Different policy effects of Ramsey and overlapping generations models

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Different policy effects of Ramsey and overlapping generations models[†]

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Abstract

Effects of taxation are examined in many studies. For such studies, the model economy assumes a logarithmic utility function. Results derived from our study indicate that attention should be devoted to using logarithm utility functions. We check the redistribution policy effect financed by capital income taxation in models of two types: a Ramsey model and an overlapping generations model. If the labor supply is inelastic, then effects of the redistribution policy financed by taxation of capital income differs between the Ramsey model and the overlapping generations model. However, if the labor supply is elastic, then the policy financed by capital income taxation is the same between the Ramsey model and the overlapping generations model. Moreover, this study presents simulation results.

Keywords: Overlapping generations model, Ramsey model, Redistribution, Taxation

JEL: E24, H20

[†] We would like to thank the seminar participants for the beneficial comments. Any errors are our responsibility.

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1. Introduction

This study was undertaken to examine how a redistribution policy affects capital stock and consumption in a steady state in a model with a logarithmic utility function. Redistribution policy initiatives are financed by wage income taxation and capital income taxation. Our study compares models of two types: a Ramsey model and an overlapping generations model. Even given identical redistribution policies of the two models, their policy effects on the capital stock can be quite different. Concretely, if the labor supply is inelastic, then an increase in the benefit financed by capital income taxation reduces the capital stock in a steady state in a Ramsey model. By contrast, when using an overlapping generations model, an increase in the benefit financed by capital income taxation raises the capital stock. If we consider the case of elastic labor supply, the result of taxation policy can be same in two models. This result emphasizes the importance of policy analysis in the model with a logarithmic utility function.

Some studies have examined effects of income taxation using a Ramsey model. Chamley (1986) derives the zero capital income tax rate to maximize social welfare in a Ramsey model. Aiyagari (1996) uses a positive capital income tax rate and Correia (1996) uses a negative capital income tax rate as the optimal capital tax rate because of precautionary saving and assumption of the production function. Jones, Manuelli and Rossi (1997) consider human capital accumulation and derive a zero wage income tax rate and a zero capital income tax rate. Barro (1990) considers government investment financed by income taxation. These studies all use a Ramsey model for their analyses.

However, many studies examine taxation effects in an overlapping generations model. Lin (2001) demonstrates that an increase in the capital income tax rate reduces the ratio of capital stock to human capital stock because of an increase in the subsidy financed by the capital income tax rate for human capital accumulation. Watanabe, Miyake and Yasuoka (2016) show how productive government investment financed by taxation affects the income growth rate. These studies examine situations using an overlapping generations model.

Some papers examine unemployment and unemployment benefits. Ono (2010) and Yasuoka (2020) specifically consider unemployment and ascertain how the unemployment benefit affects the unemployment rate. These papers all include consideration of unemployment with a decision of labor union membership. Fanti and Gori (2010) examine unemployment brought about by the minimum wage in the endogenous fertility model. Moreover, these studies all examine overlapping generations models.

The related literature includes no report of a study comparing effects of taxation policies in Ramsey and overlapping generations models. Our paper compares results obtained using models of these two types and devotes particular attention to examination of policy effects in a model with a logarithmic utility function. Although logarithmic utility functions are widely assumed in fields of theoretical economics analysis because of their simplicity for analysis, particular attention must be devoted to examination of the policy effects they entail, as described in our earlier report.[1]

[1] Hall (1988) shows that elasticity of the substitution of intertemporal consumption is 0–0.2, which shows that the logarithm utility function is not suitable for analyses. However, Campbell and Mankiw (1989) derive that the interest rate does not affect consumption, which is consistent with the logarithm utility function in the overlapping

The remainder of our paper is presented as follows. Section 2, section 3 and section 4 respectively elucidate effects of taxation in a Ramsey model and an overlapping generations model with inelastic labor supply. Section 5 presents examination of unemployment benefits for a case with elastic labor supply. Section 6 explains some simulation results. Section 7 presents consideration of unemployment brought about by the minimum wage as the other type of unemployment. Section 8 concludes our paper.

2. Ramsey Model

We consider the model economy with no population growth. There exist three types of agents: households, firms and a government.

2.1 Households

The lifetime utility function of households is assumed as

$$U_t = \sum_{s=t}^{\infty} \rho^{s-t} \ln c_s, 0 < \rho < 1. \tag{1}$$

In that equation, c_s and ρ respectively denote the consumption and discount rate.

The budget constraint in the *t* period is shown as presented below.

$$K_{t+1} = (1-\tau_w)w_t + (1-\tau_r)r_t K_t - c_t + (1-\delta)K_t + T_t \tag{2}$$

In that equation, w_t, R_t and K_t respectively denote the wage rate, the interest rate, and the capital stock. In addition, τ_w and τ_r respectively denote the wage income tax rate and capital income tax rate used for the lump-sum transfer. Also, δ denotes the capital stock depreciation rate. T_t denotes the lump-sum benefit.

We derive the optimal allocations of consumption at each period to maximize utility (1) subject to budget constraint (2). We thereby set the following Lagrange equation.

$$L = \sum_{s=t}^{\infty} \rho^{s-t} lnc_s + \sum_{s=t}^{\infty} \lambda_t\big((1-\tau_w)w_t + (1-\tau_r)R_t K_t + T_t - c_t + (1-\delta)K_t - K_{t+1}\big) \tag{3}$$

Consequently, we can obtain the Euler equation shown below as

$$\frac{c_{t+1}}{c_t} = \rho\big((1-\tau_r)R_{t+1} + (1-\delta)\big) \tag{4}$$

or

$$\frac{\Delta c_t}{c_t} = \rho\big((1-\tau_r)R_{t+1} + (1-\delta)\big) - 1. \tag{5}$$

Therein, $\Delta c_t = c_{t+1} - c_t$.

2.2 Firms

The production function is assumed to have the following Cobb–Douglas form:

$$Y_t = AK_t^{\theta} l_t^{1-\theta}, 0 < A, 0 < \theta < 1. \tag{6}$$

Therein, l_t denotes the labor input. With perfect competition, one can obtain the following profit maximizing conditions.

$$w_t = (1-\theta)AK_t^{\theta} l_t^{-\theta} \tag{7}$$

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generations model.

$$R_t = \theta A K_t^{\theta-1} l_t^{1-\theta} \tag{8}$$

In this section, because of full employment and the unity of population size, we set $l_t = 1$. Also, we assume $A = 1$.

2.3 Government

With the balanced budget, the government budget constraint can be shown as the following.

$$\tau_w w_t + \tau_r R_t K_t = T_t \tag{9}$$

3. Equilibrium in the Ramsey Model

With (2),(5), (7), (8) and (9), the equilibrium of this model is given by the following two dynamics equations.

$$\frac{\Delta c_t}{c_t} = \rho\left((1-\tau_r)\theta K_{t+1}^{\theta-1} + (1-\delta)\right) - 1 \tag{10}$$

$$\Delta K_t = K_t^{\theta} - c_t - \delta K_t \tag{11}$$

In those equations, $\Delta K_t = K_{t+1} - K_t$. With (10) and (11) we can present Fig. 1 and can show the steady state equilibrium.

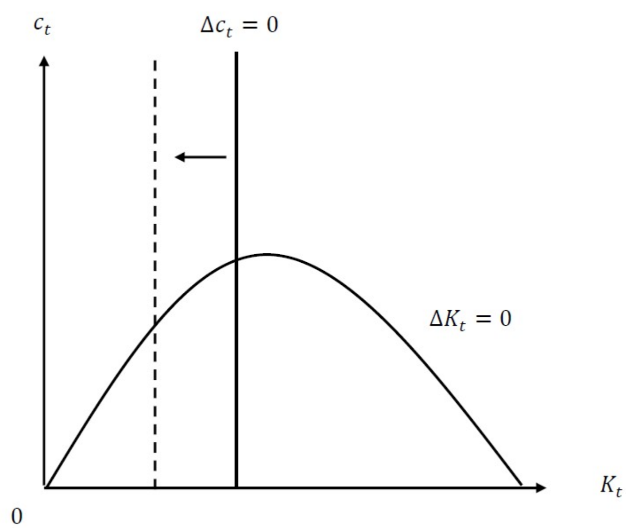

Fig. 1. Phase Diagram

Dashed line shows the case of the capital income taxation. Policy effects of an increase in the labor income tax rate and the capital income tax rate for the lump-sum transfer at the steady state are presented in Table 1.

	K	c
τ_w	No change	No change
τ_r	-	-

Table 1. Taxation effects

This result is generally known as a result of the standard Ramsey model. The capital income taxation reduces both the capital stock and the consumption level at the steady state.

4. Overlapping Generations Model

As explained in this section, we set the overlapping generations model with a lump-sum transfer policy. In this model, individuals in the household live in two periods: young and old. In the young period, individuals supply labor to obtain a wage income. The labor supply is inelastic. The lump-sum transfer is given for younger people. Then, the utility function and the budget constraint are shown as follows.

$$U_t = \ln c_{1t} + \rho \ln c_{2t+1} \tag{12}$$

$$c_{1t} + \frac{c_{2t+1}}{1 + (1 - \tau_r) r_{t+1}} = (1 - \tau_w) w_t + T_t \tag{13}$$

In those equations, c_{1t} and c_{2t+1} denote consumption in young and old periods, respectively. Also, r_{t+1} denotes the net interest rate, which is $r_{t+1} = R_{t+1} - \delta$. Then, the optimal allocations of the consumption and the saving s_t to maximize the utility (12) subject to the budget constraint (13) are shown as follows.

$$c_{1t} = \frac{1}{1+\rho}\left((1 - \tau_w) w_t + T_t\right) \tag{14}$$

$$c_{2t+1} = \frac{\rho(1 + (1 - \tau_r) r_{t+1})}{1+\rho}\left((1 - \tau_w) w_t + T_t\right) \tag{15}$$

$$s_t = \frac{\rho}{1+\rho}\left((1 - \tau_w) w_t + T_t\right) \tag{16}$$

With the assumption of the full depreciation of capital stock in a period, wage rate (7) and saving (16), the dynamics of the capital stock can be shown as presented below.

$$K_{t+1} = \frac{\rho}{1+\rho}\left((1 - \tau_w)(1 - \theta) K_t^{\theta} + T_t\right) \tag{17}$$

If the lump-sum transfer is financed by the labor income taxation, then the capital stock at the steady state is presented as shown below.

$$K = \left(\frac{\rho(1-\theta)}{1+\rho}\right)^{\frac{1}{1-\theta}} \tag{18}$$

If the lump-sum transfer is financed by the capital income taxation, then the capital stock at the steady state can be shown as presented below.

$$K = \left(\frac{\rho(1-\theta)}{1+\rho} + \tau_r \theta\right)^{\frac{1}{1-\theta}} \tag{19}$$

Taxation effects on capital stock and consumption at the steady state can be presented as Table 2.

	K	c
τ_w	No change	No change
τ_r	+	- or +

Table 2. Taxation effects

For capital income taxation, consumption during the young period rises. However, because of an increase in transfers and a decrease in the interest rate after taxation, consumption in the older period can not always be raised. Then, the effect of the consumption is shown by - or +. If one considers the logarithm utility function, then one can obtain a different effect of capital income taxation on the capital stock in a steady state between the Ramsey model and the overlapping generations model. This result is generally known. The next section presents consideration of the other type of the transfer as a benefit for unemployment.

5. Unemployment benefit

This section presents consideration of the unemployment benefit and compares results obtained using a Ramsey model with those from an overlapping generations model. This section consider the elastic labor supply.

5.1 Ramsey model

This subsection explains consideration of unemployment in a Ramsey model. In households, share l_t of individuals have a job and share $1 - l_t$ of individuals remain unemployed. They receive unemployment benefits. Then, the budget constraint is

$$K_{t+1} = (1-\tau_w)w_t l_t + (1-\tau_r)R_t K_t + (1-l_t)b_t - c_t + (1-\delta)K_t. \tag{20}$$

where b_t and l_t respectively denote the unemployment benefit and the employment rate.

The unemployment benefit is financed by income taxation. With a balanced budget, the government budget constraint can be shown as

$$\tau_w w_t l_t + \tau_r R_t k_t = (1-l_t)b_t. \tag{21}$$

Unemployment is brought about by the wage rate being higher than the wage rate of full employment in the labor market. The wage rate in this section is determined by the labor union. The labor union considers the employment rate and the wage rate to maximize the following objective function:[2]

$$v_t = (1-\tau_w)w_t l_t + (1-\tau_r)r_t K_t + b_t(1-l_t). \tag{22}$$

With wage rate (7) and interest rate (8), the employment rate l_t to maximize (20) is derived as

$$l_t = \frac{(1-\theta)\big((1-\tau_w)+(1-\tau_r)\big)}{(1-\theta)\big((1-\tau_w)+(1-\tau_r)\big)+\tau_w+\tau_r}. \tag{23}$$

An increase in tax rate τ_w and τ_r, which raises the unemployment benefit, reduces employment.

The equilibrium of this model economy is given by (21) and the two dynamics equations below.

$$\frac{\Delta c_t}{c_t} = \rho\left((1-\tau_r)\theta K_{t+1}^{\theta-1} l_{t+1}^{1-\theta} + (1-\delta)\right) - 1 \tag{24}$$

$$\Delta K_t = K_t^{\theta} l_t^{1-\theta} + R_t K_t - c_t - \delta K_t \tag{25}$$

Then, the phase diagram is portrayed in Fig. 2. Dashed line shows the case of an increase in tax rate.

[2] This objective function is similar to that explained by Ono (2010). However, Ono (2010) assumes the objective function is constructed not by the income level but also by the utility level.

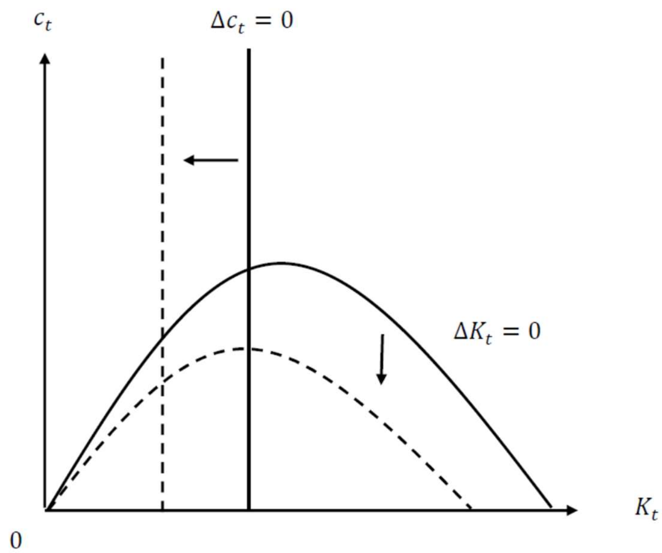

Fig. 2. Phase diagram in the case of unemployment

Effects of a tax rate increase on variables at the steady state are shown as Table 3.

	K	c	l
τ_w	-	-	-
τ_r	-	-	-

Table 3. Taxation effects

Unlike the case of lump-sum transfer, wage income taxation reduces consumption and the capital stock because of a decrease in the employment rate.

5.2 Overlapping generations model

Similarly to the discussion in the previous subsection, share l_t of individuals in a household have a job. Also, share $1 - l_t$ of individuals in households are unemployed. Then, the household budget constraint is shown as

$$c_t + \frac{c_{t+1}}{1 + (1 - \tau_r) r_{t+1}} = (1 - \tau_w) w_t l_t + b_t (1 - l_t). \tag{26}$$

Savings are given by maximizing utility function (12) subject to budget constraint (26). The dynamics of the capital stock is derived as

$$K_{t+1} = \frac{\rho}{1 + \rho} \big((1 - \theta) + \theta \tau_r\big) K_t^{\theta} l_t^{1-\theta}. \tag{27}$$

The capital stock at the steady state is given as

$$K = \left(\frac{\rho}{1 + \rho} \big((1 - \theta) + \theta \tau_r\big) \right)^{\frac{1}{1-\theta}} l. \tag{28}$$

Taxation effects on the variables at the steady state are shown by the following table.

	K	c	l
τ_w	-	- or +	-
τ_r	- or +	- or +	-

Table 4. Taxation effects

Because of decrease employment rate l, the capital stock decreases. However, the capital income tax has the effect of an increase in the capital stock. Then, effects on the capital stock at the steady state are ambiguous. Consumption in the young period decreases with wage income taxation because of a decrease in the capital stock. However, consumption during the old period can rise because of an increase in the interest rate. Even when one considers the case of capital income taxation, taxation effects on consumption during the young and old period are ambiguous. Therefore, the following proposition can be established.

Proposition 1

When considering unemployment benefits, an increase in the unemployment benefit financed by both wage income taxation and by capital income taxation reduces the capital stock, consumption, and the employment rate in a Ramsey model. However, when considered using an overlapping generations model, the capital stock and the consumption do not always decrease.

Different from a Ramsey model, the benefit financed by capital income taxation has a positive effect on capital accumulation. This effect, when conceptualized using a Ramsey model and an overlapping generations model, can yield different results. Considering the discussions presented in sections 2, 3 and 4, if the labor supply is constant over time, the results of effects of capital income taxation on the capital stock between Ramsey model and overlapping generations model differ in a steady state. However, if the labor supply is not constant, then the results of capital income taxation effects on the capital stock can be regarded as the same in a Ramsey model and in an overlapping generations model.

We can derive the condition that the capital stock is decreased by the capital income taxation in overlapping generations model. With $\theta < \frac{1}{3}$, we can obtain $\frac{dK}{d\tau_r} < 0$.[3] Then, the effect of the capital income taxation on the capital stock is same between Ramsey model and overlapping generations model.

6. Simulation

This section presents a simulation of an increase in the tax rate using a model shown as a Ramsey model in section 5.

[3] At the approximation of $\tau_w = 0, \tau_r = 0$, we obtain $\frac{dK}{K} = \frac{\theta}{(1-\theta)^2} d\tau_r + \frac{dl}{l}$ and $\frac{dl}{K} = \frac{1}{2(1-\theta)} d\tau_r$ and then $\frac{dK}{d\tau_r} = \frac{K(3\theta-1)}{2(1-\theta)^2}$.

The following equations are used.

$$\hat{l}_t = -\left(\frac{\theta}{(1-\theta)(1-\tau_w)+\tau_w} + \frac{1}{(1-\tau_w)}\right)\tilde{\tau}_{rt} \tag{29}$$

$$\hat{c}_{t+1} = \hat{c}_t + \big(1-\rho(1-\delta)\big)(\hat{r}_{t+1} - \tilde{\tau}_{rt+1}) \tag{30}$$

$$\hat{K}_{t+1} = \frac{(1-\theta)Y}{K}\big(\hat{w}_t + \hat{l}_t\big) + \frac{\theta Y}{K}\big(\hat{r}_t + \hat{K}_t\big) - \frac{c}{K}\hat{c}_t + (1-\delta)\hat{K}_t \tag{31}$$

$$\hat{w}_t = \theta\big(\hat{K}_t - \hat{l}_t\big) \tag{32}$$

$$\hat{r}_t = (1-\theta)\big(\hat{l}_t - \hat{K}_t\big) \tag{33}$$

$$\hat{Y}_t = \theta\hat{K}_t + (1-\theta)\hat{l}_t \tag{34}$$

$$\tilde{\tau}_{rt} = \phi\tilde{\tau}_{rt-1} + f \tag{35}$$

In that equations, $\hat{x}_t$ and $\tilde{x}_t$ respectively denote the deviation rate and deviation level from the steady state value. We set the first period $f = 1$ for the shock of an increase in the capital income tax rate by 1%. Also, we set 0% for the second period. In addition, ϕ shows the persistence of the policy. We set $\phi = 0.5$. Other parameters can be presented as the following table.

θ	0.3
τ_W	0.014
ρ	0.99
δ	0.06
$\frac{c}{Y}$	0.533
$\frac{Y}{K}$	0.282

Table 5. Parameter setting

For this study, θ, ρ, and δ are set as the standard Ramsey model simulation.[4] In recent years, unemployment has been at about 2%. Therefore, $\tau_W = 0.014$ and $\tau_r = 0$ because the wage income finances the unemployment benefit. Also, $\frac{c}{Y}$ and $\frac{Y}{K}$ are given by data of 2020 provided by SNA (Cabinet office, Japan). Simulation results of an increase in the unemployment benefit financed by an increase in the capital income taxation are shown as presented below.[5]

[4] For instance, Eguchi (2011) sets $\theta = 0.33$, $\rho = 0.99$, and $\delta = 0.06$.
[5] The program code of this simulation is presented at the end of this report.

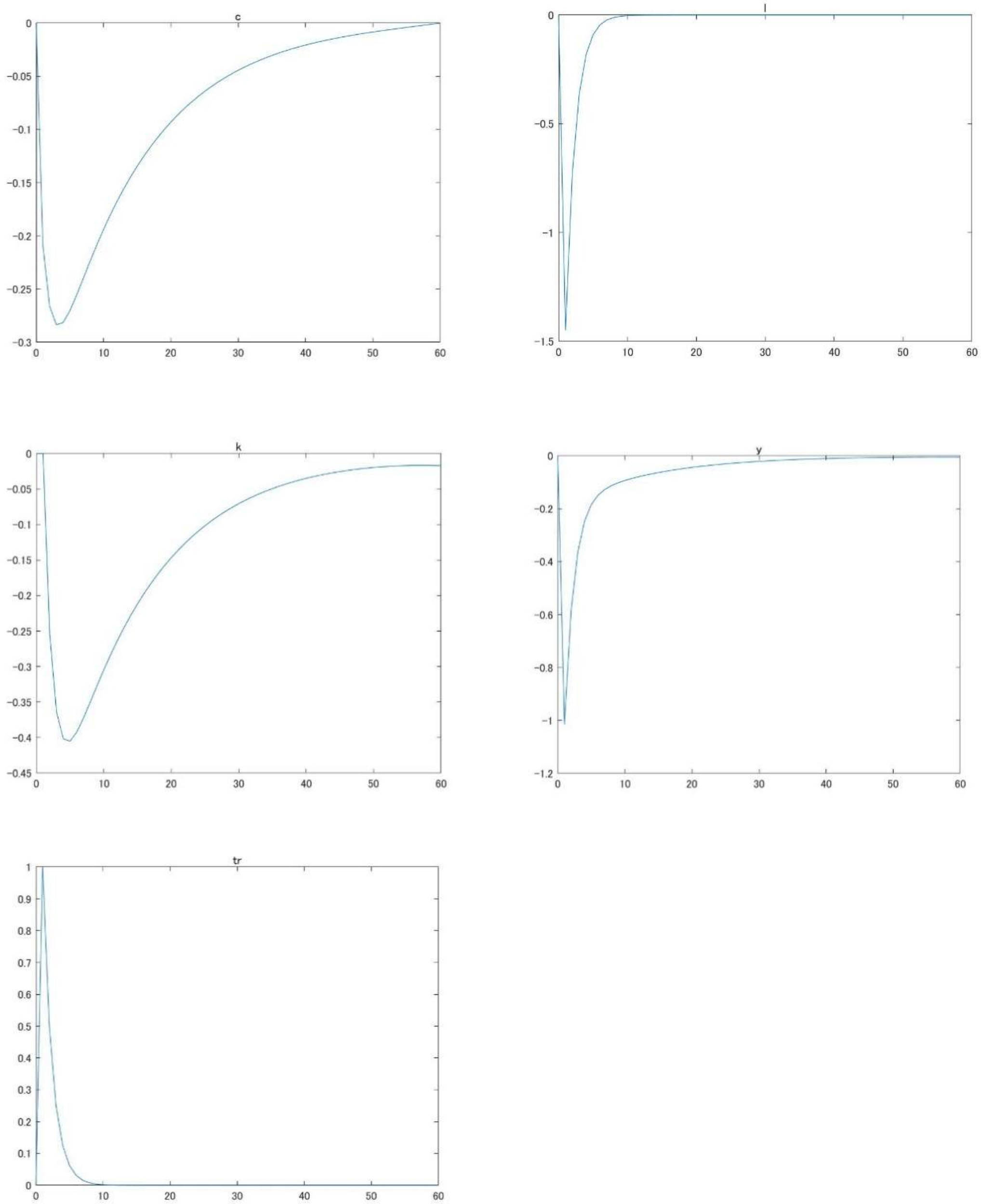

Fig. 3. Simulation results[6]

An increase in the capital income tax rate by 1% reduces production by 1% and employment by 1.5%. The results are consistent with those obtained using a theoretical approach. By comparing capital income taxation with wage income taxation, one can obtain the following Fig. 4.

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[6] x axis and the y axis in Fig.3 and Fig.4 show the period and the change rate, respectively. tr represents the capital income tax rate.

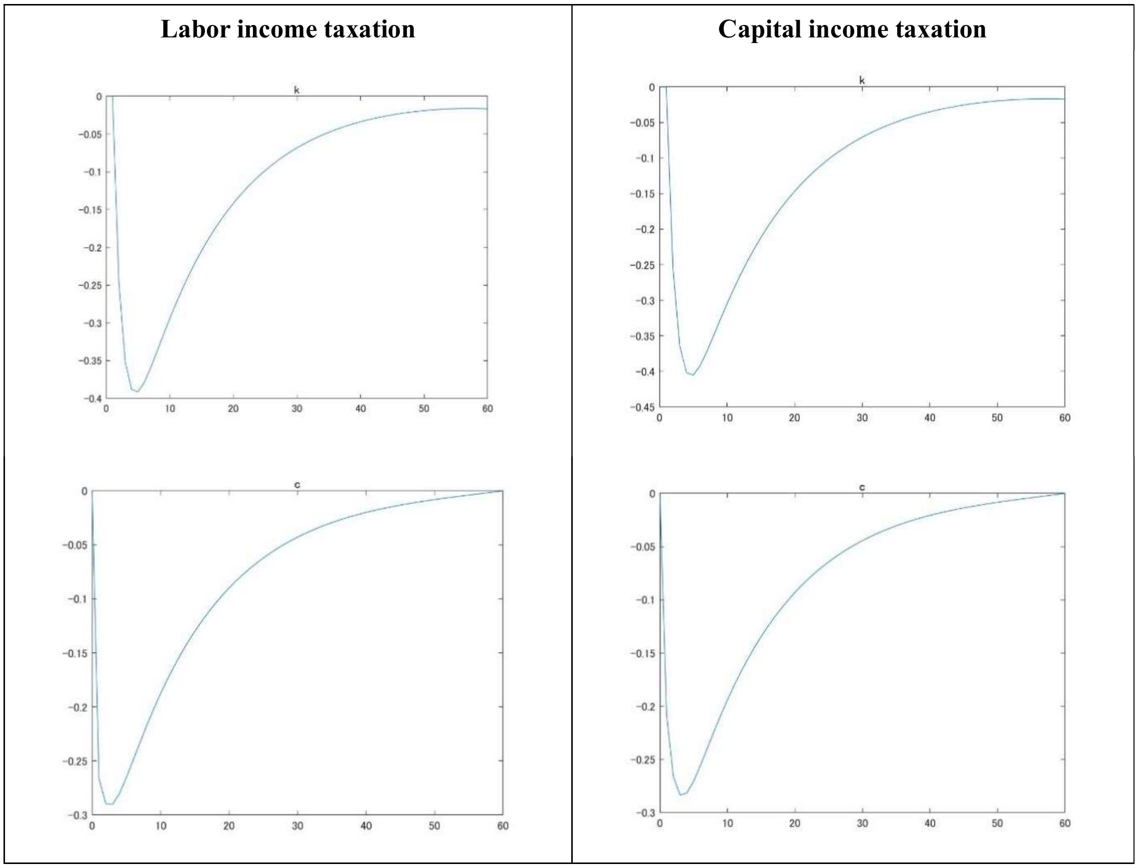

Fig. 4. Results obtained for the case of income taxation of two types.

A decrease in the capital stock in the case of the capital income taxation is greater than the case of wage income taxation. Then, an increase in the interest rate in the case of capital income taxation is greater than in the case of wage income taxation. However, a decrease occurs in the employment rate. Then the interest rate decreases. These two offsetting effects determine the consumption effects. A decrease in consumption in the case of capital income taxation is less than in the case of wage income taxation.

7. Discussion

This section presents consideration of the minimum wage as another type of unemployment. Defining w as the minimum wage, then the profit-maximizing condition can be reduced to the following labor demand.

$$l_t = \left(\frac{(1-\theta)A}{w}\right)^{\frac{1}{\theta}} K_t. \tag{36}$$

The interest rate can be presented as

$$r_t = A\theta \left(\frac{(1-\theta)A}{w}\right)^{\frac{1-\theta}{\theta}}. \tag{37}$$

In the Ramsey model, utility maximization of household decision problems can be used to derive the following dynamics of capital accumulation and consumption:

$$\frac{K_{t+1}}{K_t} = \left(\frac{(1-\theta)A}{w}\right)^{\frac{1}{\theta}} w + A\theta\left(\frac{(1-\theta)A}{w}\right)^{\frac{1-\theta}{\theta}} + 1 - \delta - \frac{c_t}{K_t}, \tag{38}$$

$$\frac{c_{t+1}}{c_t} = \rho\left((1-\tau_r)A\theta\left(\frac{(1-\theta)A}{w}\right)^{\frac{1-\theta}{\theta}} + (1-\delta)\right). \tag{39}$$

In the balanced growth path, we obtain $\frac{K_{t+1}}{K_t} = \frac{c_{t+1}}{c_t}$; then $\frac{c_t}{K_t}$ is constant over time. If the unemployment benefit is financed by labor income taxation, then no effect is exerted on the dynamics of the capital stock or of consumption. However, the capital income taxation reduces the growth rate of the consumption. Then, in the balanced growth path, the capital stock growth rate decreases.

In the case of an overlapping generations model, the dynamics of the capital stock and consumption can be presented as shown below.

$$\frac{K_{t+1}}{K_t} = (1-\tau_w)(1-\alpha)w\left(\frac{(1-\theta)A}{w}\right)^{\frac{1}{\theta}} + \frac{b_t(1-l_t)}{K_t} \tag{40}$$

$$\frac{c_{1t}}{K_t} = (1-\tau_w)\alpha w\left(\frac{(1-\theta)A}{w}\right)^{\frac{1}{\theta}} + \frac{b_t(1-l_t)}{K_t} \tag{41}$$

$$\frac{c_{2t+1}}{K_t} = (1+(1-\tau_r)r)\left((1-\tau_w)\alpha w\left(\frac{(1-\theta)A}{w}\right)^{\frac{1}{\theta}} + \frac{b_t(1-l_t)}{K_t}\right) \tag{42}$$

The benefit financed by labor income taxation does not affect the dynamics of capital because $\tau_w(1-\alpha)w\left(\frac{(1-\theta)A}{w}\right)^{\frac{1}{\theta}} = \frac{b_t(1-l_t)}{K_t}$. Capital income taxation raises consumption among young people because of an increase in $b_t(1-l_t)$. However, because of a decrease in $1+(1-\tau_r)r$, the effect on consumption during the old period is ambiguous.

If capital accumulation continues, then full employment is achieved. Finally, unemployment brought about by the minimum wage vanishes.

8. Conclusions

This paper has presented an explanation of how taxation to finance benefits affects capital accumulation and consumption. Policy effects differ between the Ramsey model and an overlapping generations model. We examine policy effects using models with a logarithmic utility function. If the labor supply is inelastic, then the effects on capital income taxation on capital accumulation differ between a Ramsey model and an overlapping generations model. However, if the labor supply is elastic, then effects of capital income taxation on capital accumulation can be equivalent in the two models. When examining policy effects with the logarithmic utility function, we devote particular attention to results derived from our study.

References

Aiyagari S. R. (1995) "Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting," Journal of Political Economy, vol.103(6), pp.1158-1175.

Barro R. J. (1990) "Government spending in a simple model of endogenous growth," Journal of Political Economy, vol.98(S5), pp.103-125.

Campbell J. Y. and N. G. Mankiw (1989) "Consumption, income and interest rates: Reinterpreting the time series evidence," NBER Macroeconomics Annual 1989, vol.4, pp.185-246.

Chamley C. P. (1986) "Optimal taxation of capital income in general equilibrium with infinite lives, " Econometrica, vol.54(3), pp.607-622.

Correia I. H. (1996) "Should capital income be taxed in the steady state?" Journal of Public Economics, vol.60(1), pp.147-151.

Eguchi M. (2011) "Dōgakuteki ippan kinkō moderu ni yoru zaisei seisaku no bunseki (Analysis of Fiscal Policy Through Dynamic General Equilibrium Modeling), " in Japanese, Mitsubishi Economic Research Institute

Fanti L. and L. Gori (2010) "Child policy solutions for the unemployment problem," Economics Letters, vol.109, pp.147–149.

Hall R. (1988) "Intertemporal substitution in consumption," Journal of Political Economy, vol. 96(2), pp.339-357.

Jones L. E., R.E. Manuelli and P. E. Rossi (1997) "On the optimal taxation of capital income," Journal of Economic Theory, vol.73(1), pp.93-117.

Lin S. (2001) "Taxation, human capital accumulation and economic growth," Japanese Economic Review, vol.52(2), pp.185–197.

Ono, T. 2010. "Growth and unemployment in an OLG economy with public pensions," Journal of Population Economics, vol. 23(2), pp. 737-767, March.

Watanabe M., Y. Miyake and M. Yasuoka (2016) "Public investment financed by consumption tax in an aging society," Singapore Economic Review, vol. 60(05), pp.1-17.

Yasuoka M. (2021) “How should a government finance for pension benefit?” Australian Economic Papers, vol.60(1), pp. 138-152.

Appendix Calibration

In the simulation section, the parameters were not derived through our estimation. However, in this appendix, we check the estimated parameters carefully. The analyses presented in this paper use Bayesian estimation for parameters. For estimation, we consider the following equations.

$$\hat{l}_t = \left(\frac{\theta(2-\tau_w)}{(1-\theta)(2-\tau_w)+\tau_w} - \frac{\theta}{1-\theta}\right)\hat{\theta}_t \tag{A.1}$$

$$\hat{c}_{t+1} = \hat{c}_t + \big(1-\rho(1-\delta)\big)\hat{r}_{t+1} \tag{A.2}$$

$$\hat{K}_{t+1} = \frac{(1-\theta)Y}{K}\left(\hat{w}_t + \hat{l}_t\right) + \frac{\theta Y}{K}\left(\hat{r}_t + \hat{K}_t\right) - \frac{c}{K}\hat{c}_t + (1-\delta)\hat{K}_t \tag{A.3}$$

$$\hat{w}_t = \hat{A}_t + \theta\left(\hat{K}_t - \hat{l}_t\right) \tag{A.4}$$

$$\hat{r}_t = \hat{A}_t + (1-\theta)\left(\hat{l}_t - \hat{K}_t\right) \tag{A.5}$$

$$\hat{Y}_t = \hat{A}_t + \theta\hat{K}_t + (1-\theta)\hat{l}_t \tag{A.6}$$

$$\hat{A}_t = \phi_1\hat{A}_{t-1} + f_1 \tag{A.7}$$

$$\hat{\theta}_t = \phi_2\hat{\theta}_{t-1} + f_2 \tag{A.8}$$

The prior means, variance, and distribution are assumed as shown by the estimation code. Some parameters are given. We estimate the share of the capital share. The estimation result shows $\theta = 0.2911$, which is nearly equal to $\theta = 0.3$.

Data used for estimation are the gross domestic product (GDP), consumption, nominal interest rate, inflation rate, increase rate of wage, the share of labor income, and the unemployment rate. The real interest rate is given as the sum of nominal interest rate and the inflation rate. The employment rate is given as 1 minus the unemployment rate. Data are annual data of 1994–2019 of Cabinet Office, Japan.

The GDP and consumption change to the value of logarithm and subtract the HP filter values. The share of labor income, the real interest rate, and unemployment are subtracted from the HP filter values.

Matlab code

```
//1. variables
var l c k w r y tr;
varexo f;

//2. parameter
parameters theta tw rho delta phi CY YK;

//2.1 parametervalue
theta = 0.3;
tw=0.014;
rho=0.99;
delta=0.06;
phi=0.5;
CY=0.533;
YK=0.282;

//3.equations
model(linear);
l=-(theta/((1-theta)*(1-tw))+1/(1-tw))*tr;
c(+1)=c+(1-rho*(1-delta))*(r(+1)-tr(+1));
k=(1-theta)*YK*(w(-1)+l(-1))+theta*YK*(r(-
1)+k(-1))-CY*YK*c(-1)+(1-delta)*k(-1);
w=theta*(k-l);
r=(1-theta)*(l-k);
y=theta*k+(1-theta)*l;
tr=phi*tr(-1)+f;
end;




//steady state check
steady;
check;

//5. simulation
shocks;
var f;
periods 1;
values 0.01;
end;

//6. results
simul(periods=60);

k1=k*100;
l1=l*100;
c1=c*100;
y1=y*100;
tr1=tr*100;

figure(1)
plot(0:60, k1(1:61)); title('k')
figure(2)
plot(0:60, l1(1:61)); title('l')
figure(3)
plot(0:60, c1(1:61)); title('c')
figure(4)
plot(0:60, y1(1:61)); title('y')
figure(5)
plot(0:60, tr1(1:61)); title('tr')
```

Matlab Calibration Code

```
var l c k w r y dtheta A y_obs c_obs r_obs w_obs dtheta_obs l_obs;

varexo ea edtheta uy uc ur uw udtheta ul;

parameters theta tw rho delta psi_1 psi_2 CY YK;

//2.1 parametervalue
tw=0.014;
rho=0.99;
delta=0.06;
CY=0.533;
YK=0.282;

model(linear);
l=(theta*(2-tw)/((1-theta)*(2-tw)+tw)-theta/(1-theta))*dtheta;
c(+1)=c+(1-rho*(1-delta))*r(+1);
k=(1-theta)*YK*(w(-1)+l(-1))+theta*YK*(r(-1)+k(-1))-CY*YK*c(-1)+(1-delta)*k(-1);
w=A+theta*(k-l);
r=A+(1-theta)*(l-k);
y=A+theta*k+(1-theta)*l;
A=psi_1*A(-1)+ea;
dtheta=psi_2*dtheta(-1)+edtheta;

y_obs=y+uy;
c_obs=c+uc;
r_obs=r+ur;
w_obs=w+uw;
dtheta_obs=dtheta+udtheta;
l_obs=l+ul;
end;

estimated_params;
psi_1, beta_pdf, 0.5, 0.1;
psi_2, beta_pdf, 0.5, 0.1;
theta, normal_pdf, 0.3, 0.1;
stderr ea, inv_gamma_pdf, 0.1, inf;
stderr edtheta, inv_gamma_pdf, 0.1, inf;
stderr uy, inv_gamma_pdf, 0.1, inf;
stderr uc, inv_gamma_pdf, 0.1, inf;
stderr ur, inv_gamma_pdf, 0.1, inf;
stderr uw, inv_gamma_pdf, 0.1, inf;
stderr udtheta, inv_gamma_pdf, 0.1, inf;
stderr ul, inv_gamma_pdf, 0.1, inf;
end;

varobs y_obs c_obs r_obs w_obs dtheta_obs l_obs;

estimation(datafile = jpdata, mode_check, mh_replic =500000, mh_nblocks =2, mh_drop =0.5, mh_jscale =0.5, bayesian_irf);
```

Data File

```
data_q = [
-0.011568375	-0.031372053	0.010353105	0.015	-0.002041548	0.010505936
0.007378797	-0.014729247	0.001389498	0.011	-0.001102573	0.008139574
0.030345589	0.002242977	-0.002180579	0.011	0.007600836	0.006766645
0.043713114	0.016796825	-0.019964466	0.016	-0.014571056	0.007375497
0.029810695	0.004779282	-0.00816814	-0.013	-0.022446428	0.000950247
0.012268735	0.004204934	0.000214902	-0.015	-0.014965221	-0.004529598
0.024167716	0.008510318	0.005196267	0.001	0.001977687	-0.004085124
0.015765443	0.013086465	0.003787425	-0.016	-0.008637428	-0.006734586
0.001102874	0.007826148	0.006196603	-0.029	0.007920756	-0.010493685
-0.00102927	-0.000207662	0.000729656	-0.007	0.028368083	-0.009373916
0.008822593	0.003949424	-0.001911428	-0.005	0.032433885	-0.003380211
0.014062141	0.008394317	0.000974876	0.008	0.017709601	-0.000511647
0.018482603	0.0149802	-0.002608706	0.002	0.018615959	0.002234813
0.026717317	0.017950246	0.003239938	-0.009	0.014248546	0.003862526
0.004900146	0.011571395	-0.010475446	-0.003	-0.037644134	0.002373453
-0.058082632	-0.021555261	0.01654686	-0.038	-0.025407477	-0.009232862
-0.037609565	-0.015914264	0.009715124	0.006	0.002257004	-0.009958355
-0.057450324	-0.030238376	0.005927269	-0.003	-0.013952689	-0.005799193
-0.052430727	-0.020678266	0.003275149	-0.008	-0.011058531	-0.003745319
-0.038508191	-0.001793441	-0.000553087	-0.002	0.010065332	-0.001783052
-0.02034185	0.00759731	-0.02297134	0.005	0.005057479	0.001103631
0.009895236	0.004639915	-0.003393161	0.001	0.024029524	0.001931866
0.014463057	-0.00780531	0.005682253	0.006	0.007783969	0.003718101
0.030160895	0.003578485	0.000457827	0.004	0.008790171	0.005477575
0.028380181	0.006963239	-0.003667065	0.014	-0.010004688	0.008223203
0.037383046	0.007222399	0.002206664	-0.005	-0.02502706	0.006964479
];
y_obs = data_q(:,1);
c_obs = data_q(:,2);
r_obs = data_q(:,3);
w_obs = data_q(:,4);
dtheta_obs = data_q(:,5);
l_obs = data_q(:,6);
```