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Monetary Policy in a Schumpeterian Growth Model with Two R&D Sectors*

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Abstract

We explore the growth and welfare effects of monetary policy in a two-sector Schumpeterian economy with cash-in-advance (CIA) constrained R&D investment in both sectors. We show that a nominal interest rate increase generates two effects on equilibrium labor allocation: a *manufacturing-R&D-reallocation effect* and a *cross-R&D-sector effect*. The former reduces economic growth by shifting labor from R&D to production, whereas the latter can enhance it by shifting labor from the less productive R&D sector to the more productive one. Unless the high productivity R&D sector is severely more CIA-constrained than the low productivity one, aggregate R&D overinvestment is sufficient but not necessary for the Friedman rule of monetary policy to be suboptimal. Our benchmark parameterization suggests that a positive nominal interest rate is optimal despite that it exacerbates the aggregate R&D underinvestment problem.

JEL classification: O30; O40; E41.

Keywords: CIA constraint, Endogenous growth; Monetary policy; R&D; Creative destruction.

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1 Introduction

In this study, we explore the growth and welfare implications of monetary policy in a Schumpeterian economy with two vertically related sectors engaging in R&D investment subject to a cash-in-advance (CIA) constraint. Friedman (1969) proposed a famous monetary policy rule, known as the Friedman rule, according to which the optimal nominal interest rate is zero. Subsequently, there has been a large body of literature analyzing the optimality of Friedman rule in different economic environments.[1] While zero-nominal-interest-rate targeting was merely a theoretical possibility before the 2008 global financial crisis, it has become almost a new normal since then. The short-term nominal interest rates in major developed economies, including the US, Euro zone, UK, and Japan, have persistently stayed at the near-zero level for over a decade now (see Huang *et al.* (2017) and Wu (2021)). This important phenomenon has generated even more interest in Friedman rule. Several recent studies have examined the (sub)optimality of Friedman rule by incorporating money demand into R&D-based growth models (e.g., Chu and Cozzi (2014) and Hori (2020)). The main purpose of this study is to explore the growth and welfare effects of Friedman rule in a multi-sector economy with CIA-constrained R&D activities.

Our two-sector endogenous growth model is motivated by two stylized facts. First, existing evidence shows strongly that both downstream and upstream firms actively engage in R&D activities. For example, Nelson (1986) finds that both upstream and downstream industries have significant contributions to the US R&D investment. McLaren (1999) and Banerjee and Lin (2003) document that innovations in the automobile sector in Japan and the US are conducted by both auto makers and auto parts suppliers. Pillai (2013) shows that new capital equipment invented by upstream semiconductor equipment firms like Nikon, Canon, and ASML allows microprocessor firms like Intel and AMD to develop higher performance microprocessors. More recently, Yang (2020) shows that in the US smartphone market, downstream handset makers (such as Samsung) build on innovations by upstream chipset firms (such as Qualcomm) to develop new hardware. Furthermore, our analysis in Section 5.1 using data from the US manufacturing industries shows that the R&D-to-assets ratios of upstream and downstream firms are similar. The traditional R&D-based endogenous growth models (for example, Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992)) consider research activities only in one sector. This setting is not suitable for examining how policy affects resource allocation between R&D activities.

Second, recent studies in corporate finance show strongly that R&D investment is subject to significant financial constraints. For example, Brown *et al.* (2012) find that the increase in corporate cash flow in the 1990s was the result of firms' objective to smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Falato and Sim (2014) demonstrate that firms hold cash to finance their R&D investment due to the presence of financing frictions. Brown and Petersen (2015) find that firms allocate cash reserves to buffer R&D but do not use cash to protect fixed investment during the recent financial crisis. Furthermore, Lyandres and Palazzo (2016) show that the sharp increase in the average cash-to-assets ratio for the US firms since the mid-1980s is driven almost only by firms investing heavily in R&D, and Ma *et al.* (2020) report a positive correlation coefficient of 0.41 between the industry-level cash- and R&D-to-assets ratios in the US. Importantly, our analysis in Section 5.1 shows that the degree of R&D financial constraints differs significantly between the upstream and downstream sectors,

[1] See, for example, Ho *et al.* (2007) and Gahvari (2012).

suggesting a need to incorporate heterogeneity in CIA constraints in R&D-based growth models.

We develop a scale-invariant version of the quality-ladder growth model with the above features. In the model, both the upstream and downstream sectors engage in R&D activities, and money demand is incorporated through a CIA constraint on R&D investment. We show that when both upstream and downstream R&D are CIA constrained, a higher nominal interest rate has two effects on resource reallocation. First, it generates a *manufacturing-R&D-reallocation effect* by shifting resources (i.e., labor in this study) from the R&D sectors to the manufacturing sector due to the increased cost of R&D financing, which reduces economic growth. Second, it generates a *cross-R&D-sector effect* by reallocating resources from the more cash-constrained R&D sector to the less constrained one, which enhances economic growth if the less constrained sector is more productive, a condition supported by our empirical analysis. We show analytically that when the cross-sectoral gaps in productivity and CIA constraints are sufficiently large, the growth-enhancing effect dominates the growth-decreasing effect at low nominal interest rates, generating an inverted-U relation between economic growth and inflation documented in recent empirical studies (e.g., López-Villavicencio and Mignon (2011) and Eggoh and Khan (2014)).

To explore the welfare effects of the Friedman rule of monetary policy (i.e., zero-nominal-interest-rate targeting), we examine the necessary and sufficient conditions for the suboptimality of this rule. It is well-known that the equilibrium R&D investment in endogenous growth models may be above or below the optimal level due to the interplay between positive externalities (e.g., the intertemporal knowledge spillover effect) and negative externalities (e.g., the business-stealing effect) of innovations (or in Schumpeter's term, the "gale of creative destruction"). Chu and Cozzi (2014) consider a model with R&D only in the upstream sector and show that under inelastic labor supply, Friedman rule is suboptimal if and only if the equilibrium is characterized by R&D overinvestment.[2] We find that the relation between R&D overinvestment, defined as the total R&D labor share across the two sectors above the socially optimal R&D labor share, and the suboptimality of Friedman rule is substantially more complicated in our two-R&D-sector model. It depends crucially on the relative degree of CIA constraints and the relative R&D productivity between the two sectors. In light of our empirical evidence, we focus on the environment in which the productivity of upstream R&D is higher than that of downstream R&D. In this setting, we show that the equivalence between R&D overinvestment and the suboptimality of Friedman rule holds only if the upstream-to-downstream CIA constraint ratio is equal to the gross-to-net markup ratio of the upstream sector. When this condition holds, the two effects mentioned above completely offset each other for the downstream labor share at the zero interest rate, making it locally unaffected by an interest rate increase. As a result, our two-R&D-sector model behaves effectively like the model of Chu and Cozzi (2014).[3] As long as the relative CIA constraint of upstream R&D is below its gross-to-net markup ratio, a weak condition the generally holds, aggregate R&D overinvestment at the zero nominal interest rate is sufficient but not necessary for Friedman rule to be suboptimal. By contrast, when the relative CIA constraint of upstream R&D

[2]When labor supply is elastic, Chu and Cozzi (2014) show that overinvestment is necessary but not sufficient for the suboptimality of Friedman rule if both consumption and R&D investment are CIA-constrained. Hori (2020) extends their model by considering heterogeneity in R&D firms' productivity. He finds that if R&D firms are heterogeneous (homogeneous), Friedman rule can be suboptimal (is always optimal) under a severe financial constraint.

[3]Appendix C shows that when R&D productivity is the same in both sectors, or when the downstream markup is too low to support any positive R&D in that sector, the relation between R&D overinvestment and the suboptimality of Friedman rule also collapses into that in Chu and Cozzi (2014).

is above this threshold, R&D overinvestment becomes necessary but not sufficient for Friedman rule to be suboptimal. Importantly, because the gross-to-net markup ratio is generally far above one, even if the more productive upstream R&D sector is significantly more CIA-constrained, in which case the cross-R&D-sector effect is welfare-decreasing, a positive nominal interest rate can still be welfare-improving in the absence of aggregate R&D overinvestment. This result arises because a positive interest rate reduces the socially wasteful R&D in the low productivity sector, the benefits of which can outweigh the costs of reducing socially desirable R&D in the high productivity sector.

In addition, we perform a quantitative analysis to evaluate the growth and welfare effects of monetary policy in our model. To facilitate the model calibration, we conduct an empirical estimation of R&D characteristics, profit margins, and financial constraints in the upstream and downstream sectors using data from US manufacturing firms. Our calibration features higher productivity and weaker CIA constraint of R&D in the upstream than in the downstream. The benchmark case shows that the welfare-maximizing nominal interest rate is positive despite that R&D is underinvested at the zero nominal rate. Although a positive interest rate further exacerbates the overall underinvestment problem, it improves welfare by reducing the inefficient R&D investment in the low productivity sector. Overinvestment emerges when R&D investment in both sectors becomes less productive, in which case the welfare costs of zero-nominal-interest-rate targeting are significantly higher. We further show that when the parameter differences in productivity and CIA constraints between the sectors are sufficiently large, a positive interest rate can boost economic growth through the positive cross-R&D-sector labor reallocation effect.

Previous studies have examined extensively the growth and welfare effects of inflation and Friedman rule, but the impact of the two-R&D-sector structure is largely unexplored in this literature. To the best of our knowledge, we are the first to analyze optimal monetary policy in a growth-theoretic framework featuring CIA-constrained R&D activities in vertically related industries. Our analysis yields novel insights, including (i) a new channel for the growth and welfare effects of monetary policy through the impact of nominal interest rates on resource reallocation across R&D sectors; (ii) a more general characterization of the relation between suboptimal R&D investment and the (sub)optimality of Friedman rule that nests the existing results as special cases; (iii) a novel theoretical result that in an environment with heterogeneous R&D productivity, a positive nominal interest rate can be welfare-improving even if it exacerbates the overall R&D underinvestment problem. In addition, our analysis of data from US manufacturing firms reveals new stylized facts about R&D activities, productivity, and financial constraints in the upstream and downstream sectors. The benchmark case of our empirically-calibrated model shows that a zero-nominal-interest-rate policy maximizes economic growth but a positive nominal interest rate maximizes social welfare. These findings have strong implications for understanding the long-run effects of the low-interest-rate policy prevalent in recent years.

This study contributes to the growth-theoretic literature on optimal monetary policy that features CIA requirements. The pioneering work by Marquis and Reffett (1994) introduces a CIA constraint on consumption to the Romer (1990) type variety-expansion model to investigate the effects of monetary policy, and proves that Friedman rule is optimal. The subsequent study by Chu and Ji (2016) explores the welfare effects of monetary policy in a Schumpeterian quality-ladder model with endogenous market structure. Unlike their models, the current study analyzes optimal monetary policy via a CIA constraint on R&D.[4] As mentioned above, our study is closely related to Chu and Cozzi (2014), who show an equivalence between R&D overinvestment and the suboptimality of Friedman rule. Their model features R&D activities only in the upstream sector, so raising the nominal interest rate yields only a reallocating effect on resources from R&D to production. Thus, this lowers economic growth and increases welfare only if R&D is overinvested. We extend their interesting study by considering R&D activities in both the intermediate-good and final-good sectors. In this setting, in addition to the manufacturing-R&D-reallocation effect, raising the nominal interest rate can generate a positive growth effect by reallocating labor from the less productive R&D sector to the more productive one. Furthermore, it can improve welfare by reducing the socially wasteful R&D investment in the low productivity sector even if the overall R&D investment is below the optimal level. Therefore, R&D overinvestment is not necessary for a positive nominal interest rate to be optimal.

Additionally, this study relates to the recent literature in R&D-based models that explores the non-monotonic effect of inflation on growth. For example, Chu *et al.* (2017) find an inverted-U relation between inflation and growth in a quality-ladder model featuring a random quality improvement, whereas Arawatari *et al.* (2018) find a cutoff inflation level around which inflation and growth exhibit a negative, nonlinear relation in a variety-expansion model featuring heterogeneous R&D abilities. The current study differs from these studies by highlighting the role of the relative CIA constraint strength and productivity heterogeneity between vertically related sectors in generating the non-monotonic inflation-growth relation. Recently, Zheng *et al.* (2021) also find an inverted-U relation between inflation and growth in a two-R&D-sector Schumpeterian growth model. Unlike in the current study, R&D in their model is conducted to develop vertical and horizontal innovations instead of in upstream and downstream industries.[5]

Finally, we contribute to a growing strand of literature on the effects of government policy in endogenous growth models with two R&D sectors. Li (2000) analyzes the effectiveness of R&D subsidies in stimulating economic growth in a two-R&D-sector model with both fully endogenous growth and semi-endogenous growth. Segerstrom (2000) characterizes the long-run growth effects of R&D subsidies in an endogenous growth model with both vertical R&D and horizontal R&D. Goh and Olivier (2002) explore optimal patent protection in a variety-expansion growth model with R&D investment in two vertically related sectors, whereas Chu (2011) addresses a similar issue in a quality-ladder growth model with R&D investment in two horizontal final-good sectors. We complement this literature by focusing on the role of monetary policy.

The rest of this paper is organized as follows. Section 2 presents the model setup. Section 3 characterizes the decentralized equilibrium and analyzes the growth effect of monetary policy. Section 4 explores the welfare effects of monetary policy. Section 5 presents a quantitative analysis. Section 6 concludes.

[4]See other recent studies, such as Chu *et al.* (2015), Gil and Iglésias (2020), and Huang *et al.* (2021), for optimal monetary policy in endogenous growth models with a CIA constraint on R&D.

[5]Huang *et al.* (2021) also examine the growth implications of inflation in a Schumpeterian growth model with endogenous market structure and distinct CIA constraints on quality-improving R&D and variety-expanding R&D. They find that the short-run effect of inflation on growth differs from the long-run effect, but both effects are monotonic.

2 Model

We extend a version of the quality-ladder model in Grossman and Helpman (1991) by allowing firms to invest in R&D to develop innovations in both the upstream (i.e., intermediate-good) and downstream (i.e., final-good) sectors as in Goh and Olivier (2002) and by introducing money demand via a CIA constraint on R&D investment as in Chu and Cozzi (2014) and Huang *et al.* (2017). The nominal interest rate serves as the monetary policy instrument and the effects of monetary policy are examined by considering the implications of altering the rate of nominal interest on economic growth and social welfare.

2.1 Households

At time t, each household has a population size of N_t, which grows at the rate of $n \geq 0$ such that $\dot{N}_t = nN_t$. There is a unit continuum of identical households, and the lifetime utility function of each member is given by

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

where $\rho > 0$ represents the discount rate, and c_t is the consumption good for each member. The law of motion for assets of each household member (expressed in real terms) is

$$\dot{a}_t + \dot{m}_t = (r_t - n)a_t + w_t + i_t b_t + \tau_t - c_t - (\pi_t + n)m_t, \tag{2}$$

where a_t is the real asset value, r_t is the real interest rate, and each individual inelastically supplies one unit of labor at the real wage rate w_t. τ_t denotes the real lump-sum transfer from the government, π_t is the inflation rate that reflects the cost of holding money, and m_t is the real money balance that the household member holds in order to facilitate entrepreneurs' loans b_t, which finances R&D investment and pays a nominal interest rate i_t. We impose a cash-in-advance (CIA) constraint on the amount of loans available to entrepreneurs: $b_t \leq m_t$.

The household's optimal problem is to maximize the discounted utility in (1) subject to the budget constraint in (2) and the CIA constraint. Solving the standard dynamic optimization yields the familiar Euler equation:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - n. \tag{3}$$

Moreover, using the optimality condition for real money balances m_t, we can derive the Fisher equation: $i_t = \pi_t + r_t$.

2.2 Final Goods

The final-good sector is referred to as *the downstream sector*. The aggregate consumption good in this economy, i.e., $C_t = N_t c_t$, is produced by using a unit continuum of differentiated final goods $Y_t(j)$ such that

$$C_t = \exp\left(\int_0^1 \ln Y_t(j) dj\right). \tag{4}$$

This equation implies that the demand function of each differentiated final goods is

$$Y_t(j) = \frac{C_t}{p_{y,t}(j)}, \tag{5}$$

where $p_{y,t}(j)$ is the price of final good j relative to the consumption good. The differentiated final goods in each industry j are produced by a monopolistic leader, who holds a patent on the latest innovation and uses a unit continuum of intermediate goods indexed by $k \in [0,1]$. This leader's products are replaced by the ones of a new entrant who has a more advanced innovation due to the *Arrow replacement effect*. The current leader's production function is given by

$$Y_t(j) = z^{q_{y,t}(j)} \exp\left(\int_0^1 \ln x_t(j,k)dk\right), \tag{6}$$

where the parameter $z > 1$ measures the step size of each quality improvement, $q_{y,t}(j)$ denotes the number of innovations between time 0 and time t, and $x_t(j,k)$ is the quantity of intermediate good k used for final good j. Thus, the marginal cost of producing final good j is

$$mc_{y,t}(j) = \frac{P_{x,t}}{z^{q_{y,t}(j)}}, \tag{7}$$

where $P_{x,t} \equiv \exp\left(\int_0^1 \ln p_{x,t}(k)dk\right)$ is the price index for intermediate goods and $p_{x,t}(k)$ is the price of intermediate good k.

Following previous studies such as Goh and Olivier (2002) and Chu and Cozzi (2014), we assume that intellectual property rights protect inventions in the form of incomplete patent breadth. The degree of patent breadth in the downstream sector, which is exogenously set by the policy of patent authority, determines the markup $\mu_y > 1$ that each final-good monopolist can charge over its marginal cost.[6] Thus, the profit-maximizing price in industry j is

$$p_{y,t}(j) = \mu_y \frac{P_{x,t}}{z^{q_{y,t}(j)}}. \tag{8}$$

The monopolistic profit of each differentiated final-good producer is identical and is given by

$$\Pi_{y,t} = \Pi_{y,t}(j) = (\mu_y - 1)Y_t(j)\frac{p_{y,t}(j)}{\mu_y} = \left(\frac{\mu_y - 1}{\mu_y}\right)C_t, \tag{9}$$

where the second equality and the third equality are obtained by using (5) and (8), respectively. Equation (9) implies that given the consumption expenditure C_t, a final-good producer's profit $\Pi_{y,t}$ is increasing in markup μ_y. Then, using the definition of price index $P_{x,t}$ along with (7), (8), and (9) yields the demand for intermediate good k:

$$x_t(k) = \int_0^1 x_t(j,k)dj = \frac{C_t}{\mu_y p_{x,t}(k)}. \tag{10}$$

[6]We adopt a common assumption in this literature that the replaced leader exits the market. Therefore, there is no constraint on the markup charged by the new leader other than incomplete patent breath.

2.3 Intermediate Goods

The intermediate-good sector is referred to as *the upstream sector*. The environment of the intermediate-good sector is similar to that of the final-good sector. Each industry in this sector is temporarily dominated by a monopolist holding the latest innovation, and the industry leadership is replaced by an entrant who holds a new invention. However, as in the conventional quality-ladder framework, production structure of this sector is different from that of the previous sector since intermediate goods are produced by manufacturing labor. Specifically, the production function for the current intermediate-good producer in industry k is given by

$$x_t(k) = z^{q_{x,t}(k)} L_{x,t}(k), \tag{11}$$

where the step size of quality improvement is assumed to be identical to that in the final-good sector, $q_{x,t}(k)$ is the number of innovations as of time t,[7] and $L_{x,t}(k)$ is the employment of manufacturing labor in industry k. Given the pricing strategy of the current leaders in this sector, the profit-maximizing price is again a constant markup over the marginal cost such that

$$p_{x,t}(k) = \mu_x mc_{x,t}(k) = \mu_x \frac{w_t}{z^{q_{x,t}(k)}}, \tag{12}$$

where the markup $\mu_x > 1$ captures the degree of patent breadth in the upstream sector, which is also exogenously set by the policy of patent authority. Accordingly, the monopolistic profit of each intermediate-good producer is

$$\Pi_{x,t} = \Pi_{x,t}(k) = \frac{(\mu_x - 1)w_t}{z^{q_{x,t}(k)}} x_t(k) = \left(\frac{\mu_x - 1}{\mu_x}\right)\frac{C_t}{\mu_y}, \tag{13}$$

where the second equality and the third one are obtained by using (10) and (12), respectively. Given C_t, the impact of the upstream patent breadth μ_x on the intermediate-good producer's profit $\Pi_{x,t}$ is different from the impact of the downstream patent breadth μ_y; increasing μ_x raises $\Pi_{x,t}$ due to a larger market power, whereas increasing μ_y lowers $\Pi_{x,t}$ due to a smaller demand for intermediate goods in (10) with the exercise of the final-good producer's market power.[8] Moreover, the manufacturing-labor income in the industry for intermediate good k is

$$w_t L_{x,t}(k) = \frac{1}{\mu_x} p_{x,t}(k) x_t(k) = \frac{C_t}{\mu_x \mu_y}, \tag{14}$$

implying that the labor demand for intermediate good k is given by $L_{x,t}(k) = C_t/(\mu_x \mu_y w_t)$.

[7] Although the step size z of quality improvement is the same between the final-good and intermediate-good sectors, the process for the arrival of innovations as stated Section 2.4 implies different numbers of jumps in the quality ladder in these two sectors as time accumulates. Hence, as will be shown in Section 3.1, the levels of state-of-the-art technology in these sectors are also different.

[8] The presence of $(1 - 1/\mu_x)/\mu_y$ in (13) captures the *double-marginalization problem* as in the traditional industrial organization literature. See, for example, Chapter 17 in Belleflamme and Peitz (2015) for more details.

2.4 Innovations and R&D

The creation of innovations for the final-good and intermediate-good sectors is as follows. The expected value of owning the most recent innovation in industry j (k) in the final- (intermediate-) good sector is denoted as $v_{y,t}(j)$ ($v_{x,t}(k)$). Following the standard literature, we focus on a symmetric equilibrium (see, for example, Cozzi *et al.* (2007)). This implies that $\Pi_{y,t}(j) = \Pi_{y,t}$ ($\Pi_{x,t}(k) = \Pi_{x,t}$), and that $v_{y,t}(j) = v_{y,t}$ ($v_{x,t}(k) = v_{x,t}$). Denote by $\lambda_{y,t}$ ($\lambda_{x,t}$) the *aggregate-level* Poisson arrival rate of innovations for final (intermediate) goods. Then, the Hamilton-Jacobi-Bellman (HJB) equation for $v_{y,t}$ ($v_{x,t}$) is given by

$$r_t v_{s,t} = \Pi_{s,t} + \dot{v}_{s,t} - \lambda_{s,t} v_{s,t}, \tag{15}$$

which is the no-arbitrage condition for the asset value in sector $s = \{y, x\}$, respectively. In equilibrium, the return on the asset $r_t v_{s,t}$ equals the sum of the flow profits $\Pi_{s,t}$, the capital gain $\dot{v}_{s,t}$, and the potential losses $\lambda_{s,t} v_{s,t}$ when creative destruction takes place.

New innovations in each industry in the final- (intermediate-) good sector are generated by a unit continuum of R&D firms indexed by $\theta \in [0,1]$ ($\vartheta \in [0,1]$), respectively, and each of the R&D firms in sector y (sector x) employs R&D labor $L^y_{r,t}(\theta)$ ($L^x_{r,t}(\vartheta)$) for producing inventions. Following the existing literature such as Chu and Cozzi (2014) and Huang *et al.* (2017), we incorporate a CIA constraint on R&D investment as follows. In order to finance the wage payment, $w_t L^y_{r,t}(\theta)$, for the downstream-R&D labor, the θ-th entrepreneur has to borrow $B^y_t(\theta) = b^y_t(\theta) N_t$ at time t from households. This loan matures instantaneously and creates an extra burden of an interest payment at the nominal interest rate $i_t \geq 0$. Similarly, the ϑ-th entrepreneur has to borrow $B^x_t(\vartheta) = b^x_t(\vartheta) N_t$ from households to finance wage payment, $w_t L^x_{r,t}(\vartheta)$, for the upstream-R&D labor. Thus, the expected profit of the χ-th R&D firm, where $\chi = \{\theta, \vartheta\}$, is

$$\Pi^s_{r,t}(\chi) = v_{s,t}\lambda_{s,t}(\chi) - (1 + \xi_s i_t) w_t L^s_{r,t}(\chi); \quad s = \{y \text{ for } \theta, x \text{ for } \vartheta\}, \tag{16}$$

where $\xi_s = \{\xi_y, \xi_x\} \in [0,1]$ is the strength of the CIA constraint on downstream R&D and upstream R&D, respectively. Moreover, the *firm-level* arrival rate of innovations $\lambda_{s,t}(\chi)$ is formulated by

$$\lambda_{s,t}(\chi) = \bar{\varphi}_{s,t} L^s_{r,t}(\chi) = \frac{\varphi_s}{N_t} L^s_{r,t}(\chi); \quad s = \{y \text{ for } \theta, x \text{ for } \vartheta\}, \tag{17}$$

where the specification $\bar{\varphi}_{s,t} = \varphi_s / N_t$ captures the dilution effect that removes scale effects as in Chu and Cozzi (2014) and $\varphi_s = \{\varphi_y, \varphi_x\}$ is the productivity parameter for downstream R&D and upstream R&D, respectively. In equilibrium, the aggregate-level arrival rate of innovations is thus given by $\lambda_{s,t} = \int_0^1 \lambda_{s,t}(\chi) d\chi = \varphi_s L^s_{r,t} / N_t$, where $L^s_{r,t} = \int_0^1 L^s_{r,t}(\chi) d\chi$ is the aggregate labor devoted to the $s = \{y, x\}$ R&D sector. Then, free entry into the R&D sectors implies the following zero-expected-profit condition:

$$v_{s,t}\lambda_{s,t} = (1 + \xi_s i_t) w_t L^s_{r,t}; \quad s = \{y, x\}. \tag{18}$$

This equation is a condition pinning down the allocation of labor in the R&D sectors.

2.5 Monetary Authority

Denote the nominal money supply by M_t and its growth rate by $\Phi_t \equiv \dot{M}_t / M_t$, respectively. Accordingly, the real money balance is given by $m_t N_t = M_t / p_t$, where p_t is the price of consumption. Consider that the growth rate of money supply Φ_t serves as a policy instrument that can be controlled by monetary authority. In this case, the rate of inflation is endogenously determined by $\pi_t = \Phi_t - \dot{m}_t / m_t - n$. Additionally, combining this condition with the Fisher equation (i.e., $i_t = \pi_t + r_t$) yields the one-to-one relation between the nominal interest rate and the nominal money supply in the balanced growth path equilibrium:[9]

$$i_t = \Phi_t + \rho. \tag{19}$$

Given this result, throughout the rest of this study, we will use i_t to represent the instrument of monetary policy for simplicity. Finally, the monetary authority redistributes to the households the increase in money supply (i.e., the seigniorage revenue) in terms of a lump-sum transfer, namely $\tau_t N_t = \dot{M}_t / p_t = \Phi_t m_t N_t = [(\pi_t + n) m_t + \dot{m}_t] N_t$.

3 Decentralized Equilibrium

An equilibrium consists of a sequence of allocations $[c_t, m_t, Y_t(j), x_t(k), L_{x,t}(k), L^y_{r,t}(\theta), L^x_{r,t}(\vartheta)]_{t=0}^{\infty}$ and a sequence of prices $[r_t, p_{y,t}(j), p_{x,t}(k), w_t, v_{y,t}, v_{x,t}]_{t=0}^{\infty}$, where $\{j, k, \theta, \vartheta\} \in [0,1]$. At each instance of time,

• households choose $[c_t]$ to maximize their utility taking $[r_t, i_t, w_t]$ as given;

• monopolistic leaders for final goods produce $[Y_t(j)]$ and choose $[p_{y,t}(j)]$ to maximize profits taking $[p_{x,t}(k)]$ as given;

• monopolistic leaders for intermediate goods produce $[x_t(k)]$ and choose $[p_{x,t}(k), L_{x,t}(k)]$ to maximize profits taking $[w_t]$ as given;

• competitive downstream-R&D firms choose $[L^y_{r,t}(\theta)]$ to maximize profits taking $[w_t, v_{y,t}]$ as given;

• competitive upstream-R&D firms choose $[L^x_{r,t}(\vartheta)]$ to maximize profits taking $[w_t, v_{x,t}]$ as given;

• the consumption-good market clears: $c_t N_t = C_t$;

• the labor market clears: $L_{x,t} + L^y_{r,t} + L^x_{r,t} = N_t$;

• the innovation value adds up to households' asset value: $v_{x,t} + v_{y,t} = a_t N_t$;

• the R&D entrepreneurs finance their wage payments through borrowing: $\xi_y w_t L^y_{r,t} + \xi_x w_t L^x_{r,t} = b_t N_t$; and

• the monetary authority balances its budget: $\tau_t N_t = (i_t - \rho) m_t N_t$.

3.1 Balanced Growth Path

This section characterizes the decentralized equilibrium and shows that the economy grows along a balanced growth path (BGP) that is saddle-point stable. To facilitate this result, we first

[9]On the balanced growth path, which will be shown in Section 3.1, c_t and m_t grow at the same rate of $r_t - \rho - n$ according to the Euler equation.

derive the growth rate of aggregate technology g_t. Substituting (6) and (11) into (4) yields the consumption production function $C_t = Z_{y,t} Z_{x,t} L_{x,t}$, where $Z_{y,t}$ and $Z_{x,t}$ are defined as the level of technology in the downstream sector and in the upstream sector, with $Z_{y,t} \equiv \exp\left(\ln z \int_0^1 q_t(j)dj\right) = \exp\left(\ln z \int_0^t \lambda_{y,\iota} d\iota\right)$ and $Z_{x,t} \equiv \exp\left(\ln z \int_0^1 q_t(k)dk\right) = \exp\left(\ln z \int_0^t \lambda_{x,\kappa} d\kappa\right)$, respectively. The second equalities in these equations are obtained by the law of large numbers. Differentiating these two equations with respect to time yields the growth rate of aggregate technology given by

$$g_t = \frac{\dot{Z}_{y,t}}{Z_{y,t}} + \frac{\dot{Z}_{x,t}}{Z_{x,t}} = (\varphi_y l_{r,t}^y + \varphi_x l_{r,t}^x) \ln z, \tag{20}$$

where the second equality is obtained by using (17) and $l_{r,t}^y \equiv L_{r,t}^y / N_t$ and $l_{r,t}^x \equiv L_{r,t}^x / N_t$ are defined as downstream-R&D labor share and upstream-R&D labor share, respectively. Similarly, $l_{x,t} \equiv L_{x,t}/N_t$ is defined as manufacturing labor share. Therefore, the growth rate of per capita consumption $\dot{c}_t / c_t$ (i.e., the economic growth rate) is also given by g_t in (20).

For an arbitrary path of the nominal interest rate $[i_t]_{t=0}^{\infty}$, we obtain the following result:

Proposition 1. *Holding constant i, the economy jumps to a unique and saddle-point stable balanced growth path.*

Proof. See Appendix A.1. □

3.2 Equilibrium Allocations and the Growth Effect

As implied by Proposition 1, given a constant i, the equilibrium labor allocations $\{l_{x,t}, l_{r,t}^y, l_{r,t}^x\}$ are stationary along the BGP. Using the zero-expected-profit condition for upstream R&D (18) and the manufacturing-labor income in the intermediate-good sector (14) yields $v_{y,t}\lambda_{y,t} = (1 + \xi_y i) C_t l_{r,t}^y / (\mu_x \mu_y l_x)$, implying $\dot{v}_{y,t}/v_{y,t} = \dot{c}_t / c_t + n$. Combining this result with (3) and (15) and imposing the BGP implies $\Pi_y / v_y = \rho + \lambda_y$. Then, substituting this equation into (18) and applying (9) and (17) derives the relation between l_r^y and l_x:

$$l_r^y = \frac{\mu_x(\mu_y - 1) l_x}{1 + \xi_y i} - \frac{\rho}{\varphi_y}, \tag{21}$$

which is the first equation to solve for $\{l_x, l_r^y, l_r^x\}$. Following a similar logic, we can use (4), (13), (14), (15), (17), and (18) to derive the second equation, which is the relation between l_r^x and l_x given by

$$l_r^x = \frac{(\mu_x - 1) l_x}{1 + \xi_x i} - \frac{\rho}{\varphi_x}. \tag{22}$$

Combined with the labor-market-clearing condition $l_x + l_r^x + l_r^y = 1$, equations (21)-(22) yield the equilibrium labor allocations as follows:[10]

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{1 + \frac{\mu_x - 1}{1 + \xi_x i} + \frac{\mu_x(\mu_y - 1)}{1 + \xi_y i}}, \tag{23}$$

[10] We restrict the parameter space to ensure that labor allocation to each sector is in the [0, 1] interval.

$$l_r^y = \frac{\frac{\mu_x(\mu_y-1)}{1+\xi_y i}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)}{1+\frac{\mu_x-1}{1+\xi_x i}+\frac{\mu_x(\mu_y-1)}{1+\xi_y i}} - \frac{\rho}{\varphi_y}, \tag{24}$$

$$l_r^x = \frac{\frac{\mu_x-1}{1+\xi_x i}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)}{1+\frac{\mu_x-1}{1+\xi_x i}+\frac{\mu_x(\mu_y-1)}{1+\xi_y i}} - \frac{\rho}{\varphi_x}. \tag{25}$$

In these equilibrium labor allocations, (23) shows that manufacturing labor l_x is increasing in the nominal interest rate i, because a higher i raises the cost of borrowing for R&D investment, which reallocates the labor from R&D to manufacturing. Nevertheless, (24) reveals that there are two effects of i on the downstream-R&D labor l_r^y. On the one hand, i has a negative effect on l_r^y due to the reallocation of labor to production as aforementioned (i.e., the *manufacturing-R&D-reallocation effect*). On the other hand, i has a negative (positive) effect on l_r^y if ξ_y is greater (smaller) than ξ_x, namely downstream R&D is more (less) bound by the CIA constraint than upstream R&D. This creates another reallocative effect of labor between the two R&D sectors: the *cross-R&D-sector effect*. Whether a higher i increases or decreases l_r^y depends on the relative magnitudes of ξ_y and ξ_x and the level of markup μ_x.

Specifically, when $\xi_y > \xi_x$, the cross-R&D-sector effect is negative and thus reinforces the manufacturing-R&D-reallocation effect, causing l_r^y to be decreasing in i. When $\xi_y < \xi_x$, the cross-R&D-sector effect on l_r^y becomes positive. Nevertheless, it is easy to show that if ξ_x is smaller (larger) than $[\mu_x/(\mu_x-1)]\xi_y$, the overall effect of an increase in i on l_r^y is negative (positive) at $i=0$.[11] In other words, as long as the upstream-to-downstream CIA constraint ratio is lower than the gross-to-net markup ratio of the upstream sector, the positive cross-R&D-sector effect is dominated by the negative manufacturing-R&D-reallocation effect. Interestingly, when $\xi_x = [\mu_x/(\mu_x-1)]\xi_y$, the two effects completely offset each other at $i=0$, leaving l_r^y locally unaffected by changes in i.

Similarly, (25) shows that i also has these two effects on the upstream-R&D labor l_r^x, which reinforce each other when $\xi_x > \xi_y$ and counteract each other when $\xi_x < \xi_y$. The analysis of the overall impact is analogous to that for the impact of i on l_r^y.[12]

We summarize the above results in the following proposition:

Proposition 2. *In the zero-nominal-interest-rate equilibrium, the interaction of the manufacturing-R&D-reallocation effect and the cross-R&D-sector effect leads to the following labor share responses to a nominal interest rate increase:*

(i) If $\xi_x = [\mu_x/(\mu_x-1)]\xi_y$, the two effects on l_r^y offset each other, leaving l_r^y locally unaffected by an interest rate increase; otherwise $(\partial l_r^y/\partial i)|_{i\to 0^+} \lesseqgtr 0$ if $\xi_x \lesseqgtr [\mu_x/(\mu_x-1)]\xi_y$.

(ii) If $\xi_y = \{1+1/[\mu_x(\mu_y-1)]\}\xi_x$, the two effects on l_r^x offset each other, leaving l_r^x locally unaffected by an interest rate increase; otherwise $(\partial l_r^x/\partial i)|_{i\to 0^+} \lesseqgtr 0$ if $\xi_y \lesseqgtr \{1+1/[\mu_x(\mu_y-1)]\}\xi_x$.

Proof. Proven in the text. □

The above result and (20) imply a mixed effect of the nominal interest rate i on the equilibrium growth rate of technology $g = \ln z(\varphi_y l_r^y + \varphi_x l_r^x)$, and this effect is determined by the impacts of i

[11] More generally, we have $\text{Sign}(\partial l_r^y/\partial i) = \text{Sign}[(\mu_x-1)(\xi_x-\xi_y) - \xi_y(1+\xi_x i)^2]$.

[12] It is easy to show that $\text{Sign}(\partial l_r^x/\partial i) = \text{Sign}[\mu_x(\mu_y-1)(\xi_y-\xi_x) - \xi_x(1+\xi_y i)^2]$.

on the levels of R&D labor l_r^y and l_r^x in addition to the productivity parameters φ_y and φ_x. Using (23)-(25) and differentiating (20) with respect to i yields

$$\begin{aligned}\frac{\partial g}{\partial i} &= \left(\varphi_y \frac{\partial l_r^y}{\partial i} + \varphi_x \frac{\partial l_r^x}{\partial i}\right) \ln z \\ &= \frac{\ln z}{\Lambda^2}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)\left\{\begin{array}{l} \varphi_y \mu_x(\mu_y - 1)[(\mu_x - 1)(1+\xi_y i)\xi_x - (\mu_x + \xi_x i)(1+\xi_x i)\xi_y] \\ +\varphi_x(\mu_x - 1)[\mu_x(\mu_y - 1)(1+\xi_x i)\xi_y - (\mu_x(\mu_y-1) + (1+\xi_y i))(1+\xi_y i)\xi_x] \end{array}\right\},\end{aligned} \tag{26}$$

where $\Lambda \equiv (1+\xi_y i)(1+\xi_x i) + (\mu_x - 1)(1+\xi_y i) + \mu_x(\mu_y - 1)(1+\xi_x i)$. It can be checked that the term in the curly brackets is decreasing in i and that Λ is increasing in i, so $\partial g/\partial i$ is decreasing in i, namely g is a concave function of i. Notice that if the following condition holds,

$$\frac{\varphi_x}{\varphi_y}[\xi_x + \mu_x(\xi_y - \xi_x)] < \left(\frac{\mu_x - 1}{\mu_y - 1}\right)\left[(\mu_y - 1)(\xi_y - \xi_x) - \frac{\xi_x}{\mu_x}\right], \tag{27}$$

the term in the curly brackets in (26) is positive at $i = 0$, implying $(\partial g/\partial i)|_{i\to 0^+} > 0$. Therefore, condition (27) ensures that the relation between g and i is first positive and then negative. Further inspection of this condition reveals that by defining the term

$$\Omega \equiv \frac{\mu_x - 1}{\mu_x}\left[\frac{\frac{\xi_y - \xi_x}{\xi_x} - \frac{1}{\mu_x(\mu_y - 1)}}{\frac{\xi_y - \xi_x}{\xi_x} + \frac{1}{\mu_x}}\right], \tag{28}$$

we have the following proposition:[13]

Proposition 3. *There is an inverted-U relation between the equilibrium growth rate g and the nominal interest rate i if*
(i) $\xi_x < \xi_y$ and $\varphi_y/\varphi_x < \Omega$, or
(ii) $\xi_x > [\mu_x/(\mu_x - 1)]\xi_y$ and $\varphi_y/\varphi_x > \Omega$;
otherwise, the relation is monotonically negative.

Proof. Proven in the text. □

This proposition shows that for the nominal interest rate to have an inverted-U effect on economic growth, the more productive sector should both have a large lead in productivity and face a much weaker CIA constraint. For example, the condition $\varphi_y/\varphi_x < \Omega$ holds when φ_y/φ_x is very small and $(\xi_y - \xi_x)/\xi_x$ is very large, i.e., when upstream R&D is far more productive and less constrained than downstream R&D. These conditions pave ground for a strong cross-R&D-sector labor reallocation effect to dominate the negative effect of the manufacturing-R&D-reallocation on growth. Thus, an increase in the nominal interest rate from zero is growth-enhancing. Notwithstanding, as i increases, the dominance of the cross-R&D-sector effect diminishes, and further increases in the interest rate become growth-decreasing.

[13]Note that when $\xi_y < \xi_x < [\mu_x/(\mu_x - 1)]\xi_y$, Ω is negative. As a result, it is impossible to have a non-monotonic relation between the interest rate and growth.

3.3 Socially Optimal Allocations

Imposing balanced growth on (1) yields

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right) \tag{29}$$

where $c_0 = Z_{x,0}Z_{y,0}l_x$ and $g = \ln z(\varphi_y l_r^y + \varphi_x l_r^x)$. Dropping the exogenous terms $Z_{x,0}$ and $Z_{y,0}$ and maximizing (29) subject to the resource constraint for labor $l_x + l_r^y + l_r^x = 1$ yields the first-best allocations denoted by a superscript asterisk:

$$l_x^* = \min\left\{\frac{\rho}{\varphi_y \ln z}, \frac{\rho}{\varphi_x \ln z}\right\}, \tag{30}$$

$$l_r^{y*} = \begin{cases} 1 - \frac{\rho}{\varphi_y \ln z} & \text{if} \quad \varphi_x < \varphi_y, \\ 0 & \text{if} \quad \varphi_x > \varphi_y, \end{cases} \tag{31}$$

$$l_r^{x*} = \begin{cases} 0 & \text{if} \quad \varphi_x < \varphi_y, \\ 1 - \frac{\rho}{\varphi_x \ln z} & \text{if} \quad \varphi_x > \varphi_y. \end{cases} \tag{32}$$

The socially optimal outcome implies that technology advances in the downstream and upstream sectors are perfectly substitutable. Therefore, a corner solution arises for the first-best labor allocations in the sense that the social optimum only allocates labor to the R&D sector with a higher level of productivity.[14] Specifically, suppose $\varphi_y < \varphi_x$, that is, innovative activities in the upstream sector are more productive, so that devoting all R&D labor to this sector is socially optimal. As a result, the labor in the downstream R&D sector is zero, implying that economic growth in the social optimum only depends on upstream innovations. By contrast, for $\varphi_y > \varphi_x$, the situation reverses, and R&D labor is only allocated to the downstream sector.

Define the R&D overinvestment, η, in the zero-nominal-interest-rate equilibrium as the gap between the R&D labor share in the decentralized equilibrium (given by (24) and (25)) and the optimal R&D labor share: $\eta \equiv (l_r^x|_{i=0} + l_r^y|_{i=0}) - (l_r^{x*} + l_r^{y*}) = l_x^* - l_x|_{i=0}$. Under the assumption $\varphi_y < \varphi_x$, a condition we empirically verify in Section 5.1, equations (23) and (30) then imply

$$\eta = \frac{\rho}{\varphi_x \ln z} - \frac{1}{\mu_x \mu_y}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right). \tag{33}$$

We immediately have the following proposition:

Proposition 4. *If upstream R&D is more productive than downstream R&D, i.e., $\varphi_x > \varphi_y$, the zero-nominal-interest-rate equilibrium features overinvestment in R&D, i.e., $\eta > 0$, if and only if*

$$1 - \frac{\varphi_x \ln z}{\mu_x \mu_y \rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right) > 0 \tag{34}$$

[14] If the productivity is the same in the two R&D sectors ($\varphi_x = \varphi_y = \varphi$), the first-best manufacturing-labor share is $l_x^* = \rho/(\varphi \ln z)$, and any combination of $\{l_r^{y*}, l_r^{x*}\}$ satisfying $l_r^{y*} + l_r^{x*} = 1 - \rho/(\varphi \ln z)$ is socially optimal.

Proof. Proven in the text. □

Proposition 4 is intuitive, and it implies that R&D underinvestment arises in equilibrium (i.e., $\eta < 0$) if and only if $1 - [(\varphi_x \ln z)/(\mu_x \mu_y \rho)](1 + \rho/\varphi_x + \rho/\varphi_y) < 0$. For overinvestment to occur in equilibrium, the markups must be relatively high so that firms have strong incentives to invest in R&D. A larger innovation step size lnz and a higher φ_x increase the socially optimal investment, thus making overinvestment less likely. A higher φ_y increases the equilibrium R&D labor in the downstream sector but has no effect on the socially optimal R&D labor share as long as $\varphi_y < \varphi_x$. Therefore, it makes overinvestment more likely. A higher ρ reduces the utility of future consumption, thus also reducing the optimal investment and making overinvestment more likely.

4 Optimal Monetary Policy and Friedman Rule

In this section, we analyze the optimal monetary policy and examine the conditions under which Friedman rule is (sub)optimal. We first consider the general case, and then consider two special cases in which the CIA constraint is only present in one sector. We denote by i^* the optimal nominal interest rate that maximizes social welfare. Given that the case $\varphi_y < \varphi_x$ is empirically supported by our analysis in Section 5.1, throughout the remaining study, we focus on the case of a high relative R&D productivity in the upstream sector.[15]

4.1 The General Case

For the general case of CIA constraints with $\xi_y \geq 0$, $\xi_x \geq 0$, and $\xi_x + \xi_y > 0$, the equilibrium labor allocations are simply given by (23)-(25). Substituting these equations into (29) and evaluating $\partial U/\partial i$ as $i \to 0^+$ yields

$$\text{sign}\left(\frac{\partial U}{\partial i}\Big|_{i\to 0^+}\right) = \text{sign}\left\{\begin{array}{l} \xi_x(\mu_x - 1) + \xi_y \mu_x(\mu_y - 1) \\ -\dfrac{\ln z}{\mu_x \mu_y \rho}\left(1 + \dfrac{\rho}{\varphi_x} + \dfrac{\rho}{\varphi_y}\right)\left[\begin{array}{l} \varphi_x \xi_x(\mu_x - 1) + \varphi_y \xi_y \mu_x(\mu_y - 1) \\ +(\varphi_y - \varphi_x)(\xi_y - \xi_x)\mu_x(\mu_x - 1)(\mu_y - 1) \end{array}\right] \end{array}\right\}. \tag{35}$$

Given $\varphi_y < \varphi_x$, we analyze the relation between suboptimal R&D investment and the (sub)optimality of Friedman rule. We show that this relation depends on the relative magnitude of CIA constraints ξ_x and ξ_y.

First, suppose that the CIA constraint in the upstream sector is not stronger than in the downstream sector, i.e., $\xi_y \geq \xi_x$. Denote the right-hand side of (35) by K. In this case, if R&D overinvestment in equilibrium occurs (i.e., inequality (34) holds), we have

$$K > \frac{1}{\mu_x \mu_y}\left\{[\varphi_x \xi_x(\mu_x - 1) + \varphi_y \xi_y \mu_x(\mu_y - 1)]\left[1 - \frac{\varphi_x \ln z}{\mu_x \mu_y \rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)\right]\right\} > 0. \tag{36}$$

[15] The analytical results of a low relative productivity of upstream R&D (i.e., $\varphi_y > \varphi_x$) are available upon request. Furthermore, we show in Appendix C that the results of our model would be the same as those in the one-R&D-sector model of Chu and Cozzi (2014) if the R&D productivity is the same in both sectors, i.e., $\varphi_x = \varphi_y$.

This shows that R&D overinvestment is sufficient for Friedman rule to be suboptimal. Intuitively, if the low productivity downstream R&D is more constrained, when i is raised from the zero level, the cross-R&D-sector effect stimulates technology growth through shifting R&D labor from the less productive sector to the more productive sector (i.e., shifting l_r^y to l_r^x). In addition, the manufacturing-R&D-reallocation effect depresses the excess technology growth (i.e., reducing $l_r^x + l_r^y$ toward $l_r^{x*} + l_r^{y*}$) and increases current consumption toward its optimal level (i.e., moving l_x closer to l_x^*). Consequently, these two effects unambiguously improve the overall welfare, leading to a positive optimal nominal interest rate i^*.

One implication of this result is that R&D underinvestment is necessary but not sufficient for the optimality of Friedman rule in this case. In particular, Appendix A.2 shows that Friedman rule can be suboptimal even if R&D is underinvested. This is not surprising because the cross-R&D-sector labor reallocation is socially beneficial. Therefore, even though a reduction in the total R&D labor is undesirable in the presence of underinvestment, the overall effect of a positive interest rate may still be positive. Our model thus leads to a very novel insight: In the presence of heterogeneous productivity across sectors, a reduction in the overall investment in R&D can be welfare-improving even if the aggregate amount of R&D investment is below the optimal level.

Second, suppose that the CIA constraint in the more productive upstream sector is stronger than in the less productive downstream sector, i.e., $\xi_y < \xi_x$. In this case, when i is raised from the zero level, the cross-R&D-sector effect yields a negative impact on welfare by stifling technology growth, given that R&D labor is shifted from the more productive sector to the less productive sector (i.e., from l_r^x to l_r^y). One may conjecture Friedman rule to be suboptimal only if R&D is overinvested at $i = 0$. Interestingly, Appendix A.2 shows that this is not the case as long as upstream R&D is not severely more CIA-constrained than downstream R&D.

Specifically, as long as $\xi_x < [\mu_x/(\mu_x - 1)]\xi_y$, R&D overinvestment is sufficient but not necessary for Friedman rule to be suboptimal. Note that the condition $\xi_x < [\mu_x/(\mu_x - 1)]\xi_y$ always holds if $\xi_x < \xi_y$. Therefore, this result nests the result discussed above for the case with a weaker CIA constraint in the upstream than in the downstream. In fact, the gross-to-net markup ratio $\mu_x/(\mu_x - 1)$ is generally far above 1, allowing the result to hold even when upstream R&D is significantly more CIA-constrained than downstream R&D. Therefore, this is a general result applicable to most empirically relevant situations. The intuition for this result is as follows. As shown in Proposition 2, as long as $\xi_x < [\mu_x/(\mu_x - 1)]\xi_y$, an increase in the nominal interest rate from zero reduces R&D labor in the low productivity downstream sector, l_r^y. Since the socially optimal l_r^{y*} is zero, a reduction in l_r^y is welfare-improving. Appendix A.2 shows that in this case, if the degree of R&D underinvestment is relatively low (which is supported by relatively high markup values of μ_x and/or μ_y), the welfare benefit of decreasing the socially wasteful R&D in the low productivity sector can dominate the welfare cost of decreasing the socially desirable R&D investment in the high productivity sector. This may be true even when l_r^x is more sensitive to the interest rate increase than l_r^y, as in the case with $\xi_y < \xi_x < [\mu_x/(\mu_x - 1)]\xi_y$.

When the upstream CIA constraint is much stronger than the downstream one, i.e., $\xi_x > [\mu_x/(\mu_x - 1)]\xi_y$, R&D overinvestment becomes necessary but not sufficient for Friedman rule to be suboptimal. In other words, R&D underinvestment is sufficient but not necessary for Friedman rule to be optimal. In this case, the cross-R&D-sector effect becomes so favorable to the less productive downstream R&D sector that the downstream R&D labor share responds positively to an increase in the intererst rate from zero. As a result, a positive interest rate only decreases the labor share of the more productive upstream sector. Not surprisingly, it can be welfare-improving only if R&D is overinvested at $i = 0$. Furthermore, because a positive interest rate triggers an undesirable reallocation of labor from the high productivity sector to the low productivity one, overinvestment is not sufficient to ensure its optimality.

Finally, in the knife-edge case of $\xi_x = [\mu_x/(\mu_x - 1)]\xi_y$, the analysis of equation (24) in Section 3.2 shows that in the equilibrium with $i = 0$, l_r^y becomes locally unaffected by changes in i. Therefore, the nominal interest rate only affects the allocation between the production sector and upstream R&D. In this case, our model effectively reduces to the model of Chu and Cozzi (2014) in terms of the labor allocation effect of interest rate changes. As a result, R&D overinvestment becomes both sufficient and necessary for Friedman rule to be suboptimal.

Accordingly, we summarize the above results in Proposition 5.

Proposition 5. *Suppose that upstream R&D is more productive than downstream R&D, i.e., $\varphi_y < \varphi_x$. Then for the general case of CIA constraints with $\xi_x \geq 0$, $\xi_y \geq 0$, and $\xi_x + \xi_y > 0$, we have*

(i) if upstream R&D is not much more constrained than downstream R&D, i.e., $\xi_x < [\mu_x/(\mu_x - 1)]\xi_y$, R&D overinvestment is sufficient but not necessary for Friedman rule to be suboptimal. In other words, Friedman rule can be suboptimal even with R&D underinvestment;

(ii) if upstream R&D is much more constrained than downstream R&D, i.e., $\xi_x > [\mu_x/(\mu_x - 1)]\xi_y$, R&D overinvestment is necessary but not sufficient for Friedman rule to be suboptimal;

(iii) in the special case with $\xi_x = [\mu_x/(\mu_x - 1)]\xi_y$, R&D overinvestment is necessary and sufficient for Friedman rule to be suboptimal.

Proof. See Appendix A.2. □

Due to the complexity of the model, it is difficult to obtain the closed-form solution for the optimal nominal interest rate i^* when both R&D sectors are CIA constrained except for the special case with $\xi_y = \xi_x$. Therefore, we examine numerically the level of i^* in Section 5.[16]

4.2 Special Cases: CIA Constraint in One Sector

We next consider two special cases. The first is the case where only downstream R&D is CIA-constrained (i.e., $\xi_y > 0$ and $\xi_x = 0$). The equilibrium labor allocations in this case are obtained by imposing $\xi_x = 0$ in (23) to (25). It is easy to see that manufacturing-labor share l_x is increasing in the nominal interest rate i, whereas downstream- (upstream-) R&D labor is decreasing (increasing) in i. Given that upstream R&D is not cash-constrained (i.e., $\xi_y > \xi_x = 0$), the effect of i operates only through the constraint on the downstream R&D sector. A higher i increases the cost of downstream R&D, leading to a labor reallocation from downstream R&D to both upstream R&D and manufacturing.

[16] The analytical solution for i^* when $\xi_x = \xi_y = \xi > 0$ can be derived by differentiating U with respect to i. Solving the first-order condition yields:

$$i^* = \max\left\{\frac{1}{\xi}\left[\frac{\mu_x\mu_y - 1}{\frac{\ln z}{\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)\left(\varphi_y\mu_x(\mu_y - 1) + \varphi_x(\mu_x - 1)\right)\frac{1}{\mu_x\mu_y - 1} - 1} - 1\right], 0\right\}.$$

By differentiating U with respect to i and solving the first-order condition, we can obtain the analytical expression for the optimal interest rate i^* in this case:

$$i^* = \max\left\{\frac{1}{\xi_y}\left[\frac{\mu_y - 1}{\frac{\ln z}{\rho}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)\left(\varphi_y - \varphi_x\left(\frac{\mu_x-1}{\mu_x}\right)\right)-1} - 1\right], 0\right\}, \tag{37}$$

and the value of i^* is chosen based on the sign of $(\partial U/\partial i)|_{i=0}$.

Given that this case satisfies the condition $\xi_x = 0 < [\mu_x/(\mu_x - 1)]\xi_y$, Proposition 5(i) holds. Therefore, we have the following corollary:

Corollary 1. *Suppose that only downstream R&D is CIA-constrained. Then the optimal nominal interest rate i^* is given by (37). Furthermore, when $\varphi_y < \varphi_x$, R&D overinvestment in the zero-nominal-interest-rate equilibrium is sufficient but not necessary for Friedman rule to be suboptimal.*

The second is the case where only upstream R&D is CIA-constrained (i.e., $\xi_x > 0$ and $\xi_y = 0$). By imposing $\xi_y = 0$ on the equilibrium labor allocations (23) to (25), it is easy to see that a higher nominal interest rate increases the manufacturing labor l_x and R&D labor in the unconstrained downstream sector l_y^r, while decreasing the R&D labor in the constrained upstream sector l_x^r. Moreover, by differentiating U with respect to i and solving the first-order condition, we can obtain the analytical expression for the optimal interest rate in this case:

$$i^* = \max\left\{\frac{1}{\xi_x}\left[\frac{\mu_x - 1}{\frac{\ln z}{\rho}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)(\varphi_x - \mu_x(\mu_y-1)(\varphi_y-\varphi_x)) - \mu_x(\mu_y-1)-1} - 1\right], 0\right\}. \tag{38}$$

Because the condition $\xi_x > [\mu_x/(\mu_x - 1)]\xi_y = 0$ is satisfied in this case, Proposition 5(ii) holds. Therefore, we have the following corollary: 2.

Corollary 2. *Suppose that only upstream R&D is CIA-constrained. Then the optimal nominal interest rate i^* is given by (38). Furthermore, when $\varphi_y < \varphi_x$, R&D overinvestment in the zero-nominal-interest-rate equilibrium is necessary but not sufficient for Friedman rule to be suboptimal.*

5 Quantitative Analysis

We now provide a quantitative analysis of the growth and welfare effects of monetary policy. In Section 5.1, we show the empirical patterns of upstream and downstream firms. In Section 5.2, we calibrate our model to the US economy. Sections 5.3 to 5.5 present the quantitative results.

5.1 Upstream vs. Downstream: Empirical Patterns

To guide our model calibration, we examine the R&D characteristics, profit margins, returns on assets, and financial constraints in the upstream and downstream sectors using firm-level data from US manufacturing industries from 1985 to 2018. We focus on manufacturing firms because they account for the majority of US R&D spending and corporate patents (see, e.g., Autor *et al.* (2020)). We use the 2002 US Bureau of Economic Analysis Benchmark Input-Output

(I-O) Tables to classify firms into two sectors. These tables track the flows of intermediate goods and services across industries, as well as the sales of each industry to final users. Following Antràs *et al.* (2012) and Gofman *et al.* (2020), we compute an upstreamness index for each I-O industry.[17] We merge the industry-level upstreamness measure with the firm-level financial data from the Compustat North America database using the mapping between the I-O industry codes and the North American Industry Classification System (NAICS) codes. A firm's upstreamness is assigned based on its historical NAICS code. We then merge our sample with the patent database compiled by Kogan *et al.* (2017). We keep only firms in manufacturing industries (defined by two-digit NAICS codes 31, 32, and 33). Furthermore, we exclude R&D inactive firms with zero R&D capital (defined in Appendix B). We deflate all dollar values to 2002 dollars using the GDP deflator and exclude firms with sales or assets less than $1 million. Our final sample consists of 3,738 firms with a total of 40,411 annual observations.

We construct several variables relevant to our model: (i) A dummy variable, Upstream, which equals one if a firm's upstreamness index is above the annual cross-sectional median and zero otherwise. (ii) R&D ratio, which is the ratio of annual R&D expenditures to the book value of total assets. Following the convention in the literature, missing R&D expenditures are assumed to be zero. (iii) Profit margin, equal to the ratio of operating income before depreciation to total sales. (iv) Return on enterprise value (ROEV), computed as the ratio of operating income before depreciation to the enterprise value (book value of debt plus market value of equity minus cash holdings). (v) A widely used index of financial constraints, the Whited-Wu (WW) Index, calculated using formula (13) in Whited and Wu (2006).[18] (vi) Two R&D productivity measures, Prod(I) and Prod(II), which are estimated using two alternative patent-based measures of R&D output relative to R&D input.[19]

We run univariate regressions to uncover the differences between the upstream and downstream firms, controlling for year fixed effects, and report the results in Table 1. To account for the fact that bigger firms play a more important role in the economy than small ones, each observation is weighted by the lagged firm assets. Column (1) shows that the (asset-weighted) average R&D ratios in the two sectors are statistically indistinguishable, although the point estimate for the upstream sector is 21% higher relative to the downstream sector. This is consistent with our assumption that both upstream and downstream firms engage in R&D. Columns (2) and (3) show that profit margins and returns on enterprise value (ROEV) are also very similar across the two sectors, at around 18% and 9%, respectively. However, column 4 (5) shows that the R&D productivity is 59% or 73% higher in the upstream sector than in the downstream sector, depending on whether productivity is measured by Prod(I) or Prod(II).

To examine the potentially different degrees of financial constraints for upstream and downstream R&D, we regress the WW index on the R&D ratio for the downstream and upstream sectors separately. The results presented in the last two columns of Table 1 show that a high R&D

[17] See Appendix B for the details about the construction of this index. Intuitively, an industry's upstreamness index is the weighted average of its customer industries' upstreamness indexes plus 1. Industries producing only goods and services for personal consumption form the bottom layer of production and have the lowest upstreamness index of 1.

[18] This measure can be interpreted as the shadow cost of external financing. Technically, Formula (13) in Whited and Wu (2006) gives a number equal to the shadow cost of external financing plus an unknown constant. We adjust each firm's index by subtracting the cross-sectional minimum in each year to pin down this constant, essentially assuming that the shadow cost of external financing is zero for the least constrained firms.

[19] The difference between Prod(I) and Prod(II) lies in the adjustment for patent citations (see Appendix B).

Table 1: Upstream vs. downstream manufacturing firms: empirical patterns

The table shows the results from weighted regressions using annual observations of US manufacturing firms from 1985 to 2018. Columns (6) and (7) report the results for the downstream and upstream samples, respectively. Each observation is weighted by lagged total assets. All models include year dummies and the intercept term is computed as the mean of $\bar{y} - \hat{b}\bar{x}$. Standard errors are clustered by industry, and t-statistics are reported in parentheses.

	(1) R&D Ratio	(2) Profit Margin	(3) ROEV	(4) Prod(I)	(5) Prod(II)	(6) WW Index (Downstream)	(7) WW Index (Upstream)
Upstream	0.007	-0.011	-0.001	0.079***	0.100***		
	(0.54)	(-0.45)	(-0.14)	(3.17)	(3.34)		
R&D Ratio						0.736***	0.254***
						(3.45)	(3.57)
Constant	0.034***	0.186***	0.091***	0.134***	0.137***	0.064***	0.146***
	(3.80)	(8.92)	(16.40)	(7.61)	(5.74)	(4.59)	(19.50)
Observations	37811	37741	37678	23218	23218	18728	18902

intensity is associated with tightened financial constraints in both sectors, consistent with the idea that R&D activities tend to face significant financial constraints, an underlying assumption of our model. More interestingly, this correlation is significantly stronger for the downstream sector than for the upstream sector. Specifically, a one percentage point increase in the R&D ratio corresponds to an increase in the WW index by 73.6 (25.4) basis points for downstream (upstream) firms.[20] This suggests that downstream R&D is substantially more financially constrained than upstream R&D, potentially because upstream R&D investment generates patents more efficiently, which can serve as collateral for external financing.

5.2 Calibration

There are nine structural parameters in our model: $\{\rho, n, \mu_x, \mu_y, z, \xi_x, \xi_y, \varphi_x, \varphi_y\}$. We estimate three of them, n, μ_x, and μ_y, directly from the data. According to the data retrieved from FRED, Federal Reserve Bank of St. Louis, the annual population growth rate in the US from 2000 to 2020 is 0.8%. Therefore, we set at $n = 0.8\%$. Table 1 shows that the profit margins in the upstream and downstream sectors are 0.175 and 0.186, respectively. Accordingly, we set the values of the two markup parameters at $\mu_x = 1.175$ and $\mu_y = 1.186$.

We then calibrate the remaining six parameters $\{\rho, z, \xi_x, \xi_y, \varphi_x, \varphi_y\}$ by matching six model-implied moments in the steady state to empirical data. The empirical moments are estimated as follows. First, the growth rate of real GDP per capita in the US is 1.0% during the period 2000-2020 (according to data retrieved from FRED, Federal Reserve Bank of St. Louis). Second, Lanjouw (1998) finds that the obsolescence rate of a patented innovation is in the range of $7-12\%$. We use this as a proxy for the aggregate innovation arrival rate. The remaining four moments are inferred from our estimation results in Table 1. (i) Column (4) in Table 1 shows that the relative R&D productivity ratio of the upstream sector is $\varphi_x/\varphi_y = 1.59$ (0.213 vs. 0.134). (ii)

[20]A regression using the full sample and the upstream dummy shows that this difference is statistically significant at the 5% level (with a t-statistic of -2.16).

Table 2: Benchmark parameter values

Panel A. Parameter Values			
Directly estimated parameter values		Calibrated parameter values	
Upstream price markup (μ_x)	1.175	Subjective discount rate (ρ)	0.04
Downstream price markup (μ_y)	1.186	Step size of innovation (z)	1.105
Population growth rate (n)	0.8%	Upstream CIA constraint (ξ_x)	0.271
		Downstream CIA constraint (ξ_y)	0.775
		Upstream R&D productivity (φ_x)	0.645
		Downstream R&D productivity (φ_y)	0.451

Panel B. Empirical vs. Model Moments					
Moments	Data	Model	Moments	Data	Model
Per capita economic growth rate (g)	1.0%	0.9%	Innovation arrival rate ($\lambda_x + \lambda_y$)	7-12%	9.0%
Relative CIA constraint ($\frac{\xi_x}{\xi_y}$)	0.35	0.35	Relative R&D ratio ($\frac{\varphi_x l_r^x}{\varphi_y l_r^y}$)	1.21	1.39
Relative R&D productivity ($\frac{\varphi_x}{\varphi_y}$)	1.59	1.43	Average ROEV ($\frac{1}{2}(\frac{\Pi_x}{v_x} + \frac{\Pi_y}{v_y})$)	9.1%	8.5%

The last two columns of Table 1 show that the relative CIA constraint for upstream R&D (ξ_x/ξ_y) is 0.35 (25.4 vs. 73.6). (iii) Column (1) in Table 1 shows that the relative R&D ratio of the upstream sector is 1.21 (4.1% vs. 3.4%). This corresponds to the ratio $\varphi_x l_r^x/(\varphi_y l_r^y)$ in our model, because equation (18) shows that the R&D ratio for sector $s \in \{x, y\}$ is $w_t L_{r,t}^s(\chi)(1 + \xi_s i)/v_{s,t} = \varphi_s l_r^s$. (iv) Column (3) in Table 1 shows that the average ROEV of the upstream and downstream sectors, which corresponds to $(\Pi_x/v_x + \Pi_y/v_y)/2$ in our model based on (3) and (15), is 9.1%.

Table 2 summarizes the benchmark parameter values, as well as the theoretical moments and their empirical counterparts. Note that to determine the steady state equilibrium, we set $\pi = 2.1\%$, matching the US inflation rate from 2000 to 2020. All the moments are matched remarkably well.[21] The equilibrium economic growth rate is $g = 0.9\%$, which is close to the per capita GDP growth in the US from 2000 to 2020 (1%) and the long-run TFP growth rate from 1954 to 2019 (0.7%, according to data from Feenstra *et al.* (2015)). The implied nominal interest rate is $i = \rho + g + n + \pi = 7.8\%$, which is close to the 10-year US Treasury rate from 1970 to 2020 (6.21%) after taking into account the convenience yields of Treasury bonds.[22] Our parameter choices are also consistent with the values used or estimated in the literature. Specifically, a subjective discount rate of 4% is used in García-Peñalosa and Turnovsky (2006), among others. Furthermore, Akcigit and Kerr (2018) estimate a net step size of 11.2% for major advances in technology, which is in line with our gross step size of $z = 1.105$.

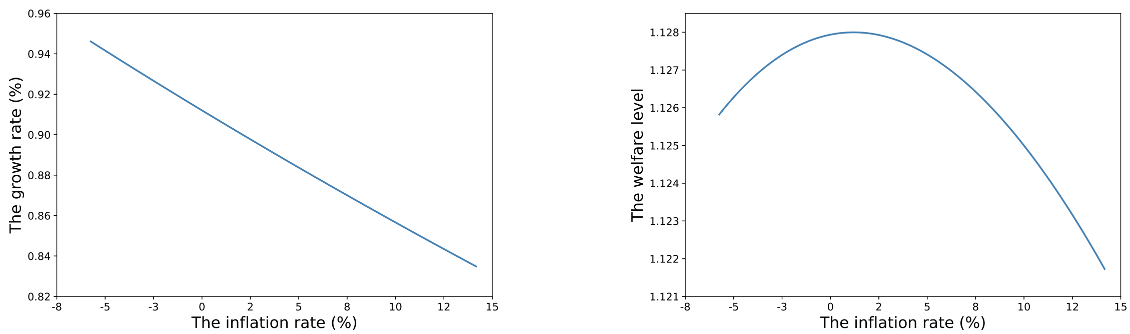
5.3 Benchmark Simulation

Based on the calibrated parameters, we proceed to evaluate the growth and welfare effects of monetary policy. Figure 1(a) indicates that in this benchmark case, the steady-state growth rate of aggregate technology (and also the rate of economic growth) g is monotonically decreasing in

[21] The small discrepancies arise because parameter values must fall in a certain range to be economically meaningful. For example, none of the parameters can be negative, and the CIA-constraint parameters must be in the [0, 1] interval.

[22] The average Moody's Seasoned AAA corporate bond yield over this period is 7.36%.

the inflation rate.[23] In particular, increasing the inflation rate from the benchmark value 2.1% to 15% causes the equilibrium growth rate to decline from 0.9% to 0.83%.[24] Intuitively, (24) and (25) show that a higher nominal interest rate i raises the costs of both downstream R&D and upstream R&D due to CIA constraints in these sectors. As discussed in Proposition 3, since the condition $\varphi_y \mu_x(\mu_y - 1)[(\mu_x - 1)\xi_x - \mu_x \xi_y] + \varphi_x(\mu_x - 1)[\mu_x(\mu_y - 1)\xi_y - (\mu_x \mu_y - \mu_x + 1)\xi_x] = -0.1033 < 0$ holds, the manufacturing-R&D-reallocation effect tends to dominate the cross-R&D-sector effect, thereby leading to a monotonic decrease in the rate of economic growth.



(a) Inflation and economic growth

(b) Inflation and social welfare

Figure 1: The benchmark case

Interestingly, the steady-state welfare has an inverted-U relation with the inflation rate, as shown in Figure 1(b). The manufacturing-R&D reallocation caused by a higher i leads to a rise in l_x but a decline in l_r^x and l_r^y, which, according to (29), increases the level of current consumption c_0 (i.e., a positive welfare effect) but decreases the growth rate g (i.e., a negative welfare effect). Another positive welfare effect arises from the cross-R&D-sector R&D labor reallocation from the less productive downstream sector to the more productive upstream sector. Together, these positive welfare effects dominate the negative effect when the inflation rate is low, but they become dominated as the inflation rate goes beyond the optimal level, which in the benchmark case is 1.2%, corresponding to an optimal nominal interest rate of 6.9%. Therefore, Friedman rule is suboptimal in our benchmark parameterization.

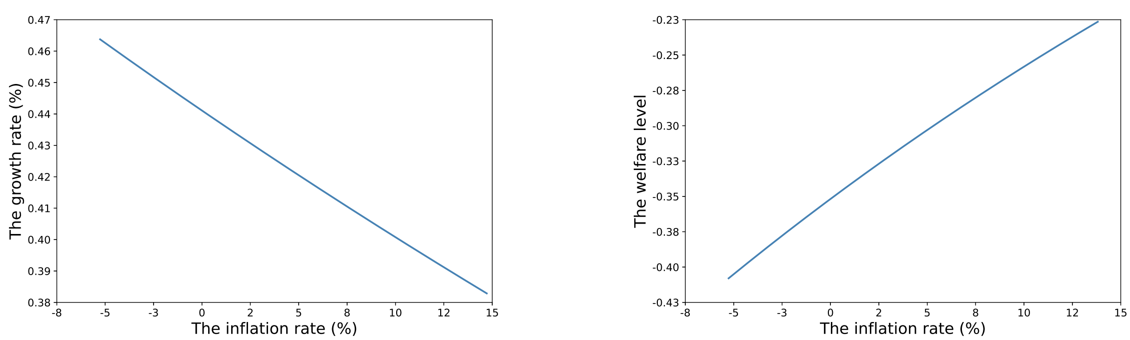
Given the higher relative productivity of upstream R&D (i.e., $\varphi_y < \varphi_x$), (30)-(32) indicate that all R&D labor should be allocated to the upstream sector to achieve the socially optimal outcome (i.e., $l_r^{y*} = 0$). By comparing the socially optimal R&D labor $l_r^{x*} + l_r^{y*} = 0.3799$ and the total R&D labor $(l_r^x + l_r^y)|_{i=0} = 0.1743$ in the zero-nominal-interest-rate equilibrium, we find that relative to the first-best allocation, R&D is underinvested in equilibrium: $(l_r^x + l_r^y)|_{i=0} < l_r^{y*} + l_r^{x*}$. Proposition 5(i) states that that R&D overinvestment is sufficient but not necessary for Friedman rule to be suboptimal. Our benchmark case, which features both underinvestment and a positive

[23] Vaona (2012) and Barro (2013) document a monotonically negative effect of inflation on economic growth.

[24] Throughout the quantitative analysis, we focus on an empirically realistic case of the inflation rate where $\pi \leq 15\%$. According to FRED, the maximum of annual inflation rate for the US from 1960 to 2020 is 13.5% . Thus, we consider 0.15 as the upper bound of the inflation rate. Correspondingly, the upper bound of the nominal interest rate is 20.63% in the benchmark case based on the condition $i - g(i) - \rho - n = 15\%$.

optimal nominal interest rate, supports this result.[25] Although a positive nominal interest rate exacerbates the underinvestment problem in aggregate R&D in this case, it improves welfare by reducing the inefficient R&D investment in the low productivity sector.

To allow for overinvestment to arise in equilibrium, we consider a lower R&D productivity level in both sectors by setting $\varphi_x = 0.44$ and $\varphi_y = \varphi_x/1.43$ while keeping other parameters at the benchmark level. This reduces the optimal R&D labor. The equilibrium then features R&D overinvestment at the zero nominal interest rate. Specifically, we have $(l_r^x + l_r^y)|_{i=0} = 0.1239 > l_r^{y*} + l_r^{x*} = 0.0909$. In this case, Proposition 5(i) predicts that given $\varphi_x > \varphi_y$ and $\xi_x < \xi_y$, R&D overinvestment is sufficient for Friedman rule to be suboptimal. This is confirmed in Figure 2, in which the inflation rate and the welfare level exhibit a positive relation within the range of i, implying a positive optimal rate of nominal interest i^*. In fact, because both the reduction in total R&D and the labor reallocation from downstream R&D to upstream R&D are welfare-improving in this case, the welfare cost of Friedman rule is substantially larger than in the benchmark case: the welfare difference between the welfare-maximizing equilibrium at the optimal interest rate ($i = 20.18\%$ with $\pi = 15\%$, the upper bound of the inflation rate) and the equilibrium at $i = 0$ amounts to 0.767% of annual consumption in the steady state.



(a) Inflation and economic growth

(b) Inflation and social welfare

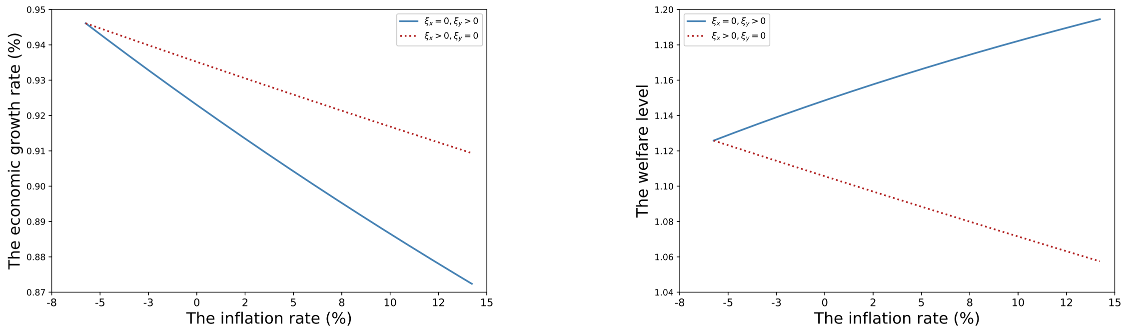
Figure 2: The overinvestment case ($\varphi_x = 0.44$, $\varphi_y = \varphi_x/1.43$)

5.4 Cases with One CIA-Constrained R&D Sector

We now consider the cases in which CIA constraint is present only in one sector, corresponding to our theoretical analysis in Section 4.2. First, Figure 3(a) shows that the rate of economic growth g still decreases in the inflation rate in the cases where the CIA constraint is only present in the downstream sector (i.e., the blue solid line for $\xi_x = 0$) or the upstream sector (i.e., the red dotted line for $\xi_y = 0$). Given the calibrated parameters, according to Proposition 3, the growth-enhancing effect of a higher inflation rate, which stems from the cross-R&D-sector effect,

[25] Define by $\exp(\rho\Delta U) - 1$ the change in steady-state welfare by the usual equivalent variation in consumption flow. Within the range of i, we find that the welfare gain is approximately 0.03% of consumption per annum by moving the equilibrium inflation rate from 15% (the upper bound of inflation) to 1.2% (the welfare-maximizing level). This welfare gain is much smaller than the results in Chu and Cozzi (2014) without vertically integrated R&D activities.

is strictly dominated by the growth-decreasing effect, which stems from the manufacturing-R&D-reallocation effect. Thus, the economic growth rate in these cases continues to be decreasing in the inflation rate as in our benchmark case.



(a) Inflation and economic growth

(b) Inflation and social welfare

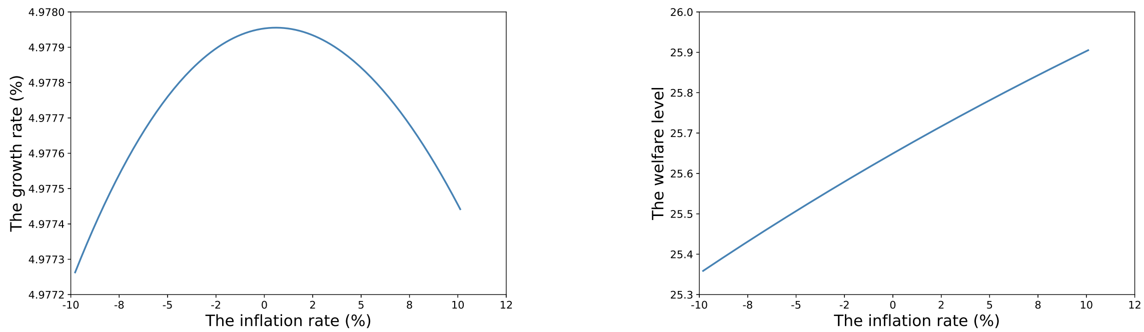
Figure 3: The cases with one constrained sector: $\{\xi_x = 0, \xi_y = 0.775\}$ and $\{\xi_x = 0.271, \xi_y = 0\}$

Figure 3(b) shows that in the case with a CIA constraint only in the downstream sector (i.e., the blue solid line for $\xi_x = 0$), the welfare increases monotonically within the range of inflation rate. It indicates that Friedman rule continues to be suboptimal as in the benchmark case. However, the welfare result is different if the CIA constraint is only present in the upstream sector (i.e., the red dotted line for $\xi_y = 0$): in this case, the welfare decreases monotonically within the range of inflation rate, indicating that Friedman rule is optimal.

To examine the relation between R&D underinvestment (or overinvestment) and the optimality of Friedman rule, we compare the first-best labor allocations and the steady-state counterparts in the zero-nominal-interest-rate equilibrium. In the case of $\xi_x = 0$, we have $(l_r^x + l_r^y)|_{i=0} = 0.1668 < l_r^{y*} + l_r^{x*} = 0.3799$, implying that R&D is underinvested in equilibrium. The suboptimality of Friedman rule in this case is consistent with Corollary 1, in that overinvestment is not necessary for Friedman rule to be suboptimal when only downstream R&D is CIA-constrained. Moreover, in the case of $\xi_y = 0$, we find that R&D is still underivested: $(l_r^x + l_r^y)|_{i=0} = 0.1721 < l_r^{x*} = 0.3799$. The optimality of Friedman rule in this case is in line with Corollary 2, in that R&D underinvestment in equilibrium is sufficient for Friedman rule to be optimal when only upstream R&D is CIA-constrained.

5.5 Inverted-U Relation Between Inflation and Growth

The last exercise is performed to examine the possibility of an inverted-U relation between the inflation rate and the economic growth rate. Such a connection has been observed in recent empirical studies such as Bick (2010) and Kremer *et al.* (2013). Intuitively, this can happen in our model if the gaps in both CIA constraints and productivity between two sectors are sufficiently large. Therefore, we consider cases with a higher φ_x and a lower ξ_x than in the benchmark case. These changes make the upstream sector even less constrained and more productive. As a result, the cross-R&D-sector effect is more likely to outweigh the manufacturing-R&D-reallocation effect



(a) Inflation and economic growth

(b) Inflation and social welfare

Figure 4: The non-monotonic inflation-growth relation case ($\xi_x = 0.02, \varphi_x = 3.64$)

at low levels of nominal interest rates. We find that when raising the upstream R&D productivity to $\varphi_x = 3.64$ and reducing the upstream CIA constraint parameter to $\xi_x = 0.02$, while fixing other parameters, the rate of economic growth g becomes an inverted-U function of inflation, as shown in Figure 4(a). In addition, the growth-maximizing inflation rate is found to be around 2.59%, which is in line with the empirical estimates of Ghosh and Phillips (1998) (i.e., 2.5%) and López-Villavicencio and Mignon (2011) (i.e., 2.7%). Admittedly, the parameter combination in this case is quite extreme, but it provides a concrete example in support of Proposition 3.[26] Not surprisingly, Figure 4(b) shows that the level of welfare is increasing in inflation, indicating that Friedman rule is suboptimal in this case.[27]

6 Conclusion

This study analyzes the growth and welfare effects of monetary policy in a Schumpeterian growth model in which both vertical sectors engage in R&D activities under a CIA constraint. We find that a higher nominal interest rate reallocates resources from the more cash-constrained R&D sector to the less constrained one. In addition to the usual growth-decreasing effect of a high interest rate due to increased R&D costs, the cross-sector reallocation of R&D labor can generate a growth-enhancing effect, which could lead to an inverted-U relation between the nominal interest rate and economic growth.

Moreover, we examine the necessary and sufficient conditions for the (sub)optimality of Friedman rule in relation to the underinvestment and overinvestment of R&D in the decentralized equilibrium. We find that this relation crucially depends on the relative strength of CIA constraints and the relative productivity between the R&D sectors. These factors determine the interaction among the welfare effects brought about by the reallocation of different types of labor, and thereby determine the optimal design of monetary policy. We show analytically that

[26] Note that under this new set of parameter values, the expression described in Proposition 3 is positive.

[27] This exercise features R&D underinvestment at $i = 0$ (with $(l_r^x + l_r^y)|_{i=0} = 0.2036 < l_r^{x*} = 0.8895$), supporting Proposition 5(i) in that overinvestment is not necessary for the suboptimality of Friedman rule.

R&D overinvestment is sufficient but not necessary for the zero nominal rate policy to be suboptimal as long as the more productive R&D sector is not severely more CIA-constrained than the less productive R&D sector.

Finally, we calibrate our model using the US data. We show empirically that upstream R&D investment are associated with higher patent-based measures of productivity and face lower degrees of financial constraints than downstream R&D does. In our benchmark parameterization, the growth-maximizing nominal interest rate is zero, but the welfare-maximizing nominal interest rate is positive despite aggregate R&D underinvestment at the zero lower bound.

Our study shows both analytically and quantitatively the importance of considering multi-sector R&D investment in the analysis of monetary-policy effects on growth and welfare. Our results highlight the complexity of trade-offs in the monetary-policy choice in an environment with strong sectoral heterogeneity. While we focus on the heterogeneity in CIA constraints and R&D productivity, many other sectoral heterogeneities affect the real effects of monetary policy, for example, the cross-sectoral differences in price stickiness (see, for example, Nakamura and Steinsson (2008)). Incorporating such heterogeneities into the endogenous growth theoretic framework is a fruitful venue for future research on monetary policy.

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Appendix A. Proofs of Propositions

In this appendix, we show the proof of Proposition 1 for the stability of the BGP equilibrium in the model and the proof of Proposition 5 for the relation between R&D overinvestment (or underinvestment) and the (sub)optimality of Friedman rule in the case with CIA constraints on both R&D sectors.

A.1 Proof of Proposition 1

First, we define a transformed variable $\Psi_{y,t} \equiv C_t / v_{y,t}$, and its law of motion is given by

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = n + \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_{y,t}}{v_{y,t}}. \tag{A.1}$$

Similarly, we define another transformed variable $\Psi_{x,t} \equiv C_t / v_{x,t}$, and its law of motion is given by

$$\frac{\dot{\Psi}_{x,t}}{\Psi_{x,t}} = n + \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_{x,t}}{v_{x,t}}. \tag{A.2}$$

From equations (9), (15), and (17), we can derive the following law of motion for $v_{y,t}$:

$$\frac{\dot{v}_{y,t}}{v_{y,t}} = r_t + \varphi_y l_{r,t}^y - \left(\frac{\mu_y - 1}{\mu_y}\right) \Psi_{y,t}. \tag{A.3}$$

Likewise, from equations (13), (15), and (17), we can derive the law of motion for $v_{x,t}$:

$$\frac{\dot{v}_{x,t}}{v_{x,t}} = r_t + \varphi_x l_{r,t}^x - \left(\frac{\mu_x - 1}{\mu_x \mu_y}\right) \Psi_{x,t}. \tag{A.4}$$

Plugging (A.3) and (A.4) into (A.1) and (A.2), respectively, along with the Euler equation (3), yields

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = \left(\frac{\mu_y - 1}{\mu_y}\right) \Psi_{y,t} - \varphi_y l_{r,t}^y - \rho, \tag{A.5}$$

and

$$\frac{\dot{\Psi}_{x,t}}{\Psi_{x,t}} = \left(\frac{\mu_x - 1}{\mu_x \mu_y}\right) \Psi_{x,t} - \varphi_x l_{r,t}^x - \rho. \tag{A.6}$$

Moreover, because $i_t = i$ for all t, using the manufacturing-labor share of consumption in (14) and the zero-expected-profit condition of downstream R&D (18) yields an expression for $l_{x,t}$:

$$l_{x,t} = \left(\frac{1+\xi_y i}{\mu_x \mu_y \varphi_y}\right) \Psi_{y,t}, \tag{A.7}$$

and using the zero-expected-profit condition of upstream R&D (18) to relate $l_{x,t}$ to $\Psi_{x,t}$ yields

$$l_{x,t} = \left(\frac{1+\xi_x i}{\mu_x \mu_y \varphi_x}\right) \Psi_{x,t}. \tag{A.8}$$

Therefore, $\Psi_{x,t}$ can be expressed as a function of $\Psi_{y,t}$:

$$\Psi_{x,t} = \left[\frac{(1+\xi_y i)\varphi_x}{(1+\xi_x i)\varphi_y}\right] \Psi_{y,t}. \tag{A.9}$$

Then, it is obvious that

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = \frac{\dot{\Psi}_{x,t}}{\Psi_{x,t}}. \tag{A.10}$$

Using (A.10) together with (A.5), (A.6) and (A.9), we derive a relation between $l^y_{r,t}$ and $l^x_{r,t}$:

$$l^x_{r,t} = \left(\frac{\varphi_y}{\varphi_x}\right) l^y_{r,t} + \left[\frac{(\mu_x - 1)(1+\xi_y i)\varphi_x}{\mu_x \mu_y (1+\xi_x i)\varphi_y} - \frac{\mu_y - 1}{\mu_y}\right] \frac{\Psi_{y,t}}{\varphi_x}. \tag{A.11}$$

Finally, to derive the relation between $\Psi_{y,t}$ and $l^y_{r,t}$, using the labor-market-clearing condition $l_{x,t} + l^y_{r,t} + l^x_{r,t} = 1$ and substituting (A.7) and (A.11) into it yields

$$\left(1 + \frac{\varphi_y}{\varphi_x}\right) l^y_{r,t} = 1 - \left[\frac{1+\xi_y i}{\mu_x \mu_y \varphi_y} + \frac{(\mu_x - 1)(1+\xi_y i)}{\mu_x \mu_y \varphi_y (1+\xi_x i)} - \frac{\mu_y - 1}{\mu_y \varphi_x}\right] \Psi_{y,t}. \tag{A.12}$$

Substituting (A.12) into (A.5) yields an autonomous dynamical equation for $\Psi_{y,t}$:

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = \frac{\varphi_x}{\varphi_y + \varphi_x} \left[\frac{1+\xi_y i}{\mu_x \mu_y} \left(1 + \frac{\mu_x - 1}{1+\xi_x i}\right) + \frac{\mu_y - 1}{\mu_y}\right] \Psi_{y,t} - \left(\frac{\varphi_x \varphi_y}{\varphi_y + \varphi_x} + \rho\right). \tag{A.13}$$

Given that $\Psi_{y,t}$ is a control variable and the coefficient on $\Psi_{y,t}$ is positive in (A.13), the dynamics of $\Psi_{y,t}$ is characterized by saddle-point stability in this model such that $\Psi_{y,t}$ jumps immediately to its interior steady-state value given by

$$\Psi_y = \frac{\mu_x \mu_y [\rho(1 + \varphi_y/\varphi_x) + \varphi_y]}{(1+\xi_y i)\,[1 + (\mu_x - 1)/(1+\xi_x i)] + \mu_x(\mu_y - 1)}. \tag{A.14}$$

Equations (A.7) and (A.12) imply that when $\Psi_{y,t}$ is stationary, $l_{x,t}$ and $l^y_{r,t}$ must be stationary, which in turn implies that $l^x_{r,t}$ is stationary as well according to (A.11).

A.2 Proof of Proposition 5

Suppose that CIA constraints are imposed on both R&D sectors and that upstream R&D is more productive than downstream R&D (i.e., $\varphi_y < \varphi_x$). Let K denote the expression in the right-hand side of equation (35):

$$K \equiv \xi_x(\mu_x - 1) + \xi_y\mu_x(\mu_y - 1) - \frac{\ln z}{\mu_x\mu_y\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)\begin{bmatrix} \varphi_x\xi_x(\mu_x - 1) + \varphi_y\xi_y\mu_x(\mu_y - 1) \\ +(\varphi_y - \varphi_x)(\xi_y - \xi_x)\mu_x(\mu_x - 1)(\mu_y - 1) \end{bmatrix}.$$

Therefore, we know that $\text{sign}(\partial U/\partial i|_{i=0}) = \text{sign}(K)$. We rewrite the expression of K as follows:

$$\begin{aligned} K =& [\xi_x(\mu_x - 1) + \xi_y\mu_x(\mu_y - 1)]\left[1 - \frac{\varphi_x \ln z}{\mu_x\mu_y\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)\right] \\ &+ \frac{\ln z}{\mu_y\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)(\varphi_x - \varphi_y)(\mu_y - 1)[(\xi_y - \xi_x)\mu_x + \xi_x] \\ =& [\xi_x(\mu_x - 1) + \xi_y\mu_x(\mu_y - 1)]\left(\frac{\varphi_x \ln z}{\rho}\right)\eta + M, \end{aligned} \tag{A.15}$$

where the definition of R&D overinvestment η in (33) is used and we denote $M \equiv [\ln z/(\mu_y\rho)](1 + \rho/\varphi_x + \rho/\varphi_y)(\varphi_x - \varphi_y)(\mu_y - 1)[(\xi_y - \xi_x)\mu_x + \xi_x]$. To facilitate this proof, we also denote $\tilde{\eta} \equiv (-M)/\{(\varphi_x \ln z/\rho)[\xi_x(\mu_x - 1) + \xi_y\mu_x(\mu_y - 1)]\}$.

Next, we show how the conditions under which the sign of K relates to R&D overinvestment (and underinvestment) would depend on the relative magnitude of ξ_x and ξ_y. Accordingly, two cases arise as follows.

Case A.2.1. When $\xi_y \geq \xi_x$, we have $M > 0$. It follows that $K > M > 0$ as long as $\eta > 0$. Consequently, under $\varphi_x > \varphi_y$ and $\xi_y \geq \xi_x$, R&D overinvestment is sufficient for Friedman rule to be suboptimal. In other words, for Friedman rule to be optimal (i.e., $K < 0$), R&D underinvestment (i.e., $\eta < 0$) in the zero-interest rate equilibrium is a necessary condition.

Moreover, Friedman rule can be suboptimal with R&D underinvestment. This is achieved by $\eta > \tilde{\eta}$ where $\tilde{\eta} < 0$, which also supports $K > 0$. In particular, a low degree of R&D underinvestment (i.e., η is not deeply below zero) under large markup values of μ_x and μ_y is more likely to satisfy the inequality $\eta > \tilde{\eta}$.[28]

Case A.2.2. When $\xi_y < \xi_x$, the sign of M becomes ambiguous. Then there are three subcases to be considered, depending on the value of μ_x.

(a) If $\xi_x < [\mu_x/(\mu_x - 1)]\xi_y$, we have $M > 0$ and $\tilde{\eta} < 0$. Therefore, R&D overinvestment in equilibrium (i.e., $\eta > 0$) is sufficient but not necessary for Friedman rule to be suboptimal. This is because Friedman rule can also be suboptimal (i.e., $K > 0$) in the presence of R&D underinvestment (i.e., $\eta < 0$) as long as the condition $\eta > \tilde{\eta}$ holds. It can be shown that a low degree of R&D underinvestment (i.e., η is not deeply below zero) under large markup values of μ_x and μ_y is more likely to satisfy the inequality $\eta > \tilde{\eta}$.

(b) If $\xi_x > [\mu_x/(\mu_x - 1)]\xi_y$, we have $M < 0$ and $\tilde{\eta} > 0$. Therefore, R&D overinvestment in equilibrium (i.e., $\eta > 0$) is necessary but not sufficient for Friedman rule to be suboptimal.

[28] It can be shown that there exist threshold values $\tilde{\mu}_x$ and $\tilde{\mu}_y$ of markups above which the sign of K becomes positive. Derivations are available upon request.

In other words, R&D underinvestment (i.e., $\eta < 0$) is sufficient but not necessary for Friedman rule to be optimal in this case. This is because the suboptimality of Friedman rule requires the condition $\eta > \tilde{\eta} > 0$ to hold, which implies that the degree of R&D overinvestment has to be sufficiently high. Moreover, a high degree of R&D overinvestment (i.e., η is well above zero) under large markup values of μ_x and μ_y is more likely to satisfy the inequality $\eta > \tilde{\eta}$.[29]

(c) If $\xi_x = [\mu_x/(\mu_x - 1)]\xi_y$, we have $M = \tilde{\eta} = 0$. Therefore, R&D overinvestment in equilibrium (i.e., $\eta > 0$) is necessary and sufficient for Friedman rule to be suboptimal. In other words, R&D underinvestment in equilibrium (i.e., $\eta < 0$) is necessary and sufficient for Friedman rule to be optimal.

Appendix B. Measures of Upstreamness and R&D Productivity

Following Antràs *et al.* (2012) and Gofman *et al.* (2020), we compute an industry-level upstreamness index using the following matrix equation:

$$U = A * U + l, \tag{B.1}$$

where U is a vector representing the unstreamness indexes of 417 private-sector I-O industries, A is a 417×417 adjacency matrix constructed using the Make and Use tables, in which element $A_{i,j}$ representing the share of industry i's output sold to industry j, and l is a vector of ones. Intuitively, an industry's upstreamness index is the weighted average of its customer industries' upstreamness indexes plus 1. Industries producing only goods and services for personal consumption form the bottom layer of production and have the lowest upstreamness index of 1.

Following Hirshleifer *et al.* (2013), we measure a firm's R&D productivity (or efficiency) by its R&D output relative to R&D input. Our first measure of R&D output comes from Kogan *et al.* (2017) and is calculated as

$$RDO(I)_{f,t} = \Sigma_{j \in P_{f,t}}(1 + \frac{C_j}{\overline{C}_j}), \tag{B.2}$$

where $P_{f,t}$ is all patents granted to firm f in year t, C_j is the total number of forward citations received by patent j up to year 2020, $\overline{C}_j$ is the average number of forward citations received by the patents that were granted in the same year as patent j. This scaling is used to adjust for citation truncation at the sample end, which affects patents granted in different years differently. As an alternative, we measure $\overline{C}_j$ by the the average number of citations received by the patents belonging to the same technology class and granted in the same year as patent j, following Hirshleifer *et al.* (2013). This helps to control for citation propensity attributed to differences in technology fields and leads to our second R&D output measure, $RDO(II)_{f,t}$. If a firm is not granted any patent in a given year, then both output measures are zero.

We measure a firm's R&D input by cumulative R&D expenses over five years, assuming an annual depreciation rate of 20%, following again Hirshleifer *et al.* (2013). Specifically, the R&D

[29] Similarly, there exist threshold values $\tilde{\mu}_x$ and $\tilde{\mu}_y$ of markups above which the sign of K becomes positive. Derivations are available upon request.

capital of firm f in year t is computed as

$$RDC_{f,t} = R\&D_{f,t} + 0.8 * R\&D_{f,t-1} + 0.6 * R\&D_{f,t-2} + 0.4 * R\&D_{f,t-3} + 0.2 * R\&D_{f,t-4}. \quad \text{(B.3)}$$

To account for the average two-year patent application-grant lag, the output variables in year t are divided by by R&D capital in year $t-2$ to yield the R&D output-input ratios. Because both ratios are highly skewed, we use the natural logarithms of 1 plus these ratios as our productivity measures:

$$Prod(I)_{f,t} = log(1 + \frac{RDO(I)_{f,t}}{RDC_{f,t-2}}), \quad \text{(B.4)}$$

$$Prod(II)_{f,t} = log(1 + \frac{RDO(II)_{f,t}}{RDC_{f,t-2}}). \quad \text{(B.5)}$$

Appendix C. Conditions for the (sub)optimality of Friedman rule in Comparison to Chu and Cozzi (2014)

In this appendix, we consider two additional cases in which the analytical results in the current model collapse to those in Chu and Cozzi (2014), in addition to the $\xi_x = [\mu_x/(\mu_x - 1)]\xi_y$ case considered in Section 4. We focus on the case in Chu and Cozzi (2014) with no CIA constraints on consumption and manufacturing and with inelastic labor supply.

C.1 Homogeneous R&D Productivity

Intuitively, if R&D productivity is the same in both sectors (i.e., $\varphi_x = \varphi_y = \varphi$), then our two-R&D-sector model should behave effectively the same as a one-R&D-sector model. This can be shown explicitly.

Note that because $\mu_x > 1$ and $\mu_y > 1$, equation (35) becomes

$$\text{sign}\left(\frac{\partial U}{\partial i}\Big|_{i\to 0^+}\right) = \text{sign}\left[1 - \frac{\varphi \ln z}{\mu_x \mu_y \rho}\left(1 + \frac{2\rho}{\varphi}\right)\right]. \quad \text{(C.1)}$$

It is straightforward to show that R&D overinvestment in equilibrium (i.e., $l_r^x|_{i=0} + l_r^y|_{i=0} > l_r^{x*} + l_r^{y*}$ or $1 - (\varphi \ln z)(1 + 2\rho/\varphi)/(\mu_x\mu_y\rho) > 0$) is sufficient and necessary for Friedman rule to be suboptimal. In other words, R&D underinvestment in equilibrium (i.e., $l_r^x|_{i=0} + l_r^y|_{i=0} < l_r^{x*} + l_r^{y*}$ or $1 - (\varphi \ln z)(1 + 2\rho/\varphi)/(\mu_x\mu_y\rho) < 0$) is sufficient and necessary for Friedman rule to be optimal.

Therefore, the productivity heterogeneity across sectors is a prerequisite for the cross-R&D-sector effect to have an impact on the welfare. If the sectoral productivities are identical, the cross-R&D-sector effect becomes irrelevant. Then the two R&D sectors work effectively as one, yielding only the manufacturing-R&D-reallocation effect on the welfare. As a result, the relations between suboptimal R&D investment and the suboptimality of Friedman rule in this model with two R&D sectors are the same as in Chu and Cozzi (2014) with a single R&D sector.

C.2 Degenerate Downstream R&D

Our model also yields identical results to those in Chu and Cozzi (2014) if R&D in the downstream (final-good) sector shrinks to zero. We verify this intuition by considering three different scenarios (namely, the CIA constraint is imposed in both sectors, in only upstream R&D, in only downstream R&D, respectively). To facilitate the comparison, we follow Chu and Cozzi (2014) to set the strength of the CIA constraint to unity when the constraint is present in the sector(s), and we assume $\varphi_x > \varphi_y$ as in the main text.

C.2.1 CIA constraints on two sectors

The steady-state equilibrium labor allocations (23)-(25) in the case of CIA constraints on two sectors (i.e., $\xi_x = \xi_y = 1$) are given by

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{1 + \frac{\mu_x \mu_y - 1}{1+i}}, \tag{C.2}$$

$$l_r^y = \frac{\frac{\mu_x(\mu_y - 1)}{1+i}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \frac{\mu_x \mu_y - 1}{1+i}} - \frac{\rho}{\varphi_y}, \tag{C.3}$$

$$l_r^x = \frac{\frac{\mu_x - 1}{1+i}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \frac{\mu_x \mu_y - 1}{1+i}} - \frac{\rho}{\varphi_x}. \tag{C.4}$$

Case C.2.1. Suppose that the productivity in upstream R&D is higher relative to downstream R&D, i.e., $\varphi_x > \varphi_y$. Equation (C.3) and the non-negativity of l_r^y imply that downstream R&D labor l_r^y is zero for all $i \geq 0$ if $\mu_y \leq [\mu_x(1 + \rho/\varphi_x + \rho/\varphi_y)]/[\mu_x + (1 + \rho/\varphi_x)]$ (because l_r^y increases in μ_y and decreases in i). In this case, the two-sector model reduces to a single R&D model as in Chu and Cozzi (2014), and the other labor allocations are given by

$$l_x = \frac{1+i}{\mu_x + i}\left(1 + \frac{\rho}{\varphi_x}\right), \tag{C.5}$$

and

$$l_r^x = \frac{\mu_x - 1}{\mu_x + i}\left(1 + \frac{\rho}{\varphi_x}\right) - \frac{\rho}{\varphi_x}. \tag{C.6}$$

Substituting $l_r^y = 0$ and (C.5)-(C.6) into the BGP lifetime utility function in (29), differentiating ρU with respect to i and evaluating it as $i \to 0^+$ yields

$$\text{sign}\left(\frac{\partial U}{\partial i}\Big|_{i \to 0^+}\right) = \text{sign}\left[1 - \frac{\ln z}{\mu_x}\left(1 + \frac{\rho}{\varphi_x}\right)\right]. \tag{C.7}$$

Therefore, it is straightforward to show that R&D overinvestment in equilibrium (i.e., $l_r^x|_{i=0} + l_r^y|_{i=0} > l_r^{x*}|_{i=0} + l_r^{y*}|_{i=0}$ or $1 - (\ln z/\mu_x)(1 + \rho/\varphi_x) > 0$) is sufficient and necessary for Friedman rule to be suboptimal. In other words, R&D underinvestment in equilibrium (i.e., $l_r^x|_{i=0} + l_r^y|_{i=0} <$

$l_r^{x*}|_{i=0} + l_r^{y*}|_{i=0}$ or $1 - (\ln z/\mu_x)(1+\rho/\varphi_x) < 0$) is sufficient and necessary for Friedman rule to be optimal; these are the same conditions as in Chu and Cozzi (2014).

C.2.2 CIA constraint on downstream R&D

Recall that the steady-state equilibrium labor allocations (23)-(25) in the case of a CIA constraint on downstream R&D (i.e., $\xi_x = 0$ and $\xi_y = 1$) are given by

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{\mu_x + \frac{\mu_x(\mu_y - 1)}{1+i}}, \tag{C.8}$$

$$l_r^y = \frac{\frac{\mu_x(\mu_y-1)}{1+i}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{\mu_x + \frac{\mu_x(\mu_y-1)}{1+i}} - \frac{\rho}{\varphi_y}, \tag{C.9}$$

$$l_r^x = \frac{(\mu_x - 1)\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{\mu_x + \frac{\mu_x(\mu_y-1)}{1+i}} - \frac{\rho}{\varphi_x}. \tag{C.10}$$

Case C.2.2. Suppose that the productivity in upstream R&D is higher relative to downstream R&D, i.e., $\varphi_x > \varphi_y$. Equation (C.9) and the non-negativity of l_r^y imply that downstream R&D labor l_r^y becomes zero for all $i \geq 0$ if $\mu_y \leq 1 + (\rho/\varphi_y)/(1+\rho/\varphi_x)$ (because l_r^y increases in μ_y and decreases in i). In this case, the two-sector model reduces to a single R&D model as in Chu and Cozzi (2014). Nevertheless, this is a trivial case, given that the other labor allocations are independent of i: $l_x = (1+\rho/\varphi_x)/\mu_x$ and $l_r^x = [(\mu_x - 1)/\mu_x](1+\rho/\varphi_x) - \rho/\varphi_x$. This is because when the CIA constraint is present in downstream R&D and downstream R&D is zero, monetary policy is irrelevant to the steady-state equilibrium labor allocations. Therefore, the welfare analysis of the nominal interest rate is not applied.

C.2.3 CIA constraint on upstream R&D

Recall that the steady-state equilibrium labor allocations (23)-(25) in the case of a CIA constraint on upstream R&D (i.e., $\xi_x = 1$ and $\xi_y = 0$) are given by

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{1 + \mu_x(\mu_y - 1) + \frac{\mu_x - 1}{1+i}}, \tag{C.11}$$

$$l_r^y = \frac{\mu_x(\mu_y - 1)\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \mu_x(\mu_y - 1) + \frac{\mu_x - 1}{1+i}} - \frac{\rho}{\varphi_y}, \tag{C.12}$$

$$l_r^x = \frac{\frac{\mu_x-1}{1+i}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \mu_x(\mu_y - 1) + \frac{\mu_x - 1}{1+i}} - \frac{\rho}{\varphi_x}. \tag{C.13}$$

Case C.2.3. Suppose that the productivity in upstream R&D is higher relative to downstream R&D, i.e., $\varphi_x > \varphi_y$. Consider an empirically relevant upper bound of the nominal interest rate, $\bar{i}$.

Equation (C.12) and the non-negativity of l_r^y imply that downstream R&D labor l_r^y becomes zero for all $i \in [0, \bar{i}]$ if $\mu_y \leq 1 + \{(\rho/\varphi_y)[1 + (\mu_x - 1)/(1 + \bar{i})]\}/[\mu_x(1 + \rho/\varphi_x)]$ (because l_r^y increases in both μ_y and i). In this case, the two-sector model reduces to a single R&D model as in Chu and Cozzi (2014), and the other labor allocations are given by equations (C.5) and (C.6). Substituting $l_r^y = 0$ and (C.5)-(C.6) into the BGP lifetime utility function in (29), differentiating ρU with respect to i and evaluating it at $i = 0$ yields equation (C.7). Therefore, it is straightforward to show that R&D overinvestment in equilibrium (i.e., $l_r^x|_{i=0} + l_r^y|_{i=0} > l_r^{x*}|_{i=0} + l_r^{y*}|_{i=0}$ or $1 - (\ln z/\mu_x)(1 + \rho/\varphi_x) > 0$) is sufficient and necessary for Friedman rule to be suboptimal. In other words, R&D underinvestment in equilibrium (i.e., $l_r^x|_{i=0} + l_r^y|_{i=0} < l_r^{x*}|_{i=0} + l_r^{y*}|_{i=0}$ or $1 - (\ln z/\mu_x)(1 + \rho/\varphi_x) < 0$) is sufficient and necessary for Friedman rule to be optimal; these are the same conditions as in Chu and Cozzi (2014).