The political economy theorem

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Abstract

Welfare maximisation is constrained by the ultimate frontier of efficient allocations, with a unique, interior optimum. By the second welfare theorem, such an optimum depends on a specific wealth distribution out of innumerable ones at given prices, whereby the state cannot refrain from redistributing. Such has long been known by the profession, but it never received a mathematical formalisation, which this article takes up. Building on the literature, this research also presents two simplified proofs to the two welfare theorems and a mathematical formalisation of the resolution to the compromise between equity and efficiency, for the additional constraint binds the social welfare function in equity and it originates the ultimate possibility frontier in efficiency.

JEL classification codes: D31; D51; D61; D63; I31; I38; P46; P48.
Keywords: competitive equilibrium; Pareto efficiency; political economy; social welfare; utility possibility; wealth distribution.

1. Introduction

There famously exist two fundamental theorems of welfare economics. The first welfare theorem dictates that a price equilibrium with transfers in a complete market system is a feasible Pareto efficient allocation (i.e. market capitalism); the second welfare theorem dictates that a feasible Pareto efficient allocation is a price equilibrium with transfers in a complete market system (i.e. state capitalism). Both theorems are the syntactic implication $A \longrightarrow B = \neg A \vee B$, which is true even if antecedent A is false (i.e. no existential fallacy); their failures, stressed by market and state socialism respectively together with their remedies, therefore only mean that A is not guaranteed in order to yield B (i.e. no counterexample, negative or inverse error singly taken): in the first case the state has an allocative role to guarantee market completeness and full employment; in the second case the state has a redistributive role to ensure the feasible Pareto efficient allocation optimising social welfare, because the social welfare function is constrained by the Pareto set and their mathematical structure yields a unique, interior optimum. In positive terms, however, market completion, business cycle stabilisation and wealth redistribution are difficult and knowledge of the optimal allocation and its necessary price enforcement are equally improbable.

The two welfare theorems are not therefore strictly concerned with welfare, but with efficiency, notes Fenoaltea (2001) especially, for the feasible Pareto efficient allocation of interest is exogenous: a feasible Pareto efficient allocation can be reached through exchange in a complete market system at fixed prices, preceded by wealth redistribution, but which one is it to be? The answer is only the one optimising social welfare. Such a reflexion triggers a welfare theorem of its own, which the profession has long known, see Bator (1952), but which lacks a mathematical formalisation, as proven by the updated encyclopaedic references Mas-Colell et al. (1995), Jehle and Reny (2001), Kreps (2012) and Varian (1978). Fenoaltea (2001) presented it more or less thus: if initial wealth redistribution is decentralised at given prices then

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the probability of selecting the initial level of wealth distribution optimising social welfare is infinitesimal; we formalise it mathematically and following him we call it the theorem of political economy.

Before doing so let us recall Fenoaltea (2001)'s remarks: market capitalism can at most yield efficiency, not welfare optimality, unlike state capitalism on contemplating our theorem; the object of redistribution is wealth because the optimal allocation is not known, which wealth redistribution can be unjust, we add. We in fact specify that because wealth redistribution is about aggregate utility optimisation, not unjustified egalitarianism, such an optimisation cannot be utilitarian, but ethical, which means it need not happen. We further note two points. Firstly, centralised transfers for the known feasible Pareto efficient allocation, whereby the market is avoided, are improbable as well as contradictory, because even if impossible for the direct achievement of the known feasible Pareto efficient allocation they would still be needed for its initial wealth redistribution. Secondly, state socialism conceptually resolves the second welfare theorem failure just as market socialism resolves the first's, providing knowledge of the feasible Pareto efficient allocation optimising social welfare, therefore, state socialism allows state capitalism to function, as sociologically understood.

2. Structure

In this section we lay out the building blocks for the mathematical formalisation of the political economy theorem following Mas-Colell et al. (1995), presenting direct proofs to the two welfare theorems in the process, with attendant remarks.

Let $\{X_i,\ \succsim_i\}_{i=1}^{I} \subset \mathbb{R}^K$ be a consumption set and preferences sequence with regard to I consumers, $\{Y_j\}_{j=1}^{J} \subset \mathbb{R}^K$ a production set sequence with regard to J producers and $\mathbf{e} = [e_1,\ \ldots,\ e_K]^\top \subset \mathbb{R}^K$ a K dimensional vector of initial endowments, relative to K real commodities. Preferences are complete and transitive and production is non-empty and closed: $\forall \{z_n\}_{n=1}^{\infty} \subset \mathbb{R}^K,\ z_1 \succsim_i z_2 \vee z_2 \succsim_i z_1$ and $z_1 \succsim_i z_2 \wedge z_2 \succsim_i z_3 \longrightarrow z_1 \succsim_i z_3;\ \overline{Y}_j = \left\{y \in \mathbb{R}^K :\ \forall \{y_{j_n}\}_{n=1}^{\infty} \subset Y_j,\ y_{j_n} \to y\right\} \neq \{\}$. Individual consumption and production vectors $\mathbf{x}_i = [x_{1i},\ \ldots,\ x_{Ki}]^\top \in X_i \subset \mathbb{R}^K$ and $\mathbf{y}_j = [y_{1j},\ \ldots,\ y_{Kj}]^\top \in Y_j \subset \mathbb{R}^K$ form the allocation $(\mathbf{x},\ \mathbf{y}) = (\mathbf{x}_1,\ \ldots,\ \mathbf{x}_I,\ \mathbf{y}_1,\ \ldots,\ \mathbf{y}_J) \in X_1 \times \ldots \times X_I \times Y_1 \times \ldots \times Y_J = X \times Y$. Preference characteristics are famously captured by household utility functions homogeneous of degree one, continuous, increasing and concave in consumption: $u : X \to \mathbb{R}^I$, where $X = \left\{\mathbf{x} :\ \mathbf{x} \in \mathbb{R}^{KI}\right\}$, such that $U = u(\mathbf{x});\ \forall \alpha \in \mathbb{R},\ u(\alpha \mathbf{x}) = \alpha u(\mathbf{x}),\ u \in C^2(X),\ u'(\mathbf{x}) > 0$ and $u''(\mathbf{x}) < 0$. The same holds for production and firm production functions.

A feasible allocation is such that aggregate consumption equals aggregate production: $a = \left\{(\mathbf{x},\ \mathbf{y}) \in X \times Y \subset \mathbb{R}^{K(I+J)} :\ \sum_{i=1}^{I} \mathbf{x}_i = \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}_j\right\}$. For any K dimensional price vector $\mathbf{p} = [p_1,\ \ldots,\ p_K]^\top \subset \mathbb{R}^K$, aggregate wealth is the sum of individual wealths, namely, aggregate consumption weighted at given prices, which is the sum of the initial endowment and aggregate production weighted at given prices: $\sum_{i=1}^{I} \mathbf{w}_i = \sum_{i=1}^{I} \mathbf{p}^\top \mathbf{x}_i = \mathbf{p}^\top \mathbf{e} + \sum_{j=1}^{J} \mathbf{p}^\top \mathbf{y}_j$. Centralised transfers happen by redistributing priced endowments and production. A feasible allocation is Pareto efficient if and only if there exists no other feasible allocation such that almost all agents prefer it to the given one and at least one agent strictly prefers it to the given one: $\tilde{a} \longleftrightarrow \nexists a'$ such that, $\not\forall i,\ a'_i \succsim_i a_i$ and, $\exists i,\ a'_i \succ_i a_i$. The Pareto set is the collection of all feasible Pareto efficient allocations: $\tilde{A} = \left\{\tilde{a} :\ \tilde{a} \in \tilde{X} \times \tilde{Y} \subset \mathbb{R}^{K(I+J)}\right\}$. For consumption, preferences, production, initial endowments and wealth, a price equilibrium with transfers is an allocation and a non-zero price vector pair such that profits and preferences are optimal and markets clear: $(a^*,\ \mathbf{p} \neq 0)$ such that (i) $\forall \mathbf{y}'_j \in Y_j,\ \mathbf{p}^\top \mathbf{y}^*_j \geq \mathbf{p}^\top \mathbf{y}'_j$, (ii) $\mathbf{x}^*_i \succ_i \mathbf{x}'_i$, where $\mathbf{x}^*_i,\ \mathbf{x}'_i \in B(\mathbf{p}^\top) = \left\{\mathbf{x}'_i \in X_i :\ \mathbf{w}_i \geq \mathbf{p}^\top \mathbf{x}'_i\right\}$, and (iii) $\sum_{i=1}^{I} \mathbf{x}^*_i = \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}^*_j$.

The first welfare theorem dictates that a price equilibrium with transfers in a complete market system is a feasible Pareto efficient allocation: $(a^*,\ \mathbf{p} \neq 0) \longrightarrow \tilde{a}^*$. It can be proven directly. There exists a consumption vector optimising preferences for optimal profits and all agents, therefore, it is preferred to all the others and the feasible allocation supporting it is Pareto efficient; formally: $\sum_{i=1}^{I} \mathbf{x}^*_i = \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}^*_j \succ \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}'_j = \sum_{i=1}^{I} \mathbf{x}'_i$ for $\sum_{i=1}^{I} \mathbf{p}^\top \mathbf{x}^*_i = \mathbf{p}^\top \mathbf{e} + \sum_{j=1}^{J} \mathbf{p}^\top \mathbf{y}^*_j \geq \mathbf{p}^\top \mathbf{e} + \sum_{j=1}^{J} \mathbf{p}^\top \mathbf{y}'_j = \sum_{i=1}^{I} \mathbf{p}^\top \mathbf{x}'_i$ and $a^* = \left\{(\mathbf{x}^*,\ \mathbf{y}^*) \in X \times Y \subset \mathbb{R}^{K(I+J)} :\ \sum_{i=1}^{I} \mathbf{x}^*_i = \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}^*_j\right\}$, therefore, $\nexists a'$ such that, $\not\forall i,\ a'_i \succsim_i a^*_i$ and, $\exists i,\ a'_i \succ_i a^*_i$. We remark that the use of locally non-satiated preferences is unnecessary, because one need not hypothesise a feasible Pareto dominating allocation automatically embedding a contradictory consumption vector optimising preferences.

The second welfare theorem dictates that a feasible Pareto efficient allocation is a price equilibrium with transfers in a complete market system: $\tilde{a} \longrightarrow (\tilde{a}^*,\ \mathbf{p} \neq 0)$. It can be likewise proven directly. There exists no feasible Pareto dominating allocation, therefore, it must embed a consumption vector optimising preferences for optimal profits and all agents; formally: $\nexists a'$ such that, $\not\forall i,\ a_i' \succsim_i a_i^*$ and, $\exists i,\ a_i' \succ_i a_i^*$ and $a^* = \left\{(\mathbf{x}^*,\ \mathbf{y}^*) \in X \times Y \subset \mathbb{R}^{K(I+J)}:\ \sum_{i=1}^{I} \mathbf{x}_i^* = \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}_j^*\right\}$, therefore, $\sum_{i=1}^{I} \mathbf{x}_i^* = \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}_j^* \succ \mathbf{e} + \sum_{j=1}^{J} \mathbf{y}_j' = \sum_{i=1}^{I} \mathbf{x}_i'$ for $\sum_{i=1}^{I} \mathbf{p}^\top \mathbf{x}_i^* = \mathbf{p}^\top \mathbf{e} + \sum_{j=1}^{J} \mathbf{p}^\top \mathbf{y}_j^* \geq \mathbf{p}^\top \mathbf{e} + \sum_{j=1}^{J} \mathbf{p}^\top \mathbf{y}_j' = \sum_{i=1}^{I} \mathbf{p}^\top \mathbf{x}_i'$. Observe that locally non-satiated preferences are similarly unnecessary. We additionally recall Maskin and Roberts (2008)'s remark: convex production sets (and preferences) are unnecessary for both the second welfare theorem and the existence of a price equilibrium with transfers (e.g. in large non-atomic economies they are unnecessary for its existence). Specifically, the second welfare theorem's historic proof uses convex production sets and preferences only to derive existence of a price quasi-equilibrium with transfers, not to show that a true antecedent implies a true consequent, therefore, we note, if the consequent is true through convex production sets and preferences then the second welfare theorem holds trivially. Maskin and Roberts (2008) also remarked that a counterexample to the second welfare theorem is a sufficient condition for the inexistence of any price quasi-equilibrium with transfers: by definition, a feasible Pareto efficient allocation embeds a consumption vector optimising preferences for optimal profits and all agents, thus, if such an optimisation were impossible then the defined notion of an equilibrium would not exist at all.

There exists a transformation of the Pareto set into multidimensional, real household utility: $t: \tilde{A} \rightarrow U$, where $U = \left\{\mathbf{u}:\ \mathbf{u} \in \mathbb{R}^I\right\}$, such that $T = t(\tilde{a})$. It is homogeneous of degree one, continuous and increasing in the Pareto set elements. The utility possibility frontier follows from the Pareto set through a function composition, because each household presents a maximal utility level given those of others, matching the notion of Pareto efficiency; more clearly, the utility possibility frontier maps household utility to the non-negative real line: $f \circ t: \tilde{A} \rightarrow \mathbb{R}_+;\ f: U \rightarrow \mathbb{R}_+$ such that $F = f(\mathbf{u})$. It is homogeneous of degree one, continuous, increasing and convex in utility: $f''(\mathbf{u}) > 0$, all else equal. The social welfare function maps household utility to the non-negative real line: $s: U \rightarrow \mathbb{R}_+$ such that $S = s(\mathbf{u})$. It is homogenous of degree one, continuous, increasing and concave in utility: $s''(\mathbf{u}) < 0$, all else equal. Social welfare optimisation is the maximisation of the social welfare function with respect to utility subject to the utility possibility frontier; social welfare function concavity and utility possibility frontier convexity are famously sufficient, but unnecessary, conditions for a unique, interior optimum: $\underset{\mathbf{u}}{max}\ s(\mathbf{u})\ \ s.t.\ f(\mathbf{u})$ and $n(U_o) = 1$, where $U_o = \left\{\mathbf{u}_o:\ \mathbf{u}_o \in \mathbb{R}^I\right\}$.

3. Theorem

In this section we formalise the political economy theorem mathematically; we further mathematically formalise the notion of the ultimate possibility frontier following Fenoaltea (2001), resolving the equity efficiency compromise in the process.

The social welfare optimum is a unique tangency point between the social welfare function and the utility possibility frontier and it is a feasible allocation along the Pareto set. A feasible allocation is such that aggregate consumption equals the initial endowment plus aggregate production; priced aggregate consumption is wealth, whereby price changes vary the angle at which the wealth hyperplane crosses the Pareto set. A probability density function with individual consumption vectors as arguments models the probability of randomly selecting an initial level of wealth distribution at given prices: for the probability space $(\Omega,\ X,\ \pi)$, where Ω is the sample space, $X = \left\{\mathbf{x}:\ \mathbf{x} \in \mathbb{R}^{KI}\right\} \subset \mathbb{R}^{KI} = \mathcal{P}(\Omega)$ is the σ-algebra and $\pi: X \rightarrow [0,\ 1]$ is the probability measure, $\mathscr{P} = \pi(\mathbf{x}) \geq 0$ and $\int_{-\infty}^{\infty} \pi(\mathbf{x})\, d\mathbf{x} = 1$.

By the second welfare theorem, a feasible Pareto efficient allocation can be reached by exchange in a complete market system at fixed prices given an initial level of wealth distribution. The feasible Pareto efficient allocation optimising social welfare is contingent on an initial level of wealth distribution at given prices and since a specific initial level of wealth distribution is infinitesimal the probability that it result through decentralisation is also infinitesimal: since the initial level of wealth distribution is modelled through an aggregate consumption vector taken from the real hyperplane at fixed prices the probability that a specific one result randomly is zero.

Theorem 3.1 (Political economy theorem) *Let* $\hat{\mathbf{w}} = \mathbf{p}^\top \hat{\mathbf{x}}$ *be an arbitrary level of initial wealth redistribution. If initial wealth redistribution is not centralised at fixed prices then the probability of selecting the initial level of wealth distribution optimising social welfare is zero. Formally:*

$$\mathbf{w} \neq_C \hat{\mathbf{w}} | \mathbf{p} \longrightarrow P(\mathbf{w} = \mathbf{w}_o) = 0.$$

Proof. The proof is direct. If wealth redistribution is not centralised at fixed prices then the probability that the initial level of wealth distribution equal the one optimising social welfare is the integral of the aggregate consumption probability density function evaluated at the point optimising social welfare, which is zero; formally: $\mathbf{w} = \bar{\mathbf{p}}^\top \mathbf{x}$, therefore, $P(\mathbf{w} = \mathbf{w}_o) = P\left(\bar{\mathbf{p}}^\top \mathbf{x} = \bar{\mathbf{p}}^\top \mathbf{x}_o\right) = P\left(\bar{\mathbf{p}}^\top \mathbf{x} \in \left[\bar{\mathbf{p}}^\top \mathbf{x}_o,\ \bar{\mathbf{p}}^\top \mathbf{x}_o\right]\right) = P\left(\mathbf{x} \in [\mathbf{x}_o,\ \mathbf{x}_o] | \mathbf{p}^\top\right) = \int_{\mathbf{x}_o}^{\mathbf{x}_o} \pi(\mathbf{x})\, d\mathbf{x} = |\phi(\mathbf{x})|_{\mathbf{x}_o}^{\mathbf{x}_o} = \phi(\mathbf{x}_o) - \phi(\mathbf{x}_o) = 0.$ □

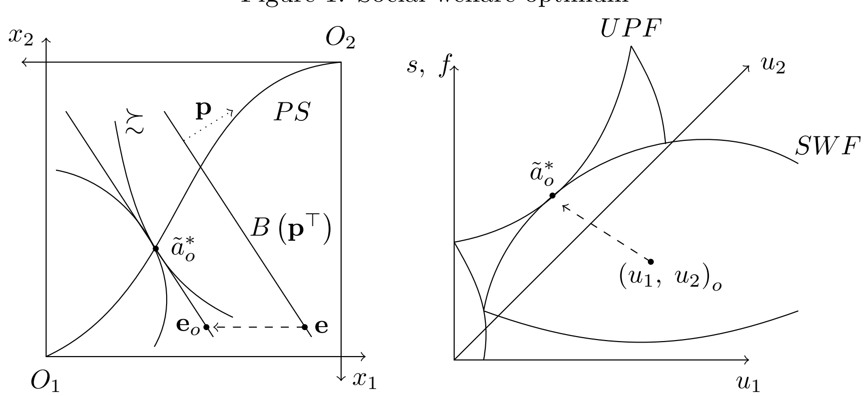
The reason for which the converse is not stated is clear from contrapositive of the converse (i.e. inverse denial), namely, if initial wealth redistribution is centralised at fixed prices then the probability of selecting the initial level of wealth distribution optimising social welfare need not be non-zero, as another initial level of wealth distribution could result, intentionally and not; formally: $\mathbf{w} =_C \hat{\mathbf{w}} | \mathbf{p} \not\longrightarrow P(\mathbf{w} = \mathbf{w}_o) \neq 0$.

Corollary 3.2 *If the probability of selecting the initial level of wealth distribution optimising social welfare is non-zero then the arbitrary level of initial wealth redistribution equals the initial level of wealth distribution optimising social welfare. Formally:*

$$P(\mathbf{w} = \mathbf{w}_o) \neq 0 \longrightarrow \hat{\mathbf{w}} = \mathbf{w}_o.$$

Proof. The proof is a tautology of Theorem 3.1's contrapositive. Theorem 3.1's contrapositive dictates that if the probability of selecting the initial level of wealth distribution optimising social welfare is non-zero then initial wealth redistribution is centralised at fixed prices: $P(\mathbf{w} = \mathbf{w}_o) \neq 0 \longrightarrow \mathbf{w} =_C \hat{\mathbf{w}} | \mathbf{p}$. It follows that the arbitrary level of initial wealth redistribution equals the initial level of wealth distribution optimising social welfare, namely, the one centrally selected: $\mathbf{w}_o = \mathbf{w} =_C \hat{\mathbf{w}}$. □

Figure 1: Social welfare optimum

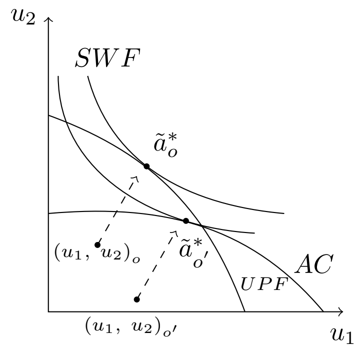


Note. The first diagram is a pure exchange Edgeworth box with two agents, O_1 and O_2, and two consumptions, x_1 and x_2 : tangency points between preferences $\succsim$ at given budget constraint $B\left(\mathbf{p}^\top\right)$, to which prices $\mathbf{p}$ are orthogonal, are Pareto set PS; the budget constraint's slope is price ratio $\frac{p_2}{p_1}$. Social welfare optimum $\tilde{a}_o^*$ is a specific feasible Pareto efficient allocation, reached at specific initial wealth $\mathbf{w}_o = \mathbf{p}^\top \mathbf{x}_o = \mathbf{p}^\top \mathbf{e}_o$ and fixed prices: if initial wealth does not yield the social welfare optimum through trade along the budget constraint's slope in a complete market system then the state can redistribute it by shifting the budget constraint at a constant slope from $\mathbf{w} = \mathbf{p}^\top \mathbf{x} = \mathbf{p}^\top \mathbf{e}$ to $\mathbf{w}_o = \mathbf{p}^\top \mathbf{x}_o = \mathbf{p}^\top \mathbf{e}_o$. The second diagram is a three dimensional graph of social welfare function SWF and of utility possibility frontier UPF with the same agents. The social welfare optimum, at specific initial wealth $\mathbf{w}_o = \mathbf{p}^\top \mathbf{x}_o = \mathbf{p}^\top \mathbf{e}_o$ and fixed prices in a complete, efficient market system, is the tangency point between the two manifolds. The probability of randomly selecting the initial wealth optimising social welfare is zero: $P(\mathbf{w}_o) = P\left(\bar{\mathbf{p}}^\top \mathbf{e}_o\right) = P\left(\mathbf{x}_o | \mathbf{p}^\top\right) = 0$. The diagrams are not to scale.

A partially more binding constraint than the utility possibility frontier for the social welfare function is such that the new tangency point between the acting social welfare function and the new constraint (i.e. second best) excludes both the former social welfare optimum and tangent allocations with lower social welfare levels on the binding segment of the utility possibility frontier. More clearly, the ultimate utility possibility frontier is the union of the binding segments of the new constraint and the utility possibility frontier about their point of intersection: new constraint function c is defined as f and, $\forall d = c,\ f$ and $\mathbf{u} \in U$ such that $U_{<\mathbf{u}} = \left\{\mathbf{u}:\ \mathbf{u} \in \left(\mathbb{R}^I \backslash G\right) \subset \mathbb{R}^I\right\}$, where $G \subset \mathbb{R}^I$ is a set subtrahend, and $U_{\geq \mathbf{u}} = U \backslash U_{<\mathbf{u}}$, $d_{<\mathbf{u}} : U_{<\mathbf{u}} \rightarrow \mathbb{R}_+$, $d_{\geq \mathbf{u}} : U_{\geq \mathbf{u}} \rightarrow \mathbb{R}_+$, $D_{<\mathbf{u}} = \{d_{<\mathbf{u}}(\mathbf{u}) :\ d_{<\mathbf{u}}(\mathbf{u}) \in \mathbb{R}_+\}$ and $D_{\geq \mathbf{u}} = \{d_{\geq \mathbf{u}}(\mathbf{u}) :\ d_{\geq \mathbf{u}}(\mathbf{u}) \in \mathbb{R}_+\}$, therefore, $F' = min\{F_{<\mathbf{u}},\ C_{<\mathbf{u}}\} \cup min\{F_{\geq \mathbf{u}},\ C_{\geq \mathbf{u}}\}$.

The fact that the second best has a higher social welfare level than the tangent allocations on the binding utility possibility frontier means there semantically exists no equity efficiency compromise: the second best is meaningfully efficient and Pareto efficiency is only Pareto stability by which exchange no longer happens. The equity efficiency compromise exists neither effectively, because the second best is the tangency point between the social welfare function and the new constraint, making it equitable (in that it optimises social welfare), and it is also the tangency point between the social welfare function and the ultimate utility possibility frontier, making it efficient (in that it accounts for Pareto efficiency). Such an observation is due to Fenoaltea (2001). Fenoaltea (2001) also added that perduring market equilibria are Pareto efficient and whenever not recognised as such they allude to unidentified binding constraints along which utilities have been maximised: it seems true, however prevaricating such further constraints be.

Figure 2: Ultimate utility possibility frontier



Note. Such a diagram graphs social welfare functions SWF, utility possibility frontier UPF and additional constraint AC with two agents in two dimensions, omitting the axis of the codomain. The diagram displays the ultimate utility possibility frontier, which is the binding union of the additional constraint and the utility possibility frontier; it highlights both the equity and efficiency of new social welfare optimum $\tilde{a}^*_{o'}$: equity because tangency point between the acting social welfare function and the binding additional constraint; efficiency because tangency point between the acting social welfare function and the ultimate utility possibility frontier. The diagram is not to scale.

4. Conclusion

Efficient allocations are competitive equilibria at given wealth and prices, but welfare is maximised subject to the ultimate frontier of efficient allocations, with a unique, interior optimum; the efficient allocation optimising welfare reached as a competitive equilibrium at given prices thus requires a specific wealth distribution, which cannot systematically arise unless it be centrally determined: if wealth redistribution is decentralised at given prices then the probability of optimising welfare is infinitesimal. Such is the political economy theorem long known by the profession, now formalised mathematically. Finally, whichever additional constraint were to bind the social welfare function would tautologically give rise to an equitable allocation, but also to an efficient one, for then partaking in the ultimate possibility frontier of the economy and settling the compromise.

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