Pareto Improvement in Monopoly Regulation Using Pre-Donation

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Abstract. Revelation principle implies that given any admissible social welfare function, the outcome of Baron and Myerson's (1982) (BM) optimal direct-revelation mechanism under incentive constraints cannot be dominated by any other mechanism in expected utilities. However, since the expected total surplus varies with a change in the social welfare function, Pareto improvements should be possible if the monopolist and consumers can agree, by means of side payments that reveal no additional information to the regulator, on the use of an alternative social welfare function which would generate a lower expected deadweight loss. We check the validity of this intuition by integrating the BM mechanism with an induced cooperative bargaining model where unilateral pre-donation by consumers or the producer is allowed. Under this new mechanism producer's pre-donation in the *ex-ante* stage always leads to *ex-ante* Pareto improvement while a certain amount of it completely eliminates the expected deadweight loss. Moreover, if optimally designed in the *interim* stage, the producer's pre-donation may also lead under some cost parameters to *interim* (and also *ex-post*) Pareto improvement. Consumers, on the other hand, have no incentive to make a unilateral pre-donation, nor to reverse the optimal pre-donation of the monopolist.

Keywords: Monopoly regulation; cooperative bargaining; pre-donation.

JEL codes: C78; D42; L51.

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1 Introduction

A seminal paper by Baron and Myerson (1982) (henceforth, BM) shows that a monopolist with unknown costs can be optimally regulated by a direct-revelation mechanism which cannot be dominated by any other mechanism in terms of the expected welfare distribution. However, since the expected total surplus implied by their mechanism varies with a change in the expected social welfare function, intuition suggests that Pareto improvements may be possible if the monopolist and consumers can agree, by means of side payments, on the use of an alternative social welfare function which would generate a lower expected deadweight loss. In this paper, we check the validity of this intuition by integrating the BM mechanism with an induced cooperative bargaining model that allows unilateral pre-donation by consumers or the producer. To explain this integrated mechanism and what it can achieve in more detail, we shall briefly re-introduce below the regulation problem considered by BM along with their solution.

Using the Revelation Principle (Dasgupta, Hammond, and Maskin, 1979; Myerson, 1979; Harris and Townsend, 1981), BM restrict themselves, for monopoly regulation, to direct revelation mechanisms that ask the monopolist to report its unknown cost parameter and that give it no incentive to lie. Using such mechanisms, BM calculate, on behalf of a benevolent and computationally able regulator, the smallest individually-rational subsidy that must be offered by consumers, at each value of the cost parameter, to the monopolist in order to induce it to truthful revelation. The information about this subsidy function allows the regulator to calculate the welfare of the monopolist (operating profit plus the subsidy received) and the welfare of consumers (consumer surplus net of the subsidy) as a function of the monopolist's possible cost reports. BM brings together these two welfare functions, representing the conflicting interests of the two parties, under a generalized social welfare function, an important novelty for the regulation literature at the time. Formally, they define the ex-post social welfare function as the sum of consumer welfare and a fraction of the producer welfare. Given the regulator's incomplete information represented by a commonly known prior belief about the unknown cost parameter, BM assume that the regulator's task is to maximize the expected value of the ex-post social welfare over the set of cost reports ensuring the operation of the monopolist.

The Bayesian approach thus introduced by BM to the regulation literature is indispensable, as it was immediately revealed by their regulatory solution that there can exist no feasible direct-revelation mechanism that can maximize the *ex-post* social welfare function unless this function treats consumers and the monopolist equally. In this restrictive case, the optimal regulatory solution that ensures marginal cost pricing coincides (in terms of the welfare allocation) with the earlier solution of Loeb and Magat (1979) (henceforth, LM) achieved by the use of a delegation mechanism where the monopolist is entitled, through an output dependent subsidy scheme, the sole right to the whole economic surplus at the output it is delegated to choose. This solution by LM, however, cannot be optimal when the social welfare function is of an asymmetric form assigning to the welfare of the monopolist a weight less than one. The reason is that marginal cost pricing would then lead to a suboptimally high level of subsidy, both in the LM and BM model, which would reduce the actual (and expected) social welfare below a level that is inevitable. The solution proposed by BM under these asymmetric forms of welfare functions requires the price of the good to be always above the marginal cost of the monopolist in order to limit the subsidy paid to the monopolist, hence its informational rent (the producer welfare).

In this study, we ask whether we can obtain a regulatory outcome which is Pareto superior to that of BM in terms of the expected or actual welfare distribution. Notice that this question is not necessarily invalidated by the Revelation Principle, which, for our problem, would state that if a socially efficient allocation rule (maximizing a given social welfare function at each cost parameter) can be implemented by an arbitrary mechanism, then the same rule can be implemented by an incentive-compatible direct mechanism. This principle merely implies that once we fix an expected social welfare function in the BM model of regulation where the welfare weights of the producer and consumers are pre-determined and do not change during the regulatory process, no mechanism of any form can generate a higher expected social welfare than the direct revelation mechanism of BM. An implication of this result is that the welfare allocations that correspond to different (expected) social welfare functions cannot be Pareto ranked, further implying that the regulator cannot have any meta preference or ranking over the set of possible social welfare functions when she tries to construct such a preference comparing the welfare allocations induced by the BM mechanism. Clearly, if she had such a meta preference, the optimality of the regulatory mechanism would require the regulator to select the best social welfare function in terms of the induced welfare allocation and announce it as part of the mechanism before the regulatory action takes place. The regulator's lack of a meta preference over the set of social welfare functions should not mean, however, a complete impartialness for her or the society on whose behalf she acts. Under the BM mechanism, the expected total surplus, or the equally weighted sum of the producer's and consumers' welfares, does vary with a change in the social welfare function. Therefore, *ex-ante* Pareto improvements should generally be possible if the monopolist and consumers can agree –by means of some constant side payments that will not harm the incentive-compatibility constraints– on the use of an alternative social welfare function that generates a lower deadweight loss than predicted by the BM model.

The desired Pareto improvements over the outcome of the BM mechanism can be achieved only if the superior mechanism we are looking for may yield welfare allocations that are not attainable by the BM mechanism. Hoping to explore such a superior mechanism, we will augment the BM mechanism with some additional elements that will map, with the help of some pre-committed side payments decided upon by consumers or the monopolist, each social welfare function that can be (initially) chosen by the regulator in the BM mechanism to a new social welfare function that will be used in the augmented mechanism. The resulting regulatory mechanism will be incentive-compatible, like the BM mechanism, only if the regulator can perfectly commit not to use any additional information revealed by the augmented mechanism to update her prior beliefs about the monopolist's private cost. We will show that in some informational situations, we do not even need this commitment on the part of the regulator since the augmented mechanism would reveal no additional information to the regulator than she would already observe under the BM mechanism.

In more detail, we make the aforementioned augmentation or modification to the BM mechanism by adding, prior to the revelation of the cost information, an initial stage involving a cooperative bargaining game between consumers and the monopolist over the possible regulatory outcomes, hence over the possible social welfare functions, under the possibility of pre-donation. We model this cooperative bargaining game as in Saglam (2021), who shows that the BM model of regulation is isomorphic to a cooperative bargaining problem a la Nash (1950) with appropriate elements. On the other hand, we borrow our insight as to the potential welfare benefits of pre-donation in a Pareto sense from a literature pioneered by Sertel (1992), who shows that in simple bargaining problems (where the bargaining set has a linear frontier) the two-person Nash bargaining rule can be manipulated via pre-donations: the bargaining party with the higher valuation can alter the bargaining set always to its benefit.[1] A more recent work by Akin et al. (2011) in the same direction even shows that in simple n-person bargaining problems the manipulation of Kalai-Smorodisnky rule through pre-donation may lead to (strong) Pareto improvements. Motivated by these results,

[1]For more in this literature, see Sertel and Orbay (1998), Orbay (2003), Akyol (2008), Akin et al. (2011), among others.

we aim to explore whether a modified mechanism bringing the regulatory bargaining idea of Saglam (2021) and the pre-donation idea of Sertel (1992) together may lead to Pareto improvements over the BM mechanism.

Since the utilities in the bargaining setup of Nash (1950), and accordingly in Saglam (2021), are von Neumann-Morgenstern (expected) utilities, any Pareto improvement which may be deduced by only inspecting the effect of pre-donation on bargaining solutions is bound to be an *ex-ante* improvement, defined in expected utilities. However, we will also deal with ex-post improvements. To make both types of improvements meaningful for the monopolist, we will consider two informational stages in our extended regulatory model. The first stage is called the *ex-ante* stage where the monopolist has not learned yet the actual value of its cost parameter and shares the regulator's beliefs about it. The second stage is the *interim* stage where the monopolist privately knows the actual value of its cost parameter. Associated with these two stages, our model will have two variants, depending upon whether pre-donations occur in the *ex-ante* stage or the *interim* stage. However, we will retain the assumption from the BM model that information revelation will occur in the *interim* stage. We will also assume that both the monopolist and consumers will be informed by the regulator as to the details of the regulatory mechanism at the beginning of the stage they are allowed to make pre-donation. Given these assumptions, we observe that if consumers should decide whether and how much to pre-donate in the *ex-ante* or *interim* stage (which they can never distinguish from each other based on their own information in the model), they should always consider the maximization of their *ex-ante* payoffs. On the other hand, the producer should take into account its *ex-ante* payoff if it makes pre-donation decisions in the *ex-ante* stage and its *interim* or equivalently *ex-post* payoff if it makes these decisions in the *interim* stage.

Our results show that any amount of pre-donation made by the producer in the *ex-ante* stage always leads to *ex-ante* Pareto improvement in the welfare allocation while a certain amount of it completely eliminates the expected deadweight loss. Moreover, pre-donation in the *ex-ante* stage reveals no information about the producer's private costs, hence it creates no commitment problem on the part of the regulator. We also show that the pre-donation of the producer, if optimally designed in the *interim* stage, may also lead under some cost parameters to *ex-post* Pareto improvement. Since the optimal pre-donation of the producer is not independent of its private cost information in the *interim* stage, the producer unintentionally reveals some part of this information regardless whether it chooses to pre-donate or not. However, since the producer can always commit to pre-donation functions that will increase the expected utility of consumers and since such increases would be verifiable before the cost revelation occurs, a benevolent regulator may find it beneficial to perfectly commit *ex-ante* not to use the information that would be revealed by pre-donation to update her prior beliefs about the producer's private cost information. Finally, we show that consumers have no incentive to make a unilateral pre-donation, nor to reverse the optimal pre-donation of the producer.

The rest of the paper is organized as follows. Section 2 introduces the basic structures, Section 3 presents our results, and finally Section 4 concludes.

2 Basic Structures

Consider a monopolist producing a single good under the inverse demand function

$$P(q) = a - q, \tag{1}$$

where $a > 0$. The monopolist is subject to a cost function

$$C(q, \theta) = \theta q \text{ if } q > 0 \text{ and } C(0, \theta) = 0, \tag{2}$$

where $q \geq 0$ denotes the quantity of supply and $\theta \in [0, a)$ denotes the constant marginal cost which is privately known by the monopolist. On the other hand, the support of θ, the demand parameter a as well as the form of the inverse demand and cost functions described are common knowledge.

The monopolist is optimally regulated by a benevolent regulator who believes that the private cost parameter of the monopolist is uniformly distributed on the interval $[0, a)$ according to the probability density function $f(\theta)$ such that $f(\theta) = 1/a$ if $\theta \in [0, a)$ and $f(\theta) = 0$ otherwise. The problem facing the regulator is to choose the optimal price of the good to maximize the expected social welfare under her beliefs. We should notice that the regulatory structure described above simplifies the structure considered by Baron and Myerson (BM) (1982), where the cost function is affinely linear, involving a fixed part as well, whereas the inverse demand function and the regulator's beliefs are not restricted to any specific forms. While we make our simplifications for the sake of clarity and tractability; it will become clear throughout our analysis that our results can be extended to other forms of regulatory structures, as well.

The solution to the regulatory problem we have described above is proposed by BM in their more general structure. According to this solution, the regulator can, with no loss of generality, restrict herself to incentive-compatible direct revelation mechanisms that ask the producer to report its parameter θ and that gives the producer no incentive for lying. These mechanisms involve functions $\langle p(.), q(.), r(.), s(.)\rangle$ such that when $\tilde{\theta}$ is the cost report of the monopolist, $p(\tilde{\theta})$ and $q(\tilde{\theta})$ become the price and quantity satisfying $p(\tilde{\theta}) = a - q(\tilde{\theta})$, $r(\tilde{\theta})$ becomes the probability that the regulated monopolist is allowed to produce and sell, and $s(\tilde{\theta})$ becomes the expected subsidy paid by consumers to the monopolist to ensure a truthful response.

Given a mechanism $\langle p(.), q(.), r(.), s(.)\rangle$, if the monopolist with the true marginal cost θ submits the cost report $\tilde{\theta}$, it obtains the regulated profit $\pi(\tilde{\theta}, \theta) = [p(\tilde{\theta})q(\tilde{\theta}) - \theta q(\tilde{\theta})]r(\theta) + s(\tilde{\theta})$. This mechanism is called feasible if (i) it is incentive-compatible; i.e. $\pi(\theta) \equiv \pi(\theta, \theta) \geq \pi(\tilde{\theta}, \theta)$ for all $\theta, \tilde{\theta} \in [0, a)$ and (ii) it is individual rational; i.e., $\pi(\theta) \geq 0$ for all $\theta \in [0, a)$. The first condition implies that the function $q(.)$ is non-increasing over $[0, a)$ and

$$\pi(\theta) = \int_0^a q(x)r(x)dx + \pi(a). \tag{3}$$

Given a feasible mechanism $\langle p(.), q(.), r(.), s(.)\rangle$, the actual welfare of consumers can be calculated as

$$CW(\theta) = \left[\int_0^{q(\theta)} (a - x)dx - p(\theta)q(\theta)\right] r(\theta) - s(\theta). \tag{4}$$

Using $s(\theta) = \pi(\theta) - [p(\theta)q(\theta) - \theta q(\theta)]r(\theta)$, the above equation can be simplified as

$$CW(\theta) = \left[\int_0^{q(\theta)} (a - x)dx - \theta q(\theta)\right] r(\theta) - \pi(\theta). \tag{5}$$

Given $\pi(\theta)$ and $CW(\theta)$, the actual social welfare can be defined, as in the BM model, by the equation

$$SW(\theta) = CW(\theta) + \alpha\pi(\theta), \tag{6}$$

where α is a fixed parameter in $[0, 1]$. The problem facing the regulator is to find a feasible mechanism $\langle p(.), q(.), r(.), s(.)\rangle$ that maximizes the expected social welfare

$$\begin{aligned} SW^e &\equiv \int_0^a SW(\theta)f(\theta)d\theta \\ &= \int_0^a \left(\left[\int_0^{q(\theta)} (a - x)dx - \theta q(\theta)\right] r(\theta) - (1 - \alpha)\pi(\theta)\right) f(\theta)d\theta. \end{aligned} \tag{7}$$

We can observe from the above equation along with (3) that any mechanism maximizing SW^e must yield $\pi(a) = 0$. To completely characterize this mechanism, we can modify the optimal mechanism of BM for the special forms of demand, cost, and belief functions in our model. This modification results in the optimal mechanism $\langle p^*(.), q^*(.), r^*(.), s^*(.)\rangle$ satisfying

$$p^*(\theta) = (2-\alpha)\theta \tag{8}$$

$$q^*(\theta) = a - (2-\alpha)\theta \tag{9}$$

$$r^*(\theta) = \begin{cases} 1 & \text{if } \theta \leq \theta^* \equiv \dfrac{a}{2-\alpha} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

and

$$s^*(\theta) = \int_0^a q^*(x)r^*(x)dx + [\theta q^*(\theta) - p^*(\theta)q^*(\theta)]r^*(\theta) \tag{11}$$

for all $\theta \in [0, a)$. The above mechanism yields to the producer the actual welfare given by

$$\begin{aligned} \pi(\theta,\alpha) \equiv \pi(\theta) &= \int_\theta^{\theta^*(\alpha)} q^*(x,\alpha)dx = \\ &= \left(\frac{2-\alpha}{2}\right)\theta^2 - a\theta + \frac{a^2}{2(2-\alpha)}, \end{aligned} \tag{12}$$

if $\theta \in [0, \theta^*(\alpha))$ and $\pi(\theta,\alpha) = 0$ otherwise. On the other hand, the consumer welfare would become

$$\begin{aligned} CW(\theta,\alpha) \equiv CW(\theta) &= \left[\int_0^{q^*(x,\alpha)} (a-x)dx - \theta q^*(x,\alpha)\right] r^*(\theta) - \pi(\theta,\alpha) \\ &= (a-\theta)\left[a-(2-\alpha)\theta\right] - \frac{1}{2}\left[a-(2-\alpha)\theta\right]^2 \\ &\quad - \left(\frac{2-\alpha}{2}\right)\theta^2 + a\theta - \frac{a^2}{2(2-\alpha)} \end{aligned} \tag{13}$$

if $\theta \in [0, \theta^*(\alpha))$ and $CW(\theta,\alpha) = 0$ otherwise. From the viewpoint of consumers and the regulator, the above welfares are unknown before θ is revealed by the producer. But, they can calculate the expected values of these welfares as

$$CW^e(\alpha) = \int_0^a CW(\theta,\alpha)r^*(\theta)f(\theta)d\theta = \frac{2(1-\alpha)a^2}{6(2-\alpha)^2} \tag{14}$$

and

$$PW^e(\alpha) = \int_0^a \pi(\theta,\alpha) r^*(\theta) f(\theta) d\theta = \frac{a^2}{6(2-\alpha)^2} \tag{15}$$

respectively. Notice that the pair $(CW^e(\alpha), PW^e(\alpha))$ denotes the expected welfare (utility) distribution generated by the BM mechanism when the producer welfare is weighted by α in the social welfare function. We will denote this pair simply by $W(\alpha)$. Likewise, we will henceforth denote $p^*(\theta), q^*(\theta), r^*(\theta), s^*(\theta)$, and θ^*, by the variables $p^*(\theta,\alpha), q^*(\theta,\alpha)$, $r^*(\theta,\alpha), s^*(\theta,\alpha)$, and $\theta^*(\alpha)$, respectively.

We can now calculate the expected economic surplus, $ES^e(\alpha) \equiv CW^e(\alpha) + PW^e(\alpha)$

$$ES^e(\alpha) = \frac{(3-2\alpha)a^2}{6(2-\alpha)^2}. \tag{16}$$

Notice that $ES^e(\alpha)$ attains its maximum value of $a^2/6$ if $\alpha = 1$, in which case $ES^e(\alpha)$ coincides with $SW^e(\alpha)$. Let V denote this maximal surplus; i.e. $V \equiv a^2/6$. Notice that V is the expected value of the actual surplus $\nu(\theta) \equiv (a-\theta)^2/2$ under the regulator's belief f; i.e., $V = E[\nu(\theta)|f]$.

Given V, we can write for any α the expected economic surplus as $ES^e(\alpha) = (3-2\alpha)V/(2-\alpha)^2$. We can also define, for any value of α, the expected deadweight loss $DW^e(\alpha) \equiv V - ES^e(\alpha)$ and calculate it as

$$DW^e(\alpha) = \frac{(1-\alpha)^2}{(2-\alpha)^2} V. \tag{17}$$

We should notice that the distribution of expected welfare, $(CW^e(\alpha), PW^e(\alpha))$, as well as the expected deadweight loss, $DW^e(\alpha)$, varies with the parameter α. In particular, we can observe that the triplet $(CW^e(\alpha), PW^e(\alpha), DW^e(\alpha))$ is equal to $(V/2, V/4, V/4)$ if $\alpha = 0$, and equal to $(0, V, 0)$ if $\alpha = 0$. We can also check that $PW^e(\alpha)$ is increasing in α, whereas $CW^a(\alpha)$ and $DW^e(\alpha)$ are decreasing. If the regulator were to choose $\alpha = 1$ to minimize (eliminate) the deadweight loss, it would unintentionally minimize the expected welfare of consumers, as well. On the other hand, if the regulator were to choose $\alpha = 0$ to maximize the expected welfare of consumers, it would unintentionally maximize the expected deadweight loss. Thus, a benevolent regulator acting on behalf of the society is confronted with a dilemma as to how to choose α in the most plausible way from the viewpoint of consumers and the social efficiency. Borrowing from Saglam (2021), we leave the solution of this dilemma to a regulatory bargaining process, between consumers and the monopolist, which integrates the bargaining model of Nash (1950) with a simplified version of BM's (1982) regulatory model, which we have described above. To define this bargaining process, we need some preliminaries.

2.1 Cooperative Bargaining

Consider a society of players $N = \{1, 2\}$, where 1 denotes consumers and 2 denotes the monopolist-producer. Following Nash (1950), we define a two-player bargaining problem for this society by a pair (S, d), where $S \subset \mathbb{R}^2$ denotes the bargaining set consisting of von Neumann-Morgenstern utility allocations, and $d \in S$ denotes the disagreement point specifying the utility each player must enjoy if they fail to agree on any other point in S. The set S is assumed to be compact and convex, and it contains a point s with $s > d$. Also, S is d-comprehensive; i.e., for all $s, s' \in \mathbb{R}^2$, $s \in S$ and $s \geq s' \geq d$ only if $s' \in S$. Let Σ^2 denote the set of all two-person bargaining problems that satisfy the assumptions above.

A bargaining rule $F : \Sigma^2 \to \mathbb{R}^2$ is a mapping such that $F(S, d) \in S$ for any $(S, d) \in \Sigma^2$. Notice that $F_1(S, d)$ and $F_2(S, d)$ are the bargaining utilities of player 1 and player 2, respectively.

Below, we define some well-known bargaining rules. The Nash (1950) rule proposes for any problem $(S, d) \in \Sigma^2$ the solution

$$N(S, d) = \text{argmax}_{x \in S}\, (x_1 - d_1)(x_2 - d_2), \tag{18}$$

at which the product of players' net utility gains from agreement attains its maximum.

The Kalai-Smorodinsky rule, proposed by Raiffa (1953) for two-person games and axiomatized by Kalai and Smorodinsky (1975), selects for any problem $(S, d) \in \Sigma^2$ the allocation

$$KS(S, d) = \max \left\{ x \in S \;\middle|\; \frac{x_1 - d_1}{x_2 - d_2} = \frac{a_1(S, d) - d_1}{a_2(S, d) - d_2} \right\}, \tag{19}$$

where for each $i = 1, 2$, $a_i(S, d) = \max\{s_i \mid s \in S \text{ and } s_{-i} = d_{-i}\}$ denotes the ideal utility player i can expect from (S, d). Accordingly, the point $a(S, d) = (a_1(S, d), a_2(S, d))$ is called the ideal point for (S, d). The Kalai-Smorodinsky rule selects the maximum point of S on the line segment connecting the points d and $a(S, d)$.

A bargaining rule is called dictatorial for player i, or Dictatorial-i, and denoted by D^i if for each $(S, d) \in \Sigma^2$

$$D^i(S, d) = \max\{x \in S \mid\; x_i \geq d_i \;\text{ and }\; x_j = d_j \text{ for } j \neq i\}. \tag{20}$$

The rule D^i chooses for player i the best point in the bargaining set, while providing to the other player its disagreement utility.

A family of solutions, known as proportional solutions (Kalai, 1977), will be sufficient for the analysis in this paper for reasons which will be explained later. Given any $\gamma \geq 0$, a bargaining rule is called γ-proportional, or simply P^γ, if it selects for any $(S,d) \in \Sigma^2$ the allocation

$$P^\gamma(S,d) = d + \Omega(S,d)(\gamma,1) \text{ and } \Omega(S,d) = \max\{t \mid d + t(\gamma,1) \in S\}. \tag{21}$$

We should notice that the rule P^γ selects the maximum point of S on the line passing through the point d and the point $(\gamma,1)$. In the definition of Kalai (1977), γ is positive. We have included $\gamma = 0$ for convenience. (Notice that when $\gamma = 0$, the proportional rule we have defined above coincides with a special rule that gives to player 2 full dictatorial power.) When $\gamma = 1$, we obtain a well-known member of the γ-proportional rules, known as the Egalitarian rule, which was first recommended by Rawls (1972). For any bargaining problem, this rule chooses an allocation at which the worst-off player's net utility gain of from agreement is maximized. Also, note that for $\gamma = 0$ and $\gamma = \infty$, the rule P^γ coincides with the dictatorial rules D^2 and D^1, respectively.

For any $S \subset \mathbb{R}^2$, we denote by $WPO(S) = \{x \in S \mid y > x \text{ implies } y \notin S\}$ the set of weakly Pareto optimal allocations in S and likewise we denote by $PO(S) = \{x \in S \mid y \geq x \text{ implies } y \notin S\}$ the set of Pareto optimal allocations in S. Below, we present some axioms for an arbitrary solution F on Σ^2.

Weak Pareto Optimality (WPO) If $(S,d) \in \Sigma^2$, then $F(S,d) \in WPO(S)$.

Pareto Optimality (PO) If $(S,d) \in \Sigma^2$, then $F(S,d) \in PO(S)$.

Nash and Kalai-Smorodinsky rules satisfy Pareto Optimality (hence Weak Pareto Optimality), whereas any γ-proportional satisfies Weak Pareto Optimality, but not Pareto Optimality.

2.2 Pre-Donation

We modify Sertel's (1992) definition of pre-donation for our model. A *pre-donation* from player i to player $j \neq i$ is a function $\boldsymbol{\lambda}^{\boldsymbol{k,i}} : \mathbb{R}^2 \to \mathbb{R}^2$, parameterized by some number $k \in [0,1)$, which transforms each $s \in \mathbb{R}^2$ into $\boldsymbol{\lambda}^{\boldsymbol{k,i}}(s)$ such that $\boldsymbol{\lambda}_i^{\boldsymbol{k,i}}(s) = (1-k)s_i$ and $\boldsymbol{\lambda}_j^{\boldsymbol{k,i}}(s) = s_j + ks_i$ if $j \neq i$. Given any bargaining set S and any pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k,i}}$, we write

$$\boldsymbol{\lambda}^{\boldsymbol{k,i}}(S) = \{\lambda^{k,i}(s) \mid s \in S\} \tag{22}$$

and for the comprehensive closure of $\boldsymbol{\lambda}^{\boldsymbol{k,i}}(S)$ we write

$$\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S) = \{s' \in \mathbb{R}^2_+ \mid s'_i \leq s_i \text{ and } s'_j \leq s_j \text{ if } j \neq i, \text{ for some } s \in \boldsymbol{\lambda}^{\boldsymbol{k,i}}(S)\}. \tag{23}$$

Notice that $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S)$ is a convex and comprehensive bargaining set as in the model of Nash (1950). Moreover if $d \in S$, then $\boldsymbol{\lambda}^{\boldsymbol{k,i}}(d) \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S)$. So, we will assume that the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k,i}}(S)$ transforms the bargaining problem (S, d) into the problem $(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k,i}}(d))$.

Given any problem $(S, d) \in \Sigma^2$, any bargaining rule F on Σ^2, any $k \in [0, 1)$, and any $i \in \{1, 2\}$, we say that the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k,i}}$ is

(i) beneficial for player $m \in \{1, 2\}$ if $F_m(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k,i}}(d)) > F_m(S, d)$,

(ii) harmful for player $m \in \{1, 2\}$ if $F_m(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k,i}}(d)) < F_m(S, d)$,

(iii) ineffective for player $m \in \{1, 2\}$ if $F_m(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k,i}}(d)) = F_m(S, d)$.

2.3 Regulatory Bargaining under Pre-donation

Now we can turn to consider the specific bargaining problem in the regulated monopolistic industry. We assume that if the monopolist and consumers fail to agree in the bargaining process, then the monopolist is not allowed to operate and consequently both parties end up with zero utilities. Accordingly, we set the disagreement point to $d^R = (0, 0)$, where the superscript R emphasizes that the bargaining payoffs are related to the 'regulatory' mechanism of BM. Notice that as the parameter α is varied on the interval $[0, 1]$, equations (14) and (15) together define a locus of points in $\mathbb{R}^2_+$. Defining $\hat{u}_1(\alpha) \equiv CW^e(\alpha)$ and $\hat{u}_2(\alpha) \equiv PW^e(\alpha)$, we can write this locus as

$$\hat{u}_1(\alpha) = 2\sqrt{V\hat{u}_2(\alpha)} - 2\hat{u}_2(\alpha). \tag{24}$$

The convex and comprehensive hull of the above locus of points defines the bargaining set, S^R, facing the players in the absence of pre-donation:

$$S^R = \left\{ u(\alpha) \;\middle|\; \begin{array}{l} u_1(\alpha) = \dfrac{2(1-\alpha)V}{(2-\alpha)^2}, \\ 0 \le u_2(\alpha) \le \dfrac{V}{(2-\alpha)^2}, \quad \alpha \in [0, 1] \end{array} \right\} \tag{25}$$

Notice that $PO(S^R)$ is the locus of points that satisfy (24). The pair (S^R, d^R) is the (regulatory) bargaining problem in the absence of pre-donation. With pre-donation, the problem (S^R, d^R) is transformed into a new problem which we will describe next.

First recall that we denote consumers and the producers by the indices 1 and 2, respectively. Thus, $\boldsymbol{\lambda}^{\boldsymbol{k,1}}$ ($\boldsymbol{\lambda}^{\boldsymbol{k,2}}$) denotes the pre-donation from consumers to the producer (from the producer to consumers), realized at the rate $k \in [0, 1)$. We should observe that given any $k \in [0,1)$, the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}$ transforms the bargaining problem (S^R, d^R) into the problem $(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S), \boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}(d))$ such that $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}(d^R) = (0,0) = d^R$ and

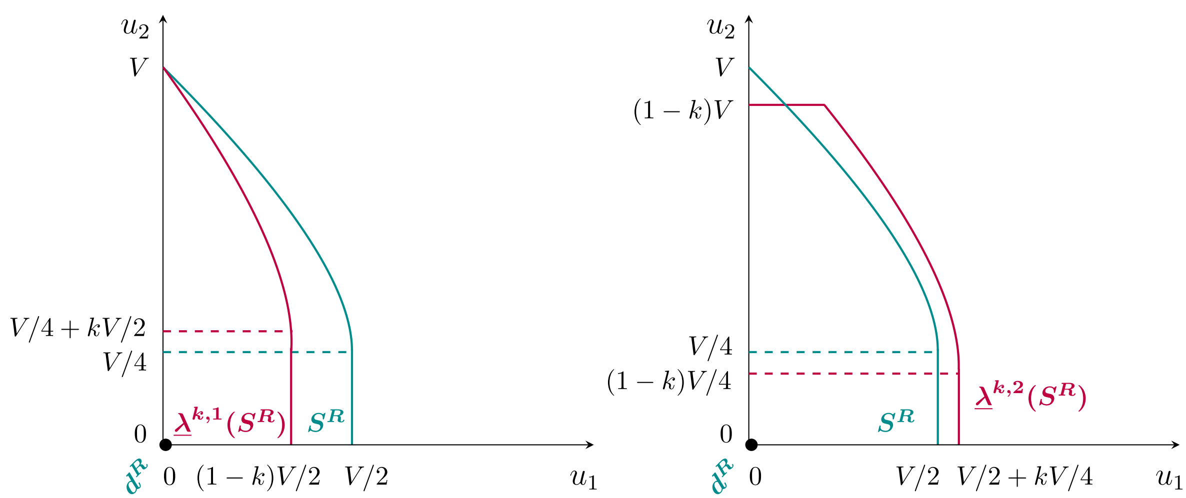
$$\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S) = \left\{ u(\alpha) \;\middle|\; \begin{array}{l} u_1(\alpha) = \dfrac{[2(1-k)(1-\alpha)]\,V}{(2-\alpha)^2}, \\[2ex] 0 \leq u_2(\alpha) \leq \dfrac{[1+2k(1-\alpha)]\,V}{(2-\alpha)^2}, \quad \alpha \in [0,1] \end{array} \right\}. \tag{26}$$

Likewise, given any $k \in [0,1)$, the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{2}}$ transforms the bargaining problem (S^R, d^R) into the problem $(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S), \boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{2}}(d))$ such that $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{2}}(d^R) = (0,0) = d^R$ and

$$\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S) = \left\{ u(\alpha) \;\middle|\; \begin{array}{l} 0 \leq u_1(\alpha) \leq \dfrac{[2(1-\alpha)+k]\,V}{(2-\alpha)^2}, \\[2ex] 0 \leq u_2(\alpha) \leq \dfrac{(1-k)\,V}{(2-\alpha)^2}, \quad \alpha \in [0,1] \end{array} \right\}. \tag{27}$$

In Figure 1, we illustrate the effect of pre-donation on a bargaining problem.

Figure 1. Bargaining Problems Under One-Sided Pre-donation



(i) Player 1 (Consumers) Pre-donates

(ii) Player 2 (Producer) Pre-donates

2.4 The Modified BM Mechanism

We will modify the regulatory mechanism of BM by allowing consumers and the monopolist to collectively choose the expected social welfare function that will be maximized by the regulator over the direct-revelation mechanisms proposed by BM. Consumers and the monopolist will solve this choice problem using a cooperative bargaining game under pre-donation (utility transfers from the pre-donating side to the receiver). We will consider this bargaining game separately under two informational situations, namely the *ex-ante* stage and the interim *stage*. The *interim* stage reflects the assumed informational state in the BM model where the producer privately knows its marginal cost parameter (which is unknown to consumers and the regulator until the end of the implementation of the regulatory mechanism). What we introduce in this study is an *ex-ante* stage where even the producer does not know about its marginal cost parameter, yet. Associated with the two informational stages, our model, and hence our modified regulatory mechanism, will have two variants, in one of which pre-donations occur in the *ex-ante* stage and in the other pre-donations occur in the *interim* stage. However, we will retain one important feature of the BM model assuming that information revelation will always occur in the *interim* stage. We will also assume that the regulator will inform both the monopolist and consumers about the details of the modified regulatory mechanism at the beginning of the stage they are allowed to make pre-donation. Given these assumptions, if consumers should decide whether and how much to pre-donate in the *ex-ante* or *interim* stage, they should always consider the maximization of their *ex-ante* payoffs. In contrast, the producer should take into account its *ex-ante* payoff only if it makes pre-donation decision in the *ex-ante* stage. When it is allowed to pre-donate in the *interim* stage, the producer should always consider the maximization of its *interim* (equivalently *ex-post*) payoff.

After these observations, we are ready to describe the modified BM mechanism under pre-donation.

The Modified BM Mechanism

Step 1: The regulator picks, and announces, from the interval $[0, 1]$ a value, α, to be used for the initial value of the social welfare weight of the producer.

Step 2: Given the announced α value, all parties (the regulator, the producer, and consumers) calculate the induced expected utility allocation $W(\alpha) = (CW^e(\alpha), PW^e(\alpha))$ implied by the BM mechanism. They also calculate the problem (S^R, d^R) and select a proportional rule P^γ with $\gamma \geq 0$ such that $P^\gamma(S^R, d^R) = W(\alpha)$.

Step 3: The regulator announces the index of the player, say i, which is allowed to make a unilateral pre-donation.

Step 4: The regulator announces a function $\tilde{\alpha} : [0,1]^2 \to [0,1]$ such that if player i were to announce pre-donation parameter, as any $k' \in [0,1)$, the regulator would run the BM mechanism with $\tilde{\alpha}(\alpha, k')$, instead of α, to ensure that $(1-k)W_i(\tilde{\alpha}(\alpha, k')) = P_i^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k',i}}(S^R), d^R)$. (Due to the geometries of the bargaining sets S^R and $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k',i}}(S^R)$ and the fact that $W(\alpha) \in PO(S^R)$, we know that $\tilde{\alpha}(\alpha, k')$ exists for all $k' \in [0,1)$.)

Step 5: Given the announced function $\tilde{\alpha}(\alpha, .)$, player i picks, and announces, from the interval $[0,1)$ a value, k, to be used for its pre-donation rate in all relevant calculations.

Step 6: Given the announced k value, all parties calculate the pre-donation function $\boldsymbol{\lambda}^{\boldsymbol{k,i}}$, the social welfare weight $\tilde{\alpha}(\alpha, k)$, the bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S^R)$, and the disagreement point $\boldsymbol{\lambda}^{\boldsymbol{k,i}}(d^R) = d^R$. They also calculate the induced bargaining solution $P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S^R), d^R)$. This is the expected utility allocation of the modified BM mechanism and denoted by $\tilde{W}^i(\alpha, k)$.

Recall that for any α chosen by the regulator, the BM mechanism consists of the list of functions $\langle(p^*(., \alpha), q^*(., \alpha), r^*(., \alpha), s^*(., \alpha)\rangle$. We will denote this mechanism by $\Gamma(\alpha)$. We can then denote the modified BM mechanism we have described above by $\tilde{\Gamma}^i(\alpha, k)$ which is equal to $\Gamma(\tilde{\alpha}(\alpha, k)) \cup \{\boldsymbol{\lambda}^{\boldsymbol{k,i}}, P^\gamma, \tilde{\alpha}\}$. Notice that the BM mechanism, $\Gamma(\alpha)$ generates the welfare allocation $W(\alpha)$ whereas the modified BM mechanism by $\tilde{\Gamma}^i(\alpha, k)$ generates $\tilde{W}^i(\alpha, k)$.

Now we turn to consider the problem of the producer in the above bargaining game. Notice that the modified BM mechanism $\tilde{\Gamma}^i(\alpha, k)$ operates through the BM mechanism $\Gamma(\tilde{\alpha}(\alpha, k))$ to extract the private information of the producer. Thus, it satisfies ex-post incentive compatibility and individual rationality conditions. Consequently, the producer will obtain in the ex-post stage, after the revelation of its private information is realized, the actual profit $\pi(\theta, \tilde{\alpha}(\alpha, k))$. In the *interim* stage, the producer can precisely calculate this profit since it completely knows the actual value of θ. In fact, it can calculate the actual gross utility $\pi(\theta, \tilde{\alpha}(\alpha, k))$ it would get under any pre-donation rate $k \in [0,1)$. When pre-donation occurs, the actual net utility of the producer would be

$$\pi^a(\theta, \alpha, k) = \pi(\theta, \tilde{\alpha}(\alpha, k)) - kW_2(\tilde{\alpha}(\alpha, k)). \tag{28}$$

When the producer is allowed to make pre-donation in the *interim* stage, it must choose k, in the interval $[0, 1)$, to maximize this actual net utility.

In the *ex-ante* stage, the objective of the producer is inevitably different. Since the producer does not (yet) know in this stage what the actual value of θ is, it cannot calculate its actual net utility resulting from any pre-donation. We assume that in the *ex-ante* stage the producer has the same (incomplete) information about θ as do the regulator and consumers. Thus, it shares their beliefs $f(.)$ about the distribution of θ. Because the producer can calculate $\pi^a(\theta, \alpha, k)$ for all possible values of $\theta \in (0, a]$ and $k \in [0, 1)$, it can calculate its expected value under the beliefs $f(.)$. Notice that the expected value of $\pi(\theta, \tilde{\alpha}(\alpha, k))$ is just equal to $W_2(\tilde{\alpha})$. Accordingly, the expected net utility of the producer from the bargaining game becomes

$$E[\pi^a(\theta, \alpha, k)|f] = (1 - k)W_2(\tilde{\alpha}(\alpha, k)), \tag{29}$$

for any $k \in [0, 1)$. So, if the producer is allowed to make pre-donation only in the *ex-ante* stage, it should maximize the above expected net utility over possible values of k in $[0, 1)$.

On the other hand, consumers who can learn about θ only after the cost-revelation occurs in the *interim* stage, always consider the maximization of their expected utility whenever they are allowed to pre-donate in the *ex-ante* or *interim* stage. This expected utility simply becomes $(1 - k)W_1(\tilde{\alpha}(\alpha, k))$ if consumers choose the pre-donation rate as $k \in [0, 1)$.

3 Results

In this section, our goal is to explore whether there exist any $\alpha \in [0, 1]$ and $i \in \{1, 2\}$ such that the modified BM mechanism $\tilde{\Gamma}^i(\alpha, k)$ can Pareto dominate, in the *ex-ante* or *interim* stage, the BM mechanism $\Gamma(\alpha)$. To achieve this goal, we will first restrict our attention to the bargaining problems with and without pre-donation and explore the effect of pre-donation by consumers or the producer on the solutions implied by some bargaining rules that are relevant for our purpose.

Notice that the regulatory outcome that is determined by the BM mechanism is always *ex-ante* Pareto optimal. Since the bargaining solution in the absence of pre-donation must be equivalent to the expected utility allocation generated by the BM mechanism, we will restrict our attention to bargaining rules that respect Weak Pareto Optimality and has the potential to select a Pareto Optimal solution for the problem (S^R, d^R). On this account, we can restrict ourselves to the class of proportional rules with no loss of generality. To see why that is so, consider any problem $(S, d) \in \Sigma^2$ and any bargaining rule F on Σ^2 that satisfies Weak Pareto Optimality. Define $\gamma \equiv F_1(S, d)/F_2(S, d)$. By the definition of the rule P^γ, we have $P^\gamma(S, d) \in WPO(S)$. Also, $P_1^\gamma(S, d)/P_2^\gamma(S, d) = F_1(S, d)/F_2(S, d)$. Moreover, $F(S, d) \in WPO(S)$ since F satisfies Weak Pareto Optimality. Therefore, we must have $P^\gamma(S, d) = F(S, d)$. So, in order to study the implications of bargaining rules that satisfy Weak Pareto Optimality in any fixed bargaining problem, it is sufficient to consider only the set of proportional bargaining rules.

Before moving to our results, we will borrow, as a preliminary, two helpful results from Saglam (2021).

Proposition 1. (Saglam, 2021) *Given the bargaining problem (S^R, d^R), the bargaining rule P^γ yields the utilities*

$$u_1 = \begin{cases} \dfrac{4\gamma}{(2+\gamma)^2} V & \text{if } \gamma \in (0, 2] \\ \dfrac{1}{2} V & \text{if } \gamma > 2 \end{cases} \quad \text{and} \quad u_2 = \begin{cases} \dfrac{4}{(2+\gamma)^2} V & \text{if } \gamma \in (0, 2] \\ \dfrac{1}{2\gamma} V & \text{if } \gamma > 2. \end{cases} \tag{30}$$

Proof. See the proof of Proposition 10 in Saglam (2021). ■

The above result leads to the following simple corollary.

Corollary 1. (Saglam, 2021) *Given the bargaining problem (S^R, d^R), the bargaining rule P^γ and the BM mechanism lead to the same utility allocation if and only if $\gamma = 2(1 - \alpha)$.*

The proof of the above corollary, which was stated as Corollary 7 in Saglam (2021), rests on the observation that the utility ratio u_1/u_2 is equal to γ under the bargaining rule P^γ while it is equal to $2(1 - \alpha)$ under the BM mechanism, as can be observed from equations (14) and (15).

Now we turn to consider the problem of pre-donation. The following lemma shows that a positive amount of pre-donation from consumers to the producer, $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}$ with $k \in (0, 1]$, contracts the bargaining set S^R so that $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S^R))$ is always below

$WPO(S^R)$ except for the point $(V, 0)$ where the two frontiers intersect.

Lemma 1. *For any $k \in (0,1]$ and $u \in WPO(S^R)$ it is true that $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}(u) = u$ if $u_1 = 0$ and $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}(u) \in S^R \setminus WPO(S^R)$ if $u_1 > 0$.*

Proof. Pick any $k \in (0,1)$ and $u \in WPO(S^R)$. If $u_1 = 0$ then $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}(u) = u$, by (26). So, let $u_1 > 0$. Again by (26), we know that $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}(u)$ is on the (half-open) line segment $[b, u[$ where $b \equiv (0, u_1 + u_2)$. Consider the line segment $[d, u[$ where $d \equiv (0, V)$. In order to prove $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}(u) \in S^R \setminus WPO(S^R)$, it is sufficient to show that b is below d implying $u_1 + u_2 < V$. By (25), there exists a unique $\alpha \in [0,1]$ such that $u_1 = 2(1-\alpha)V/(2-\alpha)^2$. Since $u_1 > 0$, we know that $\alpha \neq 1$; thus $u_2 < V$. Moreover, $u_2 = V/(2-\alpha)^2$ if $u \in PO(S^R)$, and $u_2 \in [0, V/(2-\alpha)^2)$ if $u \in WPO(S^R) \setminus PO(S^R)$ (which occurs when $\alpha = 0$). Therefore, for any $u \in WPO(S^R)$ with $u_1 > 0$, we have $u_1 + u_2 \leq (3-2\alpha)V/(2-\alpha)^2$, and we know that the right-hand-side of this inequality is less than V for all $\alpha \in [0,1)$. ■

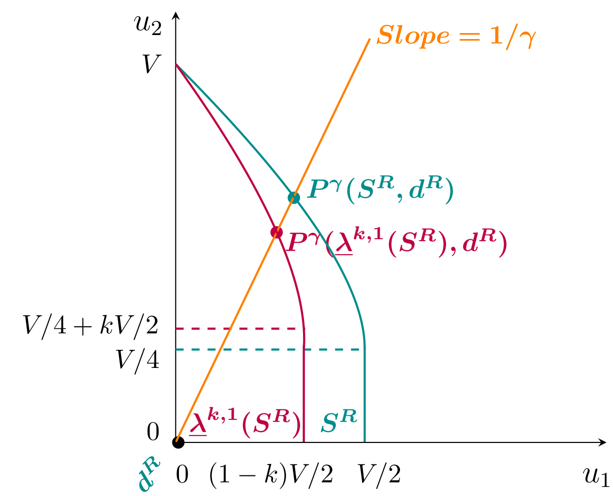
Lemma 2. *Given any bargaining rule P^γ with $\gamma > 0$, the pre-donation from consumers to the producer via $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}$ with any $k \in (0,1]$ is harmful in terms of expected utilities for both consumers and the producer.*

Proof. Pick any $\gamma > 0$ and consider the bargaining rule P^γ. Let $u = P^\gamma(S^R, d^R)$ and $u' = P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S^R), d^R)$. By (21), u and u' are on the line connecting d^R and the point $(\gamma, 1)$, and also $u \in WPO(S^R)$ and $u' \in WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S^R))$. By Lemma 1, the set $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S^R))$ is always below $WPO(S^R)$ except for the point $(V, 0)$ where the two sets intersect. Moreover, $\{u, u'\} \cap \{(V, 0)\} = \emptyset$ since $\gamma > 0$. Therefore, we must have $u'_1 < u_1$ and $u'_2 < u_2$. ■

In Figure 2, we illustrate the welfare effect predicted by Lemma 2. This result implies that if consumers make pre-donation the modified BM mechanism becomes always inferior, in terms of expected utilities, to the BM mechanism for both consumers and the producer.

Proposition 2. *For any $\alpha \in [0,1]$ and $k \in (0,1]$, the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ under the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}$ is always* ex-ante *Pareto inferior to the BM mechanism $\Gamma(\alpha)$.*

Figure 2. Bargaining under the Rule P^γ and the Pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{1}}$



Proof. Pick any $\alpha \in [0,1]$ and $k \in (0,1]$. Let $\gamma = 2(1-\alpha)$. By Corollary 1, the BM mechanism yields the expected utility allocation $W(\alpha) = P^\gamma(S^R, d^R)$. We also know that the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ yields the expected utility allocation $W^1(\alpha, k) = P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S^R), d^R)$. Moreover, Lemma 2 implies that $P_i^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{1}}(S^R), d^R) < P_i^\gamma(S^R, d^R)$ for each $i = 1, 2$. So, $\tilde{\Gamma}^1(\alpha, k)$ is *ex-ante* Pareto inferior to $\Gamma(\alpha)$. ■

Since the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ and the BM mechanism $\Gamma(\alpha)$ coincide only if $k = 0$, Proposition 2 implies that consumers would choose not to pre-donate under the modified BM mechanism. It also implies that any improvement by the modified mechanism $\tilde{\Gamma}^i(\alpha, k)$ should not be expected unless $i = 2$, i.e., the pre-donating party in the bargaining process is the producer. The following lemma shows that the pre-donation from the producer to consumers, $\boldsymbol{\lambda}^{\boldsymbol{k},\mathbf{2}}$ for any $k \in (0,1]$, twists the bargaining set S^R around a point u in $WPO(S^R)$ with $u_2 = V(1-k)$.

Lemma 3. *For any $k \in (0,1]$ it is true that*

(i) if $u \in S^R$ is such that $u_2 > (1-k)V$, then $u \notin \underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R)$,

(ii) if $u \in S^R$ is such that $u_2 \leq (1-k)V$, then there exists $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R)$ such that $u'_1 > u_1$ and $u'_2 = u_2$.

Proof. Consider any $k \in (0,1]$.

(i). Pick any $u \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$. By (26), there exists $u' \in S^R$ such that $u = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(u')$. Notice that $u'_2 \leq V$, implying $(1-k)u'_2 \leq (1-k)V$. Since $u_2 = (1-k)u'_2$, we have $u_2 \leq (1-k)V$, completing the proof of part (i).

(ii). Now pick any $u \in S^R$ such that $u_2 \leq (1-k)V$. First assume that $u \notin WPO(S^R)$. Pick any $u' \in S^R$ such that $u'_1 > u_1$ and $u'_2 = u_2$. Let $u''_2 = u'_1/(1-k)$ and $u''_1 = u'_1 - ku''_2$. Clearly, $u'' \in S^R$ and $u' = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(u'')$. Thus, $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$. Now, assume that $u \in WPO(S^R) \setminus PO(S^R)$. By (26), $u_1 = V/2$ and $u_2 \in [0, V/4)$. Let $\hat{u} \in S^R$ be such that $\hat{u}_1 = u_1$ and $\hat{u}_2 = u_2/(1-k)$. Also, let $u' \in \mathbb{R}^2_+$ be such that $u'_1 = \hat{u}_1 + k\hat{u}_2$ and $u'_2 = (1-k)\hat{u}_2$. Notice that $u' = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(\hat{u})$, hence $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$. Also, $u'_1 > u_1$ and $u'_2 = u_2$. Finally, assume that $u \in PO(S^R)$. Recall that $u_2 \leq (1-k)V$ by assumption. To prove that there exists $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$ such that $u'_1 > u_1$ and $u'_2 = u_2$, it is sufficient to show that for any $x \in PO(S^R)$ the line segment $(x, \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)]$ is outside S^R. This can be true if the slope of $[x, \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)]$ (in absolute value), which is 1, is smaller than the slope of $PO(S^R)$ (in absolute value) at any $y \in Y$ where Y is a subset of $PO(S^R)$ satisfying $\max\{y_2 \,|\, y \in Y\} = x_2$ and $\min\{y_2 \,|\, y \in Y\} = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)$. For any $y \in PO(S^R)$, we know by (24) and (25) that $y_1 = 2\sqrt{Vy_2} - 2y_2$. Thus, we have $|dy_2/dy_1| = |\sqrt{V/y_2} - 2|^{-1}$. We also know that $y_2 \in [V/4, V]$. Therefore, $|dy_2/dy_1| \in (1, \infty)$ for any $y \in Y$, implying that the line segment $(x, \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)]$ is outside S^R, which completes the proof. ■

Lemma 4. *Given any bargaining rule P^γ with $\gamma > 0$, the pre-donation from the producer to consumers via $\boldsymbol{\lambda}^{\boldsymbol{k,2}}$ is*

(i) ex-ante beneficial for the producer and consumers if $k < \bar{k}(\gamma)$,

(ii) ex-ante harmful for the producer and consumers if $k > \bar{k}(\gamma)$,

(iii) ex-ante ineffective for the producer and consumers if $k = \bar{k}(\gamma)$,

where

$$\bar{k}(\gamma) = \begin{cases} 1 - \dfrac{4}{(2+\gamma)^2} & \text{if } \gamma \in (0,2] \\[2ex] 1 - \dfrac{1}{2\gamma} & \text{if } \gamma > 2. \end{cases} \tag{31}$$

Proof. Pick any $\gamma > 0$ and consider the bargaining rule P^γ. Let $u(\gamma) \equiv P^\gamma(S^R)$ and $u'(\gamma) \equiv P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R))$. If $\gamma \in (0,2]$, then equations (21), (24), and (25) would imply that $u_1(\gamma) = 2(1-\alpha^*)/(2-\alpha^*)^2$ and $u_2(\gamma) = 1/(2-\alpha^*)^2$ for some α^* such that

$u_1(\gamma)/u_2(\gamma) = \gamma = 2(1-\alpha^*)$. It follows that $\alpha^* = 1-\gamma/2$, implying $u_2(\gamma) = 4V/(2+\gamma)^2$. On the other hand, if $\gamma > 2$, then $u_2(\gamma) = V/(2\gamma)$. Now, define $\bar{k}(\gamma) \equiv 1 - u_2(\gamma)/V$ for each $\gamma > 0$. Notice that $u(\gamma)$ and $u'(\gamma)$ are on the same line passing through the points $d^R = (0,0)$ and $(\gamma, 1)$ and they are the maximal points of $WPO(S^R)$ and $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{2}}(S^R))$ on this line. Also, since this line is positively sloped, any pre-donation by the producer makes both of the bargaining parties better off if it makes any of them better off. Thus, we observe from (27) and Lemma 3 that $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{2}}$ is (i) *ex-ante* beneficial for all parties, i.e., $u_i'(\gamma) > u_i(\gamma)$ for each $i = 1, 2$, if $u_2(\gamma) < (1-k)V$ or $k < \bar{k}(\gamma)$, (ii) *ex-ante* harmful for all parties, i.e., $u_i'(\gamma) < u_i(\gamma)$ for each $i = 1, 2$, if $u_2(\gamma) > (1-k)V$ or $k > \bar{k}(\gamma)$, and (iii) *ex-ante* ineffective for all parties, i.e., $u_i'(\gamma) = u_i(\gamma)$ for each $i = 1, 2$, if $u_2(\gamma) = (1-k)V$ or $k = \bar{k}(\gamma)$. ■

The welfare effect in Lemma 4 is illustrated in Figure 3. Recall that when $\alpha = 1$, the BM mechanism produces the utility allocation $W(1) = (0, V)$, under which player 2 (the producer) has no incentive to pre-donate. On the other hand, when $\alpha < 1$, there is always an expected deadweight loss generated by the BM mechanism, as calculated in equation (17). Below, we will explore whether this loss can be reduced by the modification of the BM mechanism under the producer's pre-donation, even when it is not optimal.

Proposition 3. *For any $\alpha \in [0,1]$ and $k \in (0,1]$, the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k)$ under the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{2}}$ is*

(i) ex-ante Pareto superior to the BM mechanism $\Gamma(\alpha)$ if $k < \bar{k}(\alpha)$,

(ii) ex-ante Pareto inferior to the BM mechanism $\Gamma(\alpha)$ if $k > \bar{k}(\alpha)$,

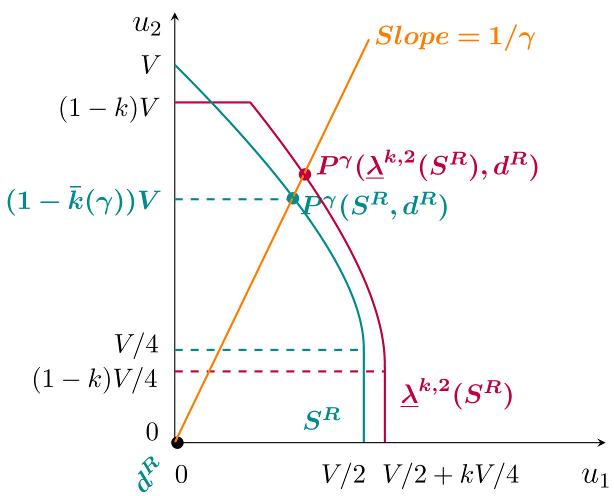
(iii) ex-ante Pareto equivalent to the BM mechanism $\Gamma(\alpha)$ if $k = \bar{k}(\alpha)$,

where $\bar{k}(\alpha) = 1 - 1/(2-\alpha)^2$.

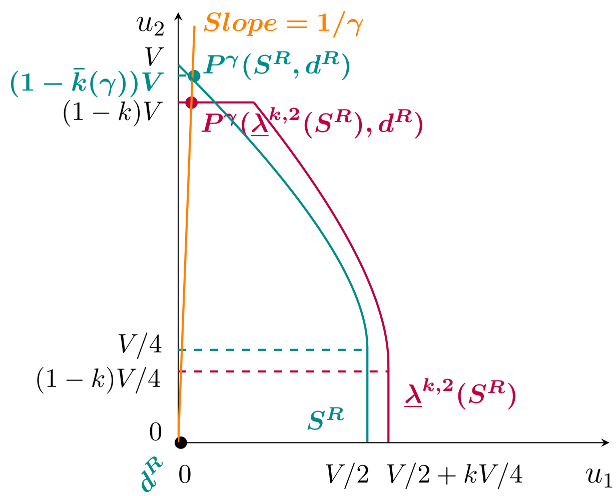
Proof. Pick any $\alpha \in [0,1]$ and $k \in (0,1]$. Let $\gamma = 2(1-\alpha)$. Notice that $\gamma \in [0,2]$. By Corollary 1, $W(\alpha) = P^\gamma(S^R, d^R)$. We also know that the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ yields the expected utility allocation $W^2(\alpha, k) = P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{2}}(S^R), d^R)$. Notice that the threshold in equation (31) reduces to $\bar{k}(\gamma) = 1 - 4/(2+\gamma)^2$ since $\gamma \in [0,2]$. Notice also that the equality $\gamma = 2(1-\alpha)$ implies that $k \geq \bar{k}(\gamma)$ if and only if $k \geq \bar{k}(\alpha) = 1 - 1/(2-\alpha)^2$. Thus, Lemma 4 implies that (i) $P_i^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{2}}(S^R), d^R) > P_i^\gamma(S^R), d^R)$ for each $i = 1, 2$ and $\tilde{\Gamma}^2(\alpha, k)$ is *ex-ante* Pareto superior to $\Gamma(\alpha)$ if $k < \bar{k}(\alpha)$, (ii)

$P_i^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R) < P_i^{\gamma}(S^R), d^R)$ for each $i = 1, 2$ and $\tilde{\Gamma}^2(\alpha, k)$ is *ex-ante* Pareto inferior to $\Gamma(\alpha)$ if $k > \bar{k}(\alpha)$, (iii) $P_i^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R) = P_i^{\gamma}(S^R), d^R)$ for each $i = 1, 2$ and $\tilde{\Gamma}^2(\alpha, k)$ is *ex-ante* Pareto equivalent to $\Gamma(\alpha)$ if $k > \bar{k}(\alpha)$. ■

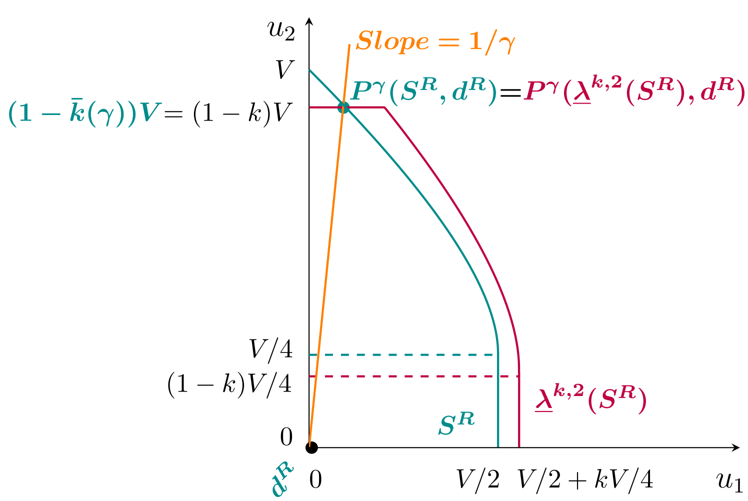
Figure 3. Bargaining under the Rule $\boldsymbol{P^{\gamma}}$ with $\boldsymbol{\gamma \leq 2}$ and the Pre-donation $\boldsymbol{\lambda^{k,2}}$



(i) $k < \bar{k}(\gamma)$



(ii) $k > \bar{k}(\gamma)$



(iii) $k = \bar{k}(\gamma)$

Notice that the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k)$ becomes *ex-ante* inferior to the BM mechanism for the producer (and consumers) when the pre-donation fraction k exceeds the threshold $\bar{k}(\alpha)$ for any given $\alpha \in [0, 1]$, Thus, we should never expect an equilibrium where the producer pre-donates more than the amount implied by this threshold. Given this observation, our next question is to find the optimal pre-donation by the producer in the *ex-ante* stage where it has not learned the actual value of θ, yet. Here, we assume that the producer will always prefer no pre-donation to an ineffective pre-donation, which arises if $k = \bar{k}(\alpha)$, because of the actual implementation costs of pre-donation we have assumed to be zero for simplicity. This leaves us with part (i) of Proposition 3 suggesting that the producer should restrict itself to pre-donation functions whose parameter, k, lies in the set $[0, 1 - 1/(2 - \alpha)^2)$. In this set, the producer should choose the value of k to maximize its bargaining utility $P_2^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R)$, which is equal to $(1 - k)W_2(\tilde{\alpha}(\alpha, k))$ by equation (29).

Lemma 5. *Given any bargaining rule P^γ with $\gamma > 0$, the ex-ante optimal pre-donation from the producer to consumers is a function $\boldsymbol{\lambda}^{\boldsymbol{k^*},\mathbf{2}}$ where $k^* = \gamma/(1 + \gamma)$. This yields to consumers and the firm the bargaining utilities $P_1^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*},\mathbf{2}}(S^R), d^R) = \gamma V/(1 + \gamma)$ and $P_2^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*},\mathbf{2}}(S^R), d^R) = V/(1 + \gamma)$, respectively.*

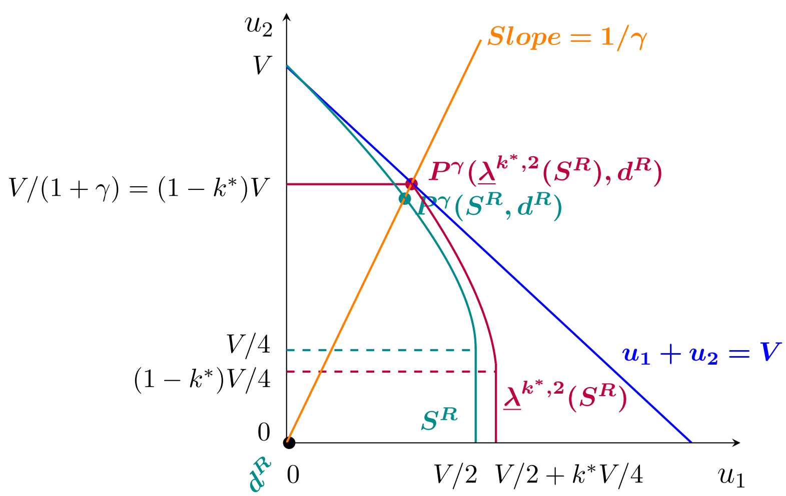
Proof. Pick any $\gamma > 0$ and consider the rule P^γ. Let $u(\gamma, k) \equiv P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R)$ for any $k \in [0, 1)$. The problem of the producer is to find the value of k that maximizes $u_2(\gamma, k)$. Notice that $u(\gamma, k)$ is the point of intersection between the set $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R))$ and the line passing through the points d^R and $(\gamma, 1)$. Notice also that the line segment $[(0, V), (V, 0)]$ is the upper envelope of the sets $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R))$ obtained when k is varied over $[0, 1]$. Therefore, $k = k^*$ maximizes $u_2(\gamma, k)$ only if $u(\gamma, k^*)$ is on the line segment $[(0, V), (V, 0)]$ or equivalently $u_1(\gamma, k^*) + u_2(\gamma, k^*) = V$. Then, using the fact that $u_1(\gamma, k^*)/u_2(\gamma, k^*) = \gamma$ by the definition of P^γ, we obtain $u_2(\gamma, k^*) = V/(1 + \gamma)$ and $u_1(\gamma, k^*) = \gamma V/(1 + \gamma)$, implying $k^* = \gamma/(1 + \gamma)$. One can easily check that $k^* < \bar{k}(\gamma)$ for all $\gamma > 0$. Thus, the pre-donation implied by k^* is optimal for the producer. ∎

Figure 4 portrays how to find the optimal pre-donation k^* by the producer. Below, we calculate the value of k^* under some well-known bargaining rules to give an insight how widely it can vary in its range when γ changes.

Remark 1. *Given the bargaining rule P^γ, the ex-ante optimal pre-donation from* *the producer to consumers via $\boldsymbol{\lambda}^{k^*,2}$ implies (i) $k^* = 2/3$ if P^γ is outcome equivalent to the Dictatorial-1 rule, (ii) $k^* = 1/2$ if P^γ is outcome equivalent to the Egalitarian rule, (iii) $k^* = 2/5$ if P^γ is outcome equivalent to the Nash rule, (iv) $k^* = 1/3$ if P^γ is outcome equivalent to the Kalai-Smorodinsky rule, and (v) $k^* = 0$ if P^γ is outcome equivalent to the Dictatorial-2 rule.*

Proof. We know that the rule P^γ is outcome equivalent to the Egalitarian rule only if $\gamma = 1$. Also, we know from Corollary 6 of Saglam (2021) that P^γ is outcome equivalent to the Nash rule only if $\gamma = 2/3$ and to the Kalai-Smorodinsky rule only if $\gamma = 1/2$. Moreover, for $\gamma = 2$ and $\gamma = 0$, P^γ becomes outcome equivalent to the Dictatorial-1 and Dictatorial-2 rules, respectively. Inserting each of these five values of γ into the equation $k^* = \gamma/(1+\gamma)$ yields the desired result. ■

Figure 4. Bargaining under the Rule P^γ and the Optimal Pre-donation $\boldsymbol{\lambda}^{k^*,2}$



Recall that for any $\alpha \in [0,1]$, the expected utility allocation produced by the BM mechanism is always Pareto optimal, i.e., $W(\alpha) \in PO(S^R)$. Moreover, Corollary 1 reveals that $W(\alpha) = P^\gamma(S^R, d^R)$ if and only if $\gamma = 2(1-\alpha)$. This implies that any proportional bargaining rule P^γ selects its solution from $PO(S^R)$ if and only if $\gamma \in [0,2]$. It follows that for any proportional rule that selects its solution from $PO(S^R)$ the optimal pre-donation by the producer must fall in the interval $[0/(1+0), 2/(1+2)] = [0, 2/3]$. Remark 1 above shows that the lower and upper bounds of this interval are induced by the Dictatorial-2 and Dictatorial-1 rules respectively, while the Egalitarian, Nash, and Kalai-Smorodisnky rules are compatible with k^* values in the interior of the same interval.

Proposition 4. *For any $\alpha \in [0,1]$, the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k^*)$ under the ex-ante optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ yields the expected utility allocation $W^2(\alpha, k^*)$ where $W_1^2(\alpha, k^*) = 2(1-\alpha)V/(3-2\alpha)$ and $W_2^2(\alpha, k^*) = V/(3-2\alpha)$.*

Proof. Pick any $\alpha \in [0,1]$. Let $\gamma = 2(1-\alpha)$. We know that the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k^*)$ yields the expected utility allocation $W^2(\alpha, k^*) = P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R), d^R)$. Also, by Lemma 5, $P_1^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R), d^R) = \gamma V/(1+\gamma)$ and $P_2^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R), d^R) = V/(1+\gamma)$. Replacing γ in the last two equations with $2(1-\alpha)$, we obtain the desired result. ■

The optimal pre-donation of the producer induced by k^* always equates the ratio between the utilities of consumers and the producer to the slope γ under any rule P^γ as can be seen from Lemma 5. Since the BM mechanism $\Gamma(\alpha)$ induced by any $\alpha \in [0,1]$ is outcome equivalent to a bargaining rule P^γ only if $\gamma = 2(1-\alpha)$, we observe that the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ induced by α and k^* equates the ratio between the utilities of consumers and the producer to $2(1-\alpha)$ as does the BM mechanism. This implies that the same ratio must exist between the utility gains of the two parties generated by the modified BM mechanism with respect to the status quo. The following result shows that these utility gains are always decreasing in α.

Corollary 2. *For any $\alpha \in [0,1]$, the expected utility gain of the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ over the BM mechanism $\Gamma(\alpha)$ is equal to $\Delta\tilde{\Gamma}^2(\alpha)$ such that*

$$\Delta\tilde{\Gamma}_1^2(\alpha) = \frac{8(1-\alpha)^3}{(3-2\alpha)(4-2\alpha)^2}V$$

and

$$\Delta\tilde{\Gamma}_2^2(\alpha) = \frac{4(1-\alpha)^2}{(3-2\alpha)(4-2\alpha)^2}V.$$

Moreover, for each $i \in \{1,2\}$ the gain $\Delta\tilde{\Gamma}_i^2(\alpha)$ is always decreasing in α.

Proof. The equations for the utility gains follow from equations (15), (14), and Proposition 4. Differentiating them with respect to α we can straightforwardly reach the desired result. ■

The result that the utility gains implied by the modified BM mechanism under the optimal pre-donation of the producer are positive as long as $\alpha < 1$ is not surprising since by Proposition 3-(i) we already know that given any $k \in [0, 1)$ and a pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{2}}$, the induced modified BM mechanism is Pareto superior to the BM mechanism if k is below the threshold $\bar{k}(\alpha)$ while by Lemma 5 and Proposition 4 we know that the optimal pre-donation by the producer satisfies this threshold condition for all $\alpha \in (0, 1]$. The result that the aforementioned utility gains are decreasing in α is not surprising either, since the expected deadweight loss in the BM mechanism is decreasing in α, as we can recall from (17). Thus, the social benefit of the modified mechanism, though it is always positive for any α less than 1, becomes lower and lower, and eventually totally diminished, as α approaches 1 from below.

Table 1. The Utilities Generated by Various Bargaining Rules under the *Ex-Ante* Optimal Pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*},\boldsymbol{2}}$

Bargaining Rule	$\boldsymbol{\alpha}$	$\boldsymbol{\gamma}$ $\parallel$ $2(1-\alpha)$	$\boldsymbol{k^*}$ $\parallel$ $\dfrac{2(1-\alpha)}{3-2\alpha}$	$\boldsymbol{P^{\gamma}(S^R, d^R)}$ $\parallel$ $\left(\dfrac{2(1-\alpha)V}{(2-\alpha)^2}, \dfrac{V}{(2-\alpha)^2}\right)$	$\boldsymbol{P^{\gamma}\left(\underline{\lambda}^{k^*,2}(S^R), \lambda^{k^*,2}(d^R)\right)}$ $\parallel$ $\left(\dfrac{2(1-\alpha)V}{3-2\alpha}, \dfrac{V}{3-2\alpha}\right)$
Dictatorial-1	0	2	20/30	$\left(\frac{1800V}{3600}, \frac{900V}{3600}\right)$	$\left(\frac{2400V}{3600}, \frac{1200V}{3600}\right)$
Egalitarian	6/12	1	15/30	$\left(\frac{1600V}{3600}, \frac{1600V}{3600}\right)$	$\left(\frac{1800V}{3600}, \frac{1800V}{3600}\right)$
Nash	8/12	8/12	12/30	$\left(\frac{1350V}{3600}, \frac{2025V}{3600}\right)$	$\left(\frac{1440V}{3600}, \frac{2160V}{3600}\right)$
KS	9/12	6/12	10/30	$\left(\frac{1152V}{3600}, \frac{2304V}{3600}\right)$	$\left(\frac{1200V}{3600}, \frac{2400V}{3600}\right)$
Dictatorial-2	1	0	0	$(0, V)$	$(0, V)$

In Table 1, we report the calculated utilities without pre-donation and with an optimal pre-donation under five distinct bargaining rules, including Dictatorial-1, Egalitarian, Nash, Kalai-Smorodinsky, and Dictatorial-2 rules. We should observe that under all five rules, the producer's optimal pre-donation, whenever positive, increases the utility of consumers as well. The percentage increase in the bargaining utilities of consumers and the producer due to the optimal pre-donation by the producer can be calculated as $(16.67, 8.33), (5.56, 5.56), (2.50, 3.75), (1.33, 2.67)$, and $(0, 0)$ for the Dictatorial-1, Egalitarian, Nash, Kalai-Smorodinsky, and Dictatorial-2 rules, respectively. It is interesting to see from Table 1 that under the optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$, the solution under the Kalai-Smorodinsky rule can be obtained from the solution under the Dictatorial-1 distribution by permuting the expected utilities of the producer and consumers. That is to say, when consumers are given the dictatorial power in the regulatory bargaining, they could obtain under the optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ only what the producer would get under the Kalai-Smorodinsky rule, instead of the diametrically opposed Dictatorial-2 rule under which the producer is entitled to the whole surplus V. This result is simply caused by the asymmetry (skewness) in the bargaining problem S^R, which remains to manifest itself in the bargaining problem $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R)$.

So far, we have implicitly assumed that when the bargaining party i makes any pre-donation within the rules of the modified BM mechanism $\tilde{\Gamma}^i(\alpha, k)$, its opponent j does not reject or reverse it. Now, we shall see the implications of relaxing this assumption. We have seen the under the mechanism $\tilde{\Gamma}^1(\alpha, k)$, the pre-donating party, consumers, have never incentive to choose the rate of pre-donation k above zero, therefore the implicit assumption that the producer never rejects pre-donation has practically no bite. On the other hand, the producer has clearly an incentive to make a reverse pre-donation under $\tilde{\Gamma}^1(\alpha, k)$. Whereas consumers would choose their pre-donation rate as $k_1 = 0$, the producer would optimally respond in turn by choosing its unasked, and formally unallowed, pre-donation rate as $k_2 = k^* = \gamma/(1+\gamma) = 2(1-\alpha)/(3-2\alpha)$, as implied by Lemma 5. Thus, by informally or illegally, yet optimally, deviating from the modified mechanism $\tilde{\Gamma}^1(\alpha, k)$, the producer has always incentive to implement the outcome of the mechanism $\tilde{\Gamma}^2(\alpha, k^*)$. Proposition 3 and the proof of Lemma 5 together imply that consumers always become better-off when the producer pre-donates at a rate $k_2 = k^*$ formally under the mechanism $\tilde{\Gamma}^2(\alpha, k_2)$, or informally under the mechanism $\tilde{\Gamma}^1(\alpha, k_1)$ after consumers optimally choose $k_1 = 0$. Thus, a benevolent regulator can be argued to serve the interests of the society by allowing the producer to make reverse pre-donation under the mechanism $\tilde{\Gamma}^1(\alpha, .)$. However, still a question remains as to whether consumers could not also improve their welfare by rejecting or reversing some part of the producer's formal pre-donation under the mechanism $\tilde{\Gamma}^2(\alpha, .)$, or equivalently some part of the producer's informal pre-donation under the mechanism $\tilde{\Gamma}^1(\alpha, .)$.

To answer the above question, notice that for any $k_1 \in [0, 1)$ a reverse pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k_1,1}}$ made by consumers under the mechanism $\tilde{\Gamma}^2(\alpha, k_2)$ when the producer make its pre-donation formally according to $\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}$ for any $k_2 \in [0, 1)$ would change the bargaining problem from $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$ to $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_1,1}}(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R))$, whereas it would have no effect on the disagreement point since $\boldsymbol{\lambda}^{\boldsymbol{k_1,1}}(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(d^R)) = d^R$.

Lemma 6. *For any $\alpha \in [0,1]$, $k_2 \in [0,1)$, and the associated mechanism $\tilde{\Gamma}^2(\alpha, k_2)$, consumers have incentive to make a reverse pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k_1,1}}$ where*

$$k_1 = \begin{cases} 0 & \text{if } k_2 \leq k^* \\ 1 - \dfrac{k^*}{k_2} & \text{if } k_2 > k^*. \end{cases} \tag{32}$$

Proof. Pick any $\alpha \in [0,1]$, $k_2 \in [0,1)$, and consider the associated mechanism $\tilde{\Gamma}^2(\alpha, k_2)$. First observe that when k_2 is equal to k^*, the optimal rate chosen by the producer in the *ex-ante* stage, consumers' expected welfare also reaches its maximal level that any value of k_2 lead to, since the equilibrium utility allocation $(k^*V, (1-k^*)V)$ implied by the mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ is on the Pareto frontier of the transformed bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R)$. Also note that if $u \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$ and $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k_1,1}}(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R))$ such that $u_2 = u'_2 \neq V$, then $u'_1 < u_1$. That is to say, reverse pre-donation of consumers always contracts the transformed bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$ for any $k_2 \in (0,1)$. Therefore, it is not optimal for consumers to contract this set even further if $k_2 \leq k^*$, implying that their optimal response must be $k_1 = 0$.

On the other hand, if $k_2 > k^*$, then the mechanism $\tilde{\Gamma}^2(\alpha, k_2)$ chooses on the bargaining set $\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R)$ the allocation $\hat{u}$ such that $\hat{u} \in WPO(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R)) \setminus PO(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R))$ with $\hat{u}_2 = (1-k_2)V$ and $\hat{u}_1 = \gamma(1-k_2)V$. Since $\gamma(1-k_2)V < \gamma(1-k^*)V = k^*V$, we have $\hat{u}_1 < k^*V$; i.e., consumers are worse off under $\tilde{\Gamma}^2(\alpha, k_2)$ than they would be under $\tilde{\Gamma}^2(\alpha, k^*)$. So, consumers have an incentive to contract $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$. Notice that consumers always get γ times what the producer obtains under the rule P^γ. Therefore, for consumers the optimal choice of $k_1 \in (0,1)$ must ensure that γ times what the producer obtains under the reverse pre-donation $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_1,1}}$ is equal to k^*V, the highest utility that consumers can obtain under P^γ. So, we must have $\gamma[k_1(k_2V) + (1-k_2)V] = k^*V$. Inserting above $\gamma = k^*/(1-k^*)$ and rearranging the equation yields $k_1 = 1-(k^*/k_2)$. ■

Proposition 5. *Consumers have no incentive to reverse the optimal pre-donation of the producer k^* under the modified BM mechanism $\tilde{\Gamma}^2$.*

Proof. Directly follows from Lemma 6. ■

Up until now, we have dealt with the possibility of pre-donation in the *ex-ante* stage. We shall henceforth consider the *interim* stage. Notice that consumers' information about the producer's private cost parameter θ is the same in the *ex-ante* and *interim* stages. Therefore, consumers make expected utility calculations in the *interim* stage, as well. Since Lemma 2 implies that the pre-donation from consumers to the producer is always harmful for consumers in the *ex-ante* stage, it must remain to be so in the *interim* stage, as well. As for the producer, however, this is not true. We know from Proposition 4 that the sum of the total utility under the modified BM mechanism $\Gamma^2(\alpha, k^*)$ is always equal to V. Therefore, the modified welfare weight is always equal to $\tilde{\alpha}(\alpha, k^*) = 1$. That is, the regulator always gives under the modified BM mechanism $\Gamma^2(\alpha, k^*)$ the whole expected surplus V to the producer, and out of this the producer pre-donates k^*V to consumers. In result, the actual net profit of the producer that optimally pre-donates in the *ex-ante* stage is equal to $\tilde{\pi}(\theta, \alpha) = \pi(\theta, \tilde{\alpha}(\alpha, k^*)) - k^*V$ or more explicitly

$$\tilde{\pi}(\theta, \alpha) = \int_{\theta}^{a} q^*(x, \tilde{\alpha}(\alpha, k^*))dx - \frac{2(1-\alpha)}{(3-2\alpha)}V = \frac{(a-\theta)^2}{2} - \frac{(1-\alpha)}{(3-2\alpha)}\frac{a^2}{3}. \tag{33}$$

(In Section 2, we saw that the upper bound of the integral in the above equation is $\theta^*(\tilde{\alpha}(\alpha, k^*)) = a/(2-\tilde{\alpha}(\alpha, k^*))$ and the optimal output function is $q^*(x, \tilde{\alpha}(\alpha, k^*)) = a - (2-\tilde{\alpha}(\alpha, k^*))x$ for any $x \in (0, a]$ which reduce to $\theta^*(\tilde{\alpha}(\alpha, k^*)) = a$ and $q^*(x, (\tilde{\alpha}(\alpha, k^*))) = a - x$ since $\tilde{\alpha}(\alpha, k^*) = 1$.)

One can easily show that the expected value of $\tilde{\pi}(\theta, \alpha)$ is just equal to $\tilde{W}_2^2(1, k^*)$, i.e., what the producer expects to earn from the modified BM mechanism $\tilde{\Gamma}^2(1, k^*)$ in the *ex-ante* stage. However, in the *interim* stage the expected value of $\tilde{\pi}(\theta, \alpha)$ is nothing but itself for the producer, as it has learned the true value of θ. Thus, in the *interim* stage the producer must only be interested in maximizing its actual profit, and this profit does not have to be equal to the actual profit $\tilde{\pi}(\theta, \alpha)$ it would obtain in the *ex-ante* stage. Since the optimal pre-donation rate of the producer in the *ex-ante* stage is independent of θ (as shown by Lemma 5), the actual profit it would induce runs the risk of becoming negative if θ is sufficiently close to a, or more formally if $\theta > \underline{\theta}(\alpha) = a[1 - \sqrt{2(1-\alpha)/(3(3-2\alpha))}]$, as can be observed from (33). When $\alpha = 0$, this condition reduces to $\theta > \underline{\theta}(0) = a[1-\sqrt{2}/3] \sim 0.53\, a$, which becomes never binding since we also know that the firm is allowed to operate only if $\theta \leq \theta^*(\alpha) = a/(2-\alpha)$ and this second condition reduces to $\theta \leq 0.50\, a$ when $\alpha = 0$. When $\alpha = 1$, pre-donation is not observed ($k^* = 0$). In this limiting case, the threshold $\underline{\theta}(\alpha)$ reduces to a, implying that the actual profit of the firm is always non-negative. On the other hand, if $\alpha \in (0.157, 1)$, then $\underline{\theta}(\alpha) < \theta^*(\alpha) < 1$ implying that $\tilde{\pi}(\theta, \alpha) < 0$ for all $\theta \in [\underline{\theta}(\alpha), \theta^*(\alpha)]$.

That is, for most values of α, there exists a non-zero measure of θ values where the producer will find that the optimal pre-donation it would make in the *ex-ante* stage can no longer be optimal in the *interim* stage.

Remark 2. For any $\alpha \in [0, 1)$ and $\theta \in [0, a)$, the producer finds that the outcome of the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ with the *ex-ante* optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ is

(i) *interim* superior to the BM mechanism $\Gamma(\alpha)$ if $\theta < \bar{\theta}(\alpha)$,

(ii) *interim* inferior to $\Gamma(\alpha)$ if $\theta > \bar{\theta}(\alpha)$, and

(iii) *interim* equivalent to $\Gamma(\alpha)$ if $\theta = \bar{\theta}(\alpha)$,

where $\bar{\theta}(\alpha) = a[1 - \sqrt{2(1-\alpha)/(3(3-2\alpha))}]$.

The observations in the above remark directly imply the following.

Proposition 6. *For any $\alpha \in [0, 1)$, the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ under the* ex-ante *optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ is interim Pareto non-comparable to the BM mechanism $\Gamma(\alpha)$.*

Our next question is to find the *interim* optimal pre-donation for the producer under the modified BM mechanism. Notice that Remark 2-(i) only shows that the producer *interim* prefers the *ex-ante* optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ to no pre-donation $\boldsymbol{\lambda}^{\boldsymbol{0,2}}$; it does not imply that $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ is *interim* optimal. As we have already discussed in Section 2.4, the objective of the producer when it chooses the pre-donation rate in the interim stage is to maximize the actual net profit $\pi^a(\theta, \alpha, k)$ given by equation (28). For any choice of pre-donation rate $k \in [0, 1)$, the modified BM mechanism will be $\tilde{\Gamma}^2(\alpha, k)$. The regulator, to whom θ is yet unknown, will determine the function $\tilde{\alpha}(.,.)$ to satisfy

$$(1-k)W_2(\tilde{\alpha}(\alpha, k)) = P_2^{\gamma}((\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R)) \tag{34}$$

at each $k \in [0, 1)$ using the conversion $\gamma = 2(1-\alpha)$. From the viewpoint of the regulator and consumers, the expected value of the gross profit from the modified BM mechanism, $\pi(\theta, \tilde{\alpha}(\alpha, k))$, is still equal to $W_2(\tilde{\alpha}(\alpha, k))$ and the expected value of the actual net profits, $\pi^a(\theta, \alpha, k)$, is therefore still $P_2^{\gamma}((\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R))$ as in the *ex-ante* stage.

The producer, on the other hand, can fully observe $\pi^a(\theta, \alpha, k)$ for any choice of k. However, the producer has now an additional constraint in the *interim* stage. Recall that in the *ex-ante* stage, the interests of the producer and consumers were aligned as shown by Lemma 4. A pre-donation by the producer is *ex-ante* beneficial (harmful) for itself if and only if it is also so for consumers. In the *interim* stage, this alignment does not necessarily exist. The producer may improve its actual profits at the expense of a deterioration in the expected utility of consumers. So, the regulator should be expected to use a modified BM mechanism allowing the producer to pre-donate in the *interim* stage only if the outcome of this mechanism yields a higher expected utility to consumers than obtained under the BM mechanism. This however can be true only if the pre-donation rate k is less than the threshold level $k < \bar{k}(\alpha)$, as implied by Proposition 3-(i). In that case, $P^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R)$ would lie on $PO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R))$. Together with equation (34), this would imply

$$P_1^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R) = W_1(\tilde{\alpha}(\alpha,k)) + kW_2(\tilde{\alpha}(\alpha,k)). \tag{35}$$

Since $P_1^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R) = \gamma P_2^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R)$, equations (34) and (35) together imply

$$W_1(\tilde{\alpha}) + kW_2(\tilde{\alpha}) = \gamma(1-k)W_2(\tilde{\alpha}), \tag{36}$$

where $\tilde{\alpha} \equiv \tilde{\alpha}(\alpha,k)$. The above equation is satisfied only if

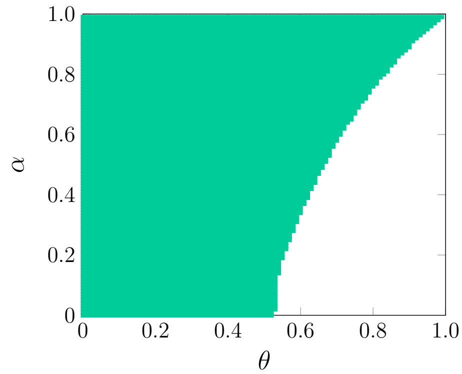
$$\tilde{\alpha} = \alpha + k\left(\frac{3-2\alpha}{2}\right). \tag{37}$$

So, using (12) along with (28), we can write the problem of the producer as

$$\max_{k\in[0,1]} \pi^a(\theta,\alpha,k) = \left(\frac{2-\tilde{\alpha}}{2}\right)\theta^2 - a\theta + \frac{a^2}{2(2-\tilde{\alpha})} - \frac{k}{(2-\tilde{\alpha})^2}\frac{a^2}{6}, \tag{38}$$

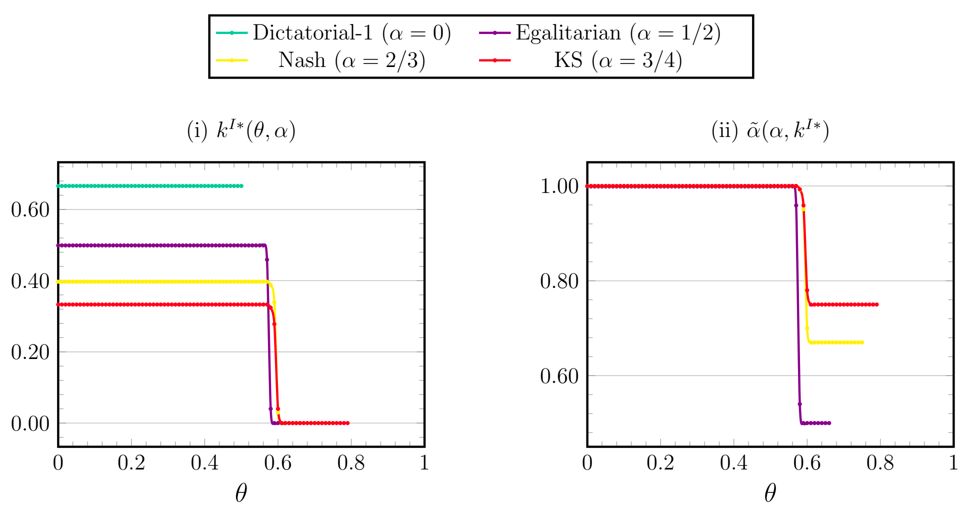
subject to the constraint $k < \bar{k}(\alpha) = 1 - 1/(2-\alpha)^2$ and equation (37) over the parameter values where $\tilde{\alpha} \in [0,1]$ is satisfied. We solve the above optimization problem with the help of a computer. In Figure 5, we plot the set of (θ,α) pairs that satisfy the aforementioned constraints faced by the producer in the *interim* stage. We observe that if α is zero, any θ inside the set $[0, 0.520]$ is consistent with the modified BM mechanism, and this set becomes wider and wider as α increases and eventually coincides with the unit interval when α becomes 1.

Figure 5. The Set of (c_1, δ) Pairs Supporting Equilibrium in the Interim State

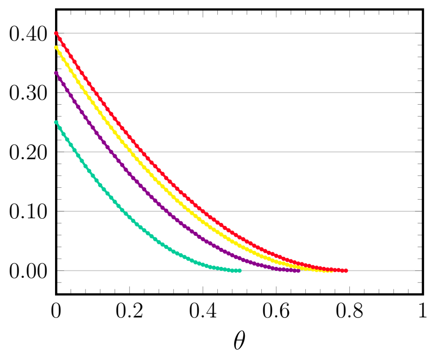


Next, we look for the solution to the constrained maximization problem in (38) over the set of (θ, α) pairs satisfying the constraints of the producer. As the first-order condition is not analytically conclusive, we will make simulations for various values of α. Here, we will denote the producer's optimal choice of pre-donation rate in the *interim* stage by k^{I*} to distinguish it from its optimal choice k^* made in the *ex-ante* stage. Recall that when $\alpha = 1$, the firm has no incentive to make pre-donation since $\tilde{\alpha}(\alpha, k)$ cannot exceed 1. So excluding $\alpha = 1$ (and the Dictatorial-2 rule associated thereof), we consider in our simulations four values of α in the set $\{0, 1/2, 2/3, 3/4\}$ corresponding to the rules in the set {Dictatorial-1, Egalitarian, Nash, Kalai-Smorodinsky}.

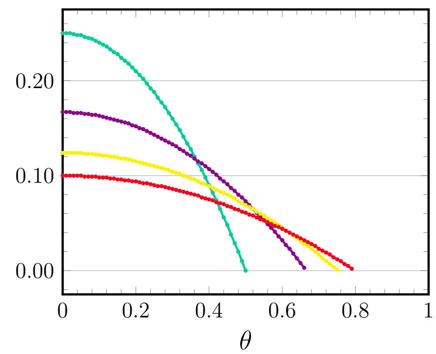
Figure 6. The Welfare Effects of Pre-donation in the Interim State



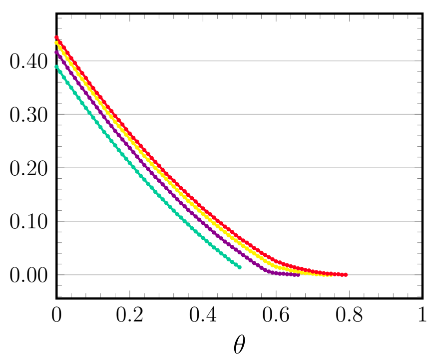
(iii) Actual Producer Welfare ($k = 0$)



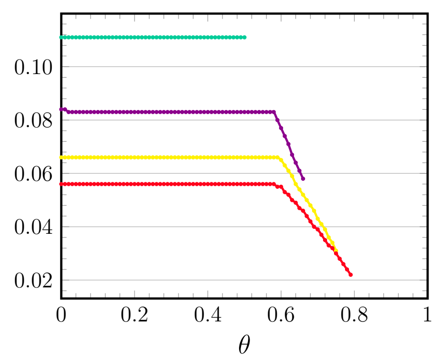
(iv) Actual Consumer Welfare ($k = 0$)



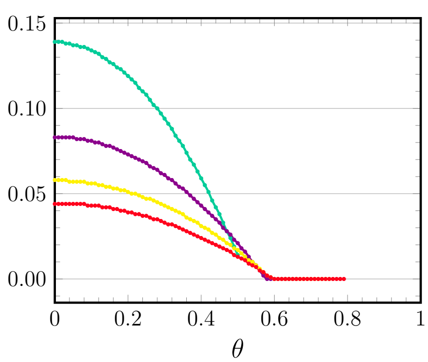
(v) Actual Producer Welfare ($k = k^{I*}$)



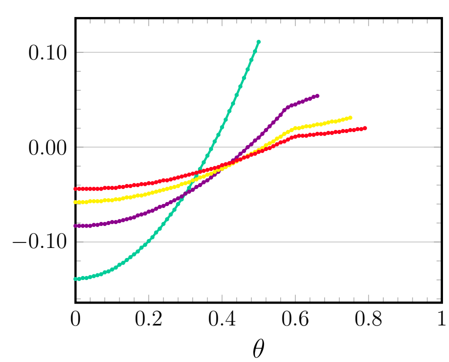
(vi) Actual Consumer Welfare ($k = k^{I*}$)

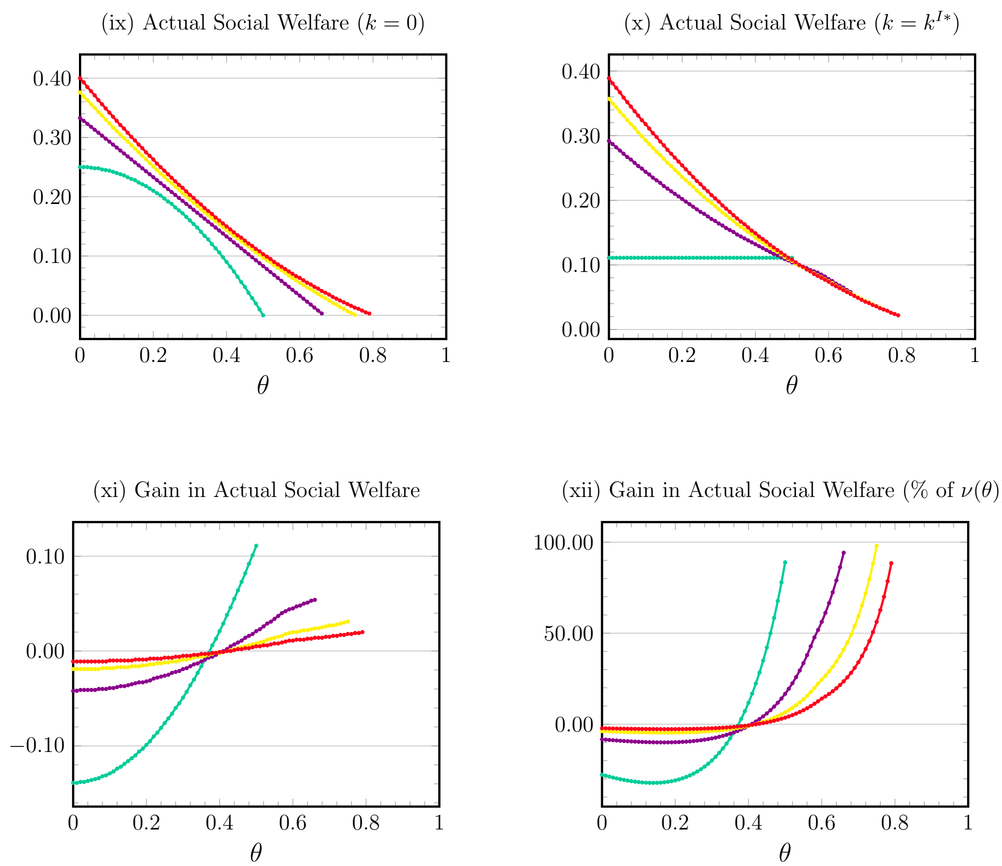


(vii) Gain in Actual Producer Welfare



(viii) Gain in Actual Consumer Welfare





The results portrayed in Figure 6 contain many findings. First, as shown by panel (i), under all four rules the producer's optimal pre-donation rate k^{I*} chosen in the *interim* stage is equal to the optimal value $k^* = 2(1-\alpha)/(3-2\alpha)$ it would choose in the *ex-ante* stage if θ is sufficiently small (not exceeding the middle values of θ in the unit interval) and zero otherwise. In result, the modified welfare weight $\tilde{\alpha}(\alpha, k^{I*})$ becomes equal to 1 when $k^{I*} = k^*$ and equal to α when $k^{I*} = 0$, as shown in panel (ii). Thus, there exists a small range of θ values which are low enough, less than $\theta^*(\tilde{\alpha}(\alpha, k^{I*})$, to warrant the operation of the producer but also high enough to imply that the producer chooses not to donate ($k^{I*} = 0$). Panels (iii) and (iv) illustrate the actual producer and consumer welfares when the pre-donation rate is zero and the modified mechanism coincides with the BM mechanism. We observe that both welfares always decrease with θ as theoretically predicted. However, the welfare effect of a change in α, and equivalently a change in the bargaining rule, is different for the two parties. The producer could rank the four bargaining rules (associated with four α values) from the best to the worst as KS, Nash, Egalitarian, and Dictatorial-1, whereas consumers would rank them in the reverse direction. Notice that these results concerning the effects of θ and α on the actual welfares remain to be observed in the *interim* stage too, as we can inspect in panels (v) and (vi). We should immediately observe that in the *interim* stage the modified BM mechanism reduces the effect of α on the actual producer welfare and eliminates the effect of θ on the actual consumer welfare when θ is not high. Comparing panels (v) and (vi) with the previous two panels, we calculate in the next two panels the actual welfare gains. Panel (vii) shows that the producer always benefits from pre-donating when θ is not too high. On the other hand, as shown in the next panel, consumers suffer from the producer's pre-donation if θ is low (nearly less than 0.4 or so) and benefit from it otherwise, as long as the producer is allowed to operate. In the last four panels, we consider the social welfare analysis. Panels (ix) and (x) together show that the modification of the BM mechanism by allowing the producer to pre-donate in the *interim* stage increases the variance of the social welfare with respect to α at low values of θ and reduces this variance otherwise. Finally, the last two panels show (in utility levels and percentage terms) that the actual social welfare always decreases (though not extremely) at low values of θ and substantially increases at high values of θ. Indeed, the last panel illustrates that when the percentage increase can be almost as high as 100% under all four bargaining rules when θ is sufficiently high. This result suggests that the lower the efficiency of the monopolist, the higher the ex-post social benefit we obtain from the modified BM mechanism. The welfare results summarized above imply the following existence result.

Proposition 7. *There exist $\alpha \in [0,1]$ and $\theta \in [0,a)$ such that by allowing the producer to optimally pre-donate in the* interim *stage under the modified BM mechanism the regulator can increase the actual (*ex-post*) welfare of both consumers and the producer.*

The simulations in Figure 6 suggest that the set of θ values for which Proposition 7 holds is a continuum. These simulations also suggest that the set of α values leading to the predicted welfare gains must contain the set $\{0, 1/2, 2/3, 3/4\}$ corresponding to four bargaining rules considered in Figure 6. Moreover, the continuity of the BM mechanism and of its modified version imply that the set of α for which Proposition 7 holds must be a continuum, too. In fact, our additional simulations not reported in this study suggest that as long as $\alpha < 1$, one can always predict to find some measurable range of θ values under which the studied modification for the BM mechanism leads to *ex-post* welfare improvements for both the producer and consumers.

Finally, we should notice that the optimal pre-donation rate k^{I*} of the producer in the interim stage is not independent of θ. Thus, it reveals information about the producer's private costs. The producer can avoid the consequences of this unintentional revelation, prior to the planned cost revelation, if it can make a contractual agreement with the regulator to prevent her from exploiting any information revealed by pre-donation. To see whether such an agreement would be plausible for the regulator, we should recall that in calculating the optimal *interim* pre-donation rate k^{I*} of the producer, we restricted ourselves to a domain where each pre-donation level was *ex-ante* admissible by the regulator and consumers, consequently ensuring $k^{I*} < \bar{k}(\alpha)$. As long as the contractual agreement between the producer and the regulator enforces the pre-donation rate in the *interim* stage to lie below the threshold level $\bar{k}(\alpha)$, the regulator may have incentive to sign this contract as it increases the expected consumer welfare whenever the producer chooses to pre-donate.

4 Conclusions

In this paper, we have proposed a Pareto-improving modification for the BM mechanism. The modification allows consumers and the monopolist to make –in the *ex-ante* or *interim* stage of the regulatory process– contingent utility transfers (pre-donations) between themselves to ensure a bilaterally beneficial improvement upon the expected social welfare function selected by the regulator in the BM mechanism.

We have proved that under the modified regulatory mechanism any amount of pre-donation by the producer in the *ex-ante* stage always leads to an *ex-ante* Pareto improvement, while a certain amount of it completely eliminates expected deadweight loss. Moreover, the optimal pre-donation of the producer in the *interim* stage may lead under some cost parameters to an *ex-post* Pareto improvement. Consumers, on the other hand, have never any incentive to make a unilateral pre-donation, nor to reverse the optimal pre-donation of the monopolist.

An important assumption we have made in modifying the BM mechanism is that the regulator can make a perfectly binding commitment preventing herself from using any information that may be revealed by the monopolist's pre-donation to update her prior beliefs about the monopolist's private cost information or to change the revelation mechanism borrowed from BM. This assumption has no bite when the monopolist makes pre-donation in the *ex-ante* stage. This is because the monopolist's pre-donation turns out to be independent of its private cost information, revealing no undesired information to the regulator. On the other hand, if the monopolist is allowed to pre-donate in the *interim* stage, its decision as to whether it should pre-donate becomes a function of its private cost information. Hence, the monopolist unintentionally reveals some cost information to the regulator before the actual revelation takes place, endangering some part of the informational rents it expects to earn from the regulatory process. However, the regulator has an incentive to be blind to any information revealed by pre-donation in the *interim* stage, as she can verifiably ensure that the optimal pre-donation of the monopolist should be increasing the expected utility of consumers. On the other hand, if the regulator chooses to exploit the information revealed by pre-donation, then the modified mechanism we propose would no longer be incentive-compatible. The monopolist would have incentives to revise its pre-donation decision strategically to limit the information revealed thereof and also to manipulate its cost report at certain values of its private cost parameter to make it comply with the announced pre-donation. We leave the characterization of the optimal regulatory mechanism in that case and the induced equilibrium pre-donation by the producer for future research.

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