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Abstract: We consider a vertically related market where one quantity setting and another price setting downstream firm negotiate the terms of a two-part tariff contract with an upstream input supplier. In contrast to the traditional belief, we show that when bargaining is decentralised, the price setting firm produces a higher output and earns a higher profit than the quantity setting firm. And, when bargaining is centralised, both firms produce the same output whereas the profit is higher under the price setting firm than the quantity setting firm.

Key Words: Bargaining; Bertrand; Cournot; Two-part tariffs; Vertical pricing; Welfare

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† The usual disclaimer applies.

1. Introduction

While a vast majority of studies in oligopoly theory are based on either quantity (Cournot) or price (Bertrand) types of competition; mixed competition (i.e., Cournot-Bertrand) has gained popularity in recent years. For example, in the small car industry, Honda and Subaru set quantities while Saturn and Scion set prices (Tremblay et al. 2013). Flath (2012) shows that in 30 out of 70 Japanese industries, companies use some form of mixed competition. Sato (1996) argues that in Japanese home electronics industry, Matsushita adopts quantity strategy whereas Sanyo employs pricing strategy.

In a seminal paper, Singh and Vives (1984) show that choosing quantity competition is optimal compared to price competition or mixed competition. Considering a setting where one firm sets quantity and the other sets price, Temblay and Tremblay (2011) show that the quantity setting firm earns higher profit than its price setting rival. In a similar setting, Semenov and Tondji (2019) gets the same profit ranking even when both firms invest in cost reducing R&D.

However, the papers above share a common ground where the input market is perfectly competitive. It is often found that in many cases input suppliers and the final goods producers are involved in two-part tariff vertical pricing contracts (Berto Villa-Boas, 2007 and Bonnet and Dubois, 2010). Given this backdrop, we consider a vertical structure where one quantity setting and another price setting downstream firm negotiate the terms of two-part tariff contract with an upstream firm either through decentralised or centralised bargaining. We formulate a demand function that accounts for both Bowley (1924) type and Shubik and Levitan (1980) type demand function and hence, measures the degree of market saturation. Our results show that when bargaining is decentralised, the price setting firm produces more and earns higher profit than the quantity setting firm whereas when bargaining is centralised both firms produce same output but the price setting firm earns a higher profit.

Our results hold both under Bowley (1924) type and Shubik and Levitan (1980) type demand function. This is in stark contrast to the existing results alluded earlier.

The rest of the paper is organised as follows. Section 2 presents the model and discusses the main results under decentralised and centralised bargaining respectively. Section 3 concludes. All proofs are relegated to the appendix.

2. The model

We consider an economy with two downstream firms, denoted by D_i producing differentiated products where $i, j = 1, 2$ and $i \neq j$. The downstream firms require a critical input for production that they purchase from a monopoly input supplier, *U*, through two-part tariff contracts involving an up-front fixed-fee and a per-unit price. *U* produces the inputs at a constant marginal cost of production, $c \geq 0$. We assume that one unit of input is required to produce one unit of the output, and D_i and D_j can convert the inputs to the final goods without incurring any further cost.

We consider a demand equation that combines the demand functions found in Bowley (1924) and Shubik and Levitan (1980). The difference between the two approaches is the degree of market-expansion effect. While in Bowley's (1924) formulation of demand function the aggregate market size increases with a higher degree of product substitutability, the formulation by Shubik and Levitan (1980) reveals that the aggregate market size is independent of the degree of product substitutability. We represent the inverse demand function as:

$$P_i = a - \theta q_i - \gamma q_j \tag{1}$$

where P_i denotes the price and q_i denotes the output of *i*th downstream firm where $i, j = 1, 2$ and $i \neq j$. We take $\gamma \in (0,1)$ to denote the degree of product differentiation. If $\gamma = 1$ the goods are perfect substitutes and if $\gamma = 0$ the goods are isolated. The parameter $\theta = 1 + \sigma(1-\gamma)$ measures the degree of market expansion, where the upper boundary, $\sigma = 1$, corresponds to no market expansion effect, i.e., the market is saturated as in Shubik and Levitan (1980) and the lower boundary, $\sigma = 0$, corresponds to full market expansion as in Bowley (1924).

We develop a model of two stage game. At stage 1, U is involved either in a decentralised bargaining or centralised bargaining with D_1 and D_2 to determine the terms of the two-part tariff contracts involving an up-front fixed-fee, F_i, and a per-unit price, w_i, $i = 1, 2$. At stage 2, D_1 competes in quantity and D_2 competes in price. We solve the game through backward induction.

2.1 Market competition stage

We begin our discussion at stage 2 where D_1 chooses quantity and D_2 chooses price. The maximisation problem of the downstream firms yields

$$\underset{q_1}{\text{Max}} \quad \Pi_1^D = \pi_1 - F_1$$

$$= \left[a - \theta q_1 - \frac{\gamma}{\theta}\left(a - P_2 - \gamma q_1\right) - w_1 \right] q_1 - F_1 \tag{2}$$

and,

$$\underset{P_2}{\text{Max}} \quad \Pi_2^D = \pi_2 - F_2$$

$$= \frac{1}{\theta}\left(P_2 - w_2\right)\left(a - P_2 - \gamma q_1\right) - F_2 \tag{3}$$

Maximising (2) and (3) and solving the first order conditions give the equilibrium quantity and price of D_1 and D_2 respectively.

$$q_1 = \frac{a\left(\gamma - 2\theta\right) + 2\theta w_1 - \gamma w_2}{3\gamma^2 - 4\theta^2} \tag{4}$$

$$P_2 = \frac{a(\gamma-\theta)(\gamma+2\theta)-\gamma\theta w_1+2(\gamma^2-\theta^2)w_2}{3\gamma^2-4\theta^2} \tag{5}$$

Hence, the profit equations in (2) and (3) reduce to $\prod_1^D = \left(\frac{\theta^2-\gamma^2}{\theta}\right)q_1^2 - F_1$ and $\prod_2^D = \theta q_2^2 - F_2$.

Next, we solve stage 1 of the game where the equilibrium contract terms are determined. We begin our discussion with decentralised bargaining and discuss the equilibrium outcomes. We repeat the same exercise under centralised bargaining in a subsequent section. For notational reasons, we use superscripts $\{d,r\}$ to denote respectively the equilibrium values under decentralised and centralised bargaining.

2.2. Decentralised bargaining

First, assume that bargaining is decentralised where U bargains with D_i over (w_i, F_i) by maximising the following generalised Nash bargaining expression:

$$\underset{F_i, w_i}{\text{Max}} \;\; [(w_i - c)q_i + F_i]^\beta \, [\pi_i - F_i]^{1-\beta} \tag{6}$$

where $(w_i - c)q_i + F_i$ and $(\pi_i - F_i)$ denote respectively the net profit of the upstream and downstream firms and β (resp. $(1-\beta)$) shows the bargaining power of the input supplier (resp. final goods producers). We restrict our analysis to $\beta \in (0,1)$.

Maximising the above with respect to F_i gives the following

$$F_i = \frac{1}{2}[\beta\pi_i - (1-\beta)(w_i - c)q_i] \tag{7}$$

Substituting (7) in (6), we get the maximisation problem as

$$\underset{w_i}{\text{Max}} \;\; [\beta(\pi_i + (w_i - c)q_i)]^\beta \, [(1-\beta)(\pi_i + (w_i - c)q_i)]^{1-\beta} \tag{8}$$

Solving (8), we obtain the equilibrium wholesale prices and fixed fees as[1]:

$$w_1^d = c + \frac{(a-c)\gamma^2(\gamma-\theta)\left(\gamma^2-2\gamma\theta-4\theta^2\right)}{5\gamma^4\theta-20\gamma^2\theta^3+16\theta^5} \tag{9}$$

$$w_2^d = c - \frac{(a-c)\gamma^2\left(4\theta^2-2\gamma\theta-\gamma^2\right)}{5\gamma^4-20\gamma^2\theta^2+16\theta^4} \tag{10}$$

$$F_1^d = \frac{2(a-c)^2(\gamma-\theta)^2\left(\gamma^2-2\gamma\theta-4\theta^2\right)^2\left[2\beta\theta^2-(1+\beta)\gamma^2\right]}{\theta\left(5\gamma^4-20\gamma^2\theta^2+16\theta^4\right)^2} \tag{11}$$

$$F_2^d = \frac{(a-c)^2\left(\gamma^2+2\gamma\theta-4\theta^2\right)^2\left(2\theta^2-\gamma^2\right)\left[(1-2\beta)\gamma^2+2\beta\theta^2\right]}{\theta\left(5\gamma^4-20\gamma^2\theta^2+16\theta^4\right)^2} \tag{12}$$

Using the above, we derive the equilibrium outputs and the profits of the downstream firms

$$q_1^d = \frac{2(a-c)(\gamma-\theta)\left(\gamma^2-2\gamma\theta-4\theta^2\right)}{5\gamma^4-20\gamma^2\theta^2+16\theta^4}\ ;\ q_2^d = \frac{(a-c)\left(\gamma^2+2\gamma\theta-4\theta^2\right)\left(\gamma^2-2\theta^2\right)}{5\gamma^4\theta-20\gamma^2\theta^3+16\theta^5}$$

$$\Pi_1^{D,d} = \frac{2(a-c)^2(1-\beta)(\gamma-\theta)^2\left(\gamma^2-2\gamma\theta-4\theta^2\right)^2\left(2\theta^2-\gamma^2\right)}{\theta\left(5\gamma^4-20\gamma^2\theta^2+16\theta^4\right)^2}$$

$$\Pi_2^{D,d} = \frac{2(a-c)^2(1-\beta)\left(\gamma^2-\theta^2\right)\left(\gamma^2+2\gamma\theta-4\theta^2\right)^2\left(\gamma^2-2\theta^2\right)}{\theta\left(5\gamma^4-20\gamma^2\theta^2+16\theta^4\right)^2}$$

The following results are immediate from the above.

Proposition 1: *(i) The upstream firm charges a lower input price to the price setting downstream firm than a quantity setting downstream firm such that* $w_2^d < c < w_1^d$.

[1] Note that, $F_1^d < 0$ for $\beta < \frac{\gamma^2}{(2-\gamma^2)+2\sigma(1-\gamma)(2+\sigma-\gamma\sigma)}\ (=\hat{\beta})$ meaning that the upstream firm subsidises the downstream firm when its bargaining power is small. We, however, assume that $\beta > \hat{\beta}$ so that it charges a positive fixed fee to the final goods producers.

(ii) The price setting downstream firm produces a higher output and earns a higher profit than the quantity setting downstream firm.

Note that the price setting downstream firm is charged a wholesale price which is less than the upstream firm's marginal cost, i.e., U subsidises the price setting firm's production. As a result, the price setting firm, D_2 sets a lower market price which in turn reduces the quantity setting firm, D_1's output and increases its own profit. This increased profit is then partly transferred to the upstream firm via the fixed fee. The opposite is true for the quantity setting downstream firm D_2, hence it is charged a wholesale price above U's marginal cost.

As the price setting downstream firm faces substantially lower wholesale price, naturally, it produces more and earns higher profit than its quantity setting rival.

2.3. Centralised bargaining

Now, assume that bargaining is centralised. The monopoly input supplier and a representative of D_1 and D_2 determine the terms of two-part tariff contract by maximising the following generalised Nash bargaining expression:

$$\underset{F_i,w_i}{\text{Max}} \left[\sum_{i=1}^{2}\big((w_i - c)q_i + F_i\big)\right]^{\beta}\left[\sum_{i=1}^{2}(\pi_i - F_i)\right]^{1-\beta} \tag{13}$$

Maximising the above with respect to F_i gives the following

$$F_i = \frac{1}{2}\left[\beta\sum_{i=1}^{2}\pi_i - (1-\beta)\sum_{i=1}^{2}(w_i - c)q_i\right] \tag{14}$$

Substituting (14) in (13), we get the maximisation problem as

$$\underset{w_i}{\text{Max}} \left[\beta\sum_{i=1}^{2}(\pi_i + (w_i - c)q_i)\right]^{\beta}\left[(1-\beta)\sum_{i=1}^{2}(\pi_i + (w_i - c)q_i)\right]^{1-\beta} \tag{15}$$

Solving (15), we obtain the equilibrium wholesale prices and fixed fees as[2]:

$$w_1^r = c + \frac{(a-c)\gamma}{2\theta} \tag{16}$$

$$w_2^r = c + \frac{(a-c)\gamma}{2(\gamma+\theta)} \tag{17}$$

$$F_1^r = F_2^r = \frac{(a-c)^2\left[2\beta\theta^2 - \gamma^2 - 2(1-\beta)\gamma\theta\right]}{8\theta(\gamma+\theta)^2} \tag{18}$$

Next, we calculate the equilibrium outputs and the profits of the downstream firms

$$q_1^r = q_2^r = \frac{a-c}{2(\gamma+\theta)}\ ;\ \Pi_1^{D,r} = \frac{(a-c)^2\left[2(1-\beta)\gamma\theta + 2(1-\beta)\theta^2 - \gamma^2\right]}{8\theta(\gamma+\theta)^2};$$

$$\Pi_2^{D,r} = \frac{(a-c)^2\left[\gamma^2 + 2(1-\beta)\gamma\theta + 2(1-\beta)\theta^2\right]}{8\theta(\gamma+\theta)^2}$$

Comparing the above we get the following.

Proposition 2: *(i) The upstream firm charges a lower input price to the price setting downstream firm than a quantity setting downstream firm such that* $c < w_2^r < w_1^r$.

(ii) The price setting downstream firm and the quantity setting downstream firm produce the same level of output.

(iii) The price setting downstream firm earns a higher profit than the quantity setting downstream firm.

As the upstream firm becomes more opportunistic when bargaining is centralised, it no longer subsidises the price setting downstream firm's production. However, analogous to

[2] Note that, $F_i^r < 0$ for $\beta < \frac{\gamma[2+\gamma+2\sigma(1-\gamma)]}{2(1+\sigma-\gamma\sigma)(1+\gamma+\sigma-\gamma\sigma)}\ (=\bar{\beta})$ meaning that the upstream firm subsidises the downstream firm when its bargaining power is small. We, however, assume that $\beta > \bar{\beta}$ so that it charges a positive fixed fee to the final goods producers.

Proposition 1(i), it still charges a lower input price to the price setting firm compared to the quantity setting firm.

To analyse Proposition 2(ii) let us recall eq(15). Note that,

$$\sum_{i=1}^{2}\left[\pi_i + \left(w_i - c\right)q_i\right] = \sum_{i=1}^{2}\left[\left(P_i - w_i\right)q_i + \left(w_i - c\right)q_i\right] = \sum_{i=1}^{2}\left(P_i - c\right)q_i$$

which is the profit of a monopoly final goods producer, producing both the products at the marginal cost of production *c*. Therefore, maximising (15) is equivalent to maximising the profit of a monopoly final goods producer. Hence, it is intuitive that the equilibrium per-unit input prices are such that they generate same total output and industry profit under Cournot and Bertrand competition. Further, in line with Proposition 1(ii), the price setting firm earns a higher profit as it faces a lower input price compared to its quantity setting rival.

3. Conclusion

We consider a vertical structure where one downstream firm sets quantity and another sets price and determine the terms of a two-part tariff contract with an upstream firm either through decentralised or centralised bargaining. We find that when bargaining is decentralised, the price setting firm produces higher output and earns higher profit than its quantity setting rival, whereas under centralised bargaining, both firms produce same output but the price setting firm earns a higher profit. This holds true regardless of the degree of market saturation.

4. Appendix

Proof of Proposition 1:

$$q_1^d - q_2^d = -\frac{(a-c)\gamma^4}{\theta\left(5\gamma^4 - 20\gamma^2\theta^2 + 16\theta^4\right)} = -\frac{(a-c)\gamma^4}{(1+\sigma-\gamma\sigma)\left[5\gamma^4 - 20\gamma^2(1+\sigma-\gamma\sigma)^2 + 16(1+\sigma-\gamma\sigma)^4\right]} < 0.$$

$$\Pi_1^{D,d} - \Pi_2^{D,d} = -\frac{4(a-c)^2(1-\beta)\gamma^5(\gamma-\theta)(\gamma^2-2\theta^2)}{\theta(5\gamma^4-20\gamma^2\theta^2+16\theta^4)^2}$$

$$= -\frac{4(a-c)^2(1-\beta)(1-\gamma)\gamma^5(1+\sigma)\left[2(1+\sigma-\gamma\sigma)^2-\gamma^2\right]}{(1+\sigma-\gamma\sigma)\left[5\gamma^4-20\gamma^2(1+\sigma-\gamma\sigma)^2+16(1+\sigma-\gamma\sigma)^4\right]^2} < 0.$$

Proof of Proposition 2:

$$w_1^r - w_2^r = \frac{(a-c)\gamma^2}{2\theta(\gamma+\theta)} = \frac{(a-c)\gamma^2}{2(1+\sigma-\gamma\sigma)(1+\gamma+\sigma-\gamma\sigma)} > 0.$$

$$\Pi_1^{D,r} - \Pi_2^{D,r} = -\frac{(a-c)^2\gamma^2}{4\theta(\gamma+\theta)^2} = -\frac{(a-c)^2\gamma^2}{4(1+\sigma-\gamma\sigma)(1+\gamma+\sigma-\gamma\sigma)^2} < 0.$$

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