady state

At the steady state, all variables as Y, c, n, a, H,[14] H_Y, H_j, H_n grow at a positive constant rate.

Proposition 1 *If u is constant then all the other variables grow at strictly positive rate*

$$g_h = g_H = g_{H_Y} = g_{H_j} = g_{H_n}, \tag{27}$$

$$g_Y = g_c = g_a = \gamma g_n + g_h, \tag{28}$$

$$g_n = Cs_n \tag{29}$$

Proof. *From the equilibrium on the skilled labor market, given by the equation (21), it easy to show that $g_H = g_{H_Y} = g_{H_j} = g_{H_n}$ if u is constant. Because of the assumption on the size of the representative household and the population growth rate, it is obviously that $g_h = g_H$. Combining these two conditions, we get $g_h = g_H = g_{H_Y} = g_{H_j} = g_{H_n}$. From the definition of the firm research process, given by the equation (8), we obtain that $g_n = Cs_n$. From the equilibrium on the product market, given by the equation (24), it is easy to find that $g_Y = g_c$. The equations (7, 10, 25, 26) implies that $g_a = \gamma g_n + g_h$. By substituting equation (6) into equation (1), then by log-differentiating the equation (1), we obtain $g_Y = \gamma g_n + g_h$. By combining the previous equations, we find the equation (28).* ■

[14] Given the assumptions on the size of the representative household and the population growth rate, $h \equiv H$ which implies that we can use g_H instead of g_h.

Using the previous equations, we can demonstrate the following steady state equilibrium values for the relevant variables of the model[15] :

$$r = \delta - \frac{\gamma(\delta + (\alpha - 1)(C + \delta)\alpha)\rho}{\delta}, \tag{30}$$

$$H_j = \frac{H\alpha(C + \delta)\alpha\rho}{C\delta}, \tag{31}$$

$$H_Y = \frac{H(C + \delta)(1 - \alpha)\rho}{C\delta}, \tag{32}$$

$$H_n = \frac{H((1 - \alpha)(C + \delta)\alpha + \delta)\rho}{C\delta}, \tag{33}$$

$$u^* = \frac{\rho}{\delta}, \tag{34}$$

$$g_h = g_H = \delta - \rho, \tag{35}$$

$$g_n = \frac{(1 - \alpha)(C + \delta)\alpha + \delta)\rho}{\delta}, \tag{36}$$

$$g_Y = g_c = \delta - \rho - \frac{\gamma(\delta + \alpha(\alpha - 1)(C + \delta))\rho}{\delta}. \tag{37}$$

According to the equation (30), the real interest rate (r) is constant. Equation (31), (32) and (33) give the amount of skilled labor in each sector at the equilibrium. Equation (34) represents the optimal and constant fraction of the household's time endowment that it will decide to devote to work (u^*) at the equilibrium. Equation (35) states that the growth rate of human capital depends on technological and preference parameters (δ and ρ).[16] Unlike the Lucas (1988), Blackburn, Hung, and Pozzolo (2000) and Bucci (2003) papers, the equation (36) show that the growth rate of the innovative activity depends not only on preference and technological parameters but also on competition. Unlike the Lucas (1988), Blackburn, Hung, and Pozzolo (2000)[17] and Bucci (2003)[18] papers, the equation (37) shows that the growth rate depends not only on the competition (α) and human capital accumulation (g_h) but also on the degree of returns from specialization (γ) and a productivity parameter (C). As in the recent Lucas (1988), Blackburn, Hung, and Pozzolo (2000) and Bucci (2003) papers, the market power enjoyed in the monopolistic sector does not play any role on the consumers' decision about how much time to invest in education and training (such a decision being solely driven by the parameters describing preferences and human capital technology).

4 The relationship between product market competition and growth

In this section, we study the long run relationship between competition and growth in the model presented above. Following most authors (Bucci (2003), Aghion, Bloom, Blundell, Griffith, and Howitt (2005), Aghion and Griffith (2005), Aghion and Howitt (2005), Bucci (2005a), Bucci (2005b), Bucci (2005c), Bianco (2006) and Bianco (2007)), we use the so-called Lerner Index to gauge the intensity of market power within a market. Such an

[15]Results (30) through (37) are demonstrated in the appendix.

[16]The condition $\delta > \rho$ also assures that $0 < u^* < 1$.

[17]In these models, output growth depends only on human capital accumulation (g_h).

[18]In this model, output growth depends not only on human capital accumulation (g_h) but also on competition (α).

index is defined by the ratio of price (P) minus marginal cost (Cm) over price. Using the definition of a mark up ($Markup{=}\frac{P}{Cm}$) and Lerner Index ($LernerIndex{=}\frac{P-Cm}{P}$), we can use (5) to define a proxy of competition as follows :

$$(1 - LernerIndex) = \alpha, \tag{38}$$

We show that in this model which consists in having the monopolistic mark-up in the intermediate goods sector and the returns to specialization treated separately, the relationship between competition and growth is inverse U shaped. This theoretical result is in line with the empirical results (Aghion and Griffith (2005)).

Proposition 2 *The relationship between competition and growth is inverse U shaped.*

Proof. *The proof is obtained by differentiating (37) with respect to α :*

$$\frac{\partial g_Y}{\partial \alpha} = -\frac{(2\alpha - 1)\gamma(C+\delta)\rho}{\delta}, \tag{39}$$

As C, $\delta, \rho > 0$ and $0 < \gamma < 1$, then we the sign of the derivative is given by the opposite sign of $2\alpha - 1$. Finally, we obtain that $\frac{\partial g_Y}{\partial \alpha} > 0$ if and only if $0 \leq \alpha < \frac{1}{2}$ and $\frac{\partial g_Y}{\partial \alpha} < 0$ if and only if $\frac{1}{2} < \alpha \leq 1$. ■

To enlighten Proposition 2, remark that any increase in competition has a non linear effect on the skilled labor allocated to the research sector (H_n). This means that the resource allocation effect seems to predict an inverted-U relationship between product market competition and growth. Moreover, the human capital accumulation which is the second source of growth in our model, has a positive and linear effect on growth. Finally, the relationship between product market competition and growth is inverse U shaped.

5 Conclusion

In this paper, we presented a generalization of the Bucci (2003) model in which we disentangle the monopolistic mark-up in the intermediate goods sector and the returns to specialization in order to have a better measure of competition. Indeed, in our model the market power parameter is not strongly related to the returns to specialization but it is completely independent. Moreover, we introduce the degree of R&D difficulty in the sense that higher values of skilled labor force require more skilled labor allocated in the research sector to achieve the same level of the growth rate of knowledge.

The results of the model can be summarized as follows. First of all, the steady-state output growth rate depends on the parameters describing preferences, human capital accumulation technology and R&D activity. Secondly, we find that the relationship between competition and growth is inverse U shaped. This result that seems to be in line this empirical results (Aghion and Griffith (2005)) is explained by the resource allocation effect.

Appendix

In this appendix, we describe the way followed in order to obtain the main results of this paper (30 through 37). Consider the representative consumer's problem (equations (12) through (21) in the main text), whose the first order conditions are stated in equations (15) through (21) with consumer's constraints and transversality conditions, we have :

$$\lambda_1 = \frac{1}{c}e^{-\rho t}, \tag{40}$$

$$-\dot{\lambda_1} = \lambda_1 r, \tag{41}$$

$$-\dot{\lambda_2} = \lambda_1 wu + \lambda_2 \delta(1-u), \tag{42}$$

$$\lambda_1 = \lambda_2 \frac{\delta}{w}. \tag{43}$$

$$\dot{a} = ra + wuh - c, \tag{44}$$

$$\dot{h} = \delta(1-u)h, \tag{45}$$

$$\lim_{t\to\infty} \lambda_{1t} a_t = 0, \tag{46}$$

$$\lim_{t\to\infty} \lambda_{2t} h_t = 0. \tag{47}$$

Combining equations (43) and (42), we obtain :

$$\frac{\dot{\lambda_2}}{\lambda_2} = -\delta. \tag{48}$$

From equation (41), we get :

$$\frac{\dot{\lambda_1}}{\lambda_1} = -r. \tag{49}$$

Equation (43) implies that :

$$\frac{\dot{\lambda_1}}{\lambda_1} = \frac{\dot{\lambda_2}}{\lambda_2} - g_w. \tag{50}$$

Combining equations (48), (49) and (50), we obtain :

$$r = \delta + g_w. \tag{51}$$

In the balanced growth path equilibrium, the growth rate of the wage accruing to human capital (g_w) is constant (see later on this appendix). This implies that the real interest rate (r) will be also constant. With a constant real interest rate and using the equation (7), the equation (10) becomes :

$$V_{nt} = A\alpha(1-\alpha)B^\alpha \int_t^\infty n_\tau^{\gamma-1} H_{j\tau}^\alpha H_{Y\tau}^{1-\alpha} e^{-r(\tau-t)} d\tau, \tau > t \tag{52}$$

In order to compute the market value of one unit of research output at time t (V_{nt}) along the balanced growth path equilibrium, we use the following equations :

$$n_\tau = n_t e^{g_n t}, \tag{53}$$

$$H_{j\tau} = H_{jt} e^{g_{H_j} t}, \tag{54}$$

$$H_{Y\tau} = H_{Yt} e^{g_{H_Y} t}. \tag{55}$$

$$\tag{56}$$

Inserting equations (53), (54) and (55) into equation (52), and after some calculations, we get :

$$V_{nt} = \frac{A\alpha(1-\alpha)B^\alpha n^{\gamma-1} H_j^\alpha H_Y^{1-\alpha}}{r - (\gamma-1)g_n - (1-\alpha)g_{H_Y} - \alpha g_{H_j}}. \tag{57}$$

Such result is obtained under the assumption that $r > (\gamma - 1)g_n - (1-\alpha)g_{H_Y} - \alpha g_{H_j}$. In a moment, we will demonstrate that this hypothesis (which assures that V_{nt} is positive for each t) is always checked along the balanced growth path equilibrium. Given V_{nt} and making use of equation (11) in the main text, we get :

$$w_n = \frac{CA\alpha(1-\alpha)B^{\alpha}n^{\gamma}H_j^{\alpha}H_Y^{1-\alpha}}{H(r-(\gamma-1)g_n-(1-\alpha)g_{H_Y}-\alpha g_{H_j})}. \tag{58}$$

From equations (3) and (6), we get the value of the wage rate accruing to human capital employed in the final good sector :

$$w_Y = (1-\alpha)AH_Y^{-\alpha}n^{\gamma}B^{\alpha}H_j^{\alpha}. \tag{59}$$

From equations (2), (5) and (6), we get the value of the wage rate accruing to human capital in the intermediate goods sector :

$$w_j = \alpha^2 AH_Y^{1-\alpha}n^{\gamma}H_j^{\alpha-1}. \tag{60}$$

Equations (58), (59) and (60) together also imply that :

$$g_{w_n} = g_{w_j} = g_{w_Y} = g_w = \gamma g_n. \tag{61}$$

Combining equations (51) and (61), we obtain :

$$r = \gamma g_n + \delta. \tag{62}$$

Using equation (23), (58), (60) and (62), we get :

$$H_j = \frac{H\alpha(Cs_n + \delta - g_h}{C(1-\alpha)}. \tag{63}$$

Combining equations (21), (58), (59) and (62), we have :

$$H_Y = \frac{H(CS_n + \delta - g_h)(\alpha - 1)}{C(\alpha-1)\alpha}. \tag{64}$$

Combining equations (40), (41) and (62), we are able to obtain the usual Euler equation, giving the optimal household's consumption path :

$$g_c = \gamma g_n + \delta - \rho. \tag{65}$$

From the equation above, we clearly see that r must be greater than ρ in order to have g_c positive. From equation (25) and using equation (57), we get :

$$g_a = g_n + g_{V_n} = g_n + (\gamma-1)g_n + \alpha g_{H_j} + (1-\alpha)g_{H_Y}. \tag{66}$$

Using equations (27), we can rewrite the equation (66) as follows :

$$g_a = \gamma g_n + g_h. \tag{67}$$

From equations (44) and (49), we have :

$$\frac{\dot{\lambda}_1}{\lambda_1} = -g_a + uw\frac{h}{a} - \frac{c}{a}. \tag{68}$$

Using equations (45) and (48), we get :

$$\frac{\dot{\lambda}_2}{\lambda_2} = -g_h - u\delta. \tag{69}$$

From equations (61), (67) (68) and (69), we obtain :

$$\frac{c}{a} = \delta u + uw\frac{h}{a}. \tag{70}$$

Using equation (66) and equation (61) and knowing that u is constant at the equilibrium, equation (70) leads to the conclusion that $\frac{c}{a}$ is constant. In other words :

$$g_c = g_a = \gamma g_n + g_h. \tag{71}$$

Combining equations (65) and (71), we get the growth rate of human capital accumulation :

$$g_h = \delta - \rho. \tag{72}$$

Combining equation (72) and (14), we get the optimal time of educating :

$$u = \frac{\rho}{\delta}. \tag{73}$$

Using equations (8), (64) and (72), we obtain :

$$H_Y = \frac{H(\alpha-1)(Cs_n+\rho)}{C(\alpha-1)\alpha}. \tag{74}$$

As $s_Y = \frac{H_Y}{H}$, we get :

$$s_Y = \frac{(\alpha-1)(Cs_n+\rho)}{C(\alpha-1)\alpha}. \tag{75}$$

Using equations (8), (63) and (72), we obtain :

$$H_j = \frac{H\alpha(Cs_n+\rho)}{C-C\alpha}. \tag{76}$$

As $s_j = \frac{H_j}{H}$, we get :

$$s_j = \frac{\alpha(Cs_n+\rho)}{C-C\alpha}. \tag{77}$$

Now, using the skilled labor market equilibrium (equation 21), we get :

$$s_n = \frac{((1-\alpha)(C+\delta)\alpha-\delta)\rho}{C\delta}. \tag{78}$$

As $s_n = \frac{H_n}{H}$, we get the skilled labor allocated in the research sector :

$$H_n = \frac{H((1-\alpha)(C+\delta)\alpha-\delta)\rho}{C\delta}. \tag{79}$$

Combining equation (74) and (78), we get the skilled labor allocated in the final good sector :

$$H_Y = \frac{H(C+\delta)(1-\alpha)\rho}{C\delta}. \tag{80}$$

Combining equation (77) and (78), we get the skilled labor allocated in the intermediate goods sector :

$$H_j = \frac{H\alpha(C+\delta)\alpha\rho}{C\delta}. \tag{81}$$

Given s_n, it is now possible to compute the growth rate of knowledge accumulation real interest by using equation (29) :

$$g_n = \frac{(1-\alpha)(C+\delta)\alpha+\delta)\rho}{\delta}. \tag{82}$$

Given g_n, it is now possible to compute the real interest by using equation (62) :

$$r = \delta - \frac{\gamma(\delta+(\alpha-1)(C+\delta)\alpha)\rho}{\delta}. \tag{83}$$

Combining equations (28), (71), (72) and (82), we get the growth rate :

$$g_Y = \delta - \rho - \frac{\gamma(\delta+\alpha(\alpha-1)(C+\delta))\rho}{\delta}. \tag{84}$$

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