Borrowing to Finance Public Investment: A Politico-economic Analysis of Fiscal Rules

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Abstract

The golden rule of public finance distinguishes public investment from consumption spending when borrowing, permitting the finance of public investment only. This study focuses on public investment in human capital and compares this rule with the balanced budget rule, which rules out debt finance, in an overlapping generations model. In the model, fiscal policy is endogenous, chosen each period by a short-lived government representing existing generations. We evaluate the government's choice and the resulting political distortions for a given fiscal rule from the long-lived planner's perspective. We find that a country with a larger preference for public consumption can minimize distortions by lowering the fraction of debt-financed public investment. We calibrate the model to a sample of OECD countries. On the one hand, we find that the golden rule of public finance in human capital is optimal and politically supported in Greece; on the other hand, the balanced budget rule (no borrowings for human capital investment) is optimal and politically supported in Germany and to a lesser extent in Japan and the United States.

- Keywords: Balanced Budget Rule; Golden Rule of Public Finance; Probabilistic Voting, Overlapping Generations
- JEL Classification: D70, E62, H63

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1 Introduction

Many developed countries have incurred large budget deficits and thereby accumulated debt in the aftermath of the global financial crisis of 2008/9 (OECD 2021). On the one hand, budget deficits enable governments to spend more than they receive in tax revenue, thereby providing citizens with higher levels of public goods and services in the short run. On the other hand, budget deficits reduce economic growth by inhibiting capital accumulation and thus, may lower the provision of public goods and services in the long run. In addition, fiscal deficits raise the issue of intergenerational equity in fiscal burdens, because they imply a transfer of the burden of public spending from the current generation to future generations.

To cope with large budget deficits and their associated problems, many developed countries have implemented various types of fiscal rules (Budina et al. (2012) ; Schaechter et al. (2012); Wyplosz (2013); Lledó et al. (2017); Caselli et al. (2018)). A well-known fiscal rule is the balanced budget rule (BB). The BB is a constitutional requirement that tax revenues must be sufficient to cover expenditures and interest payments on debt, implying zero or negative deficits (Azzimonti et al. 2016). The BB has the advantage of maintaining fiscal discipline in each year, but presents a disadvantage in terms of efficiency and intergenerational equity. In terms of efficiency, the BB does not allow for smoothing tax rates over time (Stockman 2001, 2004). In terms of intergenerational equity, the BB requires current generations to finance the entire public investment burden, even though its benefits would be received only by future generations (Bom and Ligthart 2014).

Meanwhile, the golden rule of public finance (GR) allows budget deficits to finance only public investment but not current expenditure. The GR is an alternative fiscal rule that addresses the abovementioned problems of the BB (Buiter et al. (1993); Corsetti and Roubini (1996); Robinson (1998)). Following the development of this alternative rule, many researchers conducted analysis of the GR in growth models with public capital. They showed that the GR may improve growth and welfare over time and across generations (e.g., Greiner and Semmler (2000); Ghosh and Mourmouras (2004a,b); Greiner (2008); Yakita (2008); Minea and Villieu (2009); Agénor and Yilmaz (2017); Ueshina (2018)). These studies consider tax rates and/or expenditure as given, and thus, their conclusions rely on the assumption that fiscal policy instruments are independent of changes in the fiscal rule. However, the fiscal rule has a decisive influence on governments' choice of fiscal policy (e.g., Fatás and Mihov (2006)). Therefore, considering the endogenous response of fiscal policies to the fiscal rule would provide a new perspective on the consequences of the fiscal rule for growth and welfare.

Several approaches have been attempted to investigate the effects of fiscal rules on fiscal policy formation and the resulting impacts on growth and welfare. They include Barseghyan and Battaglini (2016), Arai et al. (2018), Andersen (2019), and Uchida and Ono (2021). [1]

[1] In addition, several studies investigate the effects of fiscal rules on fiscal policy formation. These include Bisin

Barseghyan and Battaglini (2016) and Arai et al. (2018) are based on models without public capital, and thus, their framework does not touch on the GR at all. Andersen (2019) and Uchida and Ono (2021) present models with public investment in human capital that works as productive capital, but their main focus is on the debt ceiling.[2] A notable exception is Bassetto and Sargent (2006), who present a multi-period overlapping-generations model with durable public goods. They calibrate the model to the US economy and show that the GR may approximate the Pareto-efficient allocation.

Our study differs from Bassetto and Sargent (2006) in the following three aspects. First, we focus on public investment in education rather than durable public goods. This modeling choice allows us to present the role of public investment in economic growth more effectively.[3] Second, we employ probabilistic voting (Lindbeck and Weibull (1987); Persson and Tabellini (2002)) rather than majority voting. This voting scheme enables us to capture the marginal impact of demographic and political factors on growth and welfare through policymaking. Third, we calibrate the model to each Organisation for Economic Co-operation and Development (OECD) country to identify how the politically preferred and efficient fiscal rules differ across countries. The analysis also enables us to explain why fiscal rules vary significantly across countries.

To execute our analysis, we employ a two-period overlapping-generations model with physical and human capital. Each generation comprises many identical individuals who live over three periods: the young who attend school, the middle-aged who work, and the old aged who are retired. Public investment in education and parental human capital are inputs in the human capital formation process, contributing to children's human capital formation and thus, productivity in output per worker. Governments, as elected representatives, finance public investment in education and (unproductive) public goods provision through taxes on capital and labor income as well as through public debt issuance. When expenditure is constrained by fiscal rules, public goods provision must be financed solely by tax revenues, while a certain portion of the public investment, denoted by $\phi \in [0, 1]$, can be financed by public debt issuance. In particular, the rule does not allow for deficit and requires the BB when $\phi = 0$; all public investments in education are allowed to be financed with public debt issuance and thus, the GR applies when $\phi = 1$.

Under this framework, we consider the politics of fiscal policy formation for a given fiscal rule. In particular, following Song et al. (2012) and later studies, we assume probabilistic voting to demonstrate the extent to which generations face conflict over such policies. In each

et al. (2015), Halac and Yared (2018), Coate and Milton (2019), Bouton et al. (2020), Dovis and Kirpalani (2020, 2021), and Piguillem and Riboni (2020). They rely on models without physical and/or public capital, and hence, they do not focus on growth and welfare effects of fiscal rules over time and across generations.

[2] Andersen (2019) mentions the GR as an alternative rule, but the analysis is limited to a brief sketch as a possible extension.

[3] Our modeling follows Boldrin and Montes (2005), Docquier et al. (2007), Gonzalez-Eiras and Niepelt (2012), Kunze (2014), and Andersen and Bhattacharya (2020).

period, middle-aged and old-aged individuals vote on candidates.[4] The government, represented by elected politicians, maximizes the political objective function of the weighted sum of the utilities of the middle-aged and old-aged populations. In this voting environment, we start our analysis by assuming the absence of any fiscal rules and show that the current policy choice affects the decision on future policy via physical and human capital accumulation. This intertemporal effect creates the three driving forces that shape fiscal policy, namely, a general equilibrium effect through the interest rate, a disciplining effect through the capital income tax rate in the next period, and a disciplining effect through public goods provision in the next period. The three effects induce the government to finance part of its expenditure by public debt issuance.

We then move on to the case of a fiscal rule represented by ϕ, and study how changes in the fiscal rule affect the government choice of fiscal policies and the resulting voters' preferences for the fiscal rule. We show that a higher ϕ, implying that a large share of public investment financed by public debt issuance lowers physical capital accumulation through the crowding-out effect. This in turn raises the marginal cost of the labor income tax, and thus, induces the government to choose a lower labor income tax rate. We also show that a higher ϕ has two conflicting effects on the choice of public investment: a negative effect through the crowding-out effect on physical capital and a positive effect through a reduced tax burden. Given these conflicting effects, there is a threshold value of ϕ at which the two effects are balanced, and an increase in ϕ has an inverse U-shaped effect on the ratio of public investment to GDP around the threshold value of ϕ.

Our analysis also shows that the voters' preferences weight on public goods, denoted by θ, has a decisive influence on the preferred fiscal rule by voters. As θ increases, the government's incentive to increase future public goods is strengthened; this works to increase public investment in education. In addition, the crowding-out effect and the cost of reduced future public goods provision through debt issuance are highly valued as θ increases; this works to reduce public debt issuance. These two effects imply that the government, representing voters, is more likely to reduce the fraction of public investment financed by public debt issuance as θ increases. This implies that a greater preference weight for public goods is associated with stronger preferences for the BB.

To establish a normative benchmark with which to compare the political equilibrium, we describe the optimal allocation chosen by a benevolent long-lived planner who can commit to all its choices at the beginning of a period. Assuming such a planner, we focus on the deviations between the political equilibrium allocation and the planner's allocation in terms of physical and human capital accumulation, and we refer to the total deviations as political distortions.

[4]The young may also have an incentive to vote since they would benefit from public investment financed by taxing capital and labor income. However, for the tractability of analysis, we assume that politicians do not care about the young's preferences following Saint-Paul and Verdier (1993), Bernasconi and Profeta (2012), and Lancia and Russo (2016). This assumption is supported in part by the fact that a large number of the young are below the voting age.

We show that a larger preference weight on public goods (i.e., higher θ) is associated with a lower fraction of debt-financed public investment (i.e., lower ϕ) in terms of minimizing political distortions.

The mechanism behind this result is as follows. The fiscal rule, denoted by ϕ, has two opposing effects on public investment; and there is a threshold value of ϕ at which the two opposing effects are balanced and thus, the distortion is minimized in terms of human capital. This threshold depends on the preference parameter θ for public goods. In particular, as θ increases, the crowding-out effect of public debt is strengthened and thus, the threshold is lowered. Therefore, a larger preference weight on public goods is associated with a lower fraction of debt-financed public expenditure in terms of maximizing the ratio of public investment to GDP and thereby minimizing political distortions.

Given this property, we calibrate the model to a sample of OECD countries and find that Greece, the country in the sample with the lowest estimated θ, is the only country where the GR is preferred by voters and desirable from the perspective of a planner. We also find that voters prefer the BB, which is also desirable from the planner's viewpoint in most countries with high values of estimated θ, such as Germany, which has implemented a BB-like rule since 2010 (OECD 2015). This suggests that Germany has been politically successful in implementing a fiscal rule that is desirable from a long-term perspective. Furthermore, in countries with lower values of θ, the BB has little support by voters, although it is desirable from the planner's viewpoint. Such countries include Japan and the United States, which have not adopted the BB or similar rules (OECD 2013), although public debt in these countries has accumulated over the past decade. These examples illustrate the difficulty of introducing the BB in countries without political support, even where it is necessary from a long-term perspective. Overall, these results may explain why fiscal rules vary across countries, and why some countries find it difficult to achieve fiscal rules that are desirable from a long-term perspective.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 presents the politics of fiscal policy formation and characterizes the political equilibrium in the absence of fiscal rules. Section 4 introduces the fiscal rules and characterizes the political equilibrium in the presence of fiscal rules. In addition, we calibrate the model to each OECD country and identify the preferences for the fiscal rules. Section 5 characterizes the long-lived planner's allocation, compares it with the political equilibrium, and explores the fiscal rule that minimizes the deviations between the political equilibrium and the planner's allocation. Section 6 provides concluding remarks. All proofs are presented in the appendix.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle age, and old age. Each middle-aged individual has $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1): N_{t+1}=(1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their life cycles. During youth, they make no economic decisions and receive public investment in education financed by the government. In middle age, individuals work, receive market wages, and pay taxes. They use after-tax income for consumption and savings. Individuals retire in their elderly years and receive and consume returns from savings.

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with h_t units of human capital inherited from his or her parents. The individual supplies these units inelastically in the labor market and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle-aged becomes $c_t + s_t \leq (1-\tau_t)w_t h_t$.

The period $t+1$ budget constraint in old age is $d_{t+1} \leq \left(1-\tau_{t+1}^k\right) R_{t+1} s_t$, where d_{t+1} is consumption, τ_{t+1}^k is the period $t+1$ capital income tax rate, $R_{t+1}(>0)$ is the gross return from investment in physical capital, and $R_{t+1}s_t$ is the return from savings. The results are qualitatively unchanged if capital income tax is on the net return from savings rather than the gross return from savings.

Children's human capital in period $t+1$, h_{t+1}, is a function of government spending on public investment in education, x_t, and parents' human capital, h_t. In particular, h_{t+1} is formulated using the following equation:

$$h_{t+1} = D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}, \tag{1}$$

where $D(>0)$ is a scale factor and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to public investment. Private education is abstracted away from the analysis since the focus of this study is on public education expenditure as a proxy for public investment.[5] Hereafter, we use "public investment" and "public education expenditure" interchangeably.

The preferences of the middle-aged in period t are specified by the following expected utility function in the logarithmic form, $U_t^M = \ln c_t + \theta \ln g_t + \beta\left(\ln d_{t+1} + \theta \ln g_{t+1}\right)$, where g is per capita public goods provision, $\beta \in (0,1)$ is a discount factor, and $\theta(>0)$ is the degree of preferences for

[5] Parents' private investment in education may also contribute to human capital formation. For example, parents' time (Glomm and Ravikumar (1996, 2001, 2003); Glomm and Kaganovich (2008)) or spending (Glomm (2004); Lambrecht et al. (2005); Kunze (2014)) on education may complement public education. However, in the present model, parents have no incentive to invest in education privately because they exhibit no altruism toward their children. By contrast, children (i.e., the young) may have an opportunity to invest in their own human capital formation by borrowing from their parents in youth and repaying the loan in middle age. Uchida and Ono (2021) show that this possibility is ruled out if private investment in education is a perfect substitute for public investment (Lancia and Russo (2016); Bishnu and Wang (2017)).

public goods. This represents the preferences of the young in period $t-1$, U_{t-1}^{Y}, because they make no economic decision and their consumption is included in their parents' consumption. The preferences of the old in period t is given by $U_t^O = \ln d_t + \theta \ln g_t$.

We substitute the budget constraints into the utility function of the middle-aged, U_t^M, to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \theta \ln g_t + \beta\left[\ln\left(1-\tau_{t+1}^k\right) R_{t+1} s_t + \theta \ln g_{t+1}\right], \tag{2}$$

where g_t and g_{t+1} are taken as given. By solving the problem in (2), we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta}\cdot(1-\tau_t)w_t h_t, \tag{3}$$

$$c_t = \frac{1}{1+\beta}\cdot(1-\tau_t)w_t h_t, \tag{4}$$

$$d_{t+1} = \left(1-\tau_{t+1}^k\right) R_{t+1}\cdot\frac{\beta}{1+\beta}\cdot(1-\tau_t)w_t h_t. \tag{5}$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, the firms produce a final good Y_t using two inputs: aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. Aggregate output is given by $Y_t = A\left(K_t\right)^{\alpha}\left(H_t\right)^{1-\alpha}$, where $A(>0)$ is a scale parameter and $\alpha \in (0,1)$ denotes the capital share. The production function in intensive form is $y_t = y\left(k_t, h_t\right) \equiv A\left(k_t\right)^{\alpha}\left(h_t\right)^{1-\alpha}$ where $y_t \equiv Y_t/N_t$, $k_t \equiv K_t/N_t$, and $h_t \equiv H_t/N_t$ denote per capita output, physical capital, and human capital, respectively.

The first-order conditions for profit maximization with respect to H_t and K_t are

$$w_t = w\left(k_t, h_t\right) \equiv (1-\alpha)A\left(k_t\right)^{\alpha}\left(h_t\right)^{-\alpha}, \tag{6}$$

$$\rho_t = R\left(k_t, h_t\right) \equiv \alpha A\left(k_t\right)^{\alpha-1}\left(h_t\right)^{1-\alpha}, \tag{7}$$

where w_t and R_t are labor wages and the rental price of capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices. Capital is assumed to depreciate fully within each period. The rental price of capital is equal to the gross return from savings because an arbitrage condition holds.

2.3 Government Budget Constraint

The government expenditure items are public investment in education and expenditure on (unproductive) public goods provision. They are financed by taxes on capital and labor as well as public debt issues. Let B_t denote aggregate inherited debt. In the absence of fiscal rules, the government budget constraint in period t is

$$B_{t+1} + \tau_t^k R_t s_{t-1} N_{t-1} + \tau_t w_t h_t N_t = N_{t+1} x_t + G_t + R_t B_t,$$

where B_{t+1} is newly issued public bonds, $\tau_t^k R_t s_{t-1} N_{t-1}$ is aggregate capital tax revenue, $\tau_t w_t h_t N_t$ is aggregate labor tax revenue, $N_{t+1}x_t$ is aggregate public investment, G_t is aggregate public goods provision, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt. By dividing both sides of the above expression by N_t, we obtain a per-capita form of the constraint:

$$(1+n)b_{t+1} + \tau_t^k R_t \frac{s_{t-1}}{1+n} + \tau_t w_t h_t = (1+n)x_t + \frac{2+n}{1+n} g_t + R_t b_t, \tag{8}$$

where $b_t \equiv B_t/N_t$ is per-capita public debt.

Alternatively, consider a situation in which government expenditures are constrained by fiscal rules. In particular, following the literature on the GR, we focus on fiscal rules that impose constraints on how to finance public investment. In the present framework, public education expenditure corresponds to public investment, because the former contributes to the formation of human capital, which improves productivity in the future. Therefore, the fiscal rule and the associated government budget constraint can be written as follows:

$$(1+n)b_{t+1} = \phi(1+n)x_t, \tag{9}$$

$$\tau_t^k R_t \frac{s_{t-1}}{1+n} + \tau_t w_t h_t = (1-\phi)(1+n)x_t + \frac{2+n}{1+n} g_t + R_t b_t, \tag{10}$$

where $\phi \in [0,1]$ represents the fiscal rule that determines the percentage of public education expenditure to be financed by public debt issuance. We proceed with our analysis, taking ϕ as an institutionally given parameter.

When $\phi = 0$, the rule does not allow for a deficit, and requires a balanced budget. All government expenditures must be financed using tax revenues.[6] When $\phi = 1$, all public investment (i.e., public education expenditure) must be financed by public debt issuance. This rule allows a budget deficit only to finance public investment, and prohibits the financing of current expenditures (i.e., public goods provision) by budget deficit. Therefore, as ϕ increases, the fiscal burden of public investment is increasingly deferred to future generations.

2.4 Economic Equilibrium

Public bonds are traded in the domestic capital market. The market clearing condition for capital is $B_{t+1} + K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$ to the sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1} + K_{t+1}$. Using $k_{t+1} \equiv K_{t+1}/N_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, the profit-maximization condition in (6), and the savings function in (3), we

[6] Specifically, the government has a balanced budget if the third item of the expenditure in (10) is the net repayment of debt, $(R_t - 1)b_t$, rather than the gross repayment of debt, $R_t b_t$; the fiscal rule in Eq. (9) is modified to$(1+n)b_{t+1} = \phi(1+n)x_t + b_t$. However, this modification makes it difficult to obtain an analytical solution. Given this limitation, we employ the rule in Eq. (9) in the present study.

can rewrite the condition as

$$(1+n)\left(k_{t+1}+b_{t+1}\right)=s\left(\tau_t;k_t,h_t\right)\equiv\frac{\beta}{1+\beta}(1-\tau_t)w\left(k_t,h_t\right)h_t. \tag{11}$$

The following defines the economic equilibrium in the present model.

Definition 1 *Given a sequence of policies, $\left\{\tau_t^k,\tau_t,x_t,g_t\right\}_{t=0}^{\infty}$, an economic equilibrium is a sequence of allocations $\{c_t,d_t,s_t,k_{t+1},b_{t+1},h_{t+1}\}_{t=0}^{\infty}$ and prices $\{\rho_t,w_t,R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(>0),b_0(\geq 0)$ and $h_0(>0)$, such that (i) given $\left(w_t,R_{t+1},\tau_{t+1}^k,\tau_t,x_t,g_t,g_{t+1}\right)$, (c_t,d_{t+1},s_t) solves the utility-maximization problem; (ii) given (w_t,ρ_t), k_t solves a firm's profit maximization problem; (iii) given (w_t,h_t,R_t,b_t), $\left(\tau_t^k,\tau_t,x_t,b_{t+1}\right)$ satisfies the government budget constraint; and (iv) the capital market clears: $(1+n)\cdot(k_{t+1}+b_{t+1})=s_t$.*

Definition 1 allows us to reduce the economic equilibrium conditions to a system of difference equations that characterize the motion of (k_t,b_t,h_t). In the absence of fiscal rules, the system includes the capital market clearing condition in (11), the human capital formation function in (1), and the government budget constraint in (8) with the profit maximization conditions in (6) and (7), expressed as:

$$(1+n)b_{t+1}+TR^K\left(\tau_t^k;k_t,h_t\right)+TR\left(\tau_t;k_t,h_t\right)=(1+n)x_t+\frac{2+n}{1+n}g_t+R\left(k_t,h_t\right)b_t, \tag{12}$$

where $TR^K\left(\tau_t^k;k_t,h_t\right)$ and $TR\left(\tau_t;k_t,h_t\right)$ are the tax revenues from capital and labor income, respectively, defined as follows:

$$\begin{aligned}TR^K\left(\tau_t^k;k_t,h_t\right)&\equiv\tau_t^kR\left(k_t,h_t\right)\left(k_t+b_t\right),\\ TR\left(\tau_t;k_t,h_t\right)&\equiv\tau_tw\left(k_t,h_t\right)h_t.\end{aligned}$$

In the presence of the fiscal rule represented by (9), the government budget constraint in (12) is reformulated as

$$TR^K\left(\tau_t^k;k_t,h_t\right)+TR\left(\tau_t;k_t,h_t\right)=(1-\phi)(1+n)x_t+\frac{2+n}{1+n}g_t+R\left(k_t,h_t\right)b_t. \tag{13}$$

In the economic equilibrium, the indirect utility of the middle-aged population in period t, V_t^M, and that of the old-aged population in period t, V_t^O, can be expressed as functions of fiscal policy, physical and human capital, and public debt as follows:

$$\begin{aligned}V_t^M&=V^M\left(\tau_t,x_t,g_t,\tau_{t+1}^k,g_{t+1},k_{t+1},b_{t+1},h_{t+1};k_t,h_t\right)\\&\equiv\ln c\left(\tau_t,k_t,h_t\right)+\theta\ln g_t+\beta\left[\ln d\left(\tau_{t+1}^k;k_{t+1},h_{t+1},b_{t+1}\right)+\theta\ln g_{t+1}\right],\\ V_t^O&=V^O\left(\tau_t,g_t;k_t,b_t,h_t\right)\equiv\ln d\left(\tau_t^k;k_t,h_t,b_t\right)+\theta\ln g_t,\end{aligned}$$

where $c\left(\tau_t,k_t,h_t\right)$ and $d\left(\tau_t^k;k_t,h_t,b_t\right)$, representing consumption in middle and old ages, respectively, are defined as follows:

$$\begin{aligned}c\left(\tau_t,k_t,h_t\right)&\equiv\frac{1}{1+\beta}\left(1-\tau_t\right)w\left(k_t,h_t\right)h_t,\\ d\left(\tau_t^k;k_t,h_t,b_t\right)&\equiv\left(1-\tau_t^k\right)R\left(k_t,h_t\right)(1+n)\left(k_t+b_t\right).\end{aligned}$$

3 Politics

Based on the characterization of the economic equilibrium, we consider the politics of fiscal policy formation. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated by Persson and Tabellini (2002), the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters (i.e., the middle-aged and the old).

In the present framework, the young, middle-aged, and old have an incentive to vote. While the young may benefit from public investment through human capital accumulation, we assume that their preferences are not considered by politicians. We impose this assumption, which is often used in the literature (e.g., Saint-Paul and Verdier (1993); Bernasconi and Profeta (2012); Lancia and Russo (2016)), for tractability reasons. However, the assumption could be supported in part by the fact that a large number of the young are below the voting age.

The political objective is defined as the weighted sum of the utility of the middle-aged and old, given by $\tilde{\Omega}_t \equiv \omega V_t^O + (1+n)(1-\omega)V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on the old and the middle-aged, respectively, in period t. The weight on the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. We divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^O + V_t^M, \tag{14}$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^O represents the relative political weight on the old.

The political objective function suggests that the current policy choice affects the decision on future policy via physical and human capital accumulation. In particular, the period t choices of τ_t^k, x_t, g_t, and b_{t+1} affect the formation of physical and human capital in period $t+1$. This, in turn, influences the decision making on the period-$t+1$ fiscal policy. To demonstrate such an intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy in the present period depends on the current payoff-relevant state variables.

In our framework, the payoff-relevant state variables are physical capital k_t, public debt b_t, and human capital h_t. Thus, the expected rate of capital income tax for the next period, τ_{t+1}^k, is given by the function of the period-$t+1$ state variables, $\tau_{t+1}^k = T^k(k_{t+1}, b_{t+1}, h_{t+1})$. We denote the arbitrary lower limits of τ and τ^k by $-\underline{\tau}(<0)$ and $-\underline{\tau}^k(<0)$, respectively. By using recursive notation with z' denoting the next period z, we can define a Markov-perfect political equilibrium in the present framework as follows.

Definition 2 *A Markov-perfect political equilibrium is a set of functions,* $\langle \hat{T}, \hat{T}^k, \hat{X}, \hat{G}, \hat{B} \rangle$, *where* $\hat{T} : \Re_+^3 \to (-\underline{\tau}, 1)$ *is the labor income tax rule,* $\tau = \hat{T}(k,b,h)$, *and* $\hat{T}^k : \Re_+^3 \to \left(-\underline{\tau}^k, 1\right)$ *is* *a capital income tax rule, $\tau^k = \hat{T}^k(k,b,h)$, $\hat{X} : \Re^3_+ \to \Re_+$ is a public education expenditure rule, $x = \hat{X}(k,b,h)$, $\hat{G} : \Re^3_+ \to \Re_+$ is a public goods provision rule, and $\hat{B} : \Re^3_+ \to \Re_+$ is a public debt rule, $b' = \hat{B}(k,b,h)$, so that given k, b, and h, $\langle \hat{T}(k,b,h), \hat{T}^k(k,b,h), \hat{X}(k,b,h), \hat{G}(k,b,h), \hat{B}(k,b,h)\rangle$ is a solution to the problem of maximizing Ω in (14), subject to the human capital formation function in (1), the capital market clearing condition in (11), and the government budget constraint in (12) in the absence of fiscal rules or the government budget constraint in (13) and (9) in the presence of fiscal rules.*[7]

3.1 Political Equilibrium in the Absence of Fiscal Rules

We derive the equilibrium policy functions and the associated growth rate of per capita output in the absence of fiscal rules. To obtain the set of equilibrium policy functions, we conjecture the following capital tax rate and public goods provision in the next period:

$$\tau^{k\prime} = 1 - T^k \cdot \frac{1}{\alpha\left(1 + \frac{b'}{k'}\right)}, \tag{15}$$

$$g' = G \cdot y\left(k', h'\right), \tag{16}$$

where $T^k (> 0)$ and $G (> 0)$ are constant.

Given the conjecture in (15) and (16), we consider the optimization problem described in Definition 2, and obtain the following first-order derivatives:

$$\tau^k : \tfrac{\omega}{(1+n)(1-\omega)} \tfrac{d_{\tau^k}}{d} + \lambda TR^K_{\tau^k} = 0, \tag{17}$$

$$\tau : \frac{c_\tau}{c} + \beta\left(\frac{d'_\tau}{d'} + \theta\frac{g'_\tau}{g'}\right) + \lambda TR^L_\tau = 0, \tag{18}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} - \lambda\frac{2+n}{1+n} = 0, \tag{19}$$

$$x : \beta\left(\frac{d'_x}{d'} + \theta\frac{g'_x}{g'}\right) - \lambda(1+n) = 0, \tag{20}$$

$$b' : \beta\left(\frac{d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'}\right) + \lambda(1+n) \leq 0, \tag{21}$$

where $\lambda\,(\geq 0)$ is the Lagrangian multiplier associated with the government budget constraint in (12). A strict inequality holds in (21) if $b' = 0$.

The first-order conditions in (17)–(21) are summarized as follows, focusing on the marginal

[7] The state variables do not line up in compact sets because they grow across periods. To define the equilibrium more precisely, we need to redefine the equilibrium as a mapping from a compact set to a compact set by introducing the following notations: $\hat{x}_t \equiv x_t/y\,(k_t, h_t)$, $\hat{g}_t \equiv g_t/y\,(k_t, h_t)$, and $\hat{b}_{t+1} \equiv b_{t+1}/y\,(k_t, h_t)$. However, for simplicity, we define the equilibrium as in Definition 2.

benefit of public goods provision appearing on the left-hand side of (19):

$$\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{g}=-\frac{1+n}{2+n}\cdot\frac{\frac{\omega}{(1+n)(1-\omega)}\frac{d_{\tau^k}}{d}}{TR^K_{\tau^k}}, \tag{22}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{g}=-\frac{1+n}{2+n}\cdot\frac{\frac{c_\tau}{c}+\beta\left(\frac{d'_\tau}{d'}+\theta\frac{g'_\tau}{g'}\right)}{TR_\tau}, \tag{23}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{g}=(2+n)\,\beta\left(\frac{d'_x}{d'}+\theta\frac{g'_x}{g'}\right), \tag{24}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{g}\leq-(2+n)\,\beta\left(\frac{d'_{b'}}{d'}+\theta\frac{g'_{b'}}{g'}\right). \tag{25}$$

According to the expressions in (22)–(25), the government chooses policies to equate the marginal benefits of public goods provision that appear on the left-hand side with the marginal costs of capital income taxation, marginal net cost of the labor income tax, marginal cost of reduction in public investment, and marginal net cost of public debt issuance that appear on the right-hand side. However, the marginal net cost of public debt issuance may outweigh the marginal benefit of public goods provision. In such a case, the condition in (25) holds with a strict inequality: the government finds it optimal to issue no public debt and finance its expenditures solely by taxation.

A detailed interpretation of this conditions is as follows. The intuition for the marginal benefit of public goods provision appearing on the left-hand side of each equation is straightforward. An increase in public goods leads to an increase in the marginal utility of public goods for the old and for the middle-aged. The right-hand side of (22) represents the marginal cost of capital income tax. An increase in the capital income tax rate lowers the consumption of the old and thus, makes them worse off. We evaluate this effect based on the change in tax revenue through capital income taxation represented by the term $TR^K_{\tau^k}$ in the denominator. The right-hand sides of (23)–(25) include the following three inter-temporal effects: the general equilibrium effect of capital through the interest rate, $R^{'}$; the disciplining effect through the capital income tax rate, $\tau^{k\prime}$; and the disciplining effect through public goods provision, g'. These effects play crucial roles in shaping fiscal policy, which are explained in turn below.

First, consider the right-hand side of (23), showing the marginal cost of the labor income tax. A rise in the labor income tax rate causes disposable income to fall, thereby reducing the consumption of the middle-aged, as represented by the term c_τ/c. The term $\beta\left(d'_\tau/d'+\theta g'_\tau/g'\right)$ includes the marginal costs and benefits of the labor income tax that the current middle are expected to receive when they are old. The rise in the labor income tax rate decreases the disposable income of the middle-aged and thus, reduces their savings, which in turn lowers their consumption in old age. In addition, the decrease in savings works to increase the return from savings, $R^{'}$, and thus, increases their consumption in old age. At the same time, the decrease in savings works to increase the capital income tax rate in the next period, $\tau^{k\prime}$, which lowers their consumption in old age. These effects are represented by the term d'_τ/d'. The term $\theta g'_\tau/g'$

shows that the rise in the labor income tax rate lowers savings and capital, and thus, depresses public goods provision in the next period. The right-hand side evaluates these effects based on the change in the tax revenue through the labor income tax, as represented by the term TR_{τ} in the denominator.

Next, consider the right-hand side of (24), showing the marginal cost of the reduction in public investment. The cut in this investment lowers the human capital level and thus, reduces the return from savings, $R^{'}$. This leads to a decrease in consumption in old age. At the same time, the decrease in the human capital level leads to a decrease in public goods provision in the next period. The right–hand side of (24) includes these two marginal costs, which affect the middle-aged in their old age.

Finally, consider the right-hand side of (25), showing the net marginal cost of pubic debt issuance. The term $d'_{b'}/d'$ shows the marginal net benefit of public debt issuance. The issue of public debt crowds out physical capital accumulation and thus, raises the interest rate, $R^{'}$. This in turn increases consumption in old age. Simultaneously, it raises the capital income tax rate in the next period, $\tau^{k\prime}$, and thus, reduces consumption in old age. Furthermore, the issue of public debt increases the capital income tax rate in the next period, which in turn decreases consumption in old age. The term $\theta g'_{b'}/g'$ implies that the debt issue crowds out physical capital and thus, lowers public goods provision in the next period, g'. Thus, the public debt issuance creates opposing effects on what the middle-aged expect to receive when they become old.

Using the conditions in (22) and (25) alongside the government budget constraint in (10), we can verify the conjecture in (15) and (16), and obtain the following result.

Proposition 1 *There is a Markov-perfect political equilibrium such that the policy functions of* τ^k, x, g, τ, *and* b' *are given by*

$$\tau^k = 1 - \frac{1}{\Lambda} \cdot \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{\alpha \frac{k+b}{k}}, \tag{26}$$

$$(1+n)x = \frac{1}{\Lambda}\beta\eta(1-\alpha)(1+\theta)\, y(k,h), \tag{27}$$

$$\frac{2+n}{1+n}g = \frac{1}{\Lambda}\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta y(k,h), \tag{28}$$

$$\tau = \begin{cases} 1 - \frac{1}{\Lambda}\cdot\frac{1+\alpha\beta(1+\theta)}{1-\alpha} & \textit{if } 1 \le \alpha(1+\theta), \\ 1 - \frac{1}{\Lambda}\cdot\frac{1+\beta}{1-\alpha} & \textit{if } 1 > \alpha(1+\theta), \end{cases} \tag{29}$$

$$(1+n)b' = \begin{cases} 0 & \textit{if } 1 \le \alpha(1+\theta), \\ \frac{1}{\Lambda}\beta\left[1-\alpha(1+\theta)\right] y(k,h) & \textit{if } 1 > \alpha(1+\theta). \end{cases} \tag{30}$$

where Λ *is defined by*

$$\Lambda \equiv (1+\theta)\left[1+\beta\left(\alpha+\eta(1-\alpha)\right)+\frac{\omega}{(1+n)(1-\omega)}\right].$$

Proof. See Appendix A.1.

To understand the properties of the policy functions presented in Proposition 1, recall first the capital income tax rate in (26), which is rewritten as follows:

$$\left(1-\tau^{k}\right) R\left(k,h\right)\left(k+b\right)=\frac{1}{\Lambda}\cdot\frac{\omega}{(1+n)(1-\omega)}\cdot y\left(k,h\right).$$

This expression implies that the capital income tax rate is set to equal the after-tax capital income to a fixed fraction of output. The fraction, denoted by $\frac{\omega}{(1+n)(1-\omega)}/\Lambda$, represents the share of the output devoted to the old. An increase in the political weight of the old $\frac{\omega}{(1+n)(1-\omega)}$, leads to a decrease in the capital income tax rate and hence, reduces the tax burden of the old.

Second, public investment in (27) and public goods in (28) are each provided at a fixed share of output. Each share reflects the preferences of voters for each expenditure. Third, the labor income tax rate in (29) is set to be constant over time so that the share of labor income tax revenue in output remains constant. Finally, the public debt issuance in (30) indicates that θ is crucial to the government's fiscal stance. A higher θ implies that the government attaches a larger weight to the public goods expenditure and thus, has a stronger incentive to fund a part of the spending by issuing public debt. A threshold of θ at which the government decides to issue public debt satisfies $\alpha\left(1+\theta\right)=1$ as expressed in (30).

3.2 Physical and Human Capital Accumulation

Having established the policy functions, we are now ready to demonstrate the accumulation of physical and human capital in the absence of debt rules. We substitute the policy functions in Proposition 1 into the capital market clearing condition in (11) and human capital formation function in (1), and obtain

$$\frac{k'}{k}=\begin{cases}\frac{\beta[1+\alpha\beta(1+\theta)]}{(1+n)(1+\beta)\Lambda}A\left(\frac{h}{k}\right)^{1-\alpha} & \text{if } 1\leq\alpha\left(1+\theta\right),\\ \frac{\alpha\beta(1+\theta)}{(1+n)\Lambda}A\left(\frac{h}{k}\right)^{1-\alpha} & \text{if } 1>\alpha\left(1+\theta\right),\end{cases}\tag{31}$$

$$\frac{h'}{h}=D\left[\frac{\left(1+\theta\right)\beta\eta\left(1-\alpha\right)}{(1+n)\Lambda}A\left(\frac{h}{k}\right)^{-\alpha}\right]^{\eta}.\tag{32}$$

Given $\{k_0,h_0\}$, the sequence $\{k_t,h_t\}$ can be distinguished using the above two equations in (31) and (32). A steady state is defined as a political equilibrium with $k'/h'=k/h$. In other words, the ratio of physical capital to human capital is constant across periods. In the steady state, Eqs. (31) and (32) lead to

$$\frac{h'}{k'}=\begin{cases}D\left[\frac{(1+\theta)\beta\eta(1-\alpha)}{(1+n)\Lambda}\right]^{\eta}\left[A\left(\frac{h}{k}\right)^{\alpha}\right]^{1-\eta}/\left\{\frac{\beta[1+\alpha\beta(1+\theta)]}{(1+n)(1+\beta)\Lambda}\right\}, & \text{if } 1\leq\alpha\left(1+\theta\right),\\ D\left[\frac{(1+\theta)\beta\eta(1-\alpha)}{(1+n)\Lambda}\right]^{\eta}\left[A\left(\frac{h}{k}\right)^{\alpha}\right]^{1-\eta}/\left[\frac{\alpha\beta(1+\theta)}{(1+n)\Lambda}\right] & \text{if } 1>\alpha\left(1+\theta\right).\end{cases}\tag{33}$$

showing that there is a unique stable steady state for the path of $\{h/k\}$.

4 Fiscal Rules

In the previous section, we consider fiscal policy and the associated accumulation of physical and human capital in the absence of rules for public debt issues. However, in practice, many countries have introduced fiscal rules that control public debt (Budina et al. (2012); Schaechter et al. (2012)). In addition, in the present framework, public debt issuance creates a crowding-out effect on physical capital formation, which in turn triggers a welfare loss for future generations. This observation motivates us to consider the question of how fiscal rules shape the choice of fiscal policy and affect the resource allocation over time and across generations. To answer this question, in Subsection 4.1, we derive the equilibrium policy functions and the associated accumulation of physical and human capital in the presence of the fiscal rules specified in Eqs. (9). Then, in Subsection 4.2, we calibrate the model to OECD countries to quantify the effects of the fiscal rules on fiscal policy formation for each country.

4.1 Political Equilibrium in the Presence of Fiscal Rules

In line with the procedure in Section 3, we conjecture the capital income tax rate and public goods provision in the next period as in (15) and (16), respectively. Given these conjectures, we consider the optimization problem in the presence of fiscal rules described in Definition 2, and obtain the following first-order derivatives:

$$\tau^k : \tfrac{\omega}{(1+n)(1-\omega)} \tfrac{d_{\tau^k}}{d} + \lambda TR^K_{\tau^k} = 0, \tag{34}$$

$$\tau : \frac{c_\tau}{c} + \beta \left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right) + \lambda TR_\tau = 0, \tag{35}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} - \lambda \frac{2+n}{1+n} = 0, \tag{36}$$

$$x : \beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right) - \lambda (1-\phi)(1+n) = 0, \tag{37}$$

where $\lambda\,(\geq 0)$ is the Lagrangian multiplier associated with the government budget constraint in (13). The main difference from the case of the absence of fiscal rules is that the choice of x determines the level of public debt issuance according to the rule in (9). The impact of debt issuance decisions through such a rule is contained in the term $\left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right)$ of (35) and the terms $\beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right)$ and $\lambda (1-\phi)(1+n)$ of (37).

In deriving the policy functions using the first-order conditions in (34)–(37), alongside the fiscal rule in (9) and the government budget constraint in (13), we further conjecture that the policy functions of τ and x are given by

$$\tau = T, \tag{38}$$

$$(1+n)x = X \cdot y(k,h), \tag{39}$$

where T and X are constant. We can verify the conjectures in (15), (16), (38), and (39) and obtain the following result:

Proposition 2 *Given a fiscal rule in (9), there is a Markov-perfect political equilibrium such that the policy functions of τ^k, x, g, τ, and b' are given by*

$$1-\tau^k = \frac{\omega}{(1+n)(1-\omega)}(1-\alpha)\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}\frac{1}{\alpha\frac{k+b}{k}},$$

$$(1+n)x = X\cdot y(k,h),$$

$$\frac{2+n}{1+n}g = \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}(1-\alpha)\,y(k,h),$$

$$\tau = T,$$

$$(1+n)b' = \phi(1+n)x.$$

The two constants, X and T, are

$$X \equiv \frac{I(\phi)}{(1-\phi)\phi}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}[1+\alpha\beta(1+\theta)]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1},$$

$$T \equiv 1-\frac{1}{1-\alpha}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}[1+\alpha\beta(1+\theta)]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1},$$

where $I(\phi)$ is

$$I(\phi) \equiv \frac{H_1(\phi)-\sqrt{(H_1(\phi))^2-4H_2(\phi)}}{2},$$

and $H_1(\phi)$ and $H_2(\phi)$ are

$$H_1(\phi) \equiv \beta(1+\theta)(\alpha+\eta(1-\alpha))\phi+\frac{\beta}{1+\beta}[1+\alpha\beta(1+\theta)](1-\phi),$$

$$H_2(\phi) \equiv (1-\phi)\phi\beta(1+\theta)\eta(1-\alpha)\frac{\beta}{1+\beta}.$$

Proof. See Appendix A.2.

The first-order conditions with respect to τ^k in (34) and g in (36) are independent of ϕ; the two policy variables, τ^k and g, are unaffected by the choice of fiscal rules. The first-order conditions with respect to τ in (35) and x in (37) depend on ϕ, suggesting that τ and x are affected by the fiscal rule. To investigate the effects, we reformulate (35) and (37) as follows:

$$\tau: \underbrace{\frac{-1}{1-\tau}}_{\equiv\frac{c_\tau}{c}}+\underbrace{\frac{-\alpha\beta(1+\theta)\frac{\beta}{1+\beta}w(k,h)h}{\frac{\beta}{1+\beta}(1-\tau)w(k,h)h-\phi(1+n)x}}_{\equiv\beta\left(\frac{d'_\tau}{d'}+\theta\frac{g'_\tau}{g'}\right)}+\underbrace{\lambda w(k,h)h}_{\equiv\lambda TR_\tau}=0, \tag{40}$$

$$x: \underbrace{\frac{-\alpha\beta(1+\theta)\phi(1+n)}{\frac{\beta,}{1+\beta}(1-\tau)w(k,h)h-\phi(1+n)x}+\frac{\beta\eta(1+\theta)(1-\alpha)}{x}}_{\equiv\beta\left(\frac{d'_x}{d'}+\theta\frac{g'_x}{g'}\right)}-\lambda(1-\phi)(1+n)=0. \tag{41}$$

Eq. (40) shows that the fiscal rule, represented by ϕ, affects the formation of the labor income tax rate in the following way. The higher the ϕ, the greater the share of public investment financed by public debt issuance. Debt issuance lowers physical capital accumulation through crowding-out effects, reduces the next-period public goods provision, and generates the following two opposing effects on old-age consumption: a positive effect via an increase in the interest rate, and a negative effect via an increase in the next-period capital income tax rate. In the present framework, the latter effect dominates the former one. Therefore, an increase in ϕ leads to an increase in the marginal cost of τ. These effects are observed in the second term of the left-hand side in (40). To offset this increase in the marginal cost, the government has an incentive to lower the labor income tax rate as ϕ increases. Therefore, an increase in ϕ leads to a decrease in the labor income tax rate, as illustrated in Panel (a) of Figure 1. Calibration of the numerical simulation in Figure 1 is described later in this section.

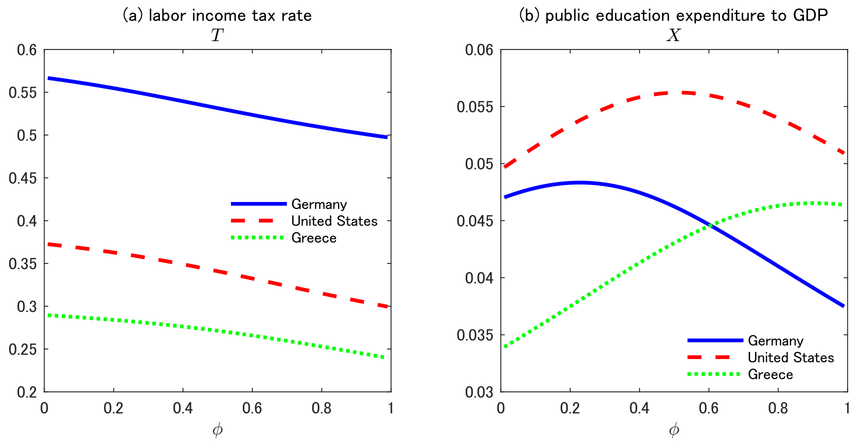

Figure 1: Effects of an increased ϕ on the labor income tax rate, T (Panel (a)), and public education expenditure to GDP, X (Panel (b)), for Germany, Greece, and the United States.

Eq. (41) shows that the fiscal rule, represented by ϕ, has two conflicting effects on the choice of public investment, x. The first is a negative effect on x observed in the first term on the left-hand side: the effect is qualitatively similar to the negative effect on τ, as mentioned previously. The second is a positive effect on x, as observed in third term on the left-hand side. This effect arises because the tax burden associated with the expenditure of x decreases as the fraction of the expenditure x financed by public debt, ϕ, increases. Given these conflicting effects, there is a threshold value of ϕ at which the two effects are balanced, and an increase in ϕ has an inverse U-shaped effect on the ratio of public investment to GDP around the threshold value, as illustrated in Panel (b) of Figure 1.

Next, we present physical and human capital accumulation in the presence of fiscal rules, in the same manner as in Section 3. We substitute the policy functions presented in Proposition into the capital market clearing condition in (11), and the human capital formation function in

(1), and obtain

$$\frac{k'}{k} = \frac{1}{1+n}\left(\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X\right) A\left(\frac{h}{k}\right)^{1-\alpha}, \tag{42}$$

$$\frac{h'}{h} = D\left[\frac{1}{1+n} X A\left(\frac{k}{h}\right)^{\alpha}\right]^{\eta}. \tag{43}$$

These equations are used to evaluate the efficiency of the political equilibrium in Section 5.

4.2 Numerical Analysis

The result in Subsection 4.1 indicates that the choice of fiscal rule ϕ has a decisive effect on the formation of fiscal policies. The effects also depend on the structural parameters of the model economy. In addition, the result suggests that, in terms of maximizing the political objective function, the type of fiscal rule preferred by voters also depends on the structural parameters. This subsection aims to clarify these effects qualitatively, based on numerical methods. In particular, we calibrate the model economy such that the steady-state equilibrium allocation and policies in the absence of fiscal rules match some key statistics of each sample country. The sample countries, as listed in Table 1, are selected from OECD countries that experienced a budget deficit in average during 1995–2016.

4.2.1 Calibration

The parameters common to all sample countries, α, β, and ω, are set in the following way. We fix the share of capital at $\alpha = 1/3$, in line with Song et al. (2012) and Lancia and Russo (2016). Each period lasts 30 years; this assumption is standard in quantitative analyses of two- or three-period overlapping-generations models (e.g., Gonzalez-Eiras and Niepelt (2008); Lancia and Russo (2016)). Our selection of β is 0.99 per quarter, which is also the standard in the literature (e.g., Kydland and Prescott (1982); de la Croix and Doepke (2003)). Since the agents in the present model plan over generations that span 30 years, we discount the future by $(0.99)^{4\times 30} = (0.99)^{120}$. Following Lancia and Russo (2016), we set the political weight of the old relative to the middle-aged, $\omega/(1-\omega)$, at 0.8, so that $\omega = 4/9$.

Country-specific parameters, n, η, and θ are calibrated in the following way.[8] The population growth rate, n, is obtained from the average of each sample country during 1995–2016. Let POP_j denote country j's annual gross population growth rate. The net population growth rate for 30 years is $(POP_j)^{30} - 1$. For η, we focus on the ratio of education expenditure to GDP in the steady state,

$$\left.\frac{N'x}{Y}\right|_j = EDU_j \equiv \frac{\beta\eta_j(1-\alpha)}{1+\beta[\alpha+\eta_j(1-\alpha)] + \frac{\omega}{(1+n_j)(1-\omega)}}, \tag{44}$$

[8] A and D are also country-specific parameters. They are not calibrated here, because they are irrelevant for the numerical analysis.

Country	POP	EDU	DEF	n	η	θ
Australia	1.014	0.0505	0.0123	0.517	0.434	1.477
Austria	1.005	0.0550	0.0269	0.145	0.525	0.983
Belgium	1.005	0.0579	0.0213	0.173	0.548	1.136
Canada	1.010	0.0521	0.0064	0.348	0.466	1.693
Czech Republic	1.001	0.0417	0.0355	0.033	0.409	0.770
Germany	1.000	0.0469	0.0210	0.012	0.466	1.116
France	1.005	0.0575	0.0369	0.177	0.544	0.766
Greece	1.001	0.0338	0.0750	0.029	0.329	0.219
Hungary	0.998	0.0483	0.0512	-0.071	0.499	0.458
Ireland	1.013	0.0475	0.0318	0.483	0.410	0.937
Iceland	1.011	0.0694	0.0035	0.382	0.627	1.821
Israel	1.021	0.0593	0.0430	0.855	0.483	0.763
Italy	1.003	0.0431	0.0341	0.096	0.412	0.815
Japan	1.001	0.0347	0.0588	0.018	0.340	0.394
Lithuania	0.989	0.0502	0.0316	-0.286	0.587	0.724
Latvia	0.989	0.0538	0.0216	-0.288	0.634	0.988
Mexico	1.014	0.0464	0.0274	0.528	0.396	1.044
Netherlands	1.005	0.0505	0.0207	0.148	0.478	1.154
Poland	0.999	0.0505	0.0425	-0.023	0.512	0.612
Portugal	1.001	0.0509	0.0523	0.043	0.502	0.477
Slovakia	1.001	0.0401	0.0510	0.018	0.395	0.497
Slovenia	1.002	0.0540	0.0376	0.054	0.531	0.724
Spain	1.008	0.0436	0.0390	0.252	0.397	0.754
Sweden	1.006	0.0683	0.0012	0.182	0.653	1.935
Switzerland	1.008	0.0504	0.0034	0.281	0.459	1.824
Turkey	1.015	0.0241	0.0122	0.559	0.200	1.497
United Kingdom	1.006	0.0490	0.0384	0.192	0.457	0.748
United States	1.009	0.0495	0.0529	0.317	0.445	0.536

Table 1: Data and calibrated parameters for sample countries. The first three columns are the annual gross population growth rate (POP), the ratio of education expenditure to GDP (EDU) and the deficit-to-GDP ratio (DEF), on average, during 1995–2016. The last three columns are calibrated parameter values of n, η, and θ.
Source: World Development Indicators (`https://databank.worldbank.org/source/world-development-indicators`, accessed on February 27, 2021) for data on POP and EDU; OECD library, General government deficit indicator (`https://doi.org/10.1787/cc9669ed-en`, accessed on February 25, 2021) for data on DEF.

where the subscript j is the country code. Given $\alpha = 1/3$, $\beta = (0.99)^{120}$, $\omega = 4/9$, and the average population growth rate for each country, we can solve Eq. (44) for η_j by using the average EDU_j during 1995–2016 for each country.

For θ, we focus on the deficit-to-GDP ratio in the steady state,

$$\left.\frac{B'}{Y}\right|_j = DEF_j \equiv \frac{\beta\left(1-\alpha\left(1+\theta_j\right)\right)}{\left(1+\theta_j\right),\left\{1+\beta\left[\alpha+\eta_j\left(1-\alpha\right)\right]+\frac{\omega}{(1+n_j)(1-\omega)}\right\}}. \tag{45}$$

Given the parameter values of α, β, ω, and n_j as well as η_j, we can solve Eq. (45) for θ_j using the average DEF_j during 1995–2016 for each country.

Based on this setup, we study how the steady-state equilibrium responds to changes in the fiscal rule represented by ϕ in (9) for each country. Figure 1 depicts the changes in the ratio of public education expenditure to GDP (Panel (a)) and the labor income tax rate (Panel (b)) for the three countries of Germany, Greece, and the United States, as typical examples. Panel (a) shows an inverse U-shaped pattern, whereas Panel (b) shows a monotone decline in response to an increase in ϕ. The mechanism behind these results is described in Subsection 4.1.

4.2.2 Voters' Preferences for Fiscal Rules

The analysis in Subsection 4.1 assumes that the parameter ϕ, representing fiscal rules, takes a value from 0 to 1. Changes in fiscal rules may affect each candidate's decision on fiscal policies, which in turn affects voters' preferences. To understand the preferences of voters for fiscal rules more effectively, we focus here on the two particular fiscal rules, the BB with $\phi = 0$ and the GR with $\phi = 1$, which have been widely proposed or implemented in many countries. Since each candidate's objective is to maximize a political objective function weighted by the utility of the middle-aged and the old, that is, Ω in (14), voters prefer the BB if the ϕ that maximizes this objective function is close to 0 (i.e., if $\Omega|_{\phi=0} > \Omega|_{\phi=1}$) and GR is preferred if it is close to 1 (i.e., if $\Omega|_{\phi=0} < \Omega|_{\phi=1}$). The result shown in this subsection may explain how preferences for fiscal rules differ among countries, depending on structural parameters.

From Eq. (9), which represents the fiscal rule, ϕ is equal to the ratio of public debt to public investment. In the absence of fiscal rules, the ratio is the consequence of maximizing the political objective through policy choices. Therefore, by focusing on the ratio with no fiscal rule, denoted by ϕ_{no}, we can determine whether the BB or the GR is preferred by the voters. The BB is preferred if ϕ_{no} is close to 0, and the GR is preferred if ϕ_{no} is close to or above 1 in the political equilibrium in the absence of fiscal rules. Table 2 presents a list of countries for which the BB (GR) is politically preferred over the GR (BB). In Figure 2, we take the same three countries (Germany, Greece, and the United States) as in Figure 1, and show how the political objective function responds to changes in the fiscal rule ϕ for each country. The results for other countries are demonstrated in Figure 4 in Appendix A.4.

The BB is politically preferred	Australia, Belgium, Canada, Switzerland, Germany, Iceland, Latvia, Netherlands, Sweden
GR is politically preferred	Austria, Czech Republic, Spain, France, United Kingdom, Greece, Hungary, Ireland, Israel, Italy, Japan, Lithuania, Mexico, Poland, Portugal, Slovakia, Slovenia, Turkey, United States

Table 2: List of countries that implement the balanced budget rule and the golden rule of public finance.

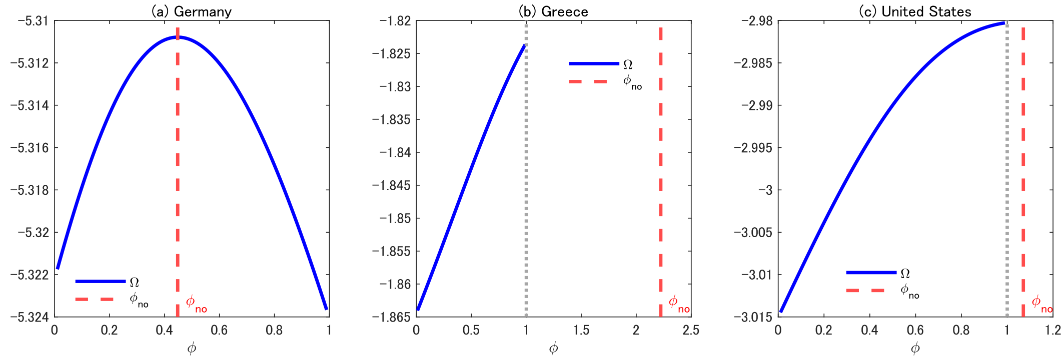

Figure 2: Effects of an increased ϕ on the political objective function for Germany (Panel (a)), Greece (Panel (b)), and the United States (Panel (c)).

From the result in Proposition 1, we find that ϕ_{no} of country j, denoted by $\phi_{no,j}$, is

$$\phi_{no,j} = \frac{(1+n_j)\, b_j'}{(1+n_j)\, x_j} = \frac{1-\alpha\,(1+\theta_j)}{\eta_j\,(1-\alpha)\,(1+\theta_j)}.$$

This equation shows that the ratio $\phi_{no,j}$ is lower for countries with stronger preferences for public goods. To understand the mechanism behind this result, recall the first-order conditions (20) for public investment, x, and (21) for public debt issuance, b'. From (20), we observe that the larger the preference weight on public goods, θ, the larger the marginal utility of x through the next period public goods provision, g', so that higher θ induces the government to increase x. Next, looking at (21), we observe that a higher θ yields a higher the marginal cost of b' through g', which induces the government to reduce b'. Thus, based on these two effects through x and b', a higher θ is associated with a lower ϕ_{no} and thus, a stronger preferences for the BB.

Two exceptions should be noted. First, in Mexico and Turkey, the GR is preferred in terms of maximizing the political objective function, although the estimated values of θ in these countries are large. This is because the elasticity of public investment, η, is small. The smaller the elasticity, η, the lower the level of public investment determined through voting, and thus, the larger the ϕ_{no} of the ratio of public debt to investment. As a result, the GR is preferred to the BB in Mexico and Turkey. Second, for Austria and Latvia, both of which are characterized by almost the same level of θ, the former chooses the GR while the latter chooses the BB. This difference is due to the different levels of η in the two countries. Thus, the preferences for the BB or the GR can be explained by focusing on the level of θ for most countries, but for some countries, η also plays an important role in determining the preferences for fiscal rules.

5 Political Distortions

To establish a normative benchmark with which to compare the political equilibrium, we begin by describing the allocation that maximizes an infinite discounted sum of generational utilities for an arbitrary social discount factor. In particular, in Subsection 5.1, we consider a benevolent planner who can commit to all its choices at the beginning of a period, subject to the human capital formation function and the resource constraint. In Subsection 5.2, assuming such a planner, we evaluate the political equilibrium by comparing it with the planner's allocation. In particular, we focus on the deviations between the political equilibrium allocation and the planner's allocation in terms of physical and human capital accumulation, and call the total deviations *political distortions*. We explore the fiscal rule that minimizes the political distortions.

5.1 Planner's Allocation

The planner is assumed to value the welfare of all generations. In particular, its objective is to maximize a discounted sum of the lifecycle utility of all current and future generations, $SW = \gamma^{-1}U_0^O + \sum_{t=0}^{\infty}\gamma^t U_t^M$, $0 < \gamma < 1$, under the human capital formation function in (1) and the resource constraint, $N_t c_t + N_{t-1}d_t + K_{t+1} + N_{t+1}x_t + (N_t + N_{t-1})\, g_t = A\,(K_t)^{\alpha}\,(H_t)^{1-\alpha}$, or,

$$c_t + \frac{d_t}{1+n} + (1+n)k_{t+1} + (1+n)x_t + \frac{2+n}{1+n}g_t = A\,(k_t)^{\alpha}\,(h_t)^{1-\alpha}\,, \tag{46}$$

where k_0 and h_0 are given. The parameter $\gamma \in (0,1)$ is the planner's discount factor. Reverse discounting, $1/\gamma(>1)$, must be applied to U_0^O (i.e., the utility of the old generation in period 0) to preserve dynamic consistency.

Solving the problem leads to the following characterization of the planner's allocation.

Proposition 3 *Given k_0 and h_0, a sequence of the planner's allocation, $\{c_t, d_t, x_t, g_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$, satisfies the human capital formation function in (1), the resource constraint in (46), and the following:*

$$c_t = \frac{\gamma\,(1-\gamma)\,[1-\alpha\gamma\,(1-\eta)]}{(1+\theta)\,(\gamma+\beta)\,[1-\gamma\,(1-\eta)]}A\,(k_t)^{\alpha}\,(h_t)^{1-\alpha}\,, \tag{47}$$

$$\frac{d_t}{1+n} = \frac{\beta\,(1-\gamma)\,[1-\alpha\gamma\,(1-\eta)]}{(1+\theta)\,(\gamma+\beta)\,[1-\gamma\,(1-\eta)]}A\,(k_t)^{\alpha}\,(h_t)^{1-\alpha}\,, \tag{48}$$

$$(1+n)\frac{k_{t+1}}{h_{t+1}} = \frac{\alpha\gamma}{D\left[\frac{\gamma\eta(1-\alpha)}{(1+n)[1-\gamma(1-\eta)]}\right]^{\eta}}\left[A\left(\frac{k_t}{h_t}\right)^{\alpha}\right]^{1-\eta},$$

$$(1+n)x_t = \frac{(1-\alpha)\,\gamma\eta}{1-\gamma\,(1-\eta)}A\,(k_t)^{\alpha}\,(h_t)^{1-\alpha}\,, \tag{49}$$

$$\frac{2+n}{1+n}g_t = \frac{\theta\,(1-\gamma)\,[1-\alpha\gamma\,(1-\eta)]}{(1+\theta)\,[1-\gamma\,(1-\eta)]}A\,(k_t)^{\alpha}\,(h_t)^{1-\alpha}\,. \tag{50}$$

Proof. See Appendix A.3.

Based on the result in Proposition 3, we demonstrate physical and human capital accumulation in the planner's allocation as follows:

$$\frac{k'}{k} = \frac{\alpha\gamma}{1+n} A \left(\frac{h}{k}\right)^{1-\alpha}, \tag{51}$$

$$\frac{h'}{h} = D\left[\frac{1}{1+n}\frac{(1-\alpha)\gamma\eta}{1-\gamma(1-\eta)} A \left(\frac{k}{h}\right)^{\alpha}\right]^{\eta}. \tag{52}$$

The derivation of (51) and (52) is provided in Appendix A.3.

5.2 Minimizing Political Distortions

The planner's allocation is characterized by the sequence of $\{k_t, h_t\}$ that satisfies (51) and (52). Similarly, the political equilibrium allocation in the presence of fiscal rules is characterized by the sequence of $\{k_t, h_t\}$ that satisfies (42) and (43). Then, given the same initial condition, $\{k_0, h_0\}$, the political equilibrium allocation in the presence of fiscal rules coincides with the planner's allocation if (42) and (43) coincide with (51) and (52), respectively. In general, the two allocations do not coincide, because the means of adjusting the political equilibrium allocation is limited to a single fiscal rule, while it is necessary to match the two sequences of physical and human capital.

Given this limitation, we consider the choice of a fiscal rule that minimizes the deviations of physical and human capital sequences in the political equilibrium allocation from those in the planner's allocation. In particular, we consider the choice of the fiscal rule, ϕ, that minimizes the political distortions specified by the following cost function, denoted by $C(\cdot)$:

$$C(\phi) \equiv \left(X - \frac{(1-\alpha)\gamma\eta}{1-\gamma(1-\eta)}\right)^2 + \left(\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X - \alpha\gamma\right)^2,$$

where the first and second terms show the deviations of physical and human capital in the political equilibrium allocation from those in the planner's allocation, respectively, for a given pair of (k, h). The deviations are assumed to be evaluated by quadratic functions to formulate the cost-minimization problem. By choosing a fiscal rule that minimizes this cost, we can bring the political equilibrium allocation closest to the planner's allocation. We should note that the cost is assessed each period for a given level of physical and human capital, so that the minimization problem is static in nature.

Figure 3 plots the impact of the fiscal rule on physical capital distortions, human capital distortions, and political distortions, for the three countries (Germany, Greece, and the United States).[9] The distortions in physical and human capital are plotted as the linear difference between the level in the political equilibrium and the level in the planner's allocation. In so doing, we can observe whether the levels of physical and human capital in the political

[9]The results for other countries are depicted in Figure 5 in Appendix A.4.

equilibrium are lower or higher than those in the planner's allocation. As observed in Figure 3, in the three country examples, the physical capital and the human capital in the political equilibrium allocations are lower than those in the planner's allocation, except for Greece's physical capital for low values of θ.

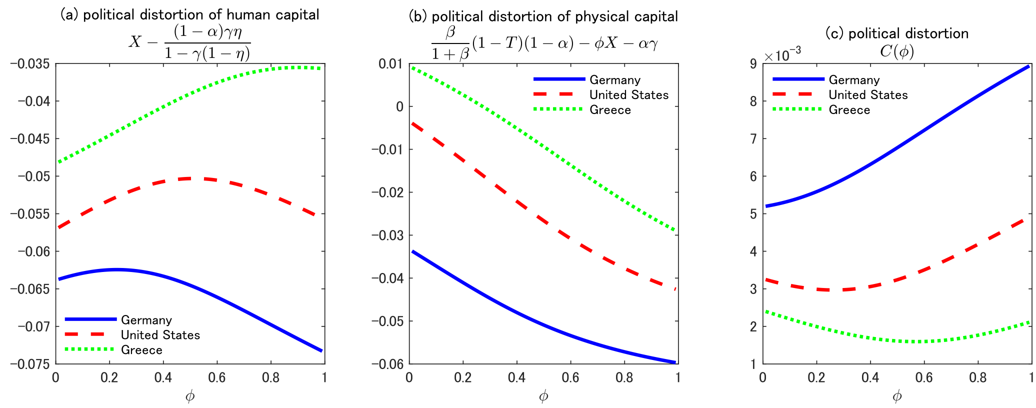

Figure 3: Effects of an increased ϕ on physical capital distortion (Panel (a)), human capital distortion (Panel (b)), and political distortions (Panel (c)) for Germany, Greece, and the United States.

First, we focus on the distortion of human capital, $X - (1-\alpha)\gamma\eta/[1-\gamma(1-\eta)]$, depicted in Panel (a) of Figure 3: as (41) shows, the fiscal rule ϕ has two opposing effects on public investment, denoted by X. There is a threshold of ϕ, at which the two opposing effects are balanced and X is maximized. In other words, at this threshold, distortion of human capital is minimized. This threshold level of ϕ depends on the preference parameter θ for public goods. As observed in the first term of the left-hand side of (41), the crowding-out effect of public debt issuance to finance public investment becomes stronger as the preference for public goods increases. Consequently, the level of ϕ that maximizes X (i.e., that minimizes the divergence of human capital) becomes lower as θ increases.

Next, we focus on the distortion of physical capital, $\beta(1-T)(1-\alpha)/(1+\beta) - \phi X - \alpha\gamma$, as shown in Panel (b) of Figure 3. In addition to the inverse U-shaped effect on X, an increase in ϕ has the effect of increasing the distortion arising from the decrease in physical capital through the crowding-out effect. This effect is observed in the term ϕ of $\phi \cdot X$. Owing to these two effects, an increase in ϕ leads to an increase in the distortion of physical capital. Meanwhile, an increase in ϕ shifts the burden of government spending from taxes to public debt. A decrease in the tax burden, T, leads to an increase in savings, and thus, an increase in physical capital. This has the effect of reducing the distortions in physical capital. Overall, the distortionary effect on physical capital through the $\phi \cdot X$ term exceeds the distortionary effect through T, and thus, the distortion of physical capital expands as ϕ increases.

We can summarize the above discussion by focusing on the preference parameter for public goods, θ, which affects the distortion of human capital but not that of physical capital. As θ

increases, the level of ϕ that minimizes distortions declines. Among the three example countries, Germany has the highest level of θ, and distortions are minimized at $\phi = 0$. However, for Greece and the United States, where the level of θ is lower than in Germany, there exists a ϕ that balances the impacts of marginal changes in ϕ on the distortions of human and physical capital; at this ϕ, the distortions are minimized. In particular, when the choice of fiscal rule is limited to $\phi = 0$ (BB) or $= 1$ (GR), $\phi = 0$ (BB) is optimal for the United States, which has a higher θ than Greece has, and $\phi = 1$ (GR) is optimal for Greece, which has a a lower θ than the United States has. Table 3 presents a list of countries in which the BB (GR) is optimal in terms of minimizing political distortions and thus, the planner's viewpoint.

The BB is optimal in terms of the planner's allocation	Australia, Austria, Belgium, Canada, Switzerland, Czech Republic, Germany, Spain, France, United Kingdom, Hungary, Ireland, Iceland, Israel, Italy, Japan, Lithuania, Latvia, Mexico, Netherlands, Poland, Portugal, Slovakia, Slovenia, Sweden, Turkey, United States
GR is optimal in terms of the planner's allocation	Greece

Table 3: List of countries for which the balanced budget rule and the golden rule of public finance is optimal in terms of the planner's allocation.

Table 4 reports the fiscal rules that are desirable in terms of the planner's allocation and those that are politically preferred for each country when the options are limited to the BB and the GR. This limitation allows us to identify whether each country's preferences for fiscal rules are more biased toward the BB or the GR. Several salient features can be identified from the results presented in Table 4. First, Greece is the only country for which the GR is desirable both in terms of the planner's allocation and maximizing the political objective. This result is because, among the sample countries, Greece has the lowest value of θ and the second lowest value of η. Second, the group of countries for which the BB is both politically preferred and optimal in terms of the planner's allocation includes Australia, Belgium, Canada, Switzerland, Germany, Iceland, Latvia, the Netherlands, and Sweden. These 9 countries are among the top 11 countries with the highest estimated θ among the sample countries.[10] In Germany, the BB rule has been in place since 2010 (OECD 2015:p.29). This suggests that Germany has been politically successful in implementing the BB, which is desirable from a long-term perspective. Halac and Yared (2018) compare Greece and Germany in terms of fiscal reform. Our results suggest that while these countries indeed have differing politically supported fiscal rules, those that they have in common are efficient for their own welfare.

Third, for the remaining countries, our results indicate that the BB has little support by voters, although it is desirable from the perspective of the planner's allocation. Among them,

[10]In the remaining two countries (Mexico and Turkey), θ is high, but the elasticity of public investment η is low; thus, the GR is politically preferred over the BB, as discussed in Section 4.

GR is optimal in terms of the planner's allocation and politically preferred	Greece
The BB is optimal in terms of the planner's allocation and politically prefered	Australia, Belgium, Canada, Switzerland, Germany, Iceland, Latvia, Netherlands, Sweden
The BB is optimal in terms of the planner's allocation but has littile support by voters	Austria, Czech Republic, Spain, France, United Kingdom, Hungary, Ireland, Israel, Italy, Japan, Lithuania, Mexico, Poland, Portugal, Slovakia, Slovenia, Turkey, United States

Table 4: Summary of the results in Tables 2 and 3.

Israel, Mexico, Poland, Spain, and the United Kingdom have introduced fiscal rules that control budget balance or debt, but public debt issuance for public investment is allowed for certain conditions (Kumar et al. 2009).[11] In Japan, construction bonds for public investment have been issued every year since 1966.[12] Furthermore, Japan and the United States, with the second and third highest ratios of general government debt to GDP among OECD countries, respectively,[13] are the only countries that have not adopted fiscal rules that control budget balance (OECD 2013). This result suggests that the introduction of the BB is necessary to control accumulated public debt in these two countries, but that there is no political support for it.

6 Conclusion

In recent years, the number of countries that have adopted fiscal rules has continued to increase, not only in developed countries, but also in developing countries. Many of those countries use public debt and budget deficit as a target for control. Deficit-financed public expenditures passes the fiscal burden on to future generations, thereby creating the issue of intergenerational equity. However, such a financing method may be supported by the principle that the beneficiary pays when expenditure contributes to the formation of future public capital, such as education and infrastructure.

The present study investigates what portion of public investment is financed by public debt issuance from the long-lived planner's viewpoint, and also analyzes what portion is preferred by short-lived voters. Calibrating the model to sample OECD countries, we find that in Greece, which has experienced a huge budget deficit and accumulated debt, voters prefer public investment fully financed by debt issuance and such a financing method is optimal from the planner's viewpoint. In Germany, which has run budget surpluses since 2012, voters prefer a balanced budget, which is also optimal from the planner's viewpoint. We show that these contrasting re-

[11] Public investment is also excluded from the application of fiscal rules in Luxembourg, the Netherlands, and Sweden.

[12] See Asako et al. (1991) for details of construction bonds in Japan.

[13] Source: OECD (2021), General government debt (indicator). doi: 10.1787/a0528cc2-en (Accessed on July 28, 2021).

sults arise from differences in preferences for public goods and the elasticity of public education expenditure to human capital formation. Our results suggest that these factors should be borne in mind when selecting fiscal rules in the future.

We also show that in some countries that have experienced accumulated public debt for the past decade, such as Japan and the United States, voters have weak preferences for the balanced budget, although it is desirable from the long-lived planner's viewpoint. This result indicates the political difficulty of financial reconstruction, and thus, suggests there is a need for some other institution or law that would allow for the introduction and maintenance of stricter fiscal rules. This research is left for future work.

A Proofs and Supplementary Explanations

A.1 Proof of Proposition 1

Under the assumptions of the logarithmic utility function and the Cobb–Douglas production function, the first-order conditions in (17) – (21) are reduced to

$$\tau^k : (-1)\tfrac{\omega}{(1+n)(1-\omega)}\tfrac{1}{1-\tau^k} + \lambda R(k,h)(k+b) = 0, \tag{A.1}$$

$$\tau : (-1)\frac{1}{1-\tau} + (-1)\frac{\alpha\beta\,(1+\theta)\,\frac{\beta}{1+\beta}w(k,h)h}{\frac{\beta}{1+\beta}\,(1-\tau)\,w(k,h)h - (1+n)b'} + \lambda w(k,h)h = 0, \tag{A.2}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} - \lambda\frac{2+n}{1+n} = 0, \tag{A.3}$$

$$x : \frac{\beta\eta\,(1+\theta)\,(1-\alpha)}{,}x - \lambda(1+n) = 0, \tag{A.4}$$

$$b' : (-1)\alpha\beta\,(1+\theta)\,\frac{1+n}{\frac{\beta}{1+\beta}\,(1-\tau)\,w(k,h)h - (1+n)b'} + \lambda(1+n) \leq 0. \tag{A.5}$$

Because the utility function fulfills the Inada condition, the choice of g and x realizes the interior solutions, and (A.3) and (A.4) are satisfied with equality. For the tax rates τ^k and τ, we allow a negative tax rate and take a sufficiently low lower bound to guarantee an interior solution. For public debt, we assume away government lending and impose non-negativity constraints: $b' \geq 0$. Thus, (A.5) holds with an equality if $b' > 0$; and with a strict inequality if $b' = 0$.

Suppose that $b' = 0$ holds. The condition in (A.5), holding with a strict inequality, is reformulated as

$$\lambda \leq \frac{\alpha\beta\,(1+\theta)}{\frac{\beta}{1+\beta}\,(1-\tau)\,w(k,h)h}. \tag{A.6}$$

The condition in (A.2) is reduced to

$$\lambda = \frac{1+\alpha\beta\,(1+\theta)}{(1-\tau)\,w(k,h)h}. \tag{A.7}$$

Substituting (A.7) into (A.6) leads to $1 \leq \alpha\,(1+\theta)$. Therefore, we conclude that

$$\begin{cases} b' = 0 & \text{if } 1 \leq \alpha\,(1+\theta)\,, \\ b' > 0 & \text{if } 1 > \alpha\,(1+\theta)\,. \end{cases}$$

In the following, we proceed by dividing the proof into two cases, $1 \leq \alpha\,(1+\theta)$ and $1 > \alpha\,(1+\theta)$.

A.1.1 Case of $1 \leq \alpha\,(1+\theta)$

First, we substitute (A.7) into (A.1), (A.3), and (A.4) and rearrange the terms to obtain

$$\tau^k R(k,h)(k+b) = R(k,h)(k+b) - \frac{1}{1+\alpha\beta\,(1+\theta)}\cdot\frac{\omega}{(1+n)(1-\omega)}\,(1-\tau)\,w(k,h)h, \tag{A.8}$$

$$(1+n)x = \frac{\beta\eta\,(1+\theta)\,(1-\alpha)\,.}{1+\alpha\beta\,(1+\theta)}\,(1-\tau)\,w(k,h)h, \tag{A.9}$$

$$\frac{2+n}{1+n}g = \frac{1}{1+\alpha\beta\,(1+\theta)}\left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\theta\,(1-\tau)\,w(k,h)h. \tag{A.10}$$

By substituting $b' = 0$ and (A.8)–(A.10) into the government budget constraint in (12), we obtain labor income tax as follows:

$$\tau = 1 - \frac{1}{1-\alpha} \cdot \frac{1+\alpha\beta(1+\theta)}{(1+\theta)\left[1+\beta(\alpha+\eta(1-\alpha))+\frac{\omega}{(1+n)(1-\omega)}\right]}. \tag{A.11}$$

With (A.8) and (A.11), we obtain the capital income tax rate as

$$\tau^k = 1 - \frac{\frac{\omega}{(1+n)(1-\omega)}}{(1+\theta)\left[1+\beta(\alpha+\eta(1-\alpha))+\frac{\omega}{(1+n)(1-\omega)}\right]} \cdot \frac{1}{\alpha\frac{k+b}{k}}, \tag{A.12}$$

which verifies the conjecture in (15).

We substitute (A.11) into (A.9) and (A.10) to obtain

$$(1+n)x = \frac{\beta\eta(1-\alpha)}{1+\beta(\alpha+\eta(1-\alpha))+\frac{\omega}{(1+n)(1-\omega)}}y(k,h), \tag{A.13}$$

$$\frac{2+n}{1+n}g = \frac{\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta}{(1+\theta)\left[1+\beta(\alpha+\eta(1-\alpha))+\frac{\omega}{(1+n)(1-\omega)}\right]}y(k,h), \tag{A.14}$$

where (A.14) verifies the conjecture in (16).

A.1.2 Case of $1 > \alpha(1+\theta)$

When $1 > \alpha(1+\theta)$, (A.5) holds with an equality. From (A.2) and (A.5), we obtain

$$(1+n)b' = \frac{\beta}{1+\beta}[1-\alpha(1+\theta)](1-\tau)w(k,h)h. \tag{A.15}$$

We substitute (A.15) into (A.2) and obtain

$$\lambda = \frac{1+\beta}{(1-\tau)w(k,h)h}. \tag{A.16}$$

Using (A.16), we can reformulate (A.1), (A.3), and (A.4) as follows:

$$1-\tau^k = \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{(1-\tau)w(k,h)h}{1+\beta} \cdot \frac{1}{R(k,h)(k+b)}, \tag{A.17}$$

$$\frac{2+n}{1+n}g = \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{1-\tau}{1+\beta}w(k,h)h, \tag{A.18}$$

$$(1+n)x = \frac{\beta}{1+\beta}\eta(1+\theta)(1-\alpha)(1-\tau)w(k,h)h. \tag{A.19}$$

We substitute (A.15), (A.17), (A.18), and (A.19) into the government budget constraint in (12) and rearrange the terms to obtain the labor income tax rate:

$$1-\tau = \frac{1}{\Lambda} \cdot \frac{1+\beta}{1-\alpha}. \tag{A.20}$$

We also obtain τ^k, g, x, and b' by substituting (A.20) into (A.17), (A.18), (A.19), and (A.15), respectively:

$$\begin{aligned} 1-\tau^k &= \frac{1}{\Lambda}\cdot\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{1}{\alpha^{\frac{k*b}{k}}}, \\ \frac{2+n}{1+n}g &= \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{\Lambda}y(k,h), \\ (1+n)x &= \frac{\beta\eta(1-\alpha)\,(1+\theta)}{\Lambda}y(k,h), \\ (1+n)b' &= \frac{\beta\left[1-\alpha\,(1+\theta)\right]}{\Lambda}y(k,h). \end{aligned}$$

The expressions in the first and second lines verify the conjectures in (15) and (16), respectively.

■

A.2 Proof of Proposition 2

Given the conjecture in (38) and (39), we can reformulate the first-order condition with respect to x in (37) as follows:

$$\lambda = \frac{\beta\,(1+\theta)}{(1-\phi)\,y\,(k,h)}\left[(-1)\,\frac{\alpha\phi\,(1+\theta,)}{\frac{\beta}{1+\beta}\,(1-\alpha)\,(1-T)-\phi X}+\frac{\eta\,(1-\alpha)}{X}\right]. \tag{A.21}$$

We can also reformulate the first-order condition with respect to τ in (35) as follows:

$$\lambda = \frac{1}{(1-\alpha)\,y\,(k,h)}\left[\frac{1}{1-T}+\frac{\alpha\beta\,(1+\theta)\,\frac{\beta}{1+\beta}\,(1-\alpha)}{\frac{\beta}{1+\beta}\,(1-\alpha)\,(1-T)-\phi X}\right]. \tag{A.22}$$

With (A.21) and (A.22), we obtain

$$\frac{\beta\,(1+\theta)\,\eta\,(1-\alpha)}{(1-\phi)}\cdot\frac{1}{X} = \frac{1}{(1-\alpha)\,(1-T)}+\frac{\alpha\beta\,(1+\theta)\left[\frac{\beta}{1+\beta}+\frac{\phi}{1-\phi}\right]}{\frac{\beta}{1+\beta}\,(1-\alpha)\,(1-T)-\phi X}, \tag{A.23}$$

where (A.23) includes two undetermined constants, T and X. Denote the left-hand (right-hand) side of (A.23) by $LHS^{(A.23)}$ ($RHS^{(A.23)}$). They have the following properties: $\partial LHS^{(A.23)}/\partial X < 0$, $\lim_{X\to 0} LHS^{(A.23)} = +\infty$, $\lim_{X\to+\infty} LHS^{(A.23)} = 0$, $\partial RHS^{(A.23)}/\partial X > 0$, $\lim_{X\to 0} RHS^{(A.23)} \in (0,+\infty)$, and $\lim_{X\to\frac{\beta}{1+\beta}(1-\alpha)(1-T)/\phi} RHS^{(A.23)} = +\infty$. These properties imply that given T, there is a unique X that satisfies (A.23): $X = X\,(T)$.

Given T and X, the first-order condition with respect to τ^k in (34) is reformulated as

$$1-\tau^k = \frac{\omega}{(1+n)(1-\omega)}\,(1-\alpha)\left[\frac{1}{1-T}+\frac{\alpha\beta\,(1+\theta)\,\frac{\beta}{1+\beta}\,(1-\alpha)}{\frac{\beta}{1+\beta}\,(1-\alpha)\,(1-T)-\phi X}\right]^{-1}\frac{1}{\alpha^{\frac{k+b}{k}}}. \tag{A.24}$$

Eq. (A.24) shows that the conjecture of τ^k in (15) is correct as long as T and X are constant.

Next, given T and X, the first-order condition with respect to g in (36) is reformulated as

$$\frac{2+n}{1+n}g = \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta\left(1+\theta\right)\frac{\beta}{1+\beta}\left(1-\alpha\right)}{\frac{\beta}{1+\beta}\left(1-\alpha\right)\left(1-T\right)-\phi X}\right]^{-1}\left(1-\alpha\right)y(k,h). \tag{A.25}$$

Eq. (A.25) shows that the conjecture of g in (16) is correct as long as T and X are constant.

The remaining task is to show that the conjectures of T in (38) and X in (39) are correct. Consider the government budget constraint in (13). We substitute the policy functions derived thus far into the constraint and rearrange the terms to obtain

$$\begin{aligned}
&\alpha-\frac{\omega}{(1+n)(1-\omega)}\left[\frac{1}{1-T}+\frac{\alpha\beta\left(1+\theta\right)\frac{\beta}{1+\beta}\left(1-\alpha\right)}{\frac{\beta}{1+\beta}\left(1-\alpha\right)\left(1-T\right)-\phi X}\right]^{-1}\left(1-\alpha\right)+T(1-\alpha)\\
&=\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta\left(1+\theta\right)\frac{\beta}{1+\beta}\left(1-\alpha\right)}{\frac{\beta}{1+\beta}\left(1-\alpha\right)\left(1-T\right)-\phi X}\right]^{-1}\left(1-\alpha\right)+\left(1-\phi\right)X.
\end{aligned} \tag{A.26}$$

The expression in (A.26) is independent of the state variables, k, h, and b, and times. Thus, we can verify that the two unknown parameters, X and T, are constant and solved for using (A.23) and (A.26).

■

A.3 Proof of Proposition 3

In the present framework, the state variable h_t does not line up in a compact set because it continues to grow along an optimal path. To reformulate the planner's problem into one in which the state variable lines up in a compact set, we undertake the following normalization:

$$\tilde{c}_t \equiv c_t/h_t,\ \tilde{d}_t \equiv d_t/h_t,\ \tilde{x}_t \equiv x_t/h_t,\ \tilde{k}_t \equiv k_t/h_t,\ \tilde{g}_t \equiv g_t/h_t.$$

Then, with the use of $h_{t+1} = D\left(h_t\right)^{1-\eta}\left(x_t\right)^{\eta}$, the resource constraint in (46) is reformulated as

$$\tilde{c}_t+\frac{\tilde{d}_t}{1+n}+(1+n)\tilde{k}_{t+1}D\left(\tilde{x}_t\right)^{\eta}+(1+n)\tilde{x}_t+\frac{2+n}{1+n}\tilde{g}_t = A\left(\tilde{k}_t\right)^{\alpha}. \tag{A.27}$$

Using the variables normalized by h, we can reformulate the utility functions of the period-0 old and the middle-aged agents as follows:

$$
\begin{aligned}
U_0^O &= \beta\left[\ln \tilde{d}_0 + \theta \ln \tilde{g}_0 + (1+\theta)\ln h_0\right],\\
U_0^M &= \ln \tilde{c}_0 + \theta \ln \tilde{g}_0 + (1+\theta)\ln h_0 + \beta\left[\ln \tilde{d}_1 + \theta \ln \tilde{g}_1 + (1+\theta)\ln D\left(\tilde{x}_0\right)^\eta h_0\right],\\
U_1^M &= \ln \tilde{c}_1 + \theta \ln \tilde{g}_1 + (1+\theta)\ln D\left(\tilde{x}_0\right)^\eta h_0 + \beta\left[\ln \tilde{d}_2 + \theta \ln \tilde{g}_2 + (1+\theta)\ln D^2\left(\tilde{x}_0\right)^\eta\left(\tilde{x}_1\right)^\eta h_0\right],\\
&\ \ \vdots\\
U_t^M &= \ln \tilde{c}_t + \theta \ln \tilde{g}_t + (1+\theta)\ln (D)^t\left(\tilde{x}_{t-1}\right)^\eta\left(\tilde{x}_{t-2}\right)^\eta \cdot \cdots \cdot \left(\tilde{x}_0\right)^\eta h_0\\
&\quad + \beta\left[\ln \tilde{d}_{t+1} + \theta \ln \tilde{g}_{t+1} + (1+\theta)\ln (D)^{t+1}\left(\tilde{x}_t\right)^\eta\left(\tilde{x}_{t-1}\right)^\eta \cdot \cdots \cdot \left(\tilde{x}_0\right)^\eta h_0\right].\\
&\ \ \vdots
\end{aligned}
$$

In particular, the utility of the period-t for the middle-aged is rewritten as

$$
\begin{aligned}
U_t^M &= \ln \tilde{c}_t + \theta \ln \tilde{g}_t + \beta \ln \tilde{d}_{t+1} + \beta\theta \ln \tilde{g}_{t+1}\\
&\quad + \eta(1+\beta)(1+\theta)\sum_{j=0}^{t-1}\ln \tilde{x}_j + \eta\beta \ln \tilde{x}_t + (1+\beta)(1+\theta)\ln h_0 + \left[t + \beta(t+1)\right](1+\theta)\ln D.
\end{aligned}
$$

Thus, by removing the terms that are irrelevant for determining the allocation, the social welfare function, denoted by SW, becomes

$$
\begin{aligned}
SW \simeq\ & \frac{\beta}{\gamma}\left(\ln \tilde{d}_0 + \theta \ln \tilde{g}_0\right)\\
&+ \ln \tilde{c}_0 + \theta \ln \tilde{g}_0 + \beta \ln \tilde{d}_1 + \beta\theta \ln \tilde{g}_1 + \eta\beta(1+\theta)\ln \tilde{x}_0\\
&+ \gamma \cdot \left[\ln \tilde{c}_1 + \theta \ln \tilde{g}_1 + \beta \ln \tilde{d}_2 + \beta\theta \ln \tilde{g}_2 + \eta(1+\beta)(1+\theta)\ln \tilde{x}_0 + \eta\beta(1+\theta)\ln \tilde{x}_1\right]\\
&+ \gamma^2 \cdot \left[\ln \tilde{c}_2 + \theta \ln \tilde{g}_2 + \beta \ln \tilde{d}_3 + \beta\theta \ln \tilde{g}_3 + \eta(1+\beta)(1+\theta)\left(\ln \tilde{x}_0 + \ln \tilde{x}_1\right) + \eta\beta(1+\theta)\ln \tilde{x}_2\right]\\
&+ \cdots,
\end{aligned}
$$

that is,

$$
SW \simeq \sum_{t=0}^{\infty}\gamma^t \cdot \left\{\ln \tilde{c}_t + \frac{\beta}{\gamma}\ln \tilde{d}_t + \theta\left(1+\frac{\beta}{\gamma}\right)\ln \tilde{g}_t + \eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]\ln \tilde{x}_t\right\}. \tag{A.28}
$$

By plugging (A.27) into (A.28), the planner's problem becomes

$$
\begin{aligned}
&\max \sum_{t=0}^{\infty}\gamma^t \cdot \left\{\ln\left[A\left(\tilde{k}_t\right)^\alpha - \frac{\tilde{d}_t}{1+n} - (1+n)\tilde{k}_{t+1}D\left(\tilde{x}_t\right)^\eta - (1+n)\tilde{x}_t - \frac{2+n}{1+n}\tilde{g}_t\right]\right.\\
&\left. + \frac{\beta}{\gamma}\ln \tilde{d}_t + \theta\left(1+\frac{\beta}{\gamma}\right)\ln \tilde{g}_t + \eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]\ln \tilde{x}_t\right\}
\end{aligned}
$$

given $\tilde{k}_0$.

We can express the Bellman equation for the problem as follows:

$$
\begin{aligned}
V(\tilde{k}) = \max_{\{\tilde{d},\tilde{x},\tilde{k}'\}} &\left\{\ln\left[A\left(\tilde{k}\right)^\alpha, -\frac{\tilde{d}}{1+n} - (1+n)\tilde{k}'D\left(\tilde{x}\right)^\eta - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}\right]\right.\\
&\left. + \frac{\beta}{\gamma}\ln \tilde{d} + \theta\left(1+\frac{\beta}{\gamma}\right)\ln \tilde{g} + \eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]\ln \tilde{x} + \gamma V(\tilde{k}')\right\}.
\end{aligned} \tag{A.29}
$$

We make the guess that $V(\tilde{k}') = z_0 + z_1 \ln \tilde{k}'$, where z_0 and z_1 are undetermined coefficients. For this guess, (A.29) becomes

$$V(\tilde{k}) = \max_{\{\tilde{d},\tilde{x},\tilde{g},\tilde{k}'\}} \left\{ \ln \left[A\left(\tilde{k}\right),^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g} \right] \right.$$
$$\left. + \frac{\beta}{\gamma} \ln \tilde{d} + \theta \left(1 + \frac{\beta}{\gamma}\right) \ln \tilde{g} + \eta(1+\theta) \left[\beta + \frac{\gamma\left(1+\beta\right)}{1-\gamma}\right] \ln \tilde{x} + \gamma \left[z_0 + z_1 \ln \tilde{k}'\right] \right\}. \quad \text{(A.30)}$$

The first-order conditions with respect to $\tilde{d}$, $\tilde{x}$, $\tilde{g}$, and $\tilde{k}'$ are

$$\tilde{d}: \frac{-\frac{1}{1+n}}{A\left(\tilde{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \frac{\beta}{\gamma}, \cdot \frac{1}{\tilde{d}} = 0, \quad \text{(A.31)}$$

$$\tilde{x}: \frac{-(1+n)\left[\eta\tilde{k}' D\left(\tilde{x}\right)^{\eta-1} + 1\right]}{A\left(\tilde{k}\right)^{\alpha}, - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \frac{\eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]}{\tilde{x}} = 0, \quad \text{(A.32)}$$

$$\tilde{g}: \frac{-\frac{2+n}{1+n}}{A\left(\tilde{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \theta\left(1 + \frac{\beta}{\gamma}\right) \cdot \frac{1}{\tilde{g}} = 0, \quad \text{(A.33)}$$

$$\tilde{k}': \frac{-(1+n)D\left(\tilde{x}\right)^{\eta}}{A\left(\tilde{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \frac{\gamma z_1}{\tilde{k}'} = 0. \quad \text{(A.34)}$$

Eqs. (A.31) and (A.34) lead to

$$\frac{\tilde{d}}{1+n} = \frac{\beta}{\gamma} \cdot \frac{\tilde{k}'(1+n)D\left(\tilde{x}\right)^{\eta}}{\gamma z_1}, \quad \text{(A.35)}$$

Eqs. (A.32) and (A.34) lead to

$$\gamma z_1 = \eta \tilde{k}' D\left(\tilde{x}\right)^{\eta-1} \left\{(1+\theta)\left[\beta + \frac{\gamma\left(1+\beta\right)}{1-\gamma}\right] - \gamma z_1\right\}, \quad \text{(A.36)}$$

and Eqs. (A.33) and (A.36) lead to

$$\frac{2+n}{1+n}\tilde{g} = (1+n)D\left(\tilde{x}\right)^{\eta} \theta \left(1 + \frac{\beta}{\gamma}\right) \frac{\tilde{k}'}{\gamma z_1}. \quad \text{(A.37)}$$

By substituting (A.35) and (A.37) into (A.34) and rearranging the terms, we obtain

$$\gamma z_1 (1+n)\tilde{x} = \gamma z_1 A\left(\tilde{k}\right)^{\alpha} - \tilde{k}'(1+n)D\left(\tilde{x}\right)^{\eta} \left[(1+\theta)\left(1 + \frac{\beta}{\gamma}\right) + \gamma z_1\right]. \quad \text{(A.38)}$$

We multiply both sides of (A.36) by $(1+n)\tilde{x}$ and rearrange the terms to obtain

$$(1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} = \frac{\gamma z_1/\eta}{(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right] - \gamma z_1}(1+n)\tilde{x}. \quad \text{(A.39)}$$

Using (A.35) – (A.39), we obtain

$$\frac{\tilde{d}}{1+n} = \frac{\frac{\beta}{\gamma\eta}}{(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma} + \frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right] - \gamma z_1\left(1-\frac{1}{\eta}\right)} A\left(\tilde{k}\right)^{\alpha}, \tag{A.40}$$

$$(1+n)\tilde{x} = \frac{(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right] - \gamma z_1}{(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma} + \frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right] - \gamma z_1\left(1-\frac{1}{\eta}\right)} A\left(\tilde{k}\right)^{\alpha}, \tag{A.41}$$

$$\frac{2+n}{1+n}\tilde{g} = \frac{\frac{\theta}{\eta}\left(1+\frac{\beta}{\gamma}\right)}{(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma} + \frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right] - \gamma z_1\left(1-\frac{1}{\eta}\right)} A\left(\tilde{k}\right)^{\alpha}, \tag{A.42}$$

$$(1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} = \frac{\frac{\gamma z_1}{\eta}}{(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma} + \frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right] - \gamma z_1\left(1-\frac{1}{\eta}\right)} A\left(\tilde{k}\right)^{\alpha}. \tag{A.43}$$

We substitute (A.40)–(A.43) into the Bellman equation in (A.30) and obtain

$$V(\tilde{k}) = \left\{\alpha(1+\theta)\left[\left(1+\frac{\beta}{\gamma}\right) + \eta\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]\right] + \alpha\gamma z_1(1-\eta)\right\}\ln\tilde{k} + C(z_0, z_1),$$

where $C(z_0, z_1)$ is the collective notation for constant terms. The guess is verified if $z_0 = C(z_0, z_1)$ and

$$z_1 = \alpha(1+\theta)\left[\left(1+\frac{\beta}{\gamma}\right) + \eta\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]\right] + \alpha\gamma z_1(1-\eta).$$

Therefore, z_1 is given by

$$z_1 = \frac{\alpha(1+\theta)\left\{\left(1+\frac{\beta}{\gamma}\right) + \eta\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]\right\}}{1-\alpha\gamma(1-\eta)},$$

and the corresponding policy functions are obtained as expressed in Proposition 3.

Using the policy functions presented in Proposition 3, we demonstrate the accumulation of physical and human capital. the physical capital accumulation, k'/k, is computed using the resource constraint in (46) and the policy functions in Proposition 1 as follows:

$$\frac{k'}{k} = \frac{\alpha\gamma}{1+n} A\left(\frac{h}{k}\right)^{1-\alpha}. \tag{A.44}$$

The human capital accumulation, h'/h, is computed using the human capital formation function in (1) and the policy function of x in Proposition 3 as follows:

$$\frac{h'}{h} = D\left[\frac{1}{1+n}\cdot\frac{(1-\alpha)\gamma\eta}{1-\gamma(1-\eta)} A\left(\frac{k}{h}\right)^{\alpha}\right]^{\eta}. \tag{A.45}$$

■

A.4 Supplement to Numerical Analysis

Figures 4 and 5 depict the results of the same numerical experiments as in Figures 2 and 3, respectively, for the remaining countries not covered in the text.

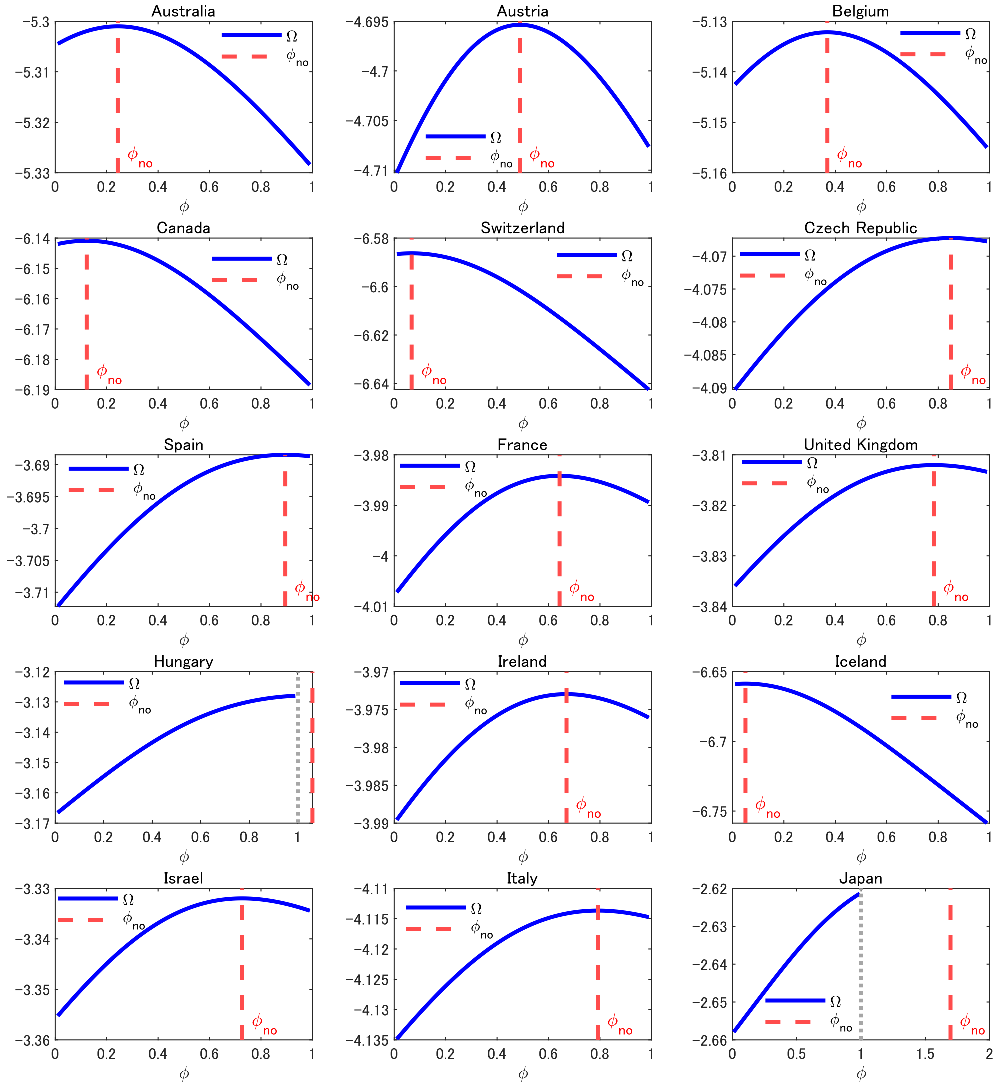

Figure 4: Effects of an increased ϕ on the political objective function for countries not covered in the text.

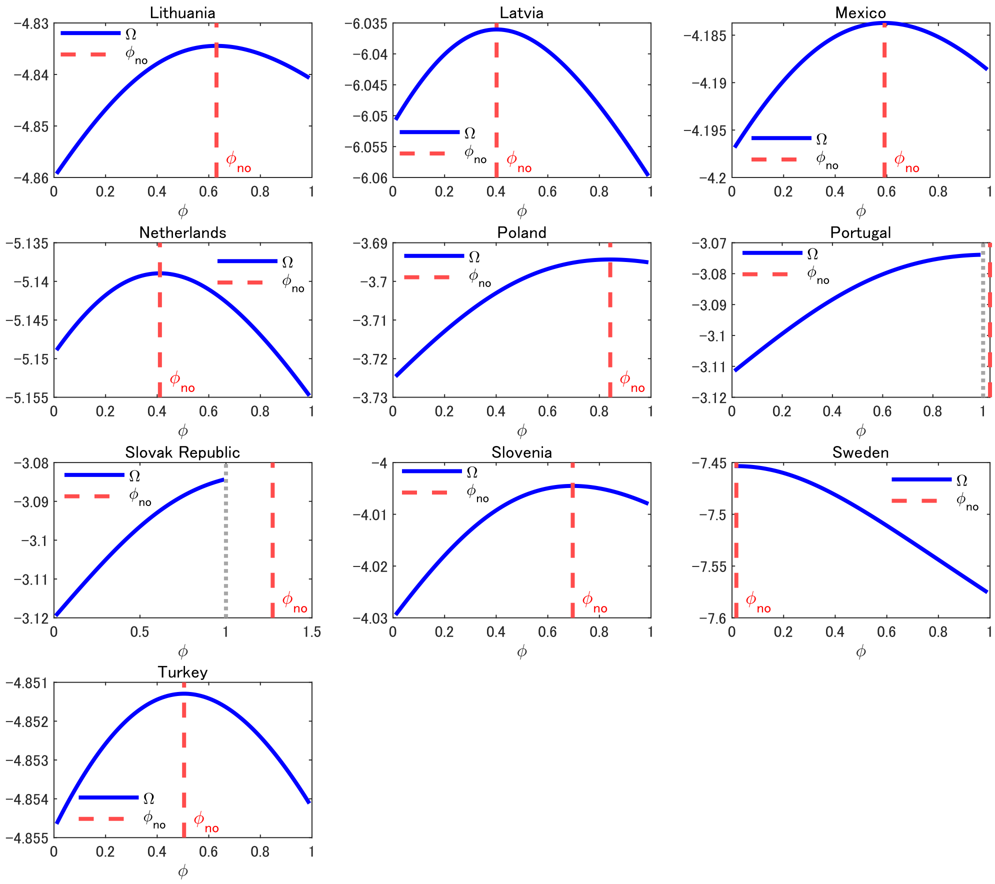

Figure 4 Continued.

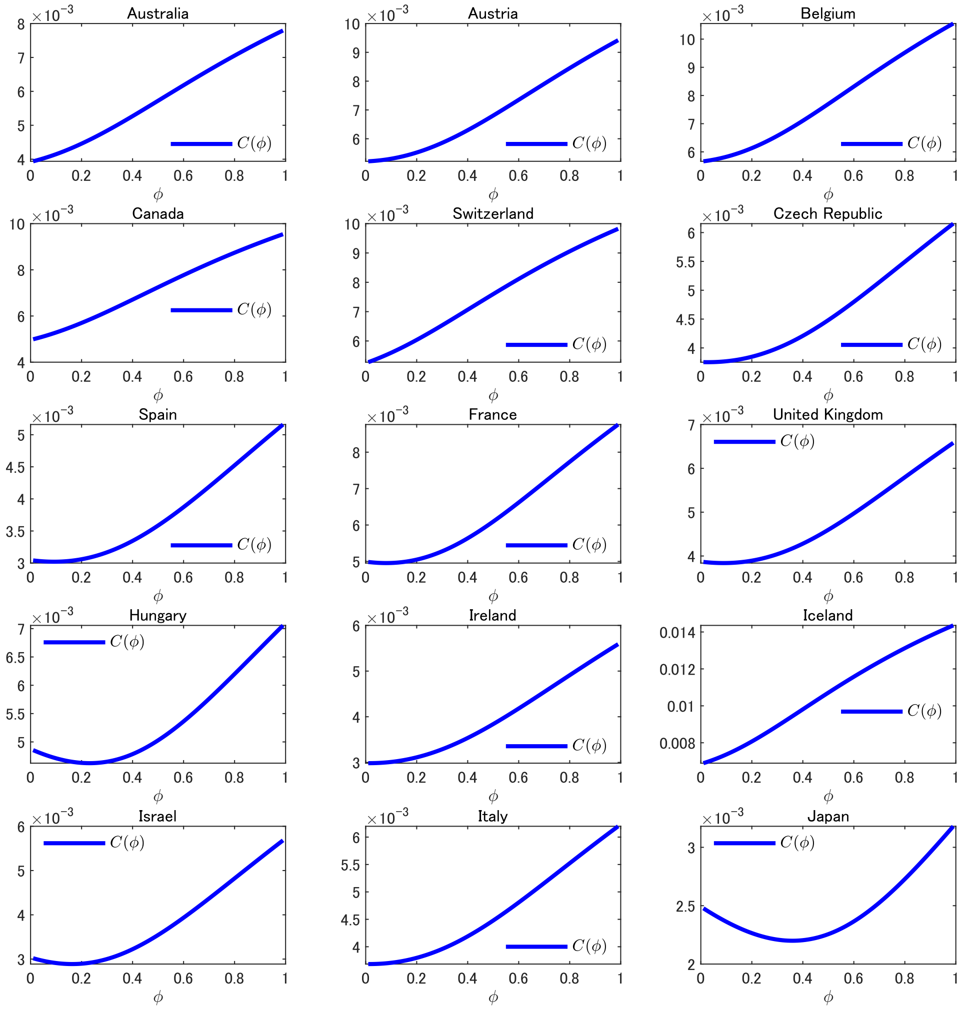

Figure 5: Effects of an increased ϕ on political distortions for countries not covered in the text.

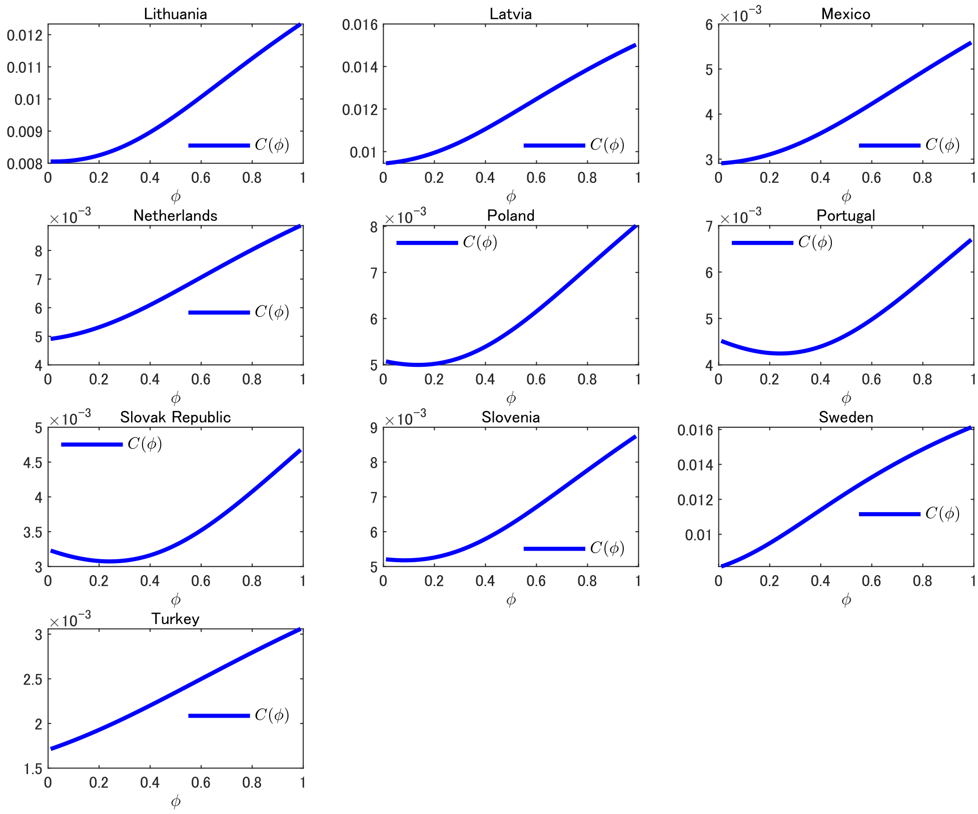

Figure 5 Continued.

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