A Baseline DSGE model of Climate Change for Climate Policy Analysis

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Abstract

This note documents a DSGE model of Climate Change. I extend the NK model with geophysical variables, such as greenhouse gas emissions, the carbon cycle, radiative forcing, and climate change. In this model, I specify five different climate policy regimes: no policy, cap, intensive, tax, and mandate.

Keywords: DSGE, climate change, climate policy

1 Introduction

I extend the NK model with geophysical variables, such as greenhouse gas emissions, the carbon cycle, radiative forcing, and climate change. In this model, the carbon emissions is a byproduct of goods production process, and GHGs raise the stock of atmospheric carbon, which increase carbon concentration in carbon cycle to strengthen radiative forcing, leading to climate change. TFP is endogenous and decreasing in the Temperature(climate change), as in the DICE-2016R of Nordhaus(2017,PNAS). My note contributes to several strands of literature. I contribute to a growing theoretical literature on climate policy. Van der Ploeg and Rezai (2020) and Rozenberg, Vogt-Schilb and Hallegatte (2020) show that unanticipated changes in climate policy may result in the stranding of carbon-intensive capital. Differently, I specify five different climate policy regimes: no policy, cap, intensive, tax, and mandate. Methodologically, my note adds an environmental component to a DSGE model (which has been called an E-DSGE model) to study climate and other environmental policies under business cycles, including Fischer and Springborn (2011), Heutel (2012), Annicchiarico and Di Dio(2015), Dissou and Karnizova (2016), Diluiso, Annicchiarico, Kalkuhl, and Minxand(2020), Carattini, Heutel, and Melkadze(2021). The most difference to above literature is the geophysical sector linking GHG emissions to the carbon cycle, radiative forcing, and climate change. The extended model and calibration follow V. N. Landi(2020)[1]. Note that I do not report all the devivations of the DSGE, which are carried out in other Landi's lecture notes.

2 The Model

The main characters of our model are:

1. representative household
2. Two types of producers: Final and Intermediate Firm
3. Physical capital accumulation
4. Price stickiness a la Rotemberg(1982)
5. Investment adjustment costs
6. Monetary policy conducted according to a Taylor rule
7. Geophysical section:
   - **a** greenhouse gas emissions
   - **b** the carbon cycle
   - **c** radiative forcing
   - **d** climate change
8. The following climate policies:
   - **a** no policy
   - **b** cap
   - **c** intensive
   - **d** tax
   - **e** mandate

Roadmap for next DSGE-CC model:

- Financial sector
- Green Financial Policy tools

2.1 Household

The household choose consumption, c_t, labor, h_t, investment, i_t, capital, k_t, and nominal governmental bond, b_t. The gross nominal price is p_t, the goss real return on bond is r_t, the real wage is w_t, the real return on capital is r_t^k. The household problem is :

[1] V. N. Landi(2020): An environmental NK model

$$
\max_{\{c_t,i_t,h_t,k_t,b_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \kappa_h \frac{h_t^{1+\varphi}}{1+\varphi} \right)
$$

$$
\text{s.t.} \begin{cases} p_t c_t + p_t i_t + b_t = r_t^k p_t k_{t-1} + r_{t-1} b_{t-1} + p_t w_t h_t - p_t t_t + p_t \Gamma_t \\ k_t = (1-\delta)k_{t-1} + \left[1 - \frac{\kappa_I}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t \end{cases}
$$

On the expenditure side of the budget constraint,

the capital accumulation equation be:

$$
k_t = (1-\delta)k_{t-1} + \left[1 - \frac{\kappa_I}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t \tag{1}
$$

This is a adjustment cost specification is based on Christiano, Eichenbaum, and Evans (2005). We refer to this is an "investment adjustment cost" (as opposed to a capital adjustment cost). Because (i) the adjustment cost is measured in units of investment, not units of capital as above, and (ii) the adjustment cost doesn't depend on the size of investment relative to the capital stock, but rather on the growth rate of investment.

Form a Lagrangian with two constraints

$$
\begin{aligned}
\mathcal{L} =& \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \kappa_h \frac{h_t^{1+\varphi}}{1+\varphi} \right) - \lambda_t \left(c_t + i_t + \frac{b_t}{p_t} - r_t^k k_{t-1} - r_{t-1}\frac{b_{t-1}}{p_t} - w_t h_t + t_t - \Gamma_t \right) \right.\right. \\
& \left.\left. -q_t \lambda_t \left\{ k_t - (1-\delta)k_{t-1} - \left[1 - \frac{\kappa_I}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t \right\} \right\} \right\}
\end{aligned}
$$

The FOCs are

$$
c_t^{-\sigma} = \lambda_t \tag{2}
$$

$$
\kappa_h h_t^{\phi} = \lambda_t w_t \tag{3}
$$

$$
q_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[r_{t+1}^k + (1-\delta) q_{t+1} \right] \right\} \tag{4}
$$

$$
1 = q_t \left[1 - \frac{\kappa_I}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \kappa_I \left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}} - 1\right) \right] + \kappa_I \beta \mathbb{E}_t \left\{ q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left[\left(\frac{i_{t+1}}{i_t}\right)^2 \left(\frac{i_{t+1}}{i_t} - 1\right) \right] \right\} \tag{5}
$$

$$
1 = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} r_t \right\} \tag{6}
$$

2.2 Firms

2.2.1 Final good firms

The final goods-producing sector is made up of a continuum of perfectly competitive firms. Firms operate a CES production technology with intermediate good inputs to produce y_t:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $y_t(i)$ is an intermediate input produced by the intermediate firm i, whose price is $p_t(i)$.

2.2.2 Intermediate good firms

There is a continuum of firms of measure unity, indexed by i, producing a differentiated input through the following Cobb-Douglas function:

$$y_t(i) = [1 - \Omega_t] a_t (k_t(i))^{\alpha} (h_t(i))^{1-\alpha} \tag{7}$$

Where Ω_t represent the damage function, which is defined:

$$\Omega_t = d_3[d_0 + d_1 T_{AT,t} + d_2 (T_{AT,t})^2] \tag{8}$$

Eq. 7 describes the economic impacts or damages of climate change. The DICE-2016R model takes globally averaged temperature change (T_{AT}) as a sufficient statistic for damages. Eq. 7 assumes that damages can be reasonably well approximated by a quadratic function of temperature change(Nordhaus, 2017).

where a_t is the exogenus component of TFP, which follows an autoregressive process:

$$\log(a_t) = (1 - \rho_a)\log(\bar{a}) + \rho_a \log(a_{t-1}) + v_t^a \tag{9}$$

Production in intermediate sector entails emissions as a byproduct. Carbon emissions are an increasing and concave function of total production:

$$e_t(i) = (1 - \mu_t(i))\, \gamma_1 y_t(i)^{1-\gamma_2} \tag{10}$$

where $\mu_t(i)$ is the abatement effort or the fraction of emission abated by firm i. $\gamma_1 > 0$ measures emissions per unit of output in the absence of abatement effort. Firm-level abatement costs z_t are, in turn, a function of the firm's abatement effort and output:

$$z_t(i) = y_t(i) \theta_1 \mu_t(i)^{\theta_2} \tag{11}$$

Firms operate in monopolistic competition, so they set the price of their own good subject to the demand of the final good firm. Firms pay quadratic adjustment costs $AC_t(i)$ in nominal terms a la Rotemberg (1982), whenever they adjust prices with respect to the inflation target$\bar{\pi}$:

$$AC_t(i) = \frac{\kappa_P}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi}\right)^2 p_t y_t \tag{12}$$

Emissions are costly to producers and the unit cost of emission τ_t depends on the climate policy regime put into place. Clearly, in this context, the marginal cost of an additional unit of output has two components: the cost associated with the extra inputs needed to manufacture the additional unit, which depends on the production technology and inputs' prices, and the costs associated with abatement and emissions, which depend on the available abatement technology and on the unit cost of emission.

From the solution of firm i's static cost minimization problem, taking the nominal wage rate w_t, the rental cost of capital r_t^k and unit cost of emission τ_t as given, I form the following Lagrangian:

$$\begin{aligned}\mathcal{L}^I =& \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[y_t (\frac{p_t(i)}{p_t})^{1-\varepsilon} \left[1 - \tau_t \left(1-\mu_t(i)\right) \gamma_1 y_t^{-\gamma_2} \right.\right.\right. \\ & - w_t h_t(i) - r_t^k k_{t-1}(i) - \frac{\kappa_P}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi}\right)^2 y_t - \\ & \left.\left.\left. + mc_t(i) \left[A_t \left(k_{t-1}(i)\right)^{\alpha} \left(h_t(i)\right)^{1-\alpha} - y_t \left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon} \right] \right] \right] \right\}\end{aligned}$$

We have the following optimality conditions for the demand of labor, capital and the abatement effort, respectively:

$$\begin{aligned} r_t^k &= mc_t(i) \alpha A_t \left(k_{t-1}(i)\right)^{\alpha-1} \left(h_t(i)\right)^{1-\alpha} \\ w_t &= mc_t(i)(1-\alpha) A_t \left(k_{t-1}(i)\right)^{\alpha} \left(h_t(i)\right)^{-\alpha} \\ \tau_t y_t^{-\gamma_2} \gamma_1 \left(\frac{p_t(i)}{p_t}\right)^{\gamma_2 \varepsilon} &= \theta_1 \theta_2 \mu_t(i)^{\theta_2 - 1} \end{aligned}$$

$$\begin{aligned}&(1-\varepsilon)\left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon}\frac{y_t}{p_t} + \varepsilon(1-\gamma_2)\tau_t\gamma_1\frac{y_t^{1-\gamma_2}}{p_t}\left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon(1-\gamma_2)-1}(1-\mu_t(i)) + \varepsilon\frac{y_t}{p_t}\left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon-1}\theta_1\mu_t(i)^{\theta_2} + \\ &+ \varepsilon mc_t(i)\frac{y_t}{p_t}\left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon-1} - \frac{\kappa_P}{p_{t-1}(i)}\left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi}\right)y_t + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\kappa_P\frac{p_{t+1}(i)}{p_t(i)^2}\left(\frac{p_{t+1}(i)}{p_t(i)} - \bar{\pi}\right)^2 y_{t+1}\right] = 0\end{aligned}$$

In a symmetric equilibrium, firms choose the same price, same inputs and same output. Using the production function it turns out:

$$r_t^k = mc_t \alpha \frac{y_t}{k_{t-1}} \tag{13}$$

$$w_t = mc_t (1-\alpha) \frac{y_t}{h_t} \tag{14}$$

$$\mu_t = \left(\frac{\tau_t \gamma_1}{\theta_1 \theta_2} y_t^{-\gamma_2} \right)^{\frac{1}{\theta_2 - 1}} \tag{15}$$

Notice that if $\tau_t = 0$,firms do not have any incentive to abate emissions and $\mu_t = 0$. This occurs because firms do not internalize their impact on TFP.

2.3 Geophysical sector

Following The DICE-2016R model, including several geophysical relationships that link the economy with the different forces affecting climate change, the geophysical sector model the carbon cycle, a radiative forcing equation, climate-change equations, and a climate-damage relationship.

2.3.1 Carbon Cycle

The carbon cycle is based upon a three-reservoir model calibrated to existing carbon-cycle models and historical data. We assume that there are three reservoirs for carbon. The variables $x_{AT,t}, x_{UP,t}, and x_{LO,t}$ represent carbon in the atmosphere, carbon in a quickly mixing reservoir in the upper oceans and the biosphere, and carbon in the deep oceans. Carbon flows in both directions between adjacent reservoirs. The mixing between the deep oceans and other reservoirs is extremely slow. The deep oceans provide a large sink for carbon in the long run. Each of the three reservoirs is assumed to be well-mixed in the short run. Equations (15) through (17) represent the equations of the carbon cycle.

$$x_{AT,t} = \phi_{11} x_{AT,t-1} + \phi_{21} x_{UP,t-1} + e_t + e^{row} \tag{16}$$

$$x_{UP,t} = \phi_{12} x_{AT,t-1} + \phi_{22} x_{UP,t-1} + \phi_{32} x_{LO,t-1} \tag{17}$$

$$x_{LO,t} = \phi_{23} x_{UP,t-1} + \phi_{33} x_{LO,t-1} \tag{18}$$

The parameters ϕ_{ij} represent the flow parameters between reservoirs. Note that emissions flow into the atmosphere. The carbon cycle is limited because it cannot represent the complex interactions of ocean chemistry and carbon absorption.

2.3.2 Climate Change

The next step concerns the relationship between the accumulation of GHGs and climate change. The climate equations are a simplified representation that includes an equation for radiative forcing and two equations for the climate system. The radiative forcing equation calculates the impact of the accumulation of

GHGs on the radiation balance of the globe. The climate equations calculate the mean surface temperature of the globe and the average temperature of the deep oceans for each time-step.

Accumulations of GHGs lead to warming at the earth's surface through increases in radiative forcing. The relationship between GHG accumulations and increased radiative forcing is derived from empirical measurements and climate models, as shown in Equation (18).

$$F_t = \eta log_2 \left(\frac{x_{AT,t}}{x_{AT,1750}} \right) + F_{EX,t} \tag{19}$$

F_t is the change in total radiative forcings of greenhouse gases since 1750 from anthropogenic sources such as CO2. $F_{EX,t}$ is exogenous forcings, and the first term is the forcings due to CO2.

Higher radiative forcing warms the atmospheric layer, which then warms the upper ocean, gradually warming the deep ocean. The lags in the system are primarily due to the diffusive inertia of the different layers. Forcings lead to warming according to a simplified two-level global climate model,

$$T_{AT,t} = T_{AT,t-1} + \xi_1 \left\{ F_{-}\xi_2 T_{AT,t-1} - \xi_3 \left[T_{AT,t-1} - T_{LO,t-1} \right] \right\} \tag{20}$$

$$T_{LO,t} = T_{LO,t-1} + \xi_4 \left\{ T_{AT,t-1} - T_{LO,t-1} \right\} \tag{21}$$

2.4 Policy

2.4.1 Monetary Policy

The monetary authority sets the nominal interest rate according to the following Taylor rule:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{y_t}{y} \right)^{\phi_y} \right]^{1-\rho_r} \exp\left(v_t^m \right) \tag{22}$$

2.4.2 Fiscal Policy

Government

nances public expenditure gt by raising lump-sum taxes and emission taxes:

$$g_t = t_t + \tau_t e_t$$

where g_t follows an autoregressive process:

$$log(g_t) = (1 - \rho_g) log(g_{t-1}) + \rho_g log(g_{t-1}) + v_g \tag{23}$$

2.4.3 Climate Policy

I specify five different climate policy regimes:

a **no policy**. This implies $\tau_t = 0$,so $\mu_t = 0$.

b **cap**. The government imposes a fixed amount of emission $e_t = \bar{e}$. In this case, τ_t can be interpreted as the price of emission permits sold by the government, determined endogenously.

c **intensive**. The government imposes an emission target per unit of output $e_t = \nu y_t$, and it sells emission permits as in 2.

d **tax**. The government taxes a constant tax on emissions.

e **mandate**. The government mandates a fixed amount of emission, but it does not sells emission permits.

2.5 Market Clearing

Clearing in the good market implies:

$$y_t = c_t + i_t + g_t + \frac{\kappa_P}{2}\left(\pi_t - \bar{\pi}\right)^2 y_t + y_t\theta_1\mu_t^\theta \tag{24}$$

Clearing in the bond market implies:

$$b_t = 0$$

3 Equilibrium

The equilibrium conditions of the model are the following:

$$k_t = (1-\delta)k_{t-1} + \left[1 - \frac{\kappa_I}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t \tag{25}$$

$$c_t^{-\sigma} = \lambda_t \tag{26}$$

$$\kappa_h h_t^\phi = \lambda_t w_t \tag{27}$$

$$q_t = \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1}^k + (1-\delta)q_{t+1}\right]\right\} \tag{28}$$

$$1 = q_t\left[1 - \frac{\kappa_I}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \kappa_I\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}} - 1\right)\right] + \kappa_I\beta\mathbb{E}_t\left\{q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{i_{t+1}}{i_t}\right)^2\left(\frac{i_{t+1}}{i_t} - 1\right)\right]\right\} \tag{29}$$

$$1 = \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}r_t\right\} \tag{30}$$

$$y_t = [1 - \Omega_t] a_t (k_t)^{\alpha} (h_t)^{1-\alpha} \tag{31}$$

$$\Omega_t = d_3 [d_0 + d_1 T_{AT,t} + d_2 (T_{AT,t})^2] \tag{32}$$

$$r_t^k = mc_t \alpha \frac{y_t}{k_{t-1}} \tag{33}$$

$$w_t = mc_t (1 - \alpha) \frac{y_t}{h_t} \tag{34}$$

$$\mu_t = \left(\frac{\tau_t \gamma_1}{\theta_1 \theta_2} y_t^{-\gamma_2} \right)^{\frac{1}{\theta_2 - 1}} \tag{35}$$

$$\begin{aligned} \pi_t \left(\pi_t - \bar{\pi} \right) =& \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1} \left(\pi_{t+1} - \bar{\pi} \right)^2 \frac{y_{t+1}}{y_t} \right] + \\ &+ \frac{\varepsilon}{\kappa_P} \left\{ \left[mc_t + (1 - \gamma_2) \gamma_1 \tau y_t^{-\gamma_2} (1 - \mu_t) + \theta_1 \mu_t^{\theta_2} \right] - \frac{\varepsilon - 1}{\varepsilon} \right\} \end{aligned}$$

$$y_t = c_t + i_t + g_t + \frac{\kappa_P}{2} \left(\pi_t - \bar{\pi} \right)^2 y_t + y_t \theta_1 \mu_t^{\theta} \tag{36}$$

$$\log(a_t) = (1 - \rho_a) \log(\bar{a}) + \rho_a \log(a_{t-1}) + v_t^a \tag{37}$$

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{y_t}{y} \right)^{\phi_y} \right]^{1-\rho_r} \exp(v_t^m) \tag{38}$$

$$log(g_t) = (1 - \rho_g) log(\bar{g}) + \rho_g log(g_{t-1}) + v_g \tag{39}$$

$$e_t(i) = (1 - \mu_t(i)) \gamma_1 y_t(i)^{1-\gamma_2} \tag{40}$$

$$x_{AT,t} = \phi_{11} x_{AT,t-1} + \phi_{21} x_{UP,t-1} + e_t + e^{row} \tag{41}$$

$$x_{UP,t} = \phi_{12} x_{AT,t-1} + \phi_{22} x_{UP,t-1} + \phi_{32} x_{LO,t-1} \tag{42}$$

$$x_{LO,t} = \phi_{23} x_{UP,t-1} + \phi_{33} x_{LO,t-1} \tag{43}$$

$$F_t = \eta log_2 \left(\frac{x_{AT,t}}{x_{AT,1750}} \right) + F_{EX,t} \tag{44}$$

$$T_{AT,t} = T_{AT,t-1} + \xi_1 \left\{ F_{-} \xi_2 T_{AT,t-1} - \xi_3 \left[T_{AT,t-1} - T_{LO,t-1} \right] \right\} \tag{45}$$

$$T_{LO,t} = T_{LO,t-1} + \xi_4 \left\{ T_{AT,t-1} - T_{LO,t-1} \right\} \tag{46}$$

a climate policy equation:

a **no policy**.

$$\tau_t = 0$$

b **cap**. The government imposes a fixed amount of emission

$$e_t = \bar{e}$$

c **intensive**. The government imposes an emission target per unit of output

$$e_t = \nu y_t$$

d **tax**. The government taxes a constant tax on emissions

$$\tau_t = \bar{\tau}$$

e **mandate**. The government mandates a fixed amount of emission, but it does not sells emission permits.

$$e_t = \bar{e} \; and \; \tau_t = 0$$

There are 24 equations for 24 endogenous variables:

$$X_t \equiv \left[\lambda_t, c_t, r_t^k, w_t, h_t, y_t, k_t, q_t, i_t, r_t, mc_t, \pi_t, \Omega_t, e_t, \mu_t, g_t, a_t, x_{AT,t}, x_{UP,t}, x_{LO,t}, F_t, T_{AT,t}, T_{LO,t}, \tau_t \right]$$

4 Seady State

Variables without time index denote the steady state level. Equation (39) in steady state imply:

$$g = \bar{g}$$

Equation (38) in steady state imply:

$$\pi = \bar{\pi}$$

Equation (30) in steady state imply:

$$r = \frac{1}{\beta}$$

Equation (29) in steady state imply:

$$q = 1$$

Equation (28) in steady state imply:

$$r^k = \frac{1}{\beta} - (1 - \delta)$$

Let's write out a bunch of other equations in steady state and work from there, given what we know and given the normalization of $y = 1$.

Price equation in steady state imply:

$$mc = \frac{\varepsilon - 1}{\varepsilon} - (1 - \gamma_2)\,\gamma_1 \tau y^{-\gamma_2}(1 - \mu) - \theta_1 \mu^{\theta_2}$$

Once we know r^k andmc, we can get the steady state of k by (33):

$$k = mc\alpha \frac{y}{r^k}$$

and in turn we get i from the law of motion of capital (25):

$$i = \delta k$$

Using (34), we can get wh:

$$wh = mc(1 - \alpha)y$$

Equation (35) in steady state imply:

$$\mu = \left(\frac{\tau\gamma_1}{\theta_1\theta_2} y^{-\gamma_2}\right)^{\frac{1}{\theta_2 - 1}}$$

Using (36), the steady-state level of consumption is:

$$c = y - [i + g + \frac{\kappa_P}{2}\left(\pi - \bar{\pi}\right)^2 y + y\theta_1\mu^{\theta}]$$

Using (26), marginal utility of consumption:

$$\lambda = c$$

Using (27), we can get

$$\kappa_h h^{\phi+1} = \lambda wh$$

And we have wh, so solve the steady state of h

$$h = \left(\frac{\lambda wh}{\kappa_h}\right)^{\frac{1}{\phi+1}}$$

Once we have h, we can get w

$$w = \frac{wh}{h}$$

Using (40), we can get e:

$$e = (1 - \mu)\,\gamma_1 y^{1-\gamma_2}$$

Once having e, we solve the following three system of three equations and three unknowns$\{x_{AT}, x_{UP}, x_{LO}\}$:

$$x_{AT} = \phi_{11}x_{AT} + \phi_{21}x_{UP} + e + e^{row}$$

$$x_{UP} = \phi_{12}x_{AT} + \phi_{22}x_{UP} + \phi_{32}x_{LO}$$

$$x_{LO} = \phi_{23}x_{UP} + \phi_{33}x_{LO}$$

Using (32), we can get Ω

$$\Omega = d_3[d_0 + d_1 T_{AT} + d_2(T_{AT})^2]$$

Now using (31), we also solve for the $\bar{a}$ consistent with our normalization of $y = 1$:

$$\bar{a} = \frac{y}{(1-\Omega)k^{\alpha}h^{1-\alpha}}$$

Next using (44)-(46), we can get the steady state of $F, T_{AT}, andT_{LO}$. At this point we have everything we need.

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A Codes

This appendix lists the dynare files used in this note. All files and the data used are contained in the file DSGE_CC2020.zip. In case simulated data was used in the estimation, one first needs to run the relevant dynare code to simulate the data, and save it.

- dsge_cc.mod - solves the above DSGEmodel and simulates data;

- dsge_cc_steadystate.m - solves the steady state values of all endogenous variables through calling the dsge_cc_solve.m file;
- dsge_cc_solve.m - using fsolve or csolve algorithm to solve the steady state values of endogenous variables;

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A Baseline DSGE model of Climate Change for Climate Policy Analysis
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% This Dynare code simulates a DSGE model of Climate Change as in Xu(2020).
% Author: Wenddy XU, 25/09/2020

close all;
warning off

 %%
 %%%%%%%%%%%%%%%%%%%%%%%Endogenous Variables %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 var


```

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