Is tax competition necessarily a Race to the bottom? Optimal tax rate trajectories in the model of tax competition for different objective functions

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Abstarct

The work is devoted research of government tax behaviour in tax competition conditions. In details we study followed issues: is necessarily tax competition lead to Race to the bottom & is possible a simultaneous optimum of tax rate for both economies? This work is continuation of research about, is it necessarily Race to the bottom is Prisoner's dilemma. Available studies of tax competition generally focus on the analysis, which countries are inherent the trend to tax rate decrease, can this trend be considered a Race to the bottom, but if not, what are the reasons, that a Race to the bottom is not observed?

The difference of the proposed work is that we do not consider additional, though important factors. The optimization model of tax competition for 2 economies evidence that even for one factor – the generalized tax burden, without the separation of income tax and "compensatory" taxes, such as taxes on consumption, labor, environment – a Race to the bottom is not necessarily. Under different conditions the trend can be directed as to decrease as to increase of tax rate.

So, it can be argued that tax competition not necessarily leads to Race to the bottom, as well as Race to the bottom is not necessarily modeled by Prisoner's dilemma. The obtained results can help understand why some countries do not always follow the general trend to tax rate decrease. In addition, it explains not always the optimal tax behavior of some countries those in this way cause a change in the trend in competitors.

Key words: tax competition; race to the bottom; prisoner's dilemma; tax rate trajectory; modelling

Introduction

The text is the continue of (Sokolovskyi, 2020)

The purpose of the previous work was to establish, whether Race to the bottom is always Prisoner's dilemma, as it considers by basic models of this process (see i.e. Polanyi (1944), pp. 57, 73, Schlesinger (1997), Kuttner (1997), Tonelson (2000), p. 15, Greider (2001)). There're applied researches that have tested this hypothesis but not always supported it. (see, (Mendoza, Tesar, 2003a), (Mendoza, Tesar, 2003b), (Abbas, Klemm, 2013), (Revilla, 2016), Kiefer, Rada (2013a, 2013b), (Rota-Graziosi, 2019)). A number of researchers (Wheeler (2001), Spar (1998), Jensen (2006), Rodric (1999), Cho (2002), Dean (1992), Jaffe at al. (1995), Tobey (1990), Ratnayake (1998), Trandafir, Brezeanu & Stanciu (2011)) emphasize that application of the main quality of Race to the bottom, namely the mutual (collective) reduction of the tax rate by economies does not necessarily lead to deterioration of labor, social, and/or environmental standards, i.e. to Prisoner's dilemma.

In (Sokolovskyi, 2020) modelling the interaction of 2 economies using game theory for different cases of the objective functions (namely the investment volume, the budget revenue, and the budget revenue + the investment volume) also confirms that Race to the bottom does not always led to Prisoner's dilemma, so optimums and equilibriums are not always got when applying pure strategies at end-points. It is established that Race to the bottom leads the presence or absence of Prisoner's dilemma not for sure; it is determined by the specific values of the interaction parameters.

It's also known that even with a pretty simple objective function "budget revenue" its maximum is usually got not at the maximum tax burden (=1), but in the middle of the interval (0; 1) (Laffer, 2004). So, when the government decrease the tax burden in the economy, it does not necessarily worsen the financial situation of the economy, even without taking into account the additional investment inflow. It's also contrary to logic Prisoner's dilemma.

In addition, observations (Mendoza, Tesar, 2005) show that during tax competition countries do not reduce rates to a minimum but find some intermediate optimal burden.

Literature review

The question of optimal taxation has been raised since F. P. Ramsey (Ramsey, 1927). Among later studies, one of the defining works is the article Diamond & Mirrlees (1971).

In general, the authors present tax optimization, first of all, as improving the structure of taxes, balancing different types of taxes, in particular, the ratio of direct and indirect taxes, the logic of establishing the tax burden.

From this perspective, it is worth noting the works of Atkinson and Stiglitz (1976 Atkinson (1977), Kaplow (2008).

Tax regulation as a way of economic behavior raises the issue of tax rates size, as well as changes ones in order to reduce the shadow segment of the economy, intensify economic activity and attract investment.

Limit tax rates were calculated by Mirrlees (1971) based on the balancing of equality and efficiency.

In that direction, Chamley (1986) and Judd (1985) used the Ramsey model (Ramsey, 1928) to analyze the optimality of a non-zero capital tax.

Capital gains taxation is the subject of research in the works of Mankiw (2000), Kones, Kitao, and Kruger (2009), Aiyagari (1994).

Related to these studies are the investigations of Kaplow (1990), Tuomala (1990), Sandmo (1993), Cremer & Gahvari (1993), Slemrod (1994, 2001), Diamond (1998), Saez (2001), Weinzierl (2009).

In particular, L. Kaplow analyzes the relationship between optimal taxation and optimal tax enforcement. The based question is which revenue should be raised through higher tax rates.

Cremer H. & Gahvari F. consider tax evasion because of Ramsey's optimal taxation problem.

Slemrod J. (1994) calculus optimal income tax progressivity and generalizes the standard model of the optimal linear income tax to include taxpayer avoidance behavior and the ability to higher marginal tax rates.

In the work of 2001 Slemrod summarizes the standard model of how taxes affect the labor-leisure choice. This model provides a conceptual structure for evaluating to what extent, and in what situations, the opportunities for tax avoidance mitigate the real substitution response to taxation.

The attractiveness of the economy by regulating tax rates can be increased both on an absolute scale, by improving the economic climate in the jurisdiction (in particular, by the tax burden optimizing), and relative to other economies, in competition with them.

Actually, international tax competition is another direction of researches of the area of the optimal taxation. By this concept we mean the use by governments of low tax rates to improve the attraction of their countries and, consequently, to attract additional investment.

A review of the theory of international tax competition see by Keen & Konrad (2013). Authors in particular analyze factors affect the choice by governments their international tax behavior.

M. Devereux, B. Lockwood, M. Redoano & S. Loretz study the possible tax competition about corporate income taxes in OECD countries (Devereux et al., 2008) and European Union countries (Devereux & Loretz, 2013). They concluded that some features of tax competition typical generally for the EU countries.

These models consider a proposition that market share of certain jurisdiction is inversely related both to its tax rate and allocative efficiency (Lee, 2009, p.20).

One of the known research models of tax competition that explains the decreasing trend of the tax burden is Race to the bottom proper.

The analysis of presence or absence Race to the bottom for different regions is contained in the works Krogstrup (2004), Dvořáková (2013), Nicodème (2006) (all – the European Union), Vezinz (2014) (Asia).

Moreover, Nicodème (2006) denies Race to the bottom.

Krogstrup (2004) notes some effect of tax competition on the corporate tax burden, however its amount is not enough to confirm for Race to the bottom.

Dvořáková (2013) fixes decreasing of corporate income tax in EU countries, moreover, the analysis let assume the presence of Race to the bottom for new members (EU-12) but does not confirm for old members (EU-15).

Ali Abbas et al. (2012) found evidence of a partial race to the bottom, notably, no race to the bottom situation for standard tax systems in the sample compared to advanced economies and the presence of race to the bottom for special tax regimes, where effective tax rates are close to zero.

Drezner (2006) identified the main forecasts for Race to the bottom in terms of regulatory standards: if some government reduces its regulatory standards to attract more investment, other open economies, according to the model Race to the bottom, must follow the same tax behavior to prevent capital flight.

Based on these studies and the model (Sokolovskyi, 2020) we're going to test the hypothesis if may the government's strategy in terms of tax competition be different from the Race to the bottom, even if we do not consider additional related tax factors except for generalized tax burden, such as taxes on consumption, labor, environment.

So, observations of the real economy evidence that tax competition does not always lead to a Race to the bottom. Moreover, competitors are affected by many factors other than income tax: taxes on labor, the environment or consumption, and various externalities that can distort the result. Therefore, there's the task to check, if a Race to the bottom is a consequence of tax competition in perfect pure conditions, without obstacles, or, conversely, the optionality of a Race to the bottom follows from the competition model itself.

Namely, when considering the single tax model, we are interested in

- ✓ if governments always only try to decrease tax burden?
- ✓ even in this case, do government tries to decrease the tax rate reduction up to 0?
- ✓ if not, what are the causes and consequences; does this correspond to the real situation?

Therefore, we're going to look for the players' tax strategies (the optimal tax rate), that maximize their objective functions, depending on the opponent's actions (for different objective functions).

We're also going to test that there is a point of mutual equilibrium at which the objective functions of both players get the maximum and, if not, determine to what results interact between economies may lead when they try to optimize their tax rates, based on the current tax rate of vis-à-vis.

Methods and models

We model the interaction of 2 economies with parametrically set values of economic productivity. It's assumed that economies are in a state of tax and investment equilibrium, i.e. a state in which all economies are the same for the potential investor in terms of productivity of investments and therefore no investment movement takes place between these economies.

To determine the dependence of economic productivity on the volume of investment there uses the exponential function. (We assume with increasing investment (investment saturation), their efficiency per unit decreases.) A sigmoid function also can be used for it, but in the work, we do not perform calculations for this type of function.

To solve the issue, we calculate the extremums of functions 1 and 2 of variables prescribed in implicit form and limited to an interval (0; 1), since the independent variables of these functions are tax rates.

When using the exponential dependence of the profitability of the economy on the volume of available investment per unit, as well as the assumption that investment is distributed only between the two economies, the profitability of the economies *i* and *j* are equal to:

$$p_i = e^{a_i - \pi_i x_i};$$

$$p_j = e^{a_j - \pi_j (1 - x_i)},$$

where e^{a_i}, e^{a_j} – multipliers, in accordance, for economies *i* and *j*, $e^{a_i} > 1$, $e^{a_j} > 1$, or, what is the same, $a_i > 0$, $a_j > 0$. Since the situation for economies *i* and *j* is symmetric, we consider the economy i is no less productive than the economy j, i.e. $a_i \ge a_j$.

π_i, π_j – the speed of decrease in profitability depending on the saturation of the economy with investment, in accordance, for economies i and j, $\pi_i>0$, $\pi_j>0$.

When taxing p_i and p_j in the volume, in accordance, τ_i and τ_j we get:

$$p_i = (1-\tau_i)e^{a_i - \pi_i x_i};$$

$$p_j = \left(1-\tau_j\right)e^{a_j - \pi_j (1-x_i)}.$$

The investment equilibrium is got for the distribution of investments $\left(\hat{x}_i;\ 1-\hat{x}_i\right)$, when $p_i = p_j$, i.e.

$$(1-\tau_i)e^{a_i - \pi_i x_i} = \left(1-\tau_j\right)e^{a_j - \pi_j (1-x_i)};$$

$$\frac{1-\tau_i}{1-\tau_j} = e^{-a_i + \pi_i x_i + a_j - \pi_j (1-x_i)};$$

$$\left(\pi_i + \pi_j\right)x_i - \pi_j - a_i + a_j = \ln\frac{1-\tau_i}{1-\tau_j};$$

$$\hat{x}_i = \frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i + \pi_j}. \qquad (1)$$

In accordance,

$$1-\hat{x}_i = \frac{\pi_i - a_i + a_j - \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i + \pi_j},$$

i.e.

$$\left(\hat{x}_i;\ 1-\hat{x}_i\right) = \left(\frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i + \pi_j};\ \frac{\pi_i - a_i + a_j - \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i + \pi_j}\right). \qquad (2)$$

It's economically advisable to consider 3 based goals, that the government strives to reach: maximizing the revenue from investments in total; maximizing the budget revenue; maximizing the budget revenue, and the revenue from investments in total.

Maximizing the revenue from investments in total as the objective function

The total volume on all investments in economies *i* and *j* is:

$$P_i = x_i\left(1-\tau_i\right)e^{a_i-\pi_i x_i};$$

$$P_j = \left(1-x_i\right)\left(1-\tau_j\right)e^{a_j-\pi_j\left(1-x_i\right)}.$$

For the state of investment equilibrium, it's equal to:

$$\hat{P}_i = \hat{x}_i\left(1-\tau_i\right)e^{a_i-\pi_i\hat{x}_i} = \frac{\pi_j + a_i - a_j + \ln\dfrac{1-\tau_i}{1-\tau_j}}{\pi_i+\pi_j}\left(1-\tau_i\right)e^{a_i-\frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{1+\frac{\pi_j}{\pi_i}}} =$$

$$= \frac{\pi_j + a_i - a_j + \ln\dfrac{1-\tau_i}{1-\tau_j}}{\pi_i+\pi_j}\left(1-\tau_i\right)e^{a_i-\frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}\cdot\frac{1-\tau_i}{1-\tau_j}^{-\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)};$$

$$\hat{P}_i = \frac{e^{a_i-\frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1-\tau_i\right)^{\left(1-\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}; \qquad (3)$$

Maximizing the budget revenue from the taxation of investment income as the objective function

If the total investment volume in the economy *i* is $P_i = x_i\left(1-\tau_i\right)e^{a_i-\pi_i x_i}$, then the volume of budget revenue for the economy *i* is

$$B_i = x_i\tau_i e^{a_i-\pi_i x_i}.$$

In accordance for the economy *j*:

$$B_j = (1 - x_i)\tau_j e^{a_j - \pi_j(1-x_i)}.$$

For the state of investment equilibrium, we have:

$$\hat{B}_i = \hat{x}_i \tau_i e^{a_i - \pi_i \hat{x}_i} = \frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i + \pi_j}\tau_i e^{a_i - \frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{1+\frac{\pi_j}{\pi_i}}} =$$

$$= \frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i + \pi_j}\tau_i e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}} \cdot \frac{1-\tau_i}{1-\tau_j}^{-\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)};$$

$$\hat{B}_i = \frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}(1-\tau_j)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{(\pi_i + \pi_j)}\left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\tau_i (1-\tau_i)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}}; \qquad (4)$$

$$\hat{B}_j = \frac{e^{a_j - \frac{\pi_i - a_i + a_j}{1+\frac{\pi_i}{\pi_j}}}(1-\tau_i)^{\left(\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)}}{(\pi_i + \pi_j)}\left(\pi_i - a_i + a_j - \ln\frac{1-\tau_i}{1-\tau_j}\right)\tau_j (1-\tau_j)^{-\frac{1}{1+\frac{\pi_i}{\pi_j}}}. \qquad (4')$$

Maximizing the total revenue of investors and the government as the objective function

Total income of investors and the government in the economy *i* (R_i) are:

$$R_i = P_i + B_i = x_i(1-\tau_i)e^{a_i - \pi_i x_i} + x_i \tau_i e^{a_i - \pi_i x_i} = x_i e^{a_i - \pi_i x_i}.$$

Respectively, in the economy *j*:

$$R_j = (1 - x_i)e^{a_j - \pi_j(1-x_i)}.$$

For the state of the investment equipment we get:

$$\hat{R}_i = \hat{x}_i\left(1-\tau_i\right)e^{a_i-\pi_i\hat{x}_i} = \frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i+\pi_j} e^{a_i - \frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{1+\frac{\pi_j}{\pi_i}}} =$$

$$= \frac{\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}}{\pi_i+\pi_j} e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}} \cdot \frac{1-\tau_i}{1-\tau_j}^{-\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)};$$

$$\hat{R}_i = \frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}}. \quad (5)$$

Since expressions (1) and (2) for economies *i* and *j* are symmetric, for the economy *j* is true

$$\hat{R}_j = \frac{e^{a_j - \frac{\pi_i - a_i + a_j}{1+\frac{\pi_i}{\pi_j}}}\left(1-\tau_i\right)^{\left(\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)}}{\left(\pi_i+\pi_j\right)}\left(\pi_i - a_i + a_j - \ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1-\tau_j\right)^{-\frac{1}{1+\frac{\pi_i}{\pi_j}}}. \quad (5')$$

Analysis of the extrema of the above functions allows us to determine their parametric maxima for τ_i and τ_j and to assess the possibility of simultaneously achieving optimal values in both economies. In the absence of such a possibility, we can simulate the alternate behavior of 2 players, aimed at maximizing their winnings at each step for different ratios of the parameters of the objective functions.

Results

Let's find the extrema of each of the objective functions for τ_i and τ_j.

Maximizing the investment volume as the objective function

Since $0 \le \tau_i < 1$ and

$$\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}>0,$$

$\ln\left(1-\tau_j\right)<0$, $1-\frac{1}{1+\frac{\pi_j}{\pi_i}}=\frac{\pi_j}{\pi_i+\pi_j}>0$, $\left(1-\tau_i\right)^{\left(1-\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}$, $\ln\left(1-\tau_i\right)$, so $\pi_j+a_i-a_j-\ln\left(1-\tau_j\right)+\ln\left(1-\tau_i\right)$ increase when τ_I decreases. Thus $\hat{P}_i$ increases when τ_i increases too. The maximum of $\hat{P}_i$ is achieved at $\tau_i=0$ and is equal to

$$\max\hat{P}_i=\hat{P}_i\Big|_{\tau_i=0}=\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\left(\pi_j+a_i-a_j-\ln\left(1-\tau_j\right)\right);$$

the maximum of $\hat{P}_j$ is achieved at $\tau_j=0$ and is equal to

$$\max\hat{P}_j=\hat{P}_j\Big|_{\tau_j=0}=\frac{e^{a_j-\frac{\pi_i-a_i+a_j}{1+\frac{\pi_i}{\pi_j}}}\left(1-\tau_i\right)^{\left(\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)}}{\left(\pi_i+\pi_j\right)}\left(\pi_i-a_i+a_j-\ln\left(1-\tau_i\right)\right).$$

In fact, these results are definitely obvious: other factors being equal, the maximum investment comes to the economy at zero tax burden.

However, the government's interest is not only in increasing investments but also in tax revenues to the budget.

Maximizing the budget revenue as the objective function

The objective functions for 2 economies like (4) & (4’). Let’s find the derivatives of $\hat{B}_i$ and $\hat{B}_j$ (see Appendix A):

$$\frac{d\hat{B}_i}{d\tau_i} = \frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}}{\left(\pi_i + \pi_j\right)} \left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)} \left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)} \times$$

$$\times \left(\left(\pi_j + a_i - a_j + \ln\left(1-\tau_i\right) - \ln\left(1-\tau_j\right)\right) \frac{1}{1+\frac{\pi_i}{\pi_j}} - 1 \right) \tau_i; \tag{6}$$

$$\frac{d\hat{B}_j}{d\tau_j} = \frac{e^{a_j - \frac{\pi_i - a_i + a_j}{1+\frac{\pi_i}{\pi_j}}}}{\pi_i + \pi_j} \left(1-\tau_i\right)^{\left(\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)} \left(1-\tau_j\right)^{-\left(1+\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)} \times$$

$$\times \left(\left(\pi_i - a_i + a_j - \ln\left(1-\tau_i\right) + \ln\left(1-\tau_j\right)\right) \frac{1}{1+\frac{\pi_j}{\pi_i}} - 1 \right) \tau_j. \tag{6'}$$

It can be shown that (see Appendix A), that

$$\frac{d\hat{B}_i}{d\tau_i} = 0 \Leftrightarrow \tau_j = 1 - \left(1-\hat{\tau}_i\right) e^{\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1}, \tag{7}$$

respectively,

$$\frac{d\hat{B}_j}{d\tau_j} = 0 \Leftrightarrow \tau_i = 1 - \left(1-\hat{\tau}_j\right) e^{\pi_i - a_i + a_j - \frac{\pi_i}{\pi_j} - 1} \tag{7'}$$

The calculation of the second-order derivatives (see Appendix A) indicates the obtained extrema are maxima.

Maximizing the total revenue of investors and the government as the objective function

The objective functions for 2 economies like 5) & (5’). Let's find the derivatives of $\hat{R}_i$ and $\hat{R}_j$ (see Appendix A):

$$\frac{d\hat{R}_i}{d\tau_i} = -\frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}}{\pi_i + \pi_j}\left(1-\tau_j\right)^{\frac{1}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)} \times \\ \times\left(1-\left(\pi_j + a_i - a_j + \ln\left(1-\tau_i\right) - \ln\left(1-\tau_j\right)\right)\frac{1}{1+\frac{\pi_j}{\pi_i}}\right). \quad (8)$$

$$\frac{d\hat{R}_i}{d\tau_i} = 0 \Leftrightarrow 1 - \left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\frac{1}{1+\frac{\pi_j}{\pi_i}} = 0;$$

It can be shown that (see Appendix A), that

$$\frac{d\hat{R}_i}{d\tau_i} = 0 \Leftrightarrow \tau_j = 1 - \left(1-\hat{\tau}_i\right)e^{\pi_j - \frac{\pi_j}{\pi_i} + a_i - a_j - 1}, \quad (9)$$

respectively,

$$\frac{d\hat{R}_j}{d\tau_j} = -\frac{e^{a_j - \frac{\pi_i - a_i + a_j}{1+\frac{\pi_i}{\pi_j}}}}{\pi_i + \pi_j}\left(1-\tau_i\right)^{\frac{1}{1+\frac{\pi_i}{\pi_j}}}\left(1-\tau_j\right)^{-\left(1+\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)} \times \\ \times\left(1-\left(\pi_i - a_i + a_j - \ln\left(1-\tau_i\right) + \ln\left(1-\tau_j\right)\right)\frac{1}{1+\frac{\pi_i}{\pi_j}}\right); \quad (8')$$

$$\frac{d\hat{R}_j}{d\tau_j} = 0 \Leftrightarrow \tau_i = 1 - \left(1-\hat{\tau}_j\right)e^{\pi_i - \frac{\pi_i}{\pi_j} - a_i + a_j - 1}. \quad (9')$$

The calculation of the second-order derivatives (see Appendix A) indicates the obtained extrema are maxima.

The conditions (9) & (9’) are completely the same as (7) & (7’). So, the conditions of maxima are the same for maximizing the budget revenue and maximizing the budget revenue & the investment volume as the objective functions. Then below we refer only to (7) & (7'), though the analysis and conclusions hold for both these cases.

Verification for the possibility of simultaneous achievement of the optimal tax rate by both players

Actually, it means verification of the presence of tax rates pair $\left(\tilde{\tau}_i;\tilde{\tau}_j\right)$ that are optimal, respectively, for economies *i* and *j*, i.e., the equations:

$$\hat{\tau}_j = 1-\left(1-\hat{\tau}_i\right)e^{\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1};$$

$$\hat{\tau}_i = 1-\left(1-\hat{\tau}_j\right)e^{\pi_i - a_i + a_j - \frac{\pi_i}{\pi_j} - 1}$$

are correct. Let's check if it's possible a common maximum $\left(\tilde{\tau}_i;\tilde{\tau}_j\right)$. (7) & (7') implies:

$$\frac{d\hat{B}_i}{d\tau_i} = 0 \wedge \frac{d\hat{B}_j}{d\tau_j} = 0 \Leftrightarrow$$
$$\Leftrightarrow \left(\tau_i, \tau_j = 1\right) \vee$$
$$\vee \left(\tau_i, \tau_j < 1\right) \wedge \left(\ln\frac{1-\tau_j}{1-\tau_i} = \pi_j + a_i - a_j - \frac{\pi_i}{\pi_j} - 1\right) \wedge \left(\ln\frac{1-\tau_i}{1-\tau_j} = \pi_i - a_i + a_j - \frac{\pi_j}{\pi_i} - 1\right);$$

$$\frac{d\hat{B}_i}{d\tau_i} = 0 \wedge \frac{d\hat{B}_j}{d\tau_j} = 0 \Leftrightarrow \pi_j + a_i - a_j - \frac{\pi_i}{\pi_j} - 1 = -\pi_i + a_i - a_j + \frac{\pi_j}{\pi_i} + 1;$$

$$\frac{d\hat{B}_i}{d\tau_i} = 0 \wedge \frac{d\hat{B}_j}{d\tau_j} = 0 \Leftrightarrow \left(\tau_i, \tau_j = 1\right) \vee \left(\tau_i, \tau_j < 1\right) \wedge \left(\pi_i + \pi_j = \frac{\pi_i}{\pi_j} + \frac{\pi_j}{\pi_i} + 2\right). \quad (10)$$

In economic view the case $\tau_i = \tau_j = 1$ is almost impossible. If τ_i, $\tau_j < 1$, then given (8), simultaneous achieving of the optimum for both economies does not depend on tax rate and is possible only for certain values of mutually independent parameters π_i and π_j, which are not directly regulated by the government. The parameters of the functioning of any economy are volatile, so stable fulfillment of the condition (10) is almost unattainable. It means that the simultaneous achievement of the optimal tax rate by both economies is almost unrealistic.

The dependences of the optimal values τ_i on τ_j and τ_j on τ_i are linear (see (7) & (7')). Their properties are as follows:

- ✓ the cross point for 2 graphs is *(1; 1)*;
- ✓ thus, there's no extremum for 2 players at one time;

Let's analyze the graphs of parametric optima for τ_i and τ_j, that illustrate the equations (7) & (7'). Based (7) & (7') can be written:

$$\tau_j = 1-\left(1-\hat{\tau}_i\right)e^{\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1} = 1 - e^{\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1} + \hat{\tau}_i e^{\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1}; \qquad (11)$$

$$\hat{\tau}_j = 1-\left(1-\tau_i\right)e^{-\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1} = 1 - e^{-\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1} + \tau_i e^{-\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1}. \qquad (12)$$

The mutual location of the corresponding lines $\tau_j(\tau_i)$ is determined by the ratio of parameters π_i and π_j. The line for parametric optimal values of τ_i , is above the line for parametric optimal values of τ_j, if the coefficient at $\hat{\tau}_i$ in (11) is less than the coefficient at τ_i in (12), i.e.,

$$e^{\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1} < e^{-\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1};$$

$$\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1 < -\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1;$$

$$\pi_i + \pi_j < \frac{\pi_j}{\pi_i} + \frac{\pi_i}{\pi_j} + 2;$$

$$\pi_i, \pi_j > 0 \Rightarrow$$

$$\left(\pi_i + \pi_j\right)\pi_i\pi_j < \pi_i^2 + \pi_j^2 + 2\pi_i\pi_j = \left(\pi_i + \pi_j\right)^2;$$

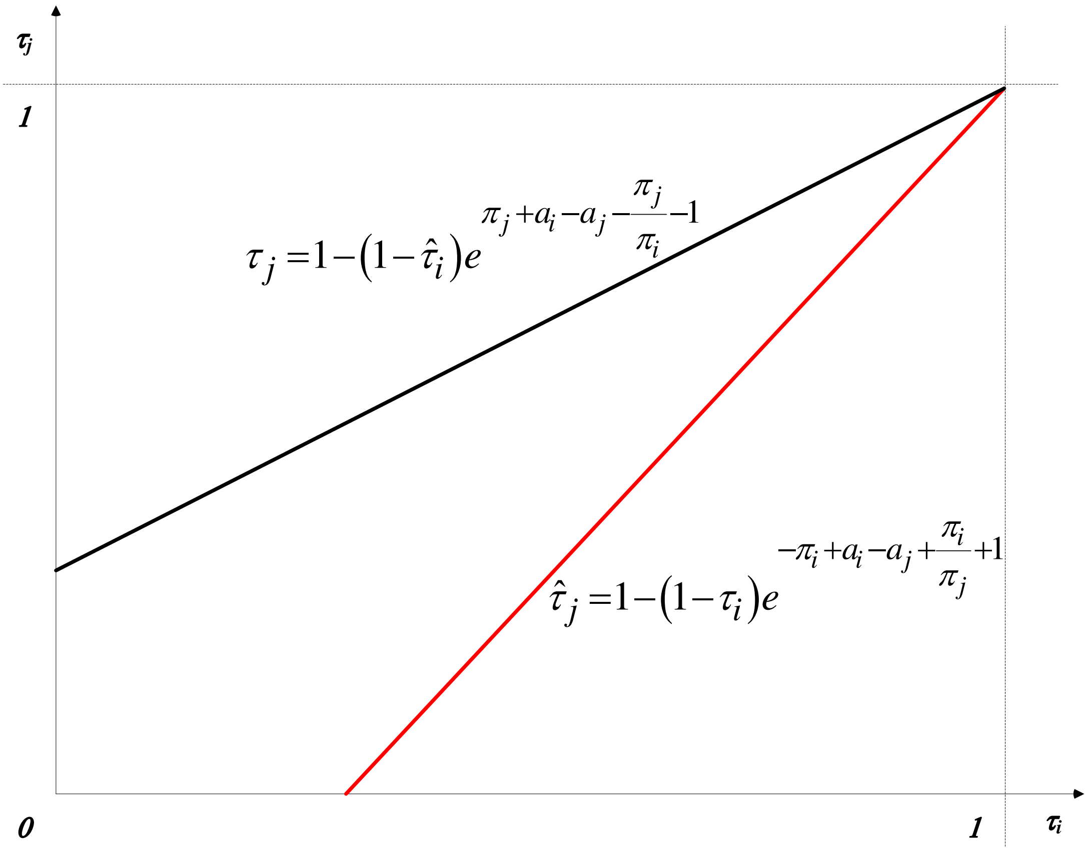
$$\pi_i\pi_j < \pi_i + \pi_j;$$

$$\frac{1}{\pi_i} + \frac{1}{\pi_j} > 1 \qquad (13)$$

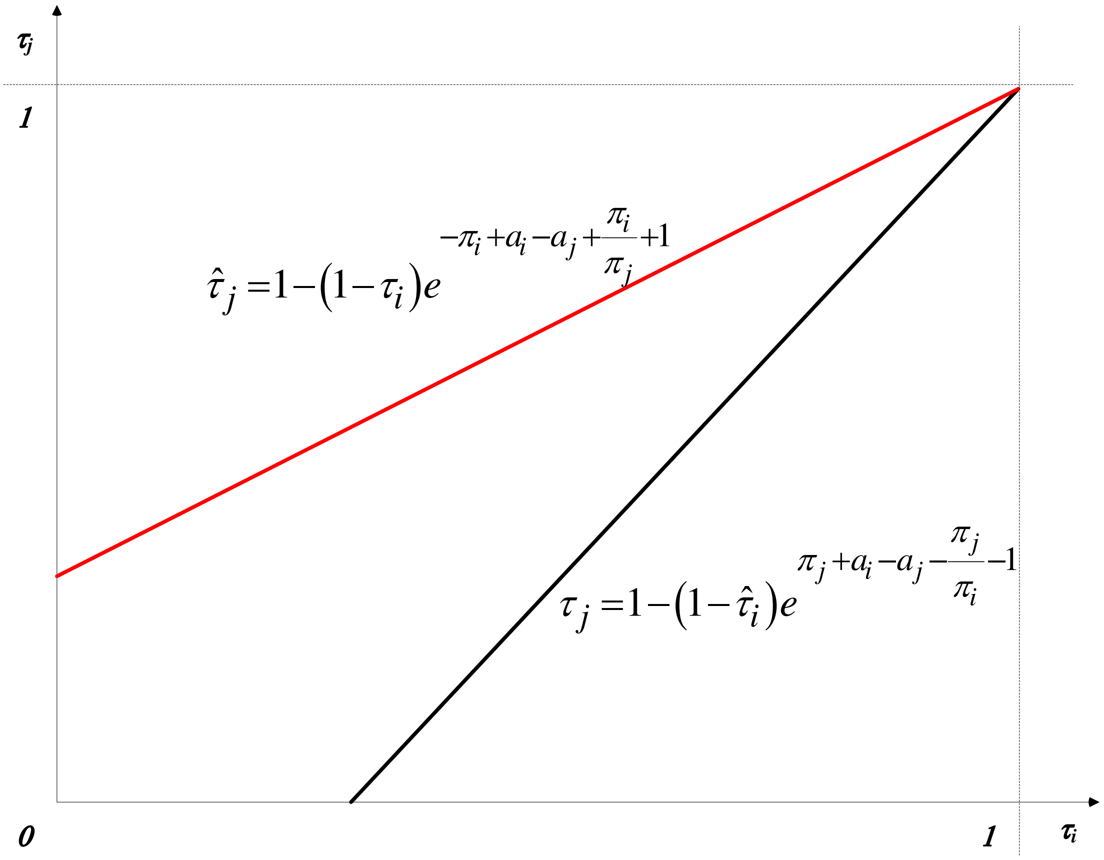
So,

- ✓ if $\frac{1}{\pi_i}+\frac{1}{\pi_j}>1$, the line for parametric optimal values of τ_i, is above the line for parametric optimal values of τ_j (fig. 1a);
- ✓ if $\frac{1}{\pi_i}+\frac{1}{\pi_j}<1$, the line for parametric optimal values of τ_i, is under the line for parametric optimal values of τ_j (fig. 1b).

Fig. 1. The lines of parametric optima for τ_i and τ_j



a) $\frac{1}{\pi_i}+\frac{1}{\pi_j}>1$



b) $\frac{1}{\pi_i} + \frac{1}{\pi_j} < 1$

It should be noted the symmetry of the above formulas, i.e., it does not matter in which of the economies the return falls faster, but there's important the sum of inverse values of returns.

For each of the 2 cases the lines of parametric optimums can intersect *X*-axis at points greater than *0*, less than *0*, or either one of them at a point greater than *0* and other one at a point less than *0*.

Since, in fact, both players cannot reach the optimum tax rate at the same time, thus it's relevant to consider step-by-step alternate player's behaviour for optimizing their own tax burden.

The scenario of players' interaction is aimed at optimizing tax rate is as follows:

Step 0: The player *i* sets an arbitrary value of his tax rate (τ_i^0);

Step 1: The player *j* calculates his tax rate value (τ_j^1) as the optimum relative to (τ_i^0);

Step 2: The player i calculates his tax rate value (τ_i^1) as the optimum relative to (τ_j^1), etc.

Based (11) & (12) we have:

$$\tau_j^1 = 1 - \left(1 - \tau_i^0\right) e^{\pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1};$$

$$\tau_i^1 = 1 - \left(1 - \tau_j^1\right) e^{\pi_i - a_i + a_j - \frac{\pi_i}{\pi_j} - 1};$$

$$\tau_i^1 = 1 - \left(1 - \tau_i^0\right) e^{\pi_i + \pi_j - \left(\frac{\pi_i}{\pi_j} + \frac{\pi_j}{\pi_i} + 2\right)}. \tag{14}$$

The condition of decreasing or increasing the optimal tax rate in the process of step-by-step optimization behavior is as follows:

$$\tau_i^1 > \tau_i^0 \Leftrightarrow 1 - \tau_i^1 < 1 - \tau_i^0 \Leftrightarrow \frac{1 - \tau_i^1}{1 - \tau_i^0} < 1 \Leftrightarrow$$

$$\Leftrightarrow e^{\pi_i + \pi_j - \left(\frac{\pi_i}{\pi_j} + \frac{\pi_j}{\pi_i} + 2\right)} < 1 \Leftrightarrow \pi_i + \pi_j - \left(\frac{\pi_i}{\pi_j} + \frac{\pi_j}{\pi_i} + 2\right) < 0 \Leftrightarrow$$

$$\Leftrightarrow \pi_i + \pi_j < \left(\frac{\pi_i}{\pi_j} + \frac{\pi_j}{\pi_i} + 2\right);$$

$$\tau_i^1 > \tau_i^0 \Leftrightarrow \frac{1}{\pi_i} + \frac{1}{\pi j} > 1. \tag{15}$$

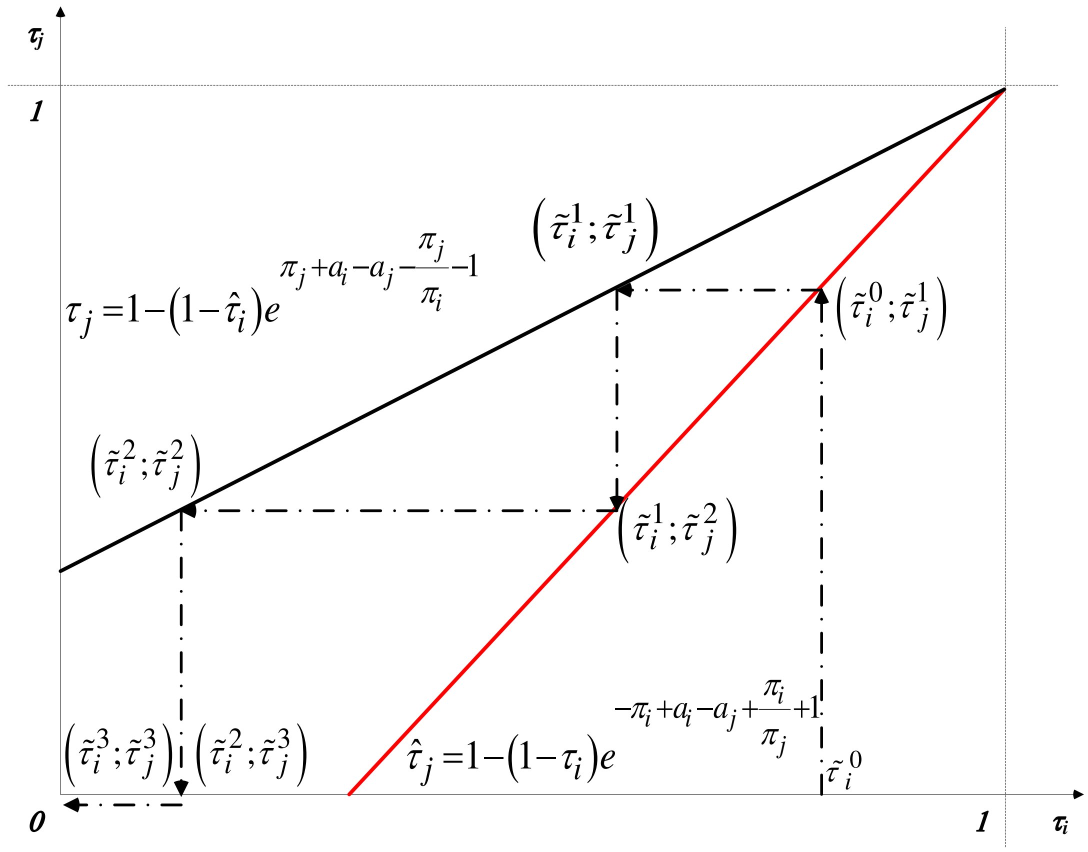
Respectively, if $\tau_i^1 < \tau_i^0$, then $\frac{1}{\pi_i} + \frac{1}{\pi_j} < 1$.

Comparing with (13), it can conclude that

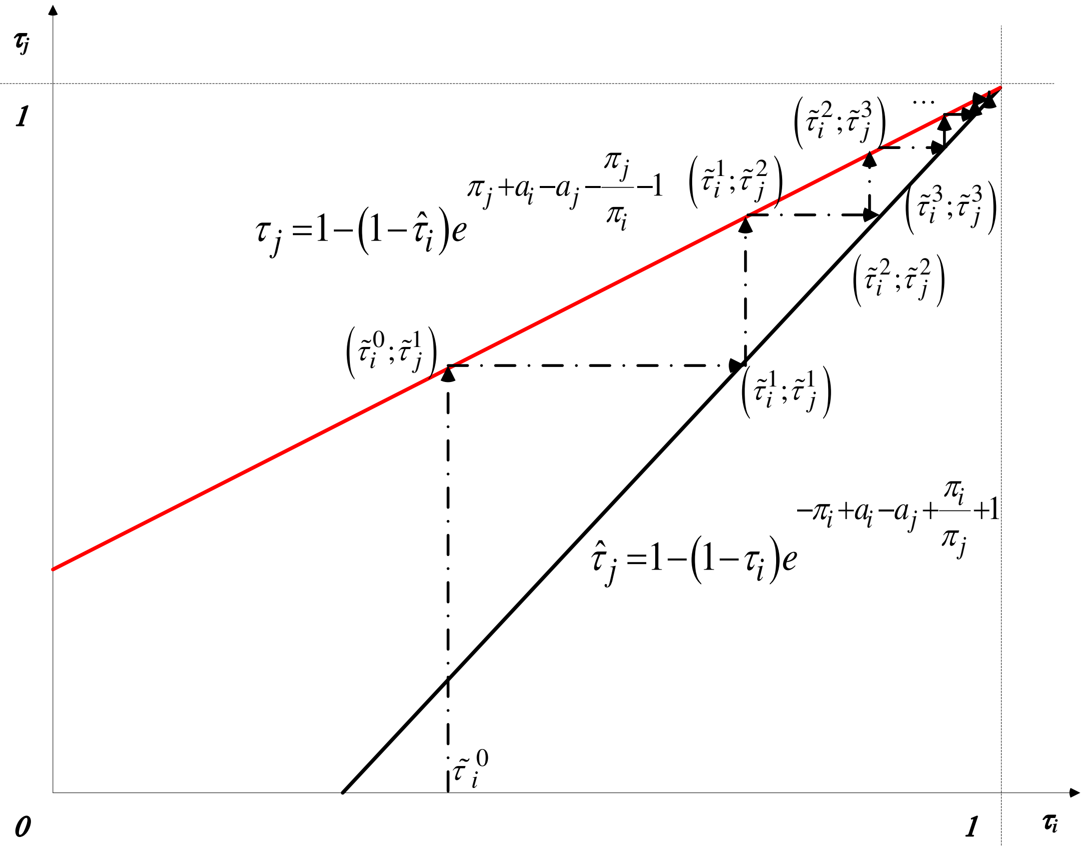
- ✓ if $\frac{1}{\pi_i} + \frac{1}{\pi_j} < 1$ and the line for parametric optimal values of τ_i is under the line for for parametric optimal values of τ_j, above government's behaviour leads to monotonic of their economy's tax rate (Fig. 2a).

✓ if $\frac{1}{\pi_i}+\frac{1}{\pi_j}>1$ and the line for parametric optimal values of τ_i is over the line for for parametric optimal values of τ_j, above government's behaviour leads to monotonic increasing of their economy's tax rate (Fig. 2b).

Fig. 2. Step-by-step alternate players' behaviour for optimizing their own tax burden



a) $\frac{1}{\pi_i}+\frac{1}{\pi_j}<1$



b) $\frac{1}{\pi_i} + \frac{1}{\pi_j} > 1$

2 variants of players' behaviour, depending on location of optimal lines for τ_i and τ_j: as it can see from fig. 2b, such acts lead or to a fixed decreasing tax rates until the tax rate of one of them becomes zero, or to fixed increasing tax rates until the rates of both players converge in the point *(1; 1)*.

As a result, there're possible such cases of the mutual location the maximum lines for τ_i and τ_j:

- ✓ $\frac{1}{\pi_i} + \frac{1}{\pi_j} > 1$, the optimizing of players' step-by-step strategy leads to increasing tax rates in both economies;
- ✓ $\frac{1}{\pi_i} + \frac{1}{\pi_j} < 1$, the optimizing of players' step-by-step strategy leads to decreasing tax rates in both economies, moreover there're possible 3 cases of the mutual location the maximum lines for τ_i and τ_j:

- ✓ both lines for parametric optimal values of τ_j and τ_j intersect *Y*-axis at points greater than *0*;
- ✓ the line for parametric optimal values of τ_j intersects *Y*-axis at points greater than *0*, and the line for parametric optimal values of τ_i, intersect *Y*-axis at points less than *0*;
- ✓ both lines for parametric optimal values of τ_i and τ_j intersect *Y*-axis at points less than *0*.

The abscissas of intersection points *X*-axis the lines for parametric optima of τ_i and τ_j are determined by equations:

$$1=\left(1-\hat{\tau}_i\right)e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1};$$

$$\hat{\tau}_i=1-e^{-\pi_j-a_i+a_j+\frac{\pi_j}{\pi_i}+1}; \qquad (16)$$

$$1=\left(1-\tau_i\right)e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1};$$

$$\tau_i=1-e^{\pi_i-a_i+a_j-\frac{\pi_i}{\pi_j}-1}. \qquad (17)$$

For decreasing trend of tax rates τ_j from (11) is less than τ_j from (12), and the coefficient on τ_i, on the contrary, is more. I.e.,

$$e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1}>e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1};$$

$$\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1>-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1;;$$

$$\pi_i+\pi_j>\frac{\pi_i}{\pi_j}+\frac{\pi_j}{\pi_i}+2.$$

3 above cases are formally described by such inequations:

$$1-e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1}<1-e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1}<0;$$

$$1-e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1}<0<1-e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1};$$

$$0<1-e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1}<1-e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1};$$

$$1<e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1}<e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1};$$

$$e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1}<1<e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1};$$

$$e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1}<e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1}<1.$$

or what the same

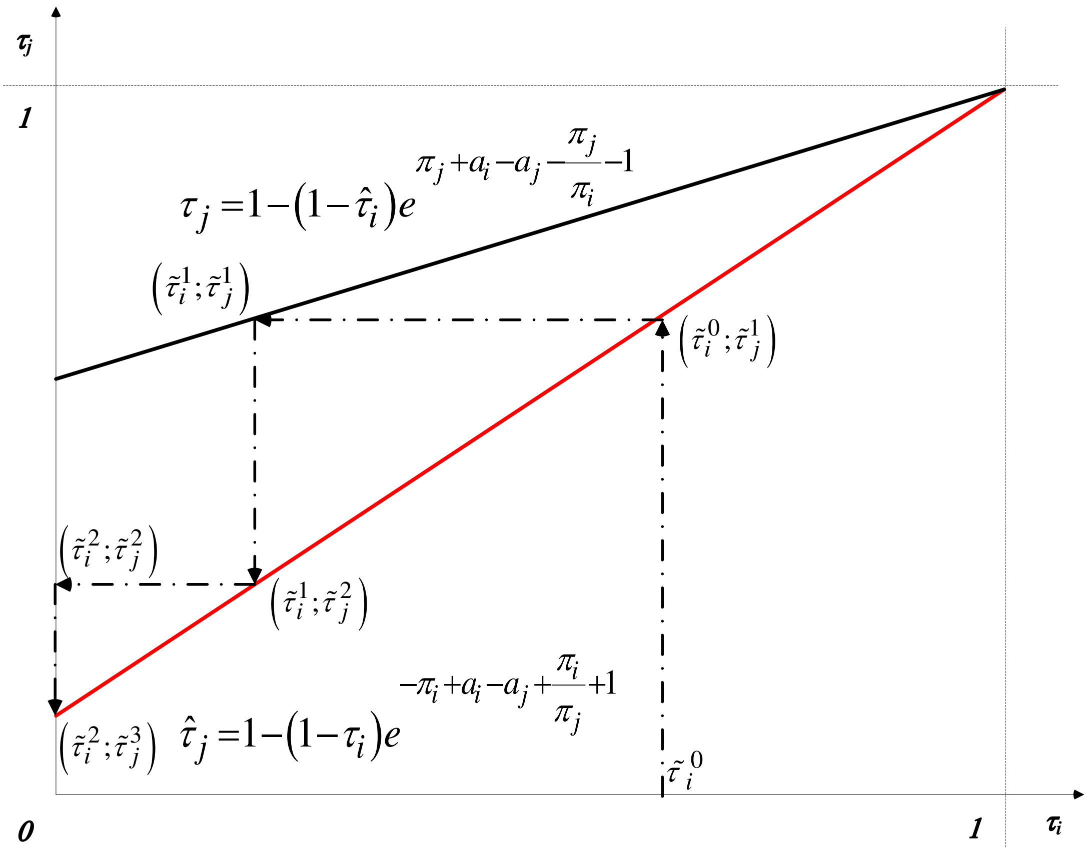
$$0<-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1<\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1;$$

$$-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1<0<\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1;$$

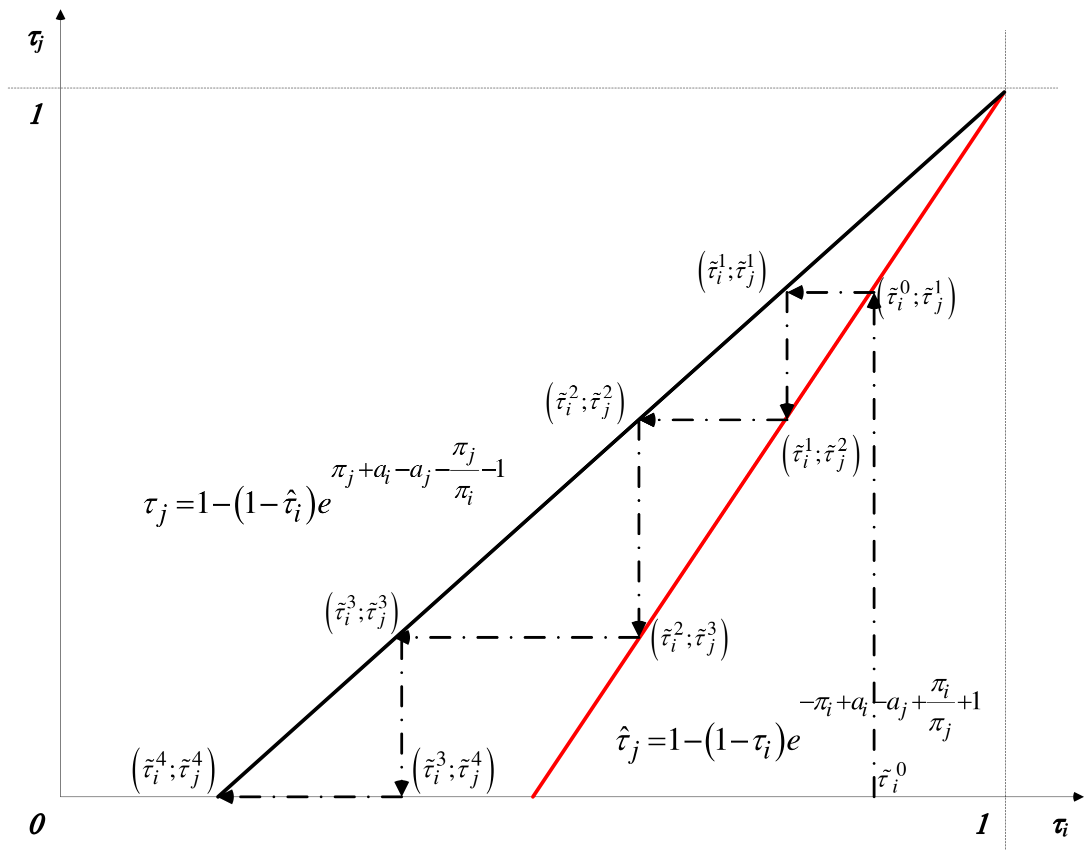
$$-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1<\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1<0$$

In case 2, the decreasing trend eventually leads both economies to zero tax rates (see fig. 2a). In case 1, the sequence of players' actions definitely leads to such limit tax rates' values as $\hat{\tau}_i=0$; $\hat{\tau}_j=1-e^{-\pi_i+a_i-a_j+\frac{\pi_i}{\pi_j}+1}$; in case 3, it leads to $\hat{\tau}_i=1-e^{-\pi_j-a_i+a_j+\frac{\pi_j}{\pi_i}+1}$; $\hat{\tau}_j=0$ (fig. 3a, b).

Fig. 3. The lines of parametric optimums for τ_i and τ_j



a) $0 < -\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1 < \pi_j + a_i - a_j - \frac{\pi_j}{\pi_i} - 1$



b) $-\pi_i + a_i - a_j + \dfrac{\pi_i}{\pi_j} + 1 < \pi_j + a_i - a_j - \dfrac{\pi_j}{\pi_i} - 1 < 0$

Discussion

The obtained results mark some main peculiar features of the model of tax competition of 2 economies.

We consider 3 objective functions: maximizing the investment, maximizing the budget revenue and maximizing the total revenue of investors and the government. Clearly, they're based on 2 factors: the investment revenue and the tax budget revenue: the first function maximizes the first factor, the second function maximizes the second factor, and the third one maximizes the sum of the first and second factors. The results for the first function are trivial, and the results for the second and third functions, despite some differences in the form of these objective functions, are completely identical. It allows for the conclusion that the main factor determining government tax behavior, is maximization the budget revenue from investments.

Moreover, we observe the following properties of government tax behavior under competitive conditions:

- ✓ in fact, both players cannot reach the optimum tax rate on the interval (0; 1) at the same time;
- ✓ on step-by-step alternate optimization, there are 2 trends: decreasing and increasing of tax rates;
- ✓ the trend direction is dependent on the mutual location of optimal lines of 2 players;
- ✓ in turn, the mutual location of optimal lines is not dependent on individual parameters, but only on their composition.

One of the questions that this study had to answer is if tax competition of economies is described by Prisoner's dilemma model?

The above model of the successive optimization tax rates of 2 economies differs from the model of Prisoner's dilemma by 2 key points.

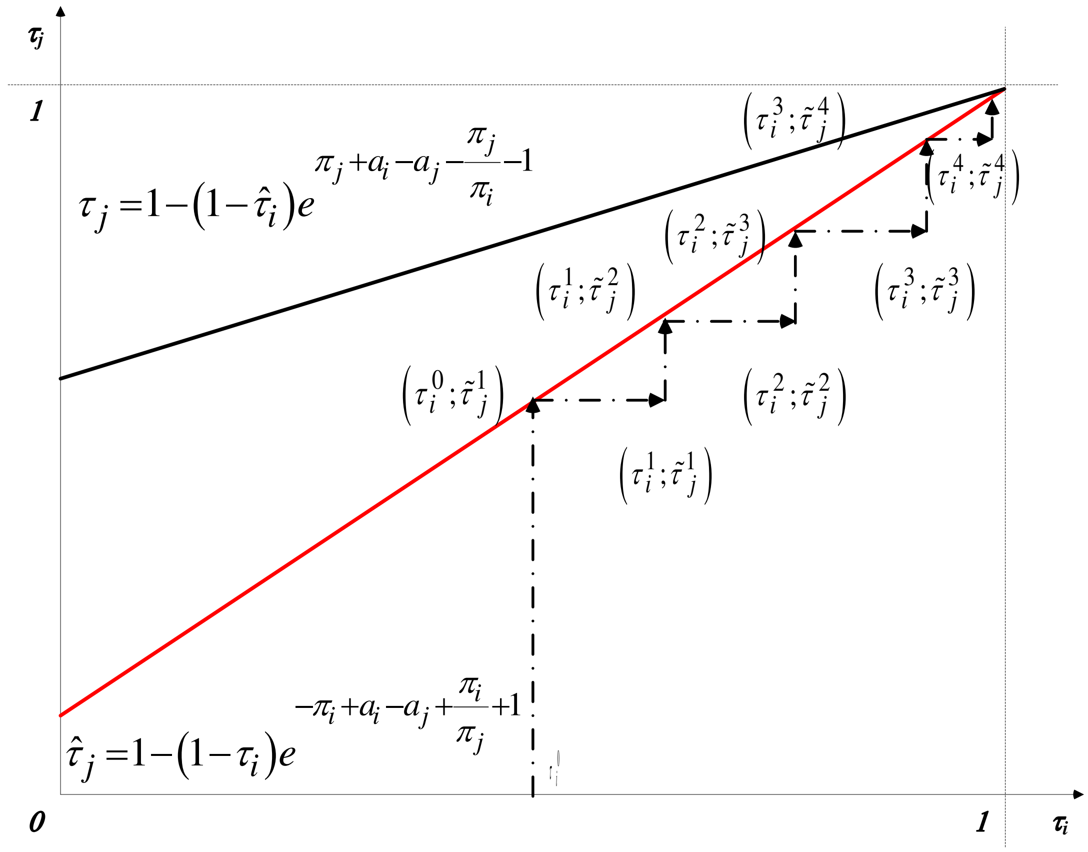
Firstly, depending on parameters of interaction the step-by-step optimization of tax rates can lead not only to minimize but to maximize ones, while tax competition like Prisoner's dilemma leads only to decreasing tax rates.

Second, in contrast to Prisoner's dilemma, a trend change is possible not only due to concerted acts (cooperation) of players but by the efforts of one of them this way. If one of the players sets the tax rate above the optimal value, while the other optimizes his tax rate, it reverses the trend towards a step-by-step increase in tax rates. I.e., there's also the possibility of an uncoordinated behavior of one of the players, that predisposes the other to decrease or increase his tax rate.

Really, based (12) $e^{-\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1} > 0$, i.e. if $\tau_i^{k+1} > \tau_i^k$ then $\tau_i^{k+1} e^{-\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1} > \tau_i^k e^{-\pi_i + a_i - a_j + \frac{\pi_i}{\pi_j} + 1}$, so $\hat{\tau}_j^{k+1} > \hat{\tau}_j^k$ (fig. 4).

Fig. 4. The step-by-step alternative to players' behavior using non-incentivizing strategy by one player

Fig. 4. The step-by-step alternative to players' behavior using non-incentivizing strategy by one player



So, the above model of an interaction of the 2 economies let reach the trend change by the efforts of one of the interacting parties, provided that the other one tries to act optimally at each step. I.e., if one of the players uses maximizing strategy, the other player can change the trend direct without its contractor's consent.

At the same time, each of the two players is available to change the trend only in one direction: one of them – from decreasing tax rate to increasing one; the other, conversely, from increasing tax rate to decreasing.

So, we can state the competition of 2 (or more) economies does not always take the form of Race to the bottom. However, even in the case of Race to the bottom synchronous decreasing tax rates do not necessarily lead to their mutual zeroing. Put it differently Race to the bottom is a particular case of competition of economies, and Prisoner's dilemma is a particular case of Race to the bottom.

Also note that the distribution of the above roles of players in no way depends on the efficiency of economies, but only on the sum of the parameters of the economic environment of 2 countries. It should be noted the symmetry of the above formulas, i.e., it does not matter, the profitability of which of the economies falls more quickly, the total value of the inverse values is important.

A lot of researches have shown the decisions of economies' governments do not always fit into the framework of classical maximizing behavior. In particular, it applies to researches on the issue of, can we classify the behavior of countries (and which exactly) as Race to the bottom, or the issue of the availability and character of the relationship between macroeconomic indicators and the dynamics of tax rates (Sokolovskiy, 2018; Sokolovskiy, 2019).

The above properties of the optimization government behavior in the interaction of economies show it does not necessarily fit into the framework of Race to the bottom. Governments can locally use non-optimal solutions to solving the strategic maximization task too. So, some non-optimization steps of a subject do not always indicate his in general non-optimization behavior. The selection of additional factors for such an analysis needs additional research.

Conclusion

We researched the issue of tax rate optimization competition of two economies for better investment conditions. In this context, we studied questions and hypotheses if it is necessarily such a competition is Race to the bottom and if it is modeled by Prisoner's dilemma.

1. On researching the issue of the optimal tax rate, we consider 3 the objective functions by that the government is ruled in its behavior:

- ✓ maximizing the investment volume,
- ✓ maximizing the budget revenue, and
- ✓ maximizing the budget revenue and the investment volume.

2. The optimal tax behaviour for the objective function "Maximizing the investment volume" is clearly.

The objective functions for both economies reach their maximums at non-zero tax rates, which is quite clear: all other things being equal, maximal investment comes to the economy at zero tax burden.

However, the government is interested not only in increasing investments but also in tax revenues.

3. For the objective function "maximizing the budget revenue" and "maximizing the budget revenue and the investment volume"

- ✓ there's exactly one extreme point in the interval of tax rate (0; 1) for each of the economies, that's depended on the current tax rate in another economy $\hat{\tau}_i\left(\tau_j\right)$ and $\hat{\tau}_j\left(\tau_i\right)$, and this extremum is the maximum;
- ✓ simultaneous achieving of the optimum for both economies is impossible in fact, because it formally depends on the changeable unregulated parameters of the economic environment, i.e. is unstable and unmanageable.

4. For both objective functions: "maximizing the budget revenue" and "maximizing the budget revenue and the investment volume" the step-by-step alternate behavior, when each player optimizes his tax rate, based tax rate of the other player, can lead to both maximally possible decrease tax rate in both economies and maximal increase ones. Moreover, the trend direction depends on the ratio of the graphs of the optimal values of tax rate both economies.

5. Depending on the parameters of the economy tax rate decreasing trend (for both above objective functions) can lead to 3 limit pairs: $\left(\hat{\tau}_i = 0;\ \hat{\tau}_j > 0\right)$, $\left(\hat{\tau}_i = 0;\ \hat{\tau}_j = 0\right)$ and $\left(\hat{\tau}_i > 0;\ \hat{\tau}_j = 0\right)$. The tax rate increasing trend leads to a unique limit pair $\left(\hat{\tau}_i = 1;\ \hat{\tau}_j = 1\right)$.

6. Competition between economies for investment attractiveness by setting optimal tax rate is not the model of Prisoner's dilemma and differs from it by two factors:

- ✓ under certain conditions, the best strategies for governments are not decreasing but increasing tax rate;

- ✓ player cooperation is not critical to achieving optimal values of objective functions; to reverse the trend there's enough purposeful non-optimization behavior of one of the players on condition of optimization behavior of the other;
- ✓ moreover, one of the players can change the tax rate decreasing trend to increasing one; another player, conversely, can change the tax rate increasing trend to decreasing one.

So, Race to the bottom is a special case of competition of economies, and Prisoner’s dilemma is a special case of Race to the bottom.

7. Above model shows the governments can consciously make tactical non-optimal decisions about current tax rates to cause a reversal of the trend. So, more powerful economies have more opportunities to vary their tax rates. The logic of such actions, the reaction to them, and the possible consequences and results can be the subject of future studies.

Also needs to be studied issue of why even when applying Race to the bottom in fact governments do not reach the limit and do not decrease tax rate to 0. I.e. what’re the reasons and factors stopping the rally. The same is true about tax rate increasing trend because, clearly, no country sets tax rate at 100%.

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Appendix A

Algebraic transformations

The objective function – the budget revenue

$$\frac{d\hat{B}_i}{d\tau_i} = -\frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i + \pi_j\right)} \cdot \left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}} \times$$

$$\times\left(\left(\frac{\tau_i}{1-\tau_i}\right) - \left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1 + \frac{1}{1+\frac{\pi_j}{\pi_i}}\frac{\tau_i}{1-\tau_i}\right)\right);$$

$$\frac{d\hat{B}_i}{d\tau_i} = \frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i + \pi_j\right)} \times$$

$$\times\left(-\frac{1}{1-\tau_i}\right)\tau_i\left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}} + \left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}} +$$

$$+\frac{1}{1+\frac{\pi_j}{\pi_i}}\left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\tau_i\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)};$$

$$\frac{d\hat{B}_i}{d\tau_i} = \frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}} \left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i + \pi_j\right)} \cdot \left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}} \times$$

$$\times\left(-\frac{\tau_i}{1-\tau_i} + \left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right) + \frac{1}{1+\frac{\pi_j}{\pi_i}}\left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\frac{\tau_i}{1-\tau_i} \right);$$

$$\frac{d\hat{B}_i}{d\tau_i} = 0 \Leftrightarrow -\frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}} \left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i + \pi_j\right)} \cdot \left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}} \times$$

$$\times\left(\left(\frac{\tau_i}{1-\tau_i}\right) - \left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1 + \frac{1}{1+\frac{\pi_j}{\pi_i}}\frac{\tau_i}{1-\tau_i}\right)\right) = 0;$$

$$\frac{d\hat{B}_i}{d\tau_i} = 0 \Leftrightarrow -\frac{e^{a_i - \frac{\pi_j + a_i - a_j}{1+\frac{\pi_j}{\pi_i}}} \left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i + \pi_j\right)} \cdot \left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}} \times$$

$$\times\left(\left(\frac{\tau_i}{1-\tau_i}\right) - \left(\pi_j + a_i - a_j + \ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1 + \frac{1}{1+\frac{\pi_j}{\pi_i}}\frac{\tau_i}{1-\tau_i}\right)\right) = 0;$$

Respectively,

$$\frac{d\hat{B}_j}{d\tau_j}=0\Leftrightarrow -\frac{e^{a_j-\frac{\pi_i-a_i+a_j}{1+\frac{\pi_i}{\pi_j}}}\left(1-\tau_i\right)^{\left(\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)}}{\left(\pi_i+\pi_j\right)}\cdot\left(1-\tau_j\right)^{-\frac{1}{1+\frac{\pi_i}{\pi_j}}}\times$$

$$\times\left(\left(\frac{\tau_j}{1-\tau_j}\right)-\left(\pi_i-a_i+a_j-\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1+\frac{1}{1+\frac{\pi_i}{\pi_j}}\frac{\tau_j}{1-\tau_j}\right)\right)=0;$$

since

$$-\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}}{\left(\pi_i+\pi_j\right)}<0;\ -\frac{e^{a_j-\frac{\pi_i-a_i+a_j}{1+\frac{\pi_i}{\pi_j}}}}{\left(\pi_i+\pi_j\right)}<0;$$

$$\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}>0\ \ \forall\tau_j:0\le\tau_j<1;\ \left(1-\tau_i\right)^{\left(\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)}>0\ \ \forall\tau_i:0\le\tau_i<1;$$

$$\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}>0\ \ \forall\tau_i:0\le\tau_i<1;\ \left(1-\tau_j\right)^{-\left(1+\frac{1}{1+\frac{\pi_i}{\pi_j}}\right)}>0\ \ \forall\tau_j:0\le\tau_j<1$$

then

$$\frac{d\hat{B}_i}{d\tau_i}=0\Leftrightarrow\left(\frac{\tau_i}{1-\tau_i}\right)-\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\frac{\tau_i}{1-\tau_i}\right)=0;$$

$$\frac{d\hat{B}_j}{d\tau_j}=0\Leftrightarrow\left(\frac{\tau_j}{1-\tau_j}\right)-\left(\pi_i-a_i+a_j-\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1+\frac{1}{1+\frac{\pi_i}{\pi_j}}\frac{\tau_j}{1-\tau_j}\right)=0.$$

$$\frac{\tau_i}{1-\tau_i}-\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\frac{\tau_i}{1-\tau_i}\right)=0;$$

$$\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\frac{\tau_i}{1-\tau_i}\right)=\frac{\tau_i}{1-\tau_i};$$

$$\left(\pi_j+a_i-a_j-\ln\left(1-\tau_j\right)+\ln\left(1-\tau_i\right)\right)\left(\frac{1-\tau_i}{\tau_i}+\frac{\pi_i}{\pi_i+\pi_j}\right)=1;$$

$$\left(\pi_j+a_i-a_j-\ln\left(1-\tau_j\right)+\ln\left(1-\tau_i\right)\right)\left(\frac{\pi_i+\pi_j-\tau_i\pi_j}{\left(\pi_i+\pi_j\right)\tau_i}\right)=1;$$

$$\pi_j+a_i-a_j-\ln\left(1-\tau_j\right)+\ln\left(1-\tau_i\right)=\frac{\left(\pi_i+\pi_j\right)\tau_i}{\pi_i+\pi_j-\tau_i\pi_j};$$

$$\ln\left(1-\tau_i\right)=\frac{\left(\pi_i+\pi_j\right)\tau_i}{\pi_i+\pi_j-\tau_i\pi_j}+\ln\left(1-\tau_j\right)-\pi_j-a_i+a_j;$$

$$\ln\left(1-\tilde{\tau}_i\right)-\frac{1}{\frac{1}{\tilde{\tau}_i}-\frac{1}{1+\frac{\pi_i}{\pi_j}}}=\ln\left(1-\tau_j\right)-\pi_j-a_i+a_j.$$

The objective function – the budget revenue + the investment volume

$$\frac{d\hat{R}_i}{d\tau_i}=\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\times$$

$$\times\frac{d}{d\tau_i}\left(\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}}\right);$$

$$\frac{d\hat{R}_i}{d\tau_i}=\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\times$$

$$\times\left(-\frac{1}{1-\tau_i}\left(1-\tau_i\right)^{-\frac{1}{1+\frac{\pi_j}{\pi_i}}}+\frac{1}{1+\frac{\pi_j}{\pi_i}}\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}\right).$$

$$\frac{d\hat{R}_i}{d\tau_i}=-\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\times$$

$$\times\left(\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}-\frac{1}{1+\frac{\pi_j}{\pi_i}}\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}\right);$$

$$\frac{d\hat{R}_i}{d\tau_i}=-\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\cdot\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}\times \tag{5}$$

$$\times\left(1-\frac{1}{1+\frac{\pi_j}{\pi_i}}\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\right);$$

$$\frac{d\hat{R}_i}{d\tau_i}=0 \Leftrightarrow -\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}\cdot\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}\times$$

$$\times\left(1-\frac{1}{1+\frac{\pi_j}{\pi_i}}\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\right)=0;$$

since

$$-\frac{e^{a_i-\frac{\pi_j+a_i-a_j}{1+\frac{\pi_j}{\pi_i}}}\left(1-\tau_j\right)^{\left(\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}}{\left(\pi_i+\pi_j\right)}<0\,;$$

$$\left(1-\tau_i\right)^{-\left(1+\frac{1}{1+\frac{\pi_j}{\pi_i}}\right)}>0 \;\; \forall\tau_i:0\le\tau_i<1$$

then

$$\frac{d\hat{R}_i}{d\tau_i}=0 \Leftrightarrow 1-\frac{1}{1+\frac{\pi_j}{\pi_i}}\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)=0.$$

$$1-\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\frac{1}{1+\frac{\pi_j}{\pi_i}}=0;$$

$$\left(\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}\right)\frac{1}{1+\frac{\pi_j}{\pi_i}}=1$$

$$\pi_j+a_i-a_j+\ln\frac{1-\tau_i}{1-\tau_j}=1+\frac{\pi_j}{\pi_i};$$

$$\ln\frac{1-\tau_i}{1-\tau_j}=-\pi_j-a_i+a_j+\frac{\pi_j}{\pi_i}+1;$$

$$\tau_j=1-(1-\tau_i)e^{\pi_j+a_i-a_j-\frac{\pi_j}{\pi_i}-1};$$

$$\tilde{\tau}_i=1-\left(1-\tau_j\right)e^{1+\frac{\pi_j}{\pi_i}-\left(\pi_j+a_i-a_j\right)}$$

or

$$\ln\frac{1-\tilde{\tau}_i}{1-\tau_j}=1+\frac{\pi_j}{\pi_i}-\left(\pi_j+a_i-a_j\right).$$

Respectively,

$$\tilde{\tau}_j=1-(1-\tau_i)e^{1+\frac{\pi_i}{\pi_j}-\left(\pi_i-a_i+a_j\right)}$$

or

$$\ln\frac{1-\tilde{\tau}_j}{1-\tau_i}=1+\frac{\pi_i}{\pi_j}-\left(\pi_i-a_i+a_j\right)$$