A Theoretical Basis for the Environmental Kuznets Curve

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2002

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1st Revision
October 4, 2004.

Abstract

This paper attempts to explain the Environmental Kuznets Curve (EKC) or inverted U-shaped relationship between income and environmental degradation in the framework of endogenous growth model. Considering a closed economy, one part of capital is used for commodity production, which generates pollution that degrades existing environment, and the remaining part is used for abating pollution (i.e., upgrading environment). Sufficient abatement activity improves / restores environmental quality. A sufficient abatement activity (associated with commodity production) could only lead optimally towards steady state. The ratio of allocation of capital between two sectors (production and abatement) is fixed along the optimal path, but it varies along the non-optimal path that exists in the off-steady state. In the economy, allocation of capital for abatement activity varies over time. Thus, a change from insufficient to sufficient allocation of capital (i.e., investment) for abatement activity is the basis for an inverted U-shaped relationship between environmental quality and economic growth.

JEL *Classification Number*: H41, O40, Q20.

Keywords: EKC, abatement activity, Environmental quality, Two-sector model, and Endogenous growth model.

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1. Introduction

Worldwide environmental degradation makes people worried about the issue of the link between economic growth and environmental degradation. A sizeable literature[1] on that subject, both theoretical as well as empirical, has grown in recent period. Among the vast empirical studies an important finding is the Environmental Kuznets Curve (*EKC*), viz., the inverted U-shaped relationship between pollution and economic growth. Environmental quality deteriorates initially and improves with economic development in later stage. The literature has mostly considered *EKC* as an empirical phenomenon. In order to strengthen connection between theoretical and empirical analyses, one needs models and stylized facts. One should analyze theoretical model(s) and observe under what conditions *EKC* is generated. An empirical regularity provides a relevant dimension for calibration of environmental aspects of growth model. This is useful to evaluate models on the basis of their analytical tractability and that of their compatibility with facts. The empirical evidence depends only on the reduced-form rather than the structural-forms. So, the question, why pollution-income follows this inverted U-shaped curve is not yet resolved fully.

Lopez (1994) and Selden and Song (1995) consider exogenous technological change and pollution is generated by production. The relationship between pollution and income level depends on the elasticities of substitution of goods and the risk preference of household (Lopez 1994). John and Pecchenino (1994) develop model based on overlapping generations where pollution is generated by consumption rather than production. Stokey (1998) allows endogenous technological change and Lieb (2002) generalizes Stokey's model and argues that satiation in consumption is needed to generate *EKC*. Andreoni and Levinson (2001) show that economies of scale in abatement are sufficient

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[1] See, for example, Agras and Chapman (1999), Bimonte (2002), Cole et al. (1997), Dinda et al. (2000), Gawande et al. (2000), Grossman and Krueger (1995), Munasinghe (1999), Pasche (2002), Rothman (1998), Selden and Song (1994), Shafik (1994), Suri and Chapman (1998), Tisdell (2001), World Bank (1992).

to generate *EKC*. They derive it directly from technological link between consumption of a desired good and abatement of its undesirable byproduct. The role of abatement expenditure is crucial to reduce the pollution in production side (Selden and Song (1994)), however, the abatement activity starts when a considerable capital stock is achieved (Selden and Song (1995)). In addition, Lopez (1994) and Bulte and Soest (2001) develop models for the depletion of natural resources[2] such as forest or fertility of land. Thus, these models generate *EKC*s under appropriate assumptions. Recently, Stern (2004) reviews the latest theories and Brock and Taylor (2004) discuss that pollution declines in high-income countries due to technological change.

This paper lays out an explanation for the *EKC*, not only that it also discusses the dynamics of the *EKC* in the framework of endogenous growth model[3]. A distinctive feature of the model is that the environmental capital enters into the utility function as well as the production function. Thus, this paper is slightly different from the existing literature[4]. Most of the existing papers have focused on the amenity value of environment without considering the environment as a productive asset. One important and unique feature of the production function is the substitutability of man-made capital (physical and human capital) and natural capital (i.e., environmental assets) that ensures long run

growth. It is closely connected to the view of Tahvonen and Kuuluvainen (1993) – 'a model of economic growth with stock pollution allows smooth substitution between emissions and capital'. Pollution is endogenous in our model and abatement activity plays a crucial role to check the environmental degradation. However, our model actually combines stock of capital,

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[2] See, Krautkraemer (1985), Tahvonen and kuuluvainen (1993).

[3] See, Aghion and howitt (1998), Mulligan and Sala-i-Martin (1993), Barro and Sala-i-Martin (1995), Bovenberg and Smulders (1995), Elbasha and Roe (1996), Michel and Rotillon (1995), Beltratti (1996) etc.

[4] There is an extensive literature on pollution and growth, including early papers by Keeler et al. (1972), D'Arge and Kogiku (1973) Forster (1973), Gruver (1976) and more recent work by Tahvonen and kuuluvainen (1993), Gradus and Smulders (1993), John and Pecchenino (1994) etc.

pollution, stock of environment (natural capital) in an endogenous growth model. In addition, it discusses the transitional dynamics rather than just steady state.

The rest of the paper is organized as follows : an endogenous growth model is built up on the basis of capital and environmental stock in section 2, for analytic tractability specific models are used in section 3, the steady state condition is discussed in section 4, the existence of *EKC* is examined in section 5 and finally, the conclusion is drawn.

2. Model Setup

Consider a one-good (closed) economy for which environment, *E*, understood as a stock variable, affects production level and utility of the representative agent. For simplicity, we consider this economy consists of a single economic agent[5] who acts as producer as well as consumer (or we can think of a central planner's problem). The representative agent maximizes her present value of utility (or welfare of the society):

$$\text{Maximize } W = \int_0^\infty e^{-\rho t} U(C(t), E(t))dt;\ U_C, U_E > 0;\ U_{CC}, U_{EE} < 0;\ U_{CE} > 0. \quad (1.1)$$

Where C, E and $\rho\,(>0)$ are composite consumption bundles, stock of environment (representing the stock of natural resources: land, air, water, flora and fauna, etc.) and rate of time preference, respectively.

Production function of the economy[6] is

$$Y = f(K_y, E);\ f_k,\ f_E > 0;\ f_{kk}, f_{EE} < 0. \qquad (1.2)$$

Where K denotes composite capital stocks (physical and human capital) and *E* is environmental stocks. In production function, E also captures the productivity effect of the

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[5] We abstract from all issues concerning population growth and labour supply by assuming a constant labour supply normalized to equal unity. Thus, all the variables should be interpreted as variables per capita.

[6] In Forster (1973) model, the production function depends only on the stock of capital in the economy, in addition to this, our model adds natural resource or environment as input.

environmental quality (for example, the effect of air and water quality on health[7], etc.). E is not the choice variable to individual agents. Representative agent views the environmental stock (E) which is given, although E(t) varies in the aggregate. The two inputs, K and E, are essential for production.

Let γ (>0) be the rate of pollution (i.e., emission or degradation of environment per unit of output produced). Pollution (or emission) is fixed[8] proportion of output or production.

$$P=\gamma Y, \quad \gamma>0. \tag{1.3}$$

In general, the quality of the environment is itself endogenous. In aggregate, E is a stock that is depleted gradually by the flow of pollutants (or emission) P, as

$$\dot{E}=-P, \text{ or } \dot{E}=-\gamma f(K_y, E) \tag{1.4}$$

This implies that stock E is decreasing over time. So, as $t \to \infty$, $E \to 0$; therefore, as $f(k_y, 0)=0$, the economy will collapse. It is true that significantly high enough pollution damages production. For example, acid rain destroys natural capital like forests, soil, river water etc and also damages (indirectly) human capital. Thus, pollution affects output by damaging the inputs used to produce the output. Here lies the importance of abatement activity which upgrades (or clean up) environment. For this abatement activity, economy needs some capital[9] (K_E, say) which helps to improve environmental quality. We assume that abatement activity depends on man-made (physical & human) capital alone, i.e.,

$$A=h(K_E) \tag{1.5}$$

So, net change in E over time is: $\dot{E}=A-P$. Now, at each point of time, representative agent allocates optimally her stock of capital for both commodity production and abatement activity.

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[7] Human health may be harmed due to pollution and thus, economic productivity may be lost. See also Gangadharan and Valenzuela 2001.

[8] γ may be a decreasing function of technology (or R & D). For simplicity, we assume a constant γ.

[9] It should be mentioned that this model is a model for a country developing technology endogenously without the possibility of adopting a ready made foreign technology. See, Parente and Prescott (2000) for foreign technology, especially for technology leader.

Given total amount of capital ($K = K_y + K_E$), optimal sectoral allocation of capital for both sectors at any moment of time is governed by $\frac{\partial Y}{\partial K_Y} = q\frac{\partial A}{\partial K_E}$, where q is the price of environment relative to physical capital. This is true only in pollution free world, whereas in the polluted world, $(1-q\gamma)\frac{\partial Y}{\partial K_Y} = q\frac{\partial A}{\partial K_E}$ is true, where $q\gamma\frac{\partial Y}{\partial K}$ is the cost of pollution (or externality). This condition states that the net marginal product of capital should be same in two sectors.

Now, $K = K_y + K_E$, or $K_y = \theta K$, where $\theta = \frac{K_y}{K}$. Therefore, $K_E = (1-\theta)K$. (1.6)

Let $\theta(t)$ $(0<\theta(t)<1)$ portion of capital stock be used for commodity production at time *t* and the remaining $(1-\theta(t))$ portion be used for upgrading (or cleaning) environment[10]. Now, Production function and environmental upgrading function can be re-expressed as $Y = f(\theta K\ ,E)$ and $A = h((1-\theta)K\ )$, respectively. So, the accumulation constraints are (for simplicity, we assume, natural depreciation rates are zero for both the stocks, i.e., $\delta_i = 0$, $i = K,E$).

$$\dot{K}(t) = f(\theta(t)K(t),E(t)) - C(t) \tag{1.7}$$

and $$\dot{E}(t) = h((1-\theta(t))K(t)) - \gamma f(\theta(t)K(t),E(t)) \tag{1.8}$$

Clearly, the first constraint (eq. 1.7) relates to physical capital accumulation while the second (eq. 1.8) relates to net environmental change due to production and environmental upgrading. The infinite time horizon inter-temporal consumption choice problem for this economy may be specified as

$$\text{Maximize } W = \int_0^\infty e^{-\rho t} U(C(t),E(t))dt$$

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[10] Our model is closely related to Gruver (1976). Using neoclassical growth model, Gruver (1976) examines the optimal division of investment between pollution control capital and directly productive capital.

subject to the accumulation constraints (1.7) & (1.8). In this economy, representative agent has control over her consumption C as well as capital allocation θ for both commodity production and upgrading environment. Treating *C(t)* and *θ (t)* as control variables and *K(t)* and *E(t)* as state variables (and suppressing the time suffixes of the variables), the current value Hamiltonian for the above optimization problem may be written as

$$H = U(C,E) + \lambda[f(\theta K,E) - C] + \mu[h((1-\theta)K) - \gamma f(\theta K,E)] \qquad (2)$$

Where $\lambda(t)$ and $\mu(t)$ are two co-state variables (i.e., the shadow prices) corresponding to state variables *K* and *E*, respectively. The necessary conditions for an optimal solution of the problem are given in appendix A. Now we rewrite first two conditions as:

$$\frac{\delta H}{\delta C} = U_C - \lambda = 0 \Rightarrow U_C = \lambda \qquad (2.1)$$

$$\frac{\delta H}{\delta \theta} = \lambda f_K - \mu(h_K + \gamma f_K) = 0 \Rightarrow \mu = \frac{\lambda f_K}{h_K + \gamma f_K} \qquad (2.2)$$

Where U_C, f_K and h_K denote the marginal utility of consumption, marginal productivity of capital in commodity production and in environment upgrading, respectively. While condition (2.1) asserts the consumption-accumulation indifference at the margin, condition (2.2) requires the value of marginal product of capital in commodity production is equal to the value marginal of environmental quality loss due to withdrawal of capital from environment upgrading. Rearranging the conditions given in appendix A, we obtain consumption path[11]

$$\frac{\dot{C}}{C} = \frac{-U_C}{CU_{CC}}\left[\frac{f_k h_k}{h_k + \gamma f_k} - \rho + \frac{EU_{CE}}{U_C}\frac{\dot{E}}{E}\right] \qquad (3a)$$

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[11] Since, the utility function and production function are strictly concave, it follows from Cass (1965) that eq. (A2.1) through eq. (A2.6), along with the transversality conditions $\lambda(t)e^{-\rho t} \to 0$ as $t \to \infty$, and $\mu(t)e^{-\rho t} \to 0$ as $t \to \infty$, are a sufficient characterization of the unique optimum path (solving the planner's problem).

Where U_{CC}, U_{CE} being the second ordered partial derivatives of $U(.)$. Note that the above condition suggests those optimal time paths of *C* and *E* should generally be interdependent. If, however, $U_{CE}(U_{CC})$ turns out to be identically zero, the optimal time path of *C* (*E*) will be autonomous and the nature of the optimal time path of *E* (*C*) will depend upon what the optimal path of the other variable is.

3. Model Specification

Now, we consider specific functions rather than general function, with a parametric specification of the functions for analytic tractability. It helps to identify some key concepts. The utility function is given by customary form of constant elasticity of marginal utility with respect to its argument. The utility function[12]:

$$U(C,E) = \frac{(C^{1-\upsilon}E^{\upsilon})^{1-\sigma} - 1}{1-\sigma} \qquad (3.1)$$

The parameter υ ($0<\upsilon<1$) and (1-υ) represent preferences towards the environment and consumption, respectively. Here, σ is the usual elasticity of the marginal utility with respect to its argument and it is also known as intertemporal elasticity of substitution. Imposing condition $U_{CE} > 0$, implies that $\sigma < 1$. Conditions $U_{EE} < 0$, $U_{CC} < 0$, imply that $\upsilon(1-\sigma) < 1$ and $\sigma > 0$.

The technology for upgrading environmental quality or abatement function is assumed to be linear; in other words, constant returns to capital are assumed. Abatement function is

$$A = A_1 K_E = A_1(1-\theta)K \qquad (3.2)$$

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[12] This is different from others, for example, Asako (1980) analyzes the interactions between capital accumulation and pollution under the constant utility, which depends on per capita consumption and pollution. See, also Becker (1982).

This is an AK type technology for upgrading the environmental quality or abatement activities[13]. With these specifications, the equilibrium growth rate of consumption (equation 3a) will be

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}[\frac{A_1 f_K}{A_1 + \gamma f_K} + \upsilon(1-\sigma)(\frac{\dot{E}}{E}) - \rho] \qquad (3b)$$

This is the optimal path of consumption of the economy. In this model, we find that economic growth ($\frac{\dot{C}}{C}$) is affected by the net change in environmental quality (or $\dot{E}$) or environmental growth rate (i.e., $\frac{\dot{E}}{E}$). Since, $\upsilon > 0$ and $\sigma < 1$ imply that $\upsilon(1-\sigma) > 0$ which is the common assumption in the literature.

Remark 1: *An improvement in Environmental quality increases the marginal utility of consumption, raising the incentive to consume at all times and thus to accumulate.*

An improvement in environmental quality ($\dot{E} > 0$) raises the equilibrium growth rates, *ceteris paribus*. This is the situation in which both equilibrium growths increase and environmental quality improves. If utility function is an additive form (i.e., $U_{CE} = 0$), or if we drop E from utility function (eq.(1.1)), then eq.(3a) is the standard economic growth rate (i.e., $\dot{E}/E$ vanishes from eq. (3a)). Now if $\dot{E}/E < 0$, following eq.(3a), the optimum economic growth rate falls. Hence, the proposition follows

Proposition 1: ***Economic growth remains at optimum provided the economy ensures the non-negativity of the environmental growth.***

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[13] It differs from J type function. Selden and Song (1995) provides a theoretical basis for recent empirical research on J curves for abatement effort and inverted U curves for pollution. The result of Andreoni and Levinson (2001) crucially depends on increasing returns in abatement function. See also Withagen (1995).

The equilibrium growth rate of capital allocation (θ) is

$$\frac{\dot{\theta}}{\theta} = \frac{-1}{(\theta K) f_{KK}} \left[\frac{U_E}{U_C} \frac{(A_1 + \gamma f_K)^2}{A_1} + (A_1 + \gamma f_K) f_E + f_{KE} \dot{E} + f_{KK} (\theta \dot{K}) - f_{KK}^2 \right] \quad (3.3)$$

For analytical purpose, let us consider Cobb-Douglas type production function,

i.e., $Y = f(\theta K, E) = B(\theta K)^{\alpha} E^{1-\alpha}$ (3.4)

4. Steady State

This model has two control variables (C and θ) and two state variables (K and E). Hence, the phase diagram of the planning problem is four-dimensional. In general, the four-dimensional problem is hard to study analytically. It will be easy to analyze the problem if we reduce four-dimensional problem to three or two-dimension. Four variables can be reduced to three variables by normalizing or simply dividing by one variable, say E. After eliminating θ , we obtain c (C/E) and k (K/E), which are analyzed in two-dimensional plane (See, figure A.1 in Appendix B). For analytical purpose some restrictions are imposed, which may help to trace out the movements of E and K variables in E- K plane (See, Appendix C). The change of the control variables C and/or θ affect the locus of $\dot{E} = 0$ and $\dot{K} = 0$ in E- K plane (see, figure A.2 in Appendix B). There are several possible positions of $\dot{E} = 0$ and $\dot{K} = 0$, taking one specific situation, we concentrate and focus on the movement of the variables E and K in E-K plane such that one possible situation produces *EKC* (other cases are shown in figure A.3a and figure A.3b in Appendix B).

In this study, we show diagrammatically one particular initial value of K and E leads optimally towards steady state. With initial capital K_0 (very low) and environment E_0 (very high), one economy starts to grow and in the process of development, economy accumulates capital and reduces environmental stock over time (see, line 2, in fig.1). In other words, there is a trade off between environment and capital, thus, economic growth is possible at the cost of environmental degradation. This is a natural phenomenon of an under developed economy which is ready to forgo environmental stock for economic growth. This developmental process continues till economy attains the steady state.

Let us consider it in different way, the capital, K, is scarce and environmental resource, E, is abandoned in under developed economy. The (shadow) price of E (i.e., μ) is unchanged but that of K (i.e., λ) increases. So, the relative price of environment, *q*, decreases. With the process of economic development in underdeveloped countries, K increases and E approximately remains constant or slowly declines, therefore, *k* (or, K/E) increases towards k^* (optimum capital stock) while *q* decreases. In the reverse situation, *k* declines with rising *q*. The economy attains a stable equilibrium at S (corresponding to k^*) following either line 2 (or 7) or line 4 in Fig.1. The usual standard proposition follows

Proposition 2: ***If k(0)<k***, k increases towards k** *as q decreases and if k(0)>k***, k decreases towards k** *while q increases towards q***.***

The steady state situation is $\frac{\dot{C}}{C}=\frac{\dot{K}}{K}=\frac{\dot{Y}}{Y}=\frac{\dot{E}}{E}=\frac{\dot{A}}{A}=\frac{\dot{\theta}}{\theta}=0$, where θ and C/K, C/E and K/E ratios are constant. In the transitional dynamics, the environment continuously falls (see, arrow line marked 2 and 7 in fig.1) and halts at steady state where no more environmental deterioration takes place. So, inverted U-curve (or *EKC*) does not arise either at steady state or in transitional dynamics moving (along optimum path) towards steady state.

Remark 2: *In transitional dynamics, a trade-off between economic growth and environmental degradation should be optimal and that is the necessary condition for the convergence towards Steady State.*

5. Existence of EKC

The economic development at the cost of environmental degradation may continue for long time (along WS in fig. 1) or after some time, both E and K may fall (see curve 1) or rise (see, curve 3 in figure). Along line 2 (or 7) $\dot{K} \succ 0$ and $\dot{E} \prec 0$, with growth it moves towards $\dot{K} = \dot{E} = 0$ at S, whereas curve 3 has no tendency towards S or $\dot{K} = \dot{E} = 0$.

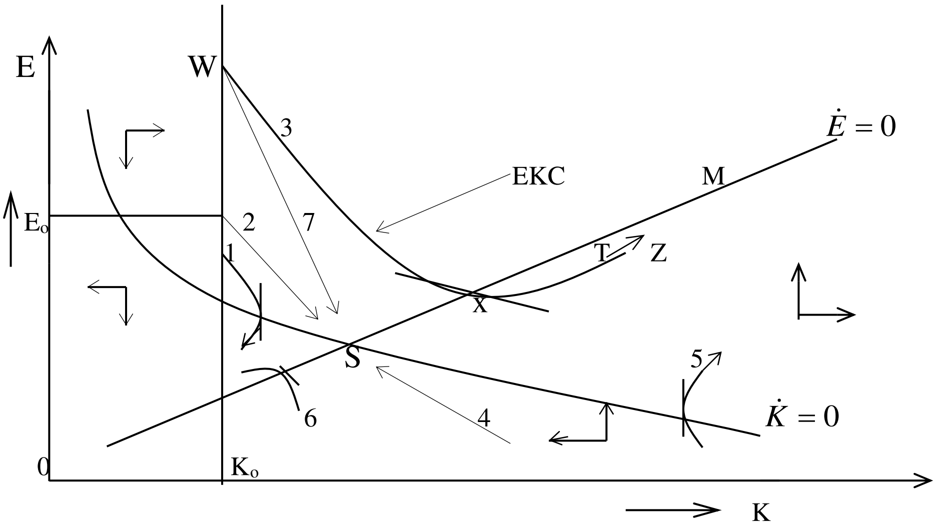

Figure 1: Transitional dynamics

From the initial point W, the economy might follow optimal allocation of capital along WS (See, line 7 in Fig.1) which moves toward equilibrium point 'S'. So, the rate of environmental degradation is fixed along WS. Thus, the optimal path of E decreases monotonically as K increases and stabilizes at S. It should be mentioned that some empirical evidences confirm it, especially the municipal waste and CO_2 emission increase monotonically whereas the access of safe drinking water and sanitation improve steadily with rising level of income in most of the countries (World Bank (1992), Shafik (1994)).

It should be noted that only saddle path leads to the steady state and all other paths either violate the transversality conditions or hit constraints. In this context, we concentrate on curve 3 (in Fig 1), which follows the law of motion but can’t be equilibrium for the reasons just mentioned.

Economy will collapse (see, curve 1 in fig.1) if the rate of environmental degradation is more compare to the optimal rate and exhibits an *EKC* (along WXTZ) if it is less. In Fig.1, the curve 3 depicts the *EKC* path in the off-steady state[14] and it is totally unstable because both the stocks (E and K) increase over time at Z point. At point X, the net change in environmental stock is zero (i.e., $\dot{E}=0$) but stock of capital still increases (i.e., $\dot{K}>0$) which is essential for continuing economic growth as well as to maintain $\dot{E}=0$. Thus, the environment is stabilized[15] with capital accumulation.

Remark 3: *A sustainable development of an economy may continue only when* $\dot{K}>0$ *and net environmental change remains unaffected (i.e.,* $\dot{E}=0$ *).*

Economic growth may be compatible with environmental protection, but this is not automatic. With an insufficient amount of resources allocation for environmental improvement, economic growth will tend to exhaust the environmental resource in long run. If society does not devote appropriate amount of resources to environmental improvement or protection, higher rate of growth[16] can only be temporary, in long run it will eventually be zero.

Suppose, WX path crosses the $\dot{E}=0$ path at X then it moves along XTZ path. At point T or Z, both stocks grow (i.e., $\dot{E}>0$ and $\dot{K}>0$), so environmental quality will improve along XTZ path[17]. Thus, in Fig.1 WXTZ path exhibits U shaped relationship between E and K in E - K

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[14] The curve 3 follows law of motion but hits some constraints before reaching the steady state. Hence it is clear that the curve 3 exists only in non-optimal/sub-optimal situation. There exists infinite number of such curves but planner chooses that path which gives him/her maximum welfare.

[15] In this context it should be mentioned that Stephens (1976) allows exponential growth of per capita income with constant environmental quality.

[16] The optimal growth rate (income is defined in narrow sense) will be lower if pollution is more because increased abatement activities crowd out investment. In broad definition of GDP, which incorporated the value of abatement activities, long run growth rate will be constant and also leads to a sustainable development in the economy.

[17] E increases maximum up to its initial level or "pristine level", then $\dot{E}=0$ but still $\dot{K}>0$, that means economy grows. Further economic growth/development may lead to fall in E, i.e., $\dot{E}<0$. Truly, there is cycle for E, which may provide multiple equilibrium (See, fig A.3a and fig A.3b in Appendix B). *EKC* is observed in one phase of a cycle. Extending to another phase, we may get 'N' shape curve.

plane. Initially environmental quality declines as the whole capital used up only in production process, nothing left for environmental protection. It is true that none the economy could thought about the environmental degradation (or constrained) in their planning at the early stages of economic development. After sometimes (at least after industrialization) economy should realize the consequence of it, and economy gradually increases the allocation of capital to protect the environment and possible to improve environment at a higher level of capital. Once the abatement expenditure is positive, according to Lieb (2002), it is sufficient to find *EKC*. It is clear that allocation of capital is insufficient for abatement activity along WX but sufficient or more along XTZ. Hence, a move from insufficient to sufficient allocation of capital for abatement activity is the basis for *EKC*. It should be mentioned that only one subset of sub-optimal situations generate *EKC*.

In this study, the policy variable θ (allocation of capital) plays the vital role for controlling environmental quality. In the process of economic development, allocation of capital for abatement activity monotonically raises over time and halts at optimum or steady state. Thus, generally the abatement activity becomes strong enough to restore the environmental quality. It will be strong only when a sufficient investment takes place in the abatement sector, then environmental quality improves. This is the basis for U-shaped relationship between environmental quality and economic growth.

Remark 4: *The abatement activity is the main force to stop the environmental degradation. So, optimal allocation of capital for abatement activity is the sufficient investment for curve down the pollution level.*

A sufficient resource has to be devoted for abatement activity and at some time it will become optimal to stop environmental degradation with capital accumulation, and economic growth may continue protecting environmental quality. It should be noted that the allocation of capital protects environment, which is crucial for the long run growth.

6. Conclusions

This paper provides a possible theoretical explanation for the Environmental Kuznets Curve in the framework of endogenous growth model. The *EKC* exists dynamically in the off-steady state. Generally, less developed economies use their whole stock of capital for commodity production that generates pollution, which damages their existing environmental stocks. This model suggests that each economy should allocate one part of their capital for abatement activity. Environmental degradation continues at early stage because of insufficient investment for abatement activity, but in later stage, sufficient investment (viz., optimal allocation of capital) prevents further degradation of environmental quality. Thus, to restore environmental quality, a sufficient investment is needed that is possible only when economy accumulates enough capital stocks. The shift from insufficient to sufficient investment for upgrading environment is the basis for curve down the pollution level, and thus, correspondingly forms the inverted U-shaped relationship between pollution and economic growth. Optimal allocation of capital for abatement activity is the sufficient investment demand to curve down the pollution level.

Acknowledgement: I am grateful to the anonymous referees of this journal who helped bringing the paper in its present form by offering valuable comments and suggestions on earlier drafts. I am also grateful to Dipankor Coondoo, Manasranjan Gupta, Dipankar Dasgupta, Charles Perrings and Kanchan Chopra for their valuable comments and suggestions on the first draft. I would also like to thank all the participants of workshop at Institute for Economic Growth (Delhi) and Jadavpur University (Kolkata) for their valuable comments. Usual disclaimers, however, apply.

References:

1. Aghion, P. and P. Howitt, (1998), *Endogenous Growth Theory*. The MIT Press, Cambridge, M.A.
2. Agras, J. and D. Chapman, (1999), A dynamic approach to the Environmental Kuznets Curve hypothesis, *Ecological Economics,* Vol.- 28(2), 267-277.
3. Andreoni, J. and A., Levinson, (2001), The Simple Analytics of the environmental Kuznets Curve, *Journal of Public Economics*, vol.-80(2), 269 – 286.
4. Asako, K., (1980), Economic Growth and Environmental Pollution under the Max-Min Principle, *Journal of Environmental Economics and Management*, vol.-7, 157-183.
5. Barro, R. J. and X. Sala-I-Martin, (1995), *Economic Growth*, McGraw-Hill, Inc., New York.
6. Becker, R. A., (1982), Intergenerational Equity: The Capital-Environment Trade-off, *Journal of Environmental Economics and Management*, vol-9, 165-185.
7. Beltratti, A., (1996), *Models of Economic Growth with Environmental Assets*, Kluwer Academic Publishers, Dordrecht.
8. Bimonte, S., (2002), Information access, income distribution, and the Environmental Kuznets Curve, *Ecological Economics,* Vol.-41(1), 145-156.
9. Bovenberg, A. L. and S. Smulders, (1995), Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model, *Journal of Public Economics*, vol.-57, 369-391.
10. Brock, W. A. and M. S. Taylor, (2004), The green Solow Model, Department of Economics, University of Wisconsin, *Working paper* 2004-16.
11. Bulte, E. H. and D. P. van Soest (2001), Environmental degradation in developing countries: Households and the (reverse) environmental Kuznets curve, *Journal of development Economics*, vol.-65, 225-235.

12. Cass, D., (1965), Optimum growth in an Aggregative Model of capital accumulation, *Review of Economics studies*, vol.-32, 233 – 240.
13. Cole, M. A., A. J. Rayner and J. M. Bates, (1997), The Environmental Kuznets Curve: an empirical analysis, *Environment and Development Economics*, vol.-2, 401-416.
14. D'Arge, R.C. and K.C. Kogiku, (1973), Economic growth and the environment, *Review of Economics Studies*, vol.-40, 61-77.
15. Dinda, S., D. Coondoo and M. Pal, (2000), Air quality and economic growth: an empirical study, *Ecological Economics*, vol.-34(3), 409 -423.
16. Elbasha, E. H. and T. L. Roe, (1996), On Endogenous Growth: The Implications of Environmental Externalities, *Journal of Environmental Economics and Management*, vol-31, 240-268.
17. Forster, B.A., (1973), Optimal capital accumulation in a polluted environment, *Southern Economic Journal*, vol.-39, 544-547.
18. Gangadharan, Lata and Valenzuela, Ma. Rebecca; (2001), Interrelationships between income, health and the environment: extending the Environmental Kuznets Curve hypothesis, *Ecological Economics,* Vol.-36(3), 513-531.
19. Gawande, Kishore; Alok K. Bohara, R. P. Berrens and P. Wang, (2000), Internal migration and the environmental Kuznets curve for US hazardous waste sites, *Ecological Economics,* Vol.- 33(1), 151-166.
20. Gradus, R. and S. Smulders, (1993), The Trade-off Between Environmental Care and Long-term Growth –Pollution in Three Prototype Growth Models, *Journal of Economics*, vol.-58(1), 25-51.
21. Grossman, G. M. and A. B. Krueger, (1995), Economic Growth and the Environment, *Quarterly Journal of Economics*, vol.-110, 353 – 377.

22. Gruver, G.W., (1976), Optimal investment in pollution control in a neoclassical growth context, *Journal of Environmental Economics and Management*, vol-3, 165-177.
23. John, A. and R. Pecchenino, (1994), An Overlapping Generations Model of Growth and the Environment, *The Economic Journal*, vol.-108, 1393 - 1410.
24. Keeler, E., M. Spence and R. Zeckhauser, (1972), The optimal control of pollution, *Journal of Economic Theory*, vol.-4, 19-34.
25. Krautkraemer, J. A., (1985), Optimal growth, Resource Amenities and the preservation of Natural Environments, *Review of Economic Studies*, vol.-LII, 153-170.
26. Lieb, C. M., (2002), The Environmental Kuznets Curve and satiation: a simple static model, *Environment and Development Economics*, vol.-7, 429 - 448.
27. Lopez, R., (1994), The environment as a factor of production: the effects of economic growth and trade liberalization, *Journal of Environmental Economics and Management*, vol.-27, 163 – 184.
28. Michel, P. and Rotillon (1995). "Disutility of Pollution and Endogenous Growth". *Environmental and Resource Economics,* vol.-6, 279-300.
29. Mulligan, C. B. and X. Sala-I-Martin, (1993), transitional Dynamics in Two-sector Models of Endogenous Growth, *Quarterly Journal of Economics*, vol.-108(3), 739-773.
30. Munasinghe, M., (1999), Is environmental degradation an inevitable consequence of economic growth: tunneling through the environmental Kuznets curve, *Ecological Economics,* Vol.-29(1), 89-109.
31. Parente S. L. and E. C. Prescott (2000) *Barriers to Riches: The Walras-Pareto Lectures*, MIT Press, Cambridge MA.
32. Pasche,M., (2002), Technical progress, structural change, and the environmental Kuznets curve, *Ecological Economics,* Vol.-42(3), 381-389.

33. Rothman, D. S., (1998), Environmental Kuznets curves—real progress or passing the buck?: A case for consumption-based approaches, *Ecological Economics,* Vol.-25(2), (*Special Issue*), 177-194.
34. Selden, T. M. and D. Song, (1994), Environmental Quality and Development: Is there a Kuznets curve for Air Pollution Emissions?, *Journal of Environmental Economics and Management*, vol.-27, 147–162.
35. Seldon T. M. and D. Song, (1995), Neoclassical growth, the J curve for abatement and the inverted U curve for pollution, *Journal of Environmental Economics and Management,* vol.-29(2), 162-168.
36. Shafik, N., (1994), Economic Development and Environmental Quality: An econometric analysis*, Oxford Economic Papers*, vol.-46, 757 – 773.
37. Stephens, J.K. (1976), A Relatively optimistic Analysis of Growth and Pollution in a Neoclassical framework, *Journal of Environmental Economics and Management,* vol.-3, 85-96.
38. Stern, D. I., (2004), The rise and fall of the Environmental Kuznets Curve, *World Development*, vol.-32(8), 1419 – 1439.
39. Stokey, N.L., (1998), Are there limits to growth? *International Economic Review*, vol.-39, 1 –31.
40. Suri, V. and D. Chapman, (1998), Economic growth, trade and energy: implications for the Environmental Kuznets Curve*, Ecological Economics*, vol.-25, 195 – 208.
41. Tahvonen, O. and J. Kuuluvainen (1993). Economic Growth, Pollution, and Renewable Resources. *Journal of Environmental Economics and Management,* vol.-24, 101—118.
42. Tisdell, C., (2001), Globalisation and sustainability: environmental Kuznets curve and the WTO, *Ecological Economics,* Vol.-39(2), 185-196.

43. Withagen, C. (1995). Pollution, Abatement and Balanced Growth. *Environmental and Resource Economics,* vol.-5, 1-8.

44. World Bank, (1992), *World Development Report*, Oxford University Press, New York.

Appendix: A

Followings are the necessary conditions for an optimal solution of the problem:

$$\frac{\delta H}{\delta C} = U_C - \lambda = 0 \Rightarrow U_C = \lambda \qquad \text{(A2.1)}$$

$$\frac{\delta H}{\delta \theta} = \lambda f_K - \mu(h_K + \gamma f_K) = 0 \Rightarrow \mu = \frac{\lambda f_K}{h_K + \gamma f_K} \qquad \text{(A2.2)}$$

$$\dot{K} = \frac{\delta H}{\delta \lambda} \Rightarrow \dot{K} = f(\theta K, E) - C \qquad \text{(A2.3)}$$

$$\dot{\lambda} = -\frac{\delta H}{\delta K} + \rho\lambda = (\mu\gamma - \lambda)\theta f_K - \mu(1-\theta)h_K + \rho\lambda \qquad \text{(A2.4)}$$

$$\dot{E} = \frac{\delta H}{\delta \mu} \Rightarrow \dot{E} = h((1-\theta)K) - \gamma f(\theta K, E) \qquad \text{(A2.5)}$$

$$\dot{\mu} = -\frac{\delta H}{\delta E} + \rho\mu = -U_E + (\mu\gamma - \lambda)f_E + \rho\mu \qquad \text{(A2.6)}$$

Where U_C, f_K and h_K denote the marginal utility of consumption, marginal productivity of capital in commodity production and in environment upgrading, respectively. While condition (A2.1) asserts the consumption-accumulation indifference at the margin, condition (A2.2) requires the value of marginal product of capital in commodity production is equal to the value marginal of environmental quality loss due to withdrawal of capital from environment upgrading. Equation (A2.3) and (A2.5) are the motion equations of the state variables (K and E). Equations (A2.4) and (A2.6) are the well-known arbitrage conditions for capital formation and environment upgrading, which ensure that the economy would be indifferent between consumption and saving, and between production and environment improvement at the margin, respectively.

Appendix B

Figure A.1: Steady State and Optimal values

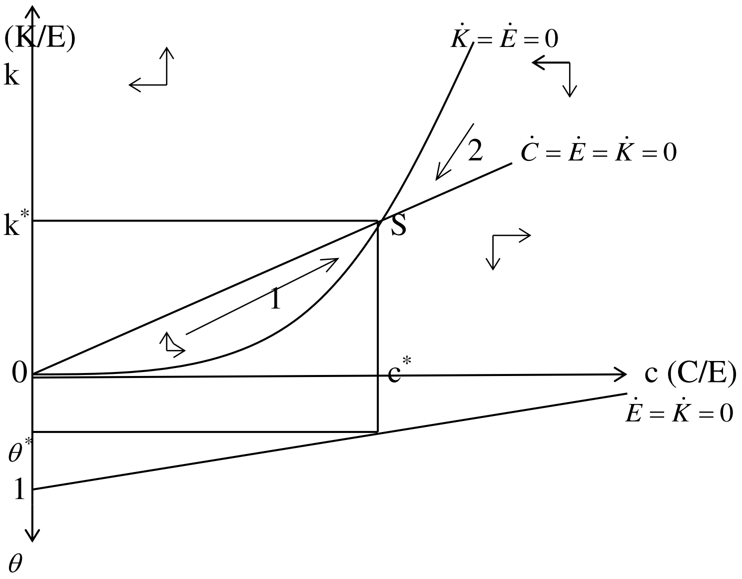


Figure A.2: Possible locus of $\dot{E} = 0$ and $\dot{K} = 0$ due to motion of other variables

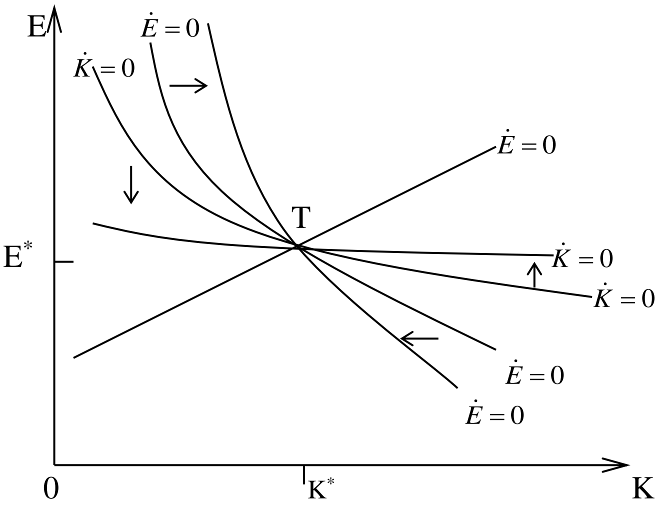
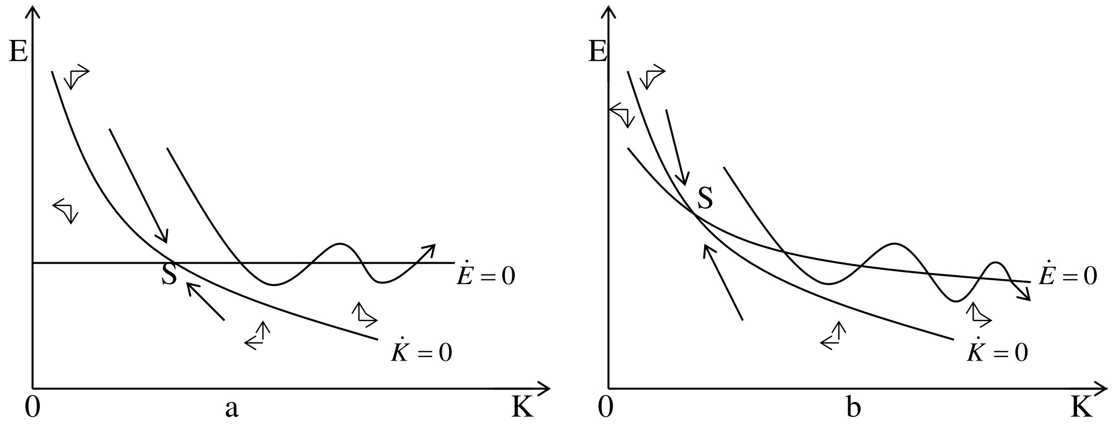


Figure A.3: Transitional dynamics and multiple equilibrium.



Appendix C

Optimum values at steady state

Now we define c and k as $c=\dfrac{C}{E}$ and $k=\dfrac{K}{E}$. Solving $\dot{K}=\dot{E}=0$ and $\dot{C}=\dot{K}=\dot{E}=0$, we get $k=\dfrac{1}{B^{\frac{1}{\alpha}}}c^{\frac{1}{\alpha}}+\dfrac{\gamma}{A_1}c$ and $k=\alpha\left(\dfrac{1}{\alpha}-\dfrac{\gamma}{A_1}\right)c$, respectively. At steady state, the optimal values are $c^*=B^{\frac{1}{1-\alpha}}\left(\dfrac{\alpha}{\rho}-(1+\alpha)\dfrac{\gamma}{A_1}\right)^{\frac{\alpha}{1-\alpha}}$ and $k^*=\alpha\left(\dfrac{1}{\alpha}-\dfrac{\gamma}{A_1}\right)c^*=\alpha B^{\frac{1}{1-\alpha}}\left(\dfrac{1}{\rho}-\dfrac{\gamma}{A_1}\right)\left[\dfrac{\alpha}{\rho}-(1+\alpha)\dfrac{\gamma}{A_!}\right]^{\frac{\alpha}{1-\alpha}}$, and the optimal value of θ is $\theta^*=1-\dfrac{\gamma}{A_1}\left(\dfrac{C}{K}\right)^*$ or, $\theta^*=1-\dfrac{\gamma}{A_1}c^*=1-\dfrac{\gamma}{A_1}\left[B^{\frac{1}{1-\alpha}}\left(\dfrac{\alpha}{\rho}-(1+\alpha)\dfrac{\gamma}{A_1}\right)^{\frac{\alpha}{1-\alpha}}\right]$.

These optimal values (θ^*, c^* and k^*) are also derived graphically (See fig A.1 in Appendix B). It should be noted that the optimal allocation of capital (θ^*) depends on the pollution rate (γ), available technologies in production (B) and abatement activity (A_1), share of capital in production (α), and the discount rate (ρ).

Existence of saddle path

It is easy to analyze the existence of a saddle path in *k-c* plane. Solving $\dot{E}=\dot{K}=\dot{C}=0$ in *k-c* plane (See line 1 and 2 in figure A.1), we obtain equation of the saddle path i.e., $A_1 k = m + \gamma c$, where $m=\left(\frac{A_1^{\alpha}}{(1-\gamma)\alpha B}\right)^{\frac{1}{\alpha-1}}$. These *k* and *c* ratios are linearly related, which is shown in figure A.1 in appendix B.

Now, we analyze the existence and stability of saddle path of four-dimensional problems in two-dimensional *E-K* plane. It is hard to analyze four variables in two-dimensional plane. Some restrictions are needed to analyze it.

$$\dot{K}=f(\theta K,E)-C$$

$$\phi_K \equiv \frac{d\dot{K}}{dK}=f_K+f_\theta\frac{d\theta}{dK}+f_E\frac{dE}{dK}-\frac{dC}{dK}$$

$$\phi_\theta \equiv \frac{d\dot{K}}{d\theta}=f_K\frac{dK}{d\theta}+f_\theta+f_E\frac{dE}{d\theta}-\frac{dC}{d\theta}$$

$$\phi_E \equiv \frac{d\dot{K}}{dE}=f_K\frac{dK}{dE}+f_\theta\frac{d\theta}{dE}+f_E-\frac{dC}{dE}$$

$$\phi_C \equiv \frac{d\dot{K}}{dC}=f_K\frac{dK}{dC}+f_\theta\frac{d\theta}{dC}+f_E\frac{dE}{dC}-1$$

$$\dot{E}=A_1(1-\theta)K-\gamma f(\theta K,E)$$

$$\psi_K \equiv \frac{d\dot{E}}{dK}=A_1(1-\theta)-\gamma f_K-(A_1K+\gamma f_\theta)\frac{d\theta}{dK}-\gamma f_E\frac{dE}{dK}$$

$$\psi_\theta \equiv \frac{d\dot{E}}{d\theta}=(A_1(1-\theta)-\gamma f_K)\frac{dK}{d\theta}-(A_1K+\gamma f_\theta)-\gamma f_E\frac{dE}{d\theta}$$

$$\psi_E \equiv \frac{d\dot{E}}{dE} = \{A_1(1-\theta) - \gamma f_K\}\frac{dK}{dE} - \{A_1 K + \gamma f_\theta\}\frac{d\theta}{dE} - \gamma f_E$$

$$\psi_C \equiv \frac{d\dot{E}}{dC} = 0$$

Now,

$\phi_K > 0$ if $f_K > f_\theta \frac{d\theta}{dC} + f_E \frac{dE}{dK} - \frac{dC}{dK}$, $\frac{d\theta}{dK}|_{\dot{K}=0} < 0$, $\frac{d\theta}{dK}|_{\dot{E}=0} < 0$,

$\phi_E > 0$ if $f_E + \frac{dC}{dK} > f_K \frac{dK}{dE} + f_\theta \frac{d\theta}{dE}$, $\frac{dC}{dK}|_{\dot{K}=0} > 0$, at low K, and <0, at high K.

$\psi_K > 0$, $\psi_E \leq 0$

Under these restrictions, in (E, K) plane, the determinant is negative, i.e.,

$$D = \begin{vmatrix} \phi_K & \psi_K \\ \phi_E & \psi_E \end{vmatrix} < 0.$$

This suggests that there exist a saddle path in *E*-*K* plane under above-mentioned restrictions. It should be noted that if once the economy attains steady state $(\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{E}}{E} = \frac{\dot{\theta}}{\theta} = 0)$, economy maintains it.