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Dinda, Soumyananda

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Soumyananda Dinda

Economic Research Unit, Indian Statistical Institute, 203, B. T. Road, Kolkata-108, India.
S. R. Fatepuria College, Beldanga, Murshidabad, W.Bengal, India.

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Abstract

The effective policy prescription depends on the actual position of the economy. This paper search for evidence of the linkage between economic growth and environment. To be specific, the paper is intended to observe how economic growth is systematically linked to the position of the economy as well as environmental quality. This systematic link is a foundation for the convergence of an economy towards steady state and the paper concentrates on growth theory and convergence highlighting on relative factor intensities of the two production sectors in determining the transitional dynamics of the system. The model predicts that average growth rate of output is inversely related to the initial level of output and directly associated with initial base level of environment.

Key Words: Convergence, Environment, Factor intensity, Growth Theory, Steady State, Transitional Dynamic,

**Address for Correspondence:* Soumyananda Dinda, Economic Research Unit, Indian Statistical Institute, 203, B. T. Road, Kolkata-108, India. Email: sdinda2000@yahoo.co.in

1. Introduction

Concern about rising economic activity and environmental quality has focussed attention on the relationship between economic growth and environmental degradation. The environmental effects due to economic growth have received increasing attention from economists in recent years. One particular aspect in the environment - versus - economic growth has evoked much discussion in the last decade (i.e.; 1990s). The relationship between economic growth and environment has been the objective of a large debate in the economic literature for many years. This debate goes back to the controversy on the limits to growth at the beginning of the 1970s. Before 1970, there was a belief that the consumption of raw materials, energy and natural resources grow almost at the same rate (viz., steady state) as economic growth. In early 1970s, the Club of Rome's ***Limits to Growth*** (Meadows et al. 1972) was forwarded about the concern for availability of natural resource on the Earth. The environmental economists of the Club of Rome argued that the finiteness of environmental resources would prevent economic growth from continuing forever and urged a zero-growth or steady state economy to avoid dramatic ecological scenarios in the future.

The motivation for the mostly empirical studies in 1990's is only to find out the cause[1] of environmental problems and policy suggestions. The effective policy prescription depends on the actual position of the economy. Our motivation for this study is to search for the evidence of the linking of economic growth and environment. To be specific, we are interested to observe how economic growth is systematically linked to the position of the economy as well as the environment. This systematic link is a foundation for the convergence of an economy towards steady state. Now we turn to concentrate on growth theory and convergence.

In this paper, we examine the convergence and growth theory where environment (as input) enters into the production function. In this study, we examine the convergence hypothesis in

[1] Before adopting any policy, it is important to understand the nature and causal relationship between economic growth and environmental quality (See, Coondoo and Dinda 2002).

endogenous growth model rather than neoclassical growth model. The endogenous growth models are characterized by the assumption of non-decreasing returns to reproducible factors of production, while the neoclassical growth models predict diminishing returns. In neoclassical growth model, the economic growth rate is systematically linked to the position of the economy. This systematic link is the basis for the convergence[2] of economic growth towards steady state.

The conditional convergence is often identified as an empirical test of alternative growth models. It is argued that if conditional convergence is found then neoclassical theory of growth is confirmed. This presumption is unfair and incorrect (See, Song 2000). It is often also argued that in the endogenous growth model, the growth rate would not be systematically linked to the position of the economy in the process of development (Mankiw et al. 1992). Song (2000) clearly shows that the above statement is incorrect. He shows that both the endogenous and neoclassical growth models are exactly the same equation for conditional convergence. So, it is not possible to distinguish between two models by estimating an equation for conditional convergence.

We analyze a two-sector model of endogenous growth, consisting of a goods sector and an abatement sector, in which each sector produces capital and environmental capital as input under constant returns to scale. The analysis of the static two-sector production model is substantially simplified by the fact that it has a block recursive structure, under the assumptions of perfect competition in goods and factor markets and the equalization of factor rewards across sectors. That means, the factor returns can be determined as a function of output price[3] alone, independently of factor supplies. An increase in the price of one sector's output leads to a more

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[2] There are two concepts -- (i) β -convergence (poor economy tends to grow faster than rich ones) (ii) σ-convergence (convergence occurs if the dispersion declines over time). β -Convergence has two concept (a) absolute convergence and (b) conditional convergence. The hypothesis that poor economies tend to grow faster than rich ones without and with conditioning on any other characteristics of economies - are referred to as absolute convergence and conditional convergence, respectively. The conditional convergence is more realistic than absolute convergence in cross-country study.

[3] The factor price equalization property of the trade models.

than proportional increase in the price of the factor used intensively in that sector[4]. An increase in the quantity of a factor leads to a more than proportional expansion of the output of the good that uses that factor intensively[5]. We show that the two sector endogenous growth model can be analyzed using these properties of static two sector models, combined with an intertemporal no-arbitrage condition which requires an equity between the net rates of return on capital and environmental capital. This approach allows us to obtain conditions under which there exists a unique steady state and also highlights the role played by the relative factor intensities of the two production sectors in determining the transitional dynamics of the system.

Bond et al. (1996) characterize the transitional dynamics, using the theorem of international trade, for the two-sector model of Rebelo (1991). Mulligan and Sala-I-Martin (1993) examine a number of two sector models, through extensive simulations, they show that steady state exists and is stable under all possible parameter values.

Mulligan and Sala-I-Martin (1993) also find that the return to capital and growth rate of output is decreasing in capital-labour ratio in all the simulation that they have conducted. Song (2000) finds that the return to capital and growth rate of output fall over time on the transition path if the initial ratio of physical capital to human capital is lower than the steady state level. He also shows that two sector endogenous growth models are consistent with the evidence on conditional convergence found by Barro (1991) and Mankiw et al. (1992).

This paper builds upon the results of Mulligan and Sala-I-Martin (1993), Bond et al. (1996) and Song (2000). This paper focuses on the behaviour of price and its implications for conditional convergence. In terms of methodology, this paper[6] is closely related to Song (2000) and Bond et al. (1996), but it is slightly different from them. Here, we use two capitals – one is physical capital and other environment, which is public capital good. However, they do not

[4] This is Stolper-Samuelson theorem.
[5] This is the Rybczynski theorem.
[6] In this study, we use the same characterization of the two-sector economy based on international trade theory (See, appendix A).

provide any empirical support for conditional convergence in two-sector endogenous growth model. We try to fill up this gap. In fact, we reconfirm their results and derive the same results analytically. The empirical findings also support our results in two-sector endogenous growth model.

This paper is organized as follows. In section 2, the two-sector model is set up. In section 3, we derive transitional dynamics and implications for conditional convergence. In section 4, empirical results and finally we conclude the paper.

2. A Two Sector Endogenous Growth Model

In this section we present a two-sector model of endogenous growth in which there are two reproducible factors of production, Capital (*K*) and Environment (*E*). An economy produces two goods, using environment and capital as inputs. One good is the final good, which is used both for consumption and for capital investment. Other good, which we will call the abatement good, is used for creating the environmental stock. These two goods are produced under the following technology and resource constraints.

$$Y = f(K_y, E)$$

$$A = h(K_E, E)$$

$$K = K_y + K_A$$

Y and *A* denote outputs of final good and abatement good, respectively. The production functions *f* and *h* satisfy the standard neoclassical properties. We assume that final good (*Y*) sector is more capital intensive than abatement (A) sector. This means $(K_Y/E)>(K_A/E)$ for all possible values of *R*, the rental price of capital, *S*, the service rate of per unit environment, and *q*, the relative price of abatement activity. For the given *q, S* and *R* are uniquely determined[7].

[7] See, Appendix A.

Representative agent views the environmental stock (E) which is given, although $E(t)$ varies in the aggregate. Now, at each point of time, representative agent should allocate optimally its stock of capital for both commodity production and abatement activity. Now, $K = K_y + K_A$, or $K_y = \theta K$, where $\theta = \frac{K_y}{K}$. Therefore, $K_A = (1-\theta)K$. Given total

amount of capital ($K = K_y + K_A$), the optimal sectoral allocation of capital for both sectors at any moment of time is governed by

$$\frac{\partial Y}{\partial K_Y} = q' \frac{\partial A}{\partial K_A},$$

where q' is the price of environment relative to physical capital. Optimality requires that marginal cost (loss of environment due to emission generated by employing capital K_Y) equals to marginal benefit (due to upgrading environment by applying capital K_A).

The infinite time horizon inter-temporal consumption choice problem for this economy may be specified as

$$\text{Maximize } W = \int_0^\infty e^{-\rho t} U(C(t), E(t)) dt$$

subject to the accumulation constraints

$$\dot{K}(t) = f(\theta(t)K(t), E(t)) - C(t)$$

and

$$\dot{E}(t) = h((1-\theta(t))K(t), E(t)) - \gamma f(\theta(t)K(t), E(t)).$$

We normalize the size of population to 1 and assume that population is not growing. Thus all variables should be interpreted as variables per capita. In this economy, representative agent has control over her consumption C as well as capital allocation parameter θ for both commodity production and upgrading environment.

Using the properties A1 to A5 (See Appendix A), we now derive the dynamic behavior of the economy. To anchor the dynamic system, we will express every quantity variable as a ratio to environment. Let us denote $c = \frac{C}{E}$, $k = \frac{K}{E}$, $y = \frac{Y}{E}$, $a = \frac{A}{E}$. Using the

homogeneity of *Y* and *A* in *E* and *K*, *y* and *a* can be expressed as functions of *q* and *k*. In other words, $y(q,k) = Y(q,1,k)$, and $a(q,k) = A(q,1,k)$.

The intertemporal no-arbitrage condition, which links the capital gain on environment to the difference between the rentals on capital and service charge of environment. Thus, the differential equation of relative price is the difference between the returns on capital and environment. So, we can write

$$\dot{q} = [R(q) - \frac{S(q)}{q}]q \tag{1}$$

Suppose U(.) is specified as $\frac{(CE^{\upsilon})^{1-\sigma} - 1}{1-\sigma}$. Using this specification, we have obtained the optimal path of consumption which is characterized by the following equation.

$$\dot{c} = \left[\frac{\Omega - \rho}{\sigma} + \left(\frac{\nu(1-\sigma)}{\sigma} - 1 \right) (a - \gamma y) \right] c \tag{2}$$

where Ω is the function of rental rate of capital (R). The motion equation of k is given as

$$\dot{k} = y - c - (a - \gamma y)k \tag{3}$$

These equations (1), (2) and (3) define a dynamic system in three variables, *q, c* and *k*. *c* and *q* are jumping variables and *k* is a state variable In the steady state[8] $\dot{q} = \dot{c} = \dot{k} = 0$. These conditions imply that

$$R(q^*) = \frac{S(q^*)}{q^*} \tag{4}$$

[8] . At steady state allocation of capital is fixed up i.e., $\dot{\theta} = 0$.

$$\frac{\Omega(R(q^*)) - \rho}{\sigma} = (1 - \frac{\nu(1-\sigma)}{\sigma})(a(q^*, k^*) - \gamma y(q^*, k^*)) \tag{5}$$

$$c^* = (1 + \gamma k^*) y(q^*, k^*) - a(q^*, k^*) \tag{6}$$

Asterisks denote steady state values. Equation (4) states that the real interest rate must be equal to the rate of return on environmental service. This condition gives us a unique value of q^*. Thus, in steady state, *q, S* and *R* are all constant. Given q^*, equation (5) determines k^* such that consumption and environment should grow at the same rate. Equation (6) determines c^*, at which capital grows at the same rate as environment. As *q* is constant and environment and capital grow at the same rate, the linear homogeneity of *V, Y* and *A* implies that $\frac{\dot{V}}{V} = \frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} = g^*$ (say).

Important feature of the system is that equation (1) is a differential equation in *q* alone. Once the initial value of *q* is known, the entire future path of *q*, hence that of the interest rate is completely determined by equation (1). Suppose *q* increases from q^*, then R(q) decreases and S(q)/q increases by the Stolper-Samuelson theorem. So, $\dot{q} < 0$ and *q* converges to q^*. For any initial value of *q, q* converges to q^*. However, the path of *q* affects the path of environment through a(q,k) and $\gamma y(q,k)$, and the path of consumption through $\Omega(R(q))$, a(q,k) and y(q,k). So, the problem is to choose the right initial value of *q* for a given state variable *k* such that the entire system converges to the steady state.

3. Transitional Dynamics and Conditional Convergence

The neoclassical growth model predicts that the partial correlation between the growth rate of output and the level of output is negative. Mankiw et al. (1992) find evidence for the conditional convergence and interpret it as supporting the neoclassical (Solow) growth model against endogenous growth models. We assume the existence, uniqueness and optimality of the path that satisfies equation (1), (2) and (3) at every moment and converges to the steady state. We restrict our analysis to the movement of the economy along the convergent path. If $k(0)<k^*$, k monotonically increases towards k^*, and if $k(0)>k^*$, k monotonically decreases towards k^*. c(k) and q(k) are the policy functions corresponding to the convergent path.

Remark1: *If $k(0)<k^*$, q monotonically increases and Ω (i.e.; $\Omega(R(q))$) monotonically decreases as the economy moves along the convergent path.*

Let we define GDP (see, A3 in Appendix) per unit of environment v as $\frac{V(q,E,K)}{E} = V(q(k),1,k)$. As q is increasing in k and V is increasing in q and k, v is an increasing function of k. This endogenous growth model could be consistent with the convergence. Suppose we have a sample of countries, which are identical except in the current level of capital (all having an identical level of environment). Then a country with a low level of output must have a low level of k, and hence its growth rate must be higher than those with higher outputs[9]. So, poor countries grow faster than rich countries if countries are identical except in the current level of output per unit of environment. This is true under the assumption that all countries have an identical level of environment. We may also think that the environment is an international public good, which acts as an exogenous good. But this is not valid for endogenous growth model, which determines the growth rate of environment endogenously.

Our endogenous growth model is consistent with conditional convergence in the sense of Barro (1991). As k approaches the steady state from below, $\frac{V}{E} = V(q(k),1,k)$ increases, while the

[9] This is the absolute β-convergence. However, it should be noted that all countries should not reach the same level of output per unit of environment. Here, a is endogenously determined and will be different across countries. So, the level of environment in the steady state will be different across countries.

growth rate of V falls. Thus, the growth rate of V is inversely related to $\frac{V}{E}$ on the transition path. In other words, if all countries have identical production functions and preferences, the growth rate of output should be negatively related to the level of output and is positively related to the level of environment. This argument can be made precisely if we linearize the system composed of equations (1), (2) and (3).

The stable arm of the linearized system is characterized by the following equations.

$$q - q^* = \phi_1 (k - k^*) \tag{7}$$

$$c - c^* = \phi_2 (k - k^*) \tag{8}$$

$$k - k^* = e^{\lambda_1 t} (k(0) - k^*) \tag{9}$$

where ϕ_1 and ϕ_2 are positive constants, and λ_1 is the negative eigenvalue of the system. λ_1 is given by the following equation.

$$\lambda_1 = \left[R'(q) - {}^{d}\!/\!_{dq}\left({}^{S}\!/\!_{q} \right) \right] q^* < 0.$$

By equation (7) and (9), $q - q^* = e^{\lambda_1 t}(q(0) - q^*)$. (10)

Let us linearize log v(q,k) around q^* and k^*. Using equations (9) and (10),

$$\log v(q,k) - \log v(q^*, k^*) \cong \frac{v_q}{v}(q - q^*) + \frac{v_k}{v}(k - k^*)$$

$$= e^{\lambda_1 t} \left[\frac{v_q}{v}(q(0) - q^*) + \frac{v_k}{v}(k(0) - k^*) \right]$$

$$\cong e^{\lambda_1 t} \left[\log v(q(0), k(0)) - \log v(q^*, k^*) \right]$$, where $\frac{v_q}{v}$ and $\frac{v_k}{v}$ are evaluated at q^* and k^*. Therefore,

$$\frac{\log v(T) - \log v(0)}{T} = \frac{1 - e^{\lambda_1 T}}{T} \log v^* - \frac{1 - e^{\lambda_1 T}}{T} \log v(0). \tag{11}$$

Now note the followings.

$V(0) = v(0)E(0)$, and $V(T) = v(T)E(T) = v(T)E(0)e^{(l(T)-m(T))T} = v(T)E(0)e^{x(T)T}$, where *l(T)* and *m(T)* are average growth rate of abatement and emission, respectively and *x(T)=l(T)-m(T)*. Now, *x(T)* is the net average growth rate of environment from 0 to T and is defined as $x(T) = \frac{\int_0^T (a(t) - \gamma y(t))dt}{T}$. Here, the sign of *x(T)* depends on the relative strength of *l(T)* and *m(T)*, i.e.; *x(T)>0* if *l(T)>m(T)* or *x(T)<0* if *l(T)<m(T)*.

Plugging these equations into equation (11), we obtain

$$\frac{\log V(T) - \log V(0)}{T} = \frac{1-e^{\lambda_1 T}}{T}\log v^* - \frac{1-e^{\lambda_1 T}}{T}\log V(0) + \frac{1-e^{\lambda_1 T}}{T}\log E(0) + x(T) \qquad (12)$$

Note that the equation (12) is exactly the same equation as derived by Song (2000) for two-sector endogenous growth model (See also Mankiw et al. (1992) for the Solow model, and Barro and Sala-I-Martin (1992) for the Ramsey model)[10]. Our model predicts that the average growth rate of GDP is inversely related to the initial level of GDP and positively related to the initial level of environment.

The natural question is whether the data support these predictions concerning the determinants of income growth. In other words, we want to investigate whether real income is higher in countries with higher environment and lower in countries with lower environment[11]. For the quantitative analysis of equation (12) we need the data on V as well as on E.

[10] All these models predict that the average growth rate of output is negatively related to the initial level of output and positively related to the initial level of human capital, controlling for the cross-country differences in variables affecting the steady state output v^*. The difference lies in that each model has a different set of parameters that determine the speed of convergence λ_1.

[11] This means that the country with higher environment has higher growth rates. This is true (under certain assumptions) if the high growth country has low emission level and low growth country has high emission (or pollution) level. For example, in the USA, annual average growth rate of income is approximately 2% with high level of emission (Rank 1 in total CO_2 emission level in the World), while in India and China, economic growth rates are high (Indian growth rate is 5% - 6% and Chinese growth rate is 8 % -10%.) with low level of emission. It should be noted that with high emission level, the USA has also very good quality of environment. This is possible because of better technology, good protection and law and order etc.

Conclusion

This paper examines the convergence hypothesis in two-sector endogenous growth model. This two-sector endogenous growth model consists of production and abatement sector, both sectors use capital and environment as inputs. The relative factor intensities of two production sectors play the vital role in determining the transitional dynamics of the system. The growth rate of output falls over time on the transitional path if the initial ratio of physical capital to environment is lower than the steady state level. We obtain an algebraic equation of the conditional convergence in two-sector model of endogenous growth with physical capital and environmental capital. This paper examines the implications for conditional convergence in two-sector endogenous growth model. Our model predicts that the average growth rate of output is negatively related to the initial level of output and positively related to the initial level of environment.

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Appendix A

For the given *q, S* and *R* are uniquely determined. In other words,

A1: *S and R are functions of q alone.* (Factor Price Equalization Theorem)

We will denote these functions by S(q) and R(q).

A2: *S(q)/q is increasing in q, and R(q) is decreasing in q.* (Stolper-Samuelson Theorem)

q determines factor prices, which in turn determines the factor intensity of each sector.

A3: *Given q, Y/K is increasing in K, and A is decreasing in K.* (Rybczynski Theorem)

The above discussions allow us to express outputs of the two sectors as functions of *q, K* and *E*. *Y* is decreasing in *q* and *A* is increasing in *q*. Now we define the following variable.

$$V(q,K,E) \equiv Y(q,K,E) + qA(q,K,E).$$

Where *V* is the GDP of the economy, which includes the value of the abatement activity produced. Since *Y* and *A* are linearly homogeneous in *K* and *E*, *V* is also linearly homogeneous in *K* and *E*.

A4: $V_K(q,K,E) = R(q)$, $V_E(q,K,E) = S(q)$, $V_q(q,K,E) = A(q,K,E)$.

If the price of *A* increases by one unit, *V* increases by *A*. The marginal contributions of environment and capital to GDP are equal to their prices in factors market. Since *V* is linearly homogenous in *E* and *K*, the value of outputs can be expressed as the total of factor income.

A5: $V(q,K,E) = S(q)E + R(q)K.$