A Markov Chain Analysis for Capitalization Dynamics in the Cryptocurrency Market

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Abstract

Utilizing all cryptocurrencies since market inception, we investigate the mobility properties of the market. Using a Markov Chain model, we estimate the Transition Matrix, describing the probabilistic structure of cross-sectional capitalization transitions. We further apply various indices providing the anatomy of cross-sectional dynamics. Additionally, we compare the early cryptocurrency market period to the more recent era, investigating whether there are any discernible changes in the mobility structure. We find that persistence, in the first decade of the crypto market's operation has been substantial. Moreover, mobility (persistence) is found to be lower (higher) in the recent era of the market. Also, we document that the exit probability monotonically decreases with the cryptocurrency's capitalization. Exit probability exhibits a clear reduction in the recent market era. Overall, the results of this study can also be interpreted as signs that the cryptocurrency market has entered into a maturity phase.

Keywords: Cryptocurrencies; Markov Chain; Transition Matrix.

JEL classification: G10, G15, G23.

Disclaimer: The views and opinions expressed in this paper are those of the authors and do not reflect those of their respective institutions.

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1. Introduction

The launch of the Bitcoin, about a decade ago, the first cryptocurrency that was an innovative and disruptive blockchain technology (Nakamoto, 2008) was followed by a proliferation of other cryptocurrencies introduced -the so-called altcoins[1]- resulting in a new ecosystem (Dwyer, 2015; Corbet *et al.*, 2018a). This ecosystem has attracted huge attention among retail and institutional investors, for whom it undoubtedly widens the investment opportunity set, but also for them and regulators alike represents uncharted waters (Bouoiyour and Selmi, 2015; Baur *et al.*, 2018).

Today the total capitalization of the cryptocurrency market stands well above USD 200bn, an amount that is distributed -quite unevenly- across 2000^{+} distinct cryptocurrencies[2]. Hence, it is apparent that the cryptocurrency market has witnessed an increase both in its intensive, as well as its extensive margins.

To an outsider, the crypto market may be synonymous with the Bitcoin and possibly to a handful of other cryptos. This is probably because concentration is high, with the top-10 cryptocurrencies steadily accounting for the lion share in terms of capitalization, and the publicity that the Bitcoin has attracted (Mai *et al.*, 2018; Urquhart, 2018). Accepting this observation may lead to the – erroneous – conclusion that the crypto market possesses stable characteristics over time. To put it plainly, if one focuses on the surface of the crypto market, it seems that nothing happens. However, a closer inspection reveals that the capitalization ranking map of cryptocurrencies constantly changes and it is not rare to observe large segments of the market being identified as big losers or gainers, in the sense

[1] Altcoins use similar cryptography technology but are based on different algorithmic structures.
[2] The all-time high capitalization of USD 830bn was recorded on January 7th, 2018.

that they advance or retreat quite substantially within the cross-sectional capitalization distribution. Thus, the explosive increase in the number of traded cryptocurrencies, as well as the rapid inflow of funds to the cryptocurrency market result in a fast-changing terrain. In fact, a recent trend of the major online data providers is to provide such lists where the biggest winners and losers are recorded, as a tool in the investors' pursuit of gain (Baur *et al.* 2017; Wei, 2018).

From a systemic point of view, the realization of losers and gainers, as well as those cryptocurrencies holding their ground is a manifestation of the cryptocurrency market's fundamental properties stemming from mobility and persistence. Mobility refers to the observed tendency of cryptocurrencies to ascent (or descend) across the cross-sectional capitalization distribution, while persistence describes the tendency to remain in the same position.

Despite the rapidly growing academic literature, to the best of our knowledge, these properties have not been formally investigated. This is exactly the gap of the academic literature we aim to fill, by investigating and measuring the mobility traits of the cryptocurrency market. In some more detail, we set out to measure the cross-sectional capitalization distribution dynamics, utilizing the whole population of cryptocurrencies, by paying special attention to mobility and persistence of the capitalization rankings. Additionally, we split the first decade of cryptocurrency operation into its early and its most recent era, to explore whether there any discernible changes in the mobility and persistence properties of the cryptocurrency market.

We will pursue a micro-level analysis by initially placing each cryptocurrency to a decile based on observed market capitalization rankings, and then track the micro-level transitions across deciles. Transitions across the capitalization distribution will formally be represented with a Transition Matrix pertinent to a Markovian model driving the cross-sectional dynamics (Noris, 1998). Furthermore, this will allow us to construct, well-known in the mobility literature, indices that will offer a clear picture of the dynamic behavior of the cross-sectional capitalization distribution.

Note that to avoid survivorship bias (Brown *et al.*, 1992; Carpenter and Lynch, 1999) we have gone at great lengths ensuring that in each period we have collected the total number of cryptocurrencies traded. This property of our dataset opens up other fruitful prospects as a byproduct. In particular, by using all traded assets in each period, we have collected information regarding the new entries (birth of cryptos) and the exits based on market data. These dimensions have been appropriately incorporated in the Markovian model, enriching the Transition Matrix to offer information over and above the standard transition probabilities. In this way, we can also estimate the probability of exiting, conditional on decile placement, as well as estimating in which decile a newly born (entry) cryptocurrency is expected to be placed.

The remainder of the paper is structured as follows. Section 2 describes the dataset and the construction of the variables used in the analysis. Section 3 provides a review of the Markov Chain model and the mobility metrics deployed in the analysis. Section 4 presents and discusses the empirical findings. Finally, Section 5 concludes.

2. Data Issues

Our dataset comprises of micro data at the level of cryptocurrency, for which capitalization levels were collected weekly, spanning the period from 28 April 2013 to 12 May 2019[3], providing us with a sample of 209,929 observations. The choice of the weekly sampling frequency was on the following grounds. Firstly, a high frequency increases sample observations and thus the degrees of freedom. Secondly, if the chosen sampling frequency is too high excessive noise in the data is introduced. Based on these, we selected the weekly frequency instead daily or annual.

Capitalization is simply the product of the circulating supply, *i.e.,* the number of coins available to users, and price. Our aim was to collect all available cryptocurrencies at each time point (week), and subsequently all "entries" and "exits" in the cryptocurrency market are included in our dataset. Where entries are the new cryptocurrencies being 'born' and exits are cases where a cryptocurrency is not reported[4].

Graph 1 depicts the time trajectory of total market capitalization since the market's inception.

*******Graph 1*******

The evolution of the number of active cryptocurrencies is shown in Graph 2.

*******Graph 2*******

The absolute number of entries and exits over time is given in Graph 3.

[3] https://www.coinmarketcap.com

[4] This exit case is a combination of "deaths" and cases where the data provider ceases to report further information on particular coins.

*******Graph 3*******

Let $C_{i,t}$ denote the capitalization of the i^{th} cryptocurrency in period t. Then for the t period's cross-sectional capitalization distribution, we calculate the deciles, denoted by j, where $j = \{1, 2, ..., 10\}$. Table 1 shows the basic sample summary statistics for each capitalization decile in the whole sample.

******* Table 1*******

Then in each period t, every cryptocurrency is allocated in one of the j deciles according to its capitalization. Let $D_{i,t,j}$ be an indicator denoting the decile j in which cryptocurrency i belongs in period t. Then in period $t+1$ based on the observed cross-sectional capitalization $C_{i,t+1}$, we recalculate the corresponding deciles and again record in which decile each cryptocurrency belongs. This process is reiterated until the end of the sample, and all movements of cryptocurrencies between any two consecutive periods t, $t+1$ are recorded.

3. Empirical Methodology

3.1 Markov Chain Analysis

We will model cryptocurrencies' movements across deciles as a Markov Chain, with each decile representing a possible state in which a cryptocurrency may fall in. Between successive time periods movements across deciles correspond to transitions across states, and thus represent mobility, while remaining in the same state represents persistence.

The parameters of the Markov model are the transition probability matrix $\mathbf{P}$ and its initial distribution $\boldsymbol{\pi}_{0i}$, $i \in j$. The typical element p_{ij} of the transition probability matrix $\mathbf{P}$ indicates the transition probability that a cryptocurrency is in state (capitalization decile) j at time t, given it was in state i in time $t-1$. By definition $p_{ij} \geq 0, \forall\ i, j$ and since the set of states is exhaustive, the row elements of the matrix $\mathbf{P}$ add up to unity: $\sum_{j=1}^{n} p_{ij} = 1, \forall i$. Matrix $\mathbf{P}$ summarizes all the n^2 transition probabilities that correspond to all possible movements across capitalization deciles (Ross, 1996).

However, to appropriately account for the actual market dynamics, two more adjustments are needed. The first deals with the cases of cryptocurrencies that cease their operation, namely while a cryptocurrency belonged to a given decile in period t, it is not traded in period $t+1$. This calls for the construction of an absorbing state, that denotes an artificial state in which all exiting cryptocurrencies fall in. By construction, an absorbing state only "accepts" members, i.e., the cryptocurrencies exiting the market, and once entering the absorbing state a cryptocurrency never "escapes". Thus, the addition of the absorbing state would capture the "death" of cryptocurrencies, and the corresponding transition probability of exiting the market. This absorbing/exiting state is typically treated as the last state in the state space. In addition to the exit state, we have amended the transition matrix with an entry state. This second adjustment deals with the launch of new cryptocurrencies, which while in period t were not traded, they enter the market in $t+1$. Thus, we construct an extra state, that "sends" cryptocurrencies to states (deciles) in period $t+1$, which they did not trade in period t and thus did not belong to any state in the previous period. Thus, the empirical transition matrix is of 12x12 dimensions (10 deciles, plus entry and exit states). It is our firm belief that by explicitly taking into account entries and exits in the cryptocurrency market we overcome any biases. Considering the above-mentioned information, the transition probability matrix takes the form:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n-1,1} & p_{n-1,2} & \cdots & p_{n-1,n} \\ 0 & 0 & \ldots & 1 \end{bmatrix} \tag{1}$$

An obvious solution for estimating the transition probabilities p_{ij} is to resort to the frequency of transitions from i to j in the data. We formalize and illustrate this estimate in this section. We will justify the estimate in terms of statistical theory below after introducing the idea of a maximum likelihood estimate.

Let $N_{ij} \equiv \sum_{l=1}^{n} 1\left(d_{l-1} = i, d_l = j\right)$ be the count of all transitions from state i to state j in the data, where $\mathbf{1}(\alpha)$ is an indicator function that takes the value 1 when the argument α is true. Let us also define $N_{i\bullet} \equiv \sum_{j\in\mathcal{S}} N_{ij} = \sum_{j=1}^{n} N_{ij}$ number of transitions starting from state i in data. Then we will estimate the transition probability p_{ij} by:

$$\hat{p}_{ij} = \frac{N_{ij}}{N_{i\bullet}} \tag{2}$$

Let us briefly explain why this is so. The likelihood function takes the form:

$$\mathbb{L} = p_0 \prod_{i\in\mathcal{S}} \prod_{j\in\mathcal{S}} p_{ij}^{N_{ij}} = p_0 \prod_{i\in\mathcal{S}} \mathbb{L}_i\left(p_{ij}\right) \tag{3}$$

where $\mathbb{L}_i = \prod_{j \in \mathcal{S}} p_{ij}^{N_{ij}}$ depends on the elements of the i-th row of the transition matrix. The log-likelihood then becomes:

$$\ln \mathbb{L} = \mathcal{L} = \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} N_{ij} \ln p_{ij} \quad \text{s.t.} \sum_{j \in \mathcal{S}} p_{ij} = \sum_{j=1}^{n} p_{ij} = 1, \; p_{ij} \geq 0 \tag{4}$$

The FOC for likelihood maximization yield:

$$\hat{p}_{ij} = \frac{N_{ij}}{\sum_{j \in \mathcal{S}} N_{ij}} = \frac{N_{ij}}{\sum_{j=1}^{n} N_{ij}} = \frac{N_{ij}}{N_{i\bullet}}, \tag{5}$$

as the asymptotically unbiased and normally distributed ML estimator of p_{ij} (Anderson and Goodman, 1957). One can think of this as multinomial likelihoods from random samples.

3.2 Measuring Mobility and Persistence

The main diagonal of the transition matrix contains important information. In particular, it denotes the probability of the chain remaining in the same state between two successive periods, therefore describing persistence. In our context persistence suggests a situation where a cryptocurrency tends to remain in the future in its current state. In contrast, the off-main diagonal elements of the transition matrix describe how likely it is that cryptos transit to different states between successive periods. Lower (higher) mobility between states is to be expected as the values of the off-main diagonal elements of the transition matrix are getting smaller (larger)[5]. In the case where the off-main diagonal elements were zero, we would encounter absolute persistence since no transitions between

[5] A characteristic of the mobility index is that is bounded between 0 and 1.

states would be feasible. In such an extreme case the transition matrix would essentially be the identity matrix.

Since there is no universally accepted mobility indicator, there is a variety of mobility indices proposed and applied in the relevant literature (Atkinson, 1970; Bigard *et al.*, 1998; Geweke *et al.*, 1986; Maasoumi and Zadvakili, 1986; Schluter, 1998; Jafry and Schuermann, 2004; Trück and Rachev, 2006). For our analysis, we consider seven different indices.

The first three, are the so-called Summary Mobility Indices (Bigard *et al.*, 1998), and offer a preliminary analysis of the empirical transition matrix under scrutiny $\mathbf{P} = [p_{ij}]_{n\times n}$. These three indices are the Immobility Ratio (IR), the Moving Up (MU), and the Moving Down (MD) ratios. The immobility ratio index captures any persistence in the system in the form of the probability for the typical cryptocurrency to remain in its current state, and takes the following form:

$$IR = \left[\frac{\sum_{i=1}^{n} p_{ii}}{\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}}\right]\times 100 = \left[\frac{\sum_{i=1}^{n} p_{ii}}{n}\right]\times 100 \qquad (6)$$

where n is the number of columns and rows as migration matrices are symmetric.

The other two indices offer us the percentage of the cryptocurrencies moving to a higher/lower state within a period (week).

$$MU = \left[\frac{\sum_{i<j} p_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}}\right]\times 100 = \left[\frac{\sum_{i<j} p_{ij}}{n}\right]\times 100 \qquad (7)$$

$$MD = \left[\frac{\sum_{i>j} p_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} p_{ij}} \right] \times 100 = \left[\frac{\sum_{i>j} p_{ij}}{n} \right] \times 100 \tag{8}$$

where n is the number of columns and rows as migration matrices are symmetric.

It should be noted that, under the coding adopted in this particular analysis, the move to a higher (lower) state designates an improvement (deterioration) of the cryptocurrency capitalization.

The other indices utilized in the analysis are the so-called Eigenvalue-based Indices, because they are expressed in terms of the eigenvalues of the empirical transition probability matrix. The first of these four, is the Prais-Shorrocks Index (Prais, 1955) and is based on the trace of the mobility matrix:

$$M_{PS} = \frac{n - tr(P)}{n-1} = \frac{n - \sum_{i=1}^{n} \lambda_i}{n-1} \tag{9}$$

where λ_i denotes the transition matrix eigenvalues in descending order (in absolute value) and where n is the number of columns and rows as migration matrices are symmetric.

Next, we have the Sommers-Conlisk Index (Sommers and Conlisk, 1979), which is based on the second largest eigenvalue (λ_2) of the empirical transition matrix:

$$M_{SC} = 1 - |\lambda_2| \tag{10}$$

The next index is the Half-Life Index (Theil, 1972), and is expressed as follows:

$$M_h = e^{-h} \tag{11}$$

where $h = \frac{-\log 2}{\log|\lambda_2|}$ and λ_2 is the second largest eigenvalue of the mobility matrix.

All indices, except M_h, take values in the [0,1] interval, with 1 denoting the highest degree of mobility and 0 the lowest. M_h describes the speed of convergence towards the equilibrium distribution $\mathbf{\Pi} = \lim_{r \to \infty} \mathbf{P}^r$, by indicating the needed time that a system needs to cover half of the deviation from equilibrium. The M_h index takes values from zero (perfect mobility) to infinity (perfect immobility).

However, we are also interested in investigating whether there are any discernible changes in the cryptocurrency market over time. In order to accommodate this investigation, we will follow the analysis of Ballis and Drakos (2021) regarding the timeline of the expansion of the cryptocurrency market, and we will estimate separate transition matrices, called $\mathbf{P}_{pre}$ (referring to the period from 28 April 2013 to 1 May 2016) and $\mathbf{P}_{post}$ (referring to the period from 1 May 2016 to 12 May 2019) and then compare them in terms of their mobility (persistence).

The final mobility index related to the eigenvalues of the transition matrix that is calculated, is the Singular Value Decomposition Index (SVD) (Jafry and Schuermann, 2004) and is expressed as follows:

$$M_{SVD} = \frac{\sum_{i=1}^{n} \sqrt{\lambda_i^*}}{n} \tag{12}$$

where λ_i^* denotes the positive eigenvalues of the matrix $(\mathbf{P}-\mathbf{I})'(\mathbf{P}-\mathbf{I})$ and n is the number of columns and rows as migration matrices are symmetric.

The comparison of the two sub-periods transition matrices calls for a new set of tools, which will depict the closeness between two matrices. One set of such tools is provided by indices known as cell-by-cell distance measures. These are the absolute deviations distance, D_{L_1} (Israel *et al.*, 2001), the Euclidean distance, D_{L_2} (Bangia *et al.*, 2002), and the maximum distance, $D_{L_{\max}}$ (Trück, 2004). These distance indices, take into account any differences between any two transition matrices $\mathbf{P}=[p_{ij}]_{n\times n}$ and $\mathbf{Q}=[q_{ij}]_{n\times n}$, are expressed as follows:

$$D_{L_1}\left(P,Q\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}\left|p_{ij}-q_{ij}\right| \tag{13}$$

$$D_{L_2}(P,Q)=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(p_{ij}-q_{ij})^2} \tag{14}$$

$$D_{L_{\max}}\left(P,Q\right)=\max_{1\le i,\, j\le n}\left|p_{ij}-q_{ij}\right| \tag{15}$$

where n is the number of columns and rows as migration matrices are symmetric.

The literature provides several variations and extensions of the above-mentioned metrics (Jackson and Murray, 2004). Most of them can be represented by a category of distance measures of the form:

$$D_{weight}\left(\mathbf{P},\mathbf{Q}\right)=\left(\sum_{i=1}^{n}\sum_{j=1}^{n}p_{ij}^{r}\left|p_{ij}-q_{ij}\right|\right)^{p} \tag{16}$$

with r varying from -1 to 1 and p from 1 to infinity. For r less than 0, the elements p_{ij} cannot be zero, or the fraction will be undefined.

In his study Lahr (2001) suggests a weighted absolute difference (WAD) where $r=1$ and $p=1$. The measure is expressed as follows:

$$D_{WAD}\left(P,Q\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}p_{ij}\cdot\left|p_{ij}-q_{ij}\right| \tag{17}$$

Obviously $D_{WAD}(P,Q)\neq D_{WAD}(Q,P)$, in order for D_{WAD} to not meet the symmetry condition. So that this anomaly can be corrected, the following metrics can be used:

$$D_{WAD}^{average}\left(P,Q\right)=0.5\left(D_{WAD}\left(Q,P\right)+D_{WAD}\left(Q,P\right)\right) \tag{18}$$

$$D_{WAD}^{\max}\left(P,Q\right)=\max\left(D_{WAD}\left(Q,P\right),D_{WAD}\left(Q,P\right)\right) \tag{19}$$

In their studies, Trück (2004) and Trück and Rachev (2006) introduced a way to measure the difference between two migration matrices in terms of mobility based on Singular Values Decomposition, by calculating:

$$D_{SVD}\left(P,Q\right)=\left|M_{SVD}\left(P\right)-M_{SVD}\left(Q\right)\right| \tag{20}$$

4. Empirical Results

4.1 Estimation of the sample transition matrices

As stated earlier, our main objective is the comparison of the early cryptocurrency market to the recent era. As a result, we need the empirical transition probability matrices referring to those two time periods. But before we do so we will estimate the transition matrix for the whole sample period to assess its properties, which is provided in Table 2.

******* Table 2*******

The inspection of the matrix for the whole sample period shows that, as expected, the main diagonal elements are markedly higher in comparison to their off-diagonal counterparts. This property indicates that persistence is the dominant feature of the chain, and therefore between successive periods, the most likely outcome is that a cryptocurrency will remain in the same decile. In addition, persistence is found to be more vigorous for cryptocurrencies belonging to higher deciles. In other words, while persistence is evident in the whole chain, it is greater for cryptocurrencies with higher capitalizations.

Another property of the matrix relates to the likelihood of transitions, where it is documented that neighboring transitions are more likely in comparison to distant transitions. That is, if a transition occurs, the probability of occurrence decreases with the distance between the initial state (decile) and the destination state.

Another interesting and intuitive finding relates to the probability of exiting the cryptocurrency market, as shown in the last column of the transition matrix. We find that the probability of exiting is not uniform and in fact depends on the decile placement of the cryptocurrency. In particular, the probability of exiting monotonically increases as the cryptocurrency's capitalization is lower. Moreover, while the exit probabilities are relatively small in magnitude, one has to bear in mind that the transition horizon is just a week. Thus, exit probabilities are indeed sizeable. Given their size, they also show considerable variation across deciles. For instance, looking at the two extremes of the capitalization distribution, the ratio of the exit probabilities between cryptos from the 1st decile (1.89) to that of cryptos from the 10th decile (0.06) is approximately 31, suggesting that on average it is 30^{+} times more likely that a crypto from the lowest capitalization decile exits the market in comparison to its counterpart from the highest capitalization decile.

Now we turn our attention to the comparison of the transition matrices that describe the properties of the Markov chain in the two sub-periods of the sample that represent approximately 50% each. In terms of calendar time, the 1st half covers the period from 28

April 2013 to 1 May 2016, which we call the early cryptocurrency market. Similarly, the 2^{nd} half covers the period from 1 May 2016 to 12 May 2019, which we call the recent era of the cryptocurrency market. The corresponding transition matrices $\mathbf{P}_{pre}$ and $\mathbf{P}_{post}$ are shown in Tables 3 and 4.

***** **Table 3*****

***** **Table 4*****

We will refrain from discussing the individual properties of the two sub-periods matrices since they are similar to that of the whole sample matrix. Thus, to put succinctly the two matrices share similar properties with the overall matrix in terms of persistence relative to mobility, as well as to the likelihood of distant relative to near transitions and in terms of the documented behavior of the exit probabilities.

What we are interested in is as to whether there are any noticeable differences between the two periods. As it turns out there are several differences between the two matrices, suggesting that the transition dynamics of the cryptocurrency market have undergone substantial changes. For instance, we document a change in the main diagonal elements, where they are clearly higher in the 2^{nd} half of the sample period. This finding suggests that persistence (mobility) in the cryptocurrency market has increased (decreased). In other words, it seems that in the latter period of the market the movement across capitalization rankings is less likely. Graph 4 depicts the main diagonal probabilities by decile (state) for the two sub-periods, that offers a pictorial representation of persistence.

*****Graph 4*****

Another important difference between the two matrices is found concerning the exit probabilities. For any given decile (state) the exit probability was higher in the early cryptocurrency period in comparison to the recent period. Graph 5 depicts the exit probabilities by decile (state) for the two sub-periods.

*******Graph 5*******

These two findings taken in conjunction could be interpreted as indicating that the cryptocurrency market has entered in a maturity phase, in the sense that extreme events such as exits (deaths) are less likely and that the population of capitalization rankings is more stable.

Another rather subtle, but equally interesting, dimension in which differences between the two sub-periods is encountered relates to the behavior of new entrants. In the 1st half of the sample, a new entrant had a 60% chance of being placed below the median of the capitalization distribution, while in the 2nd half new entrants have a 57.5% chance of being place above the median.

In the next sections, we will further investigate the persistence and mobility properties by deploying more robust metrics that will assist us in shedding more light on the issues at hand.

4.2 Calculation of the mobility indices

Recall that our research agenda is to explore, not just the properties of the cryptocurrency market's cross-sectional capitalization dynamics, but also whether there are any discernible changes across time in these properties.

We start with the calculation, for the $\mathbf{P}_{pre}$ and $\mathbf{P}_{post}$ matrices, of the summary mobility indices (Immobility Ratio Index, Moving Up Index, and Moving Down Index) and the eigenvalue-based indices (Prais-Shorrocks Index, Sommers-Conlisk Index, Half-Life Index, and Singular Value Decomposition Index), which are reported in Table 5.

******* Table 5*******

The main message from the indices is that persistence (mobility) has indeed increased (decreased) in the latter period of the cryptocurrency market. This is evident from the Immobility Ratio, which in the 1st half of the sample attained a value of 0.58, while in the 2nd half of the sample has reached the level of 0.73 indicating that persistence has increased. The Prais-Shorrocks and the Sommers-Conlisk also point to the direction of increased persistence in the 2nd half of the sample. Recall that for these indices as they approach the value of unity they indicate higher mobility. For both indices, we document a drop in their value in the 2nd half of the sample period, which supports an increase (decrease) in persistence (mobility).

Moreover, the Half-life metric is considerably higher in the 2nd half of the sample implying that the time required to achieve a hypothetical equilibrium status is much longer, which is generated by the decreased mobility resulting in a lower speed of convergence towards equilibrium.

4.3 Calculation of distance metrics and indices

The set of indices calculated and discussed in the previous section considered the individual properties of the transition matrices as estimated from the two sub-periods. In this section, we conduct a comparison of the two sub-period matrices to further comprehend the (dis-)similarities. Note that this comparison is indirect, using the identity matrix (complete persistence matrix) as a fixed reference point from which the distances of the two sub-period transition matrices are calculated. Thus, using the identity matrix as a numeraire allows an indirect comparison of the two sub-period matrices.

The Distance metrics are provided in Table 6. Inspecting the distance metrics, we find that across the board they attain lower values in the 2^{nd} half of the sample period, indicating that the transition matrix associated with the latter half of the sample exhibits greater similarity to the identity matrix. Thus, the distance metrics document an unequivocal increase (decrease) of persistence (mobility) in the cryptocurrency market.

******* Table 6*******

5. Conclusion

The cryptocurrency market is a relatively new asset class with unique traits and as such is attracting growing research in an attempt to comprehend and establish its fundamental properties. The present study contributes to the literature by investigating the persistence (mobility) features of the cross-sectional capitalization distribution of the cryptocurrency market. The research agenda consisted of establishing these properties for the whole history of the cryptocurrency market and also exploring whether there are any discernible changes in these properties over time.

Our analysis was based on a Markov Chain representation for the movement across capitalization rankings and estimated the pertinent transition matrices from the observed behavior of cryptocurrencies at the micro-level. Then a battery of mobility indices was calculated to assess the persistence (mobility) features contained in the transition matrices.

According to our empirical findings, the cryptocurrency market has unequivocally undergone a transformation that has led to higher (lower) persistence (mobility). Additionally, the probability structure of exiting the cryptocurrency market has markedly decreased. Although these findings have their idiosyncratic value as pieces of information regarding the dynamic properties of the cross-sectional capitalization distribution, one may also interpret them as signs that the cryptocurrency market has entered into a maturity phase.

Our results provide useful information both for market participants as well as academia. For investors, it gives a formal and systematic view of the average tendencies for capitalization increases/decreases and the likelihood of exits. For academia, our analysis documents the mobility-persistence properties of the cryptocurrency market based on mobility metrics that are well established in the broader economic literature.

Future directions of research could include issues such as the investigation of the transition determinants and/or the impact of business cycle effects. Additionally, behavioral aspects might also be in operation such as the attention's role on the capitalization trajectory of cryptocurrencies.

Declarations

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Conflicts of interest

No conflict of interest exits in the submission of this manuscript, and this manuscript is approved by all authors for publication.

Availability of data

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Code availability

The code that support the findings of this study are available from the corresponding author upon reasonable request.

Authors' contributions

AB: Formal Analysis, Conceptualization, Writing, Review & Editing, Data curation, Software. KD: Formal analysis, Writing & editing, Project administration, Supervision.

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Tables

Table 1. Summary Statistics for Market Capitalization by Decile for the whole sample.

Decile	Mean	Standard Deviation	Minimum	Maximum	Skewness	Kurtosis
1	11,668.78	20,077.09	0	1,162,266	11.29677	533.1537
2	58,277.91	77,465.54	538	1,424,087	2.46804	14.84228
3	165,226.3	219,342.2	1,267	1,503,099	1.952051	7.250199
4	413,406.7	546,636.5	2,292	3,431,495	1.794767	6.188263
5	972,360.8	1,324,946	5,192	8,893,903	1.997092	7.425443
6	2,019,420	2,019,420	10,754	1.96e+07	2.188827	8.593351
7	3,955,552	5,451,755	17,413	4.31e+07	2.352109	10.25153
8	8,163,804	1.13e+07	40,149	1.04e+08	2.698295	13.37977
9	2.04e+07	3.07e+07	102,744	3.21e+08	3.237223	18.10878
10	1.26e+09	9.52e+09	326,867	3.21e+11	15.35995	304.4408

Table 2. Transition probability matrix $\mathbf{P}$ for the whole sample.											
From/To	1	2	3	4	5	6	7	8	9	10	11	12
1	66.77	15.82	9.03	3.99	1.25	0.65	0.36	0.13	0.06	0.05	0.00	1.89
2	17.41	59.21	10.86	5.33	4.40	1.22	0.30	0.19	0.07	0.01	0.00	0.99
3	9.00	13.07	59.32	10.91	2.55	2.77	1.23	0.24	0.16	0.01	0.00	0.73
4	4.32	5.07	13.41	61.57	10.77	1.45	1.78	0.99	0.07	0.02	0.00	0.53
5	1.37	4.48	2.24	13.42	63.32	11.16	1.41	1.92	0.31	0.06	0.00	0.30
6	0.52	1.25	2.86	1.33	13.56	66.39	11.14	1.31	1.37	0.04	0.00	0.23
7	0.33	0.26	1.31	1.84	1.18	13.27	69.89	10.26	1.28	0.16	0.00	0.22
8	0.11	0.17	0.23	0.98	1.87	1.05	11.98	74.57	8.39	0.51	0.00	0.14
9	0.02	0.06	0.15	0.04	0.39	1.34	1.04	9.30	82.48	5.08	0.00	0.10
10	0.00	0.01	0.01	0.02	0.04	0.02	0.16	0.41	5.28	93.96	0.00	0.06
11	9.09	9.41	10.17	10.17	10.13	11.75	11.61	10.74	10.24	6.68	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Notes: 1-10 denote deciles, while 11 and 12 denote the Entry and Exit states respectively.

Table 3. Transition probability matrix $\mathbf{P}_{pre}$ for the April 2013 – May 2016 period (early cryptocurrency market)

From/To	1	2	3	4	5	6	7	8	9	10	11	12
1	57.51	17.03	9.63	6.87	2.57	1.52	1.07	0.28	0.02	0.04	0.00	3.45
2	19.46	48.73	13.44	4.88	7.19	3.42	0.66	0.49	0.18	0.00	0.00	1.56
3	10.01	16.02	46.38	13.47	3.13	4.64	4.31	0.63	0.43	0.00	0.00	0.98
4	6.98	4.90	16.07	47.31	13.76	2.45	4.16	3.43	0.22	0.06	0.00	0.65
5	3.19	6.65	2.97	16.34	48.35	13.99	2.05	4.70	1.19	0.10	0.00	0.47
6	1.17	3.95	4.69	2.31	15.66	52.69	13.50	2.10	3.62	0.04	0.00	0.26
7	0.95	0.63	4.62	3.91	2.23	14.73	56.92	11.57	3.69	0.53	0.00	0.22
8	0.39	0.45	0.61	3.66	4.30	1.91	12.70	64.88	9.18	1.65	0.00	0.26
9	0.06	0.26	0.45	0.12	1.38	3.74	3.29	9.87	75.10	5.63	0.00	0.10
10	0.00	0.00	0.02	0.08	0.08	0.04	0.64	1.38	6.03	91.62	0.00	0.10
11	13.72	11.96	12.35	11.38	10.51	10.80	8.85	7.88	6.81	5.74	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Notes: 1-10 denote deciles, while 11 and 12 denote the Entry and Exit states respectively.

Table 4. Transition probability matrix $\mathbf{P}_{post}$ for the May 2016 – May 2019 period (recent era of the cryptocurrency market).

From/To	1	2	3	4	5	6	7	8	9	10	11	12
1	69.93	15.49	8.82	3.04	0.79	0.33	0.10	0.06	0.02	0.02	0.00	1.40
2	16.73	62.61	10.04	5.49	3.51	0.52	0.17	0.08	0.03	0.01	0.00	0.80
3	8.69	12.15	63.50	10.10	2.36	2.15	0.23	0.12	0.05	0.01	0.00	0.65
4	3.47	5.10	12.57	66.19	9.81	1.12	1.02	0.21	0.01	0.01	0.00	0.50
5	0.79	3.78	2.02	12.48	68.10	10.26	1.21	1.03	0.04	0.04	0.00	0.25
6	0.30	0.36	2.27	1.02	12.87	70.81	10.40	1.05	0.65	0.04	0.00	0.23
7	0.13	0.14	0.25	1.16	0.84	12.80	74.03	9.86	0.51	0.05	0.00	0.22
8	0.03	0.07	0.10	0.12	1.09	0.78	11.76	77.66	8.14	0.14	0.00	0.10
9	0.01	0.00	0.06	0.02	0.08	0.56	0.32	9.12	84.83	4.90	0.00	0.10
10	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.11	5.05	94.70	0.00	0.05
11	6.34	7.94	8.79	9.48	9.94	12.28	13.25	12.45	12.28	7.25	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Notes: 1-10 denote deciles, while 11 and 12 denote the Entry and Exit states respectively.

Table 5. Mobility indices for transition probability matrices

Index	Whole sample	Pre (April 2013 – May 2016)	Post (May 2016 – May 2019)
Panel A: Summary Mobility Indices			
Immobility Ratio	0.69748	0.58949	0.73236
Moving Up	0.14107	0.19352	0.12404
Moving Down	0.14198	0.18990	0.12662
Panel B: Eigenvalue Based Mobility Indices			
Prais-Shorrocks (M_{PS})	0.365927	0.4641	0.334218
Sommers-Conlisk (M_{SC})	0.0052	0.0079	0.0044
Half Life (h)	132.9507	87.39312	157.1866

Table 6. Distances between the transition probability matrices

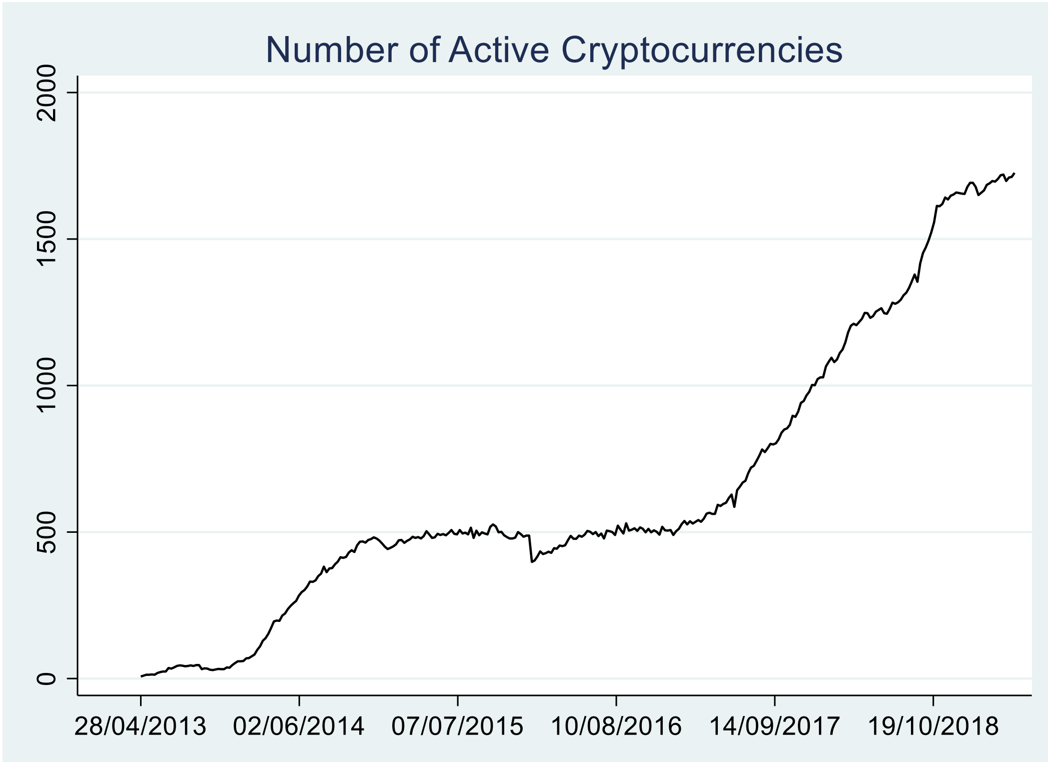
Panel A: Distances between $\mathbf{P}$, $\mathbf{P}_{pre}$, $\mathbf{P}_{post}$ from $\mathbf{I}$			
Metric	$D(\mathbf{P},\mathbf{I})$	$D(\mathbf{P}_{pre},\mathbf{I})$	$D(\mathbf{P}_{post},\mathbf{I})$
Cell by Cell Based Distances			
D_{L_1}	8.0495	10.2098	7.3527
D_{L_2}	1.5544307	1.844142869	1.472577601
$D_{L_{\max}}$	1	1	1
Difference Distances			
$D_{WAD}^{average}$	3.206171	3.928447	2.952744
$D_{WAD}^{\max}$	4.0252	5.1051	3.6764
Singular Value Based Distance			
D_{SVD}	0.3409	0.4315	0.3115

Graphs

Graph 1: The time trajectory of total market capitalization since the market's inception.

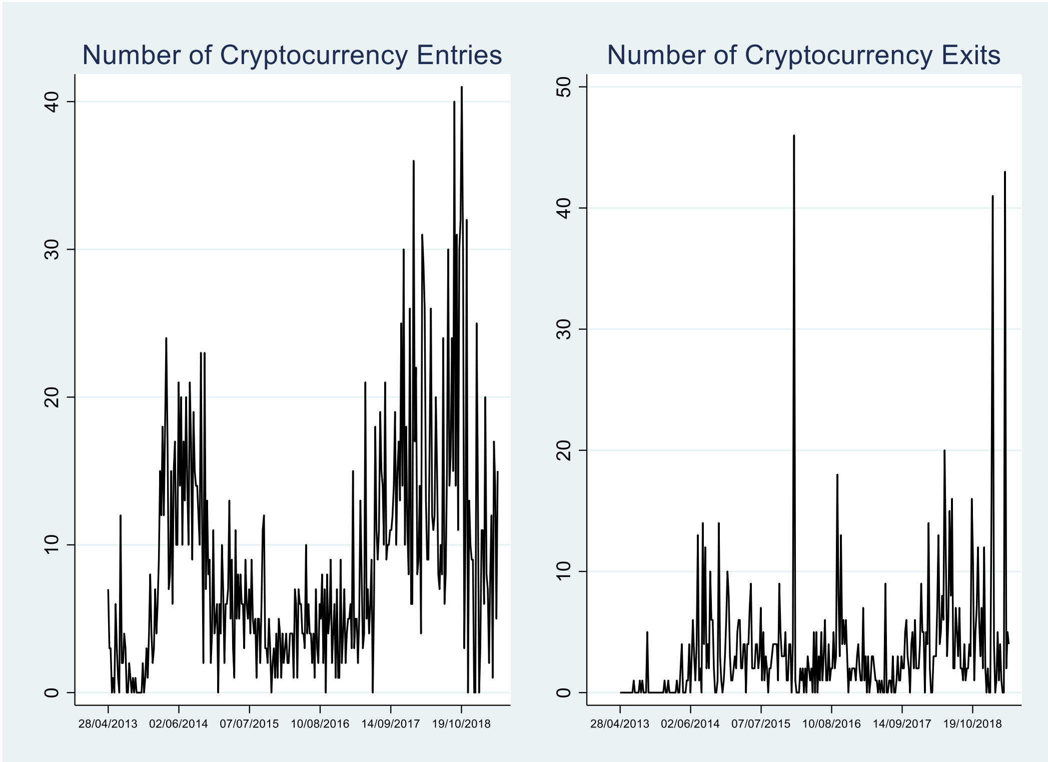
Source: Own estimations.

Graph 2: The evolution of the number of active cryptocurrencies.



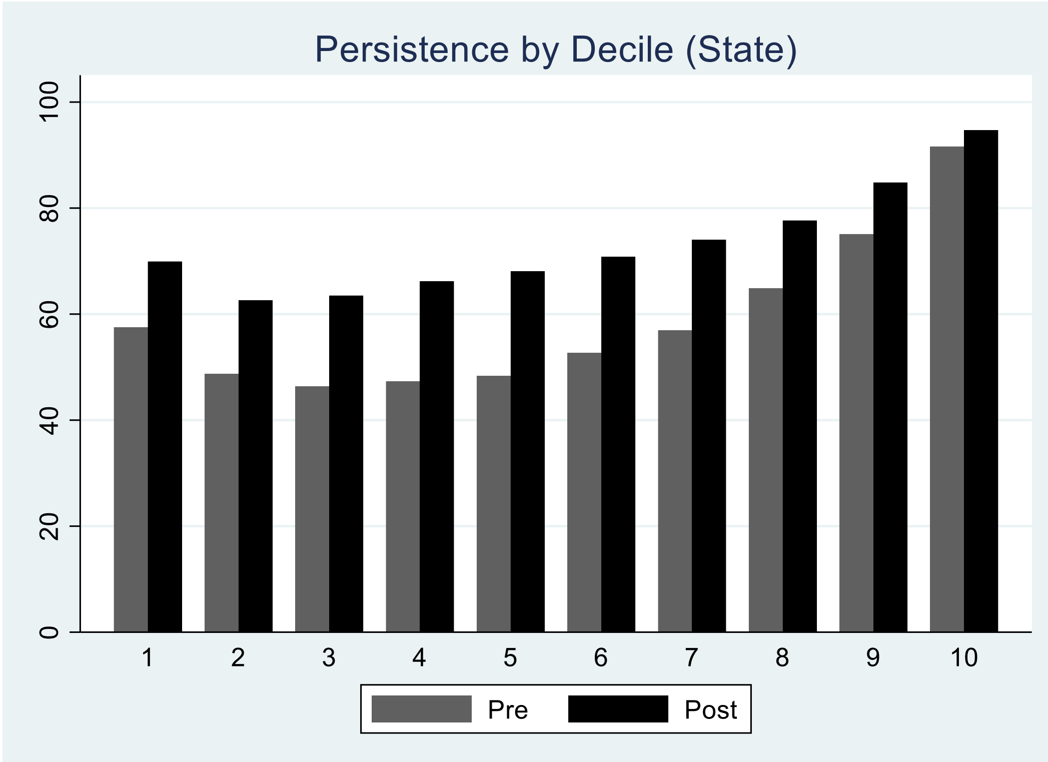
Source: Own estimations.

Graph 3: Absolute number of cryptocurrency entries and exits.



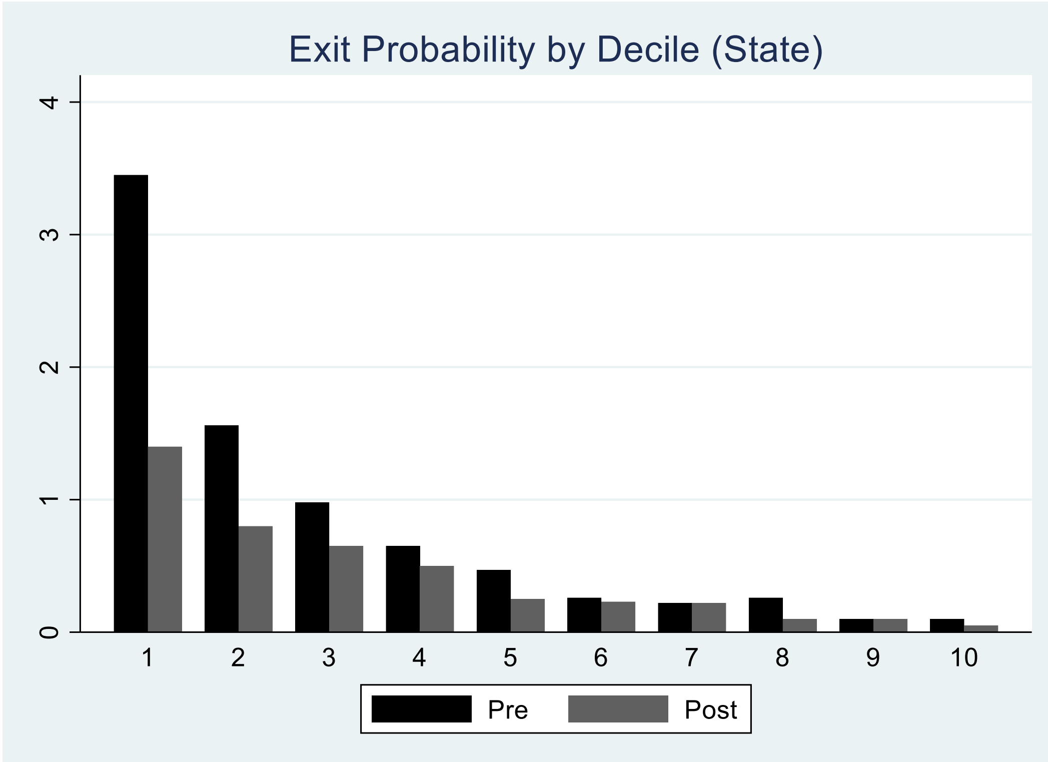
Source: Own estimations.

Graph 4: Main diagonal probabilities by decile (state) for the two sub-periods.



Source: Own estimations.

Graph 5: Exit probabilities by decile (state) for the two sub-periods.



Source: Own estimations.