# Measuring Inflation: Criticism and Solution

Laczó, Ferenc

Independent researcher (previously conducted basic research at the Institute of Economics of the Hungarian Academy of Sciences.)

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# Measuring Inflation: Criticism and Solution

Discussion Paper

Written by
**Dr. Ferenc Laczó PhD**

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**This study addresses two fundamental misunderstandings that make inflation data biased. One of those is the use of the concept of commodities in a way that ignores the value judgment of consumers. Another fundamental mistake, which is related to the previous one, is to assume that changes in price level can be described as the average of price relatives.**

I graduated from the Karl Marx University of Economics in Budapest, earned university doctorate from this university and got a PhD from the Hungarian Academy of Sciences. Now I am a member of the public body of the Hungarian Academy of Sciences.

For the theoretical basis, I needed to go back to three 19th century works. Namely to English translation of *Herman Heinrich Gossen's* The Laws of Human Relations and the Rules of Human Action Derived Therefrom, to *William Stanley Jevons'* Brief Account of a General Mathematical Theory of Political Economy and to *Alfred Marshall's* Principles of Economics.

I am grateful to my ex colleagues in the Institute of Economics Hungarian Academy of Sciences for many comments. I am also grateful to *Ragnar Frisch* and *Aleksandr Konüs* for their evaluation on an earlier version of the study it was made in 1972. They both stressed that this theory is unique and ultimately based on the introduced symmetry rule.

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If any questions or need further information, please
feel free to contact the author:
ferenc.laczo(at)gmail.com

**Abstract**

In this study the concept of commodities is formulated according to the utility theory; following the principle of price elasticity of demand, differences of uncompensated and compensated price changes will be clearly interpreted; as the uncompensated and compensated price changes have different averaging properties, so two different CPI formulas need to be defined; arbitrary price changes are broken down into uncompensated and compensated price change to obtain a complete, dual CPI formula.

**Keywords:** Economic Value of a Commodity; Uncompensated vs. Compensated Price Change; Common Units in Measurements; Dual CPI Formula; Supply-Driven and Demand-Driven Economy

**JEL classification:** E31 Price Level • Inflation • Deflation

## Introduction

In this chapter will be summarized some common features of measurement of inflation and what the basic problem is with the formulas in use.

Inflation is commonly defined as the change in the prices of a basket of goods and services that are typically purchased by population of national wide or specific groups of households. It is measured by the consumer price index (CPI) and is a key macro-economic indicator which also plays an important role in the determination of changing in the purchasing power of money. The compilation of a consumer basket requires special attention and from time to time, maintenance. The most common calculation methods (Laspeyres, Paasche, Jevons and the 'ideal' formula) have been available since the late 19th century.[1] The formula should be described the basic relationship of a market economy. Namely, a description of how changes in supply and demand effect on changes in prices. These formulas do not meet this requirement.

The indices calculated with these or any other formulas in use are not reliable, as they do not give consistent results and are not able to track changes in the market situation.

[1] The *Étienne Laspeyres's* formula was introduced in 1871, the *Hermann Paasche's* one in 1874, *William Stanley Jevons* suggested his formula in 1883 and the 'ideal' formula was originally proposed by *Arthur Lyon Bowley* in 1899.

These indices measure lower CPI and higher consumption in the normal, no-crisis state of the market economy, when it is usually demand-driven. During an economic crisis the trend is expected to reverse: the market becomes supply driven. Assuming, of course, that this crisis is a 'classic' one, i.e. that there is overproduction relative to solvent demand. In such a case the commonly used indices usually overestimate inflation while underestimating consumption compared to indicator which will be defined in this study.[2]

**The consistency criteria in calculations**

In this chapter the range of interpretation will be specified. It will be stressed that only two consistency criteria (tests) and economically interpretable are necessary and sufficient. Lastly Irving Fisher's answer will be cited when he faced with these criteria.

The stipulation in this study is that a price (is actually an average price) and a quantity used in measurement should not be zero or less or infinite. I.e. range of interpretation will be as follows $0 < p_t^i; p_s^i < \infty \;\; i = \overline{1,n}$ and $0 < q_t^i; q_s^i < \infty \;\; i = \overline{1,n}$. This only means that price and volume data should be economically interpretable. E.g. if there is a certain quantity of which the price is 0, or there is a price to which does not belong a quantity economically cannot be interpreted, i.e. economically does not exist. Of course, this does not mean that, we cannot use estimated data if necessary.

The most important aspect that should override all other considerations is that price, quantity, utility etc. should always be considered as they are, variable, if we want to give an exact definition for the CPI formula.

Following the tradition I use consistency criteria (tests) to check the formulas, but only two. They are as follows: (1) the direct indices should be the same as chain indices that is fulfillment of *transitivity* (i.e. chain/circular test); (2) the change in amounts spent on commodities (value index) must equal the product of the price index and the volume index, i.e. the fulfillment of the so-called *factor test*. No further tests are needed to determine whether the consistency requirement is met. However beyond compliance with these consistency criteria, the formulas must be economically interpretable and meaningful.[3]

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[2] Hereinafter, I will use denotation for quantity, $q_t^i$ and for the price, $p_t^i$. The superscript indices denote the commodities and the subscript ones do the time periods or places (countries). My interpretation of the demand-driven case is that if the $q_s/q_t < p_s/p_t$ relationships dominate and the supply-driven case is if the $q_s/q_t > p_s/p_t$ relationships dominate.

[3] A good example of this is that some constructed formulas such as 221 and 321 ones in Irving Fisher's book met the two basic consistency criteria, but are not economically interpretable and meaningful. By the way

As the circular test did not fit into Irving Fisher's interpretation of the index numbers, revised his earlier position[4] preferred the uniquely interpreted accuracy over circular test: "…the best formulae yield results which check under the circular test to a degree of accuracy far beyond that required for any practical use to which index numbers are now put."[5] The interpretation of the 'accuracy' is as follows: "…the accuracy of an index number has been meant its accuracy as a measure of the average movement of the given set of prices (or quantities, as the case may be)."[6]

**Problem with the definition of an index number**

In this chapter I will quote the classic definitions of an index number by Irving Fisher and W Erwin Diewert, the ones I consider inappropriate. Lastly I quote a remarkable view of Ragnar Frisch on the index numbers.

In accordance with interpretation of the 'accuracy', Irving Fisher pointed out that "An index number of the prices of a number of commodities is an average of their price relatives."[7] This definition is logically applicable for the volume index too: an index number of the quantities of a number of commodities is an average of their quantity relatives. *W Erwin Diewert* confirms Irving Fisher's definition "…a price index can be regarded as a weighted mean of the change in the relative prices of the commodities under consideration in the two situations."[8] However, the correctness of this definition is questionable.

In the case of price index calculation, the price relatives are the variable components to be averaged and the quantities are the constant components to be weights. In a volume index calculation the prices and quantities switch roles like the characters in a burlesque... Only the method remains the same, namely a statistical standardisation one. The use of this method should have raised doubts as the price and quantity are variable components.

*Ragnar Frisch* also criticises the applied method in the index theory and probably considers the method of Irving Fisher to be insufficient: "Indeed, all discussions about the

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these formulas were given a low rating because they did not fit Irving Fisher's idea of index numbers. (See Irving Fisher, The Making of Index Numbers, Houghton Mifflin Company, Boston and New York, 1922, pp. 250 and 251) Utility theory was not part of Irving Fisher's index theory, which led to the denial of requirement of transitivity.

[4] "I have found among my former conclusions has to do with the so-called "circular test" which I originally, with other writers, accepted as sound, but which, in this book, I reject as theoretically unsound." See, Irving Fisher, ibid, Preference p. XIII.

[5] Fisher, Irving: Ibid, p. 291.

[6] Fisher, Irving: Ibid, p. 330.

[7] Fisher, Irving: Idem, p. 3.

[8] Diewert, W. E.: Idem p. 6.

'best' index formula, the 'most correct' weights, etc., must be vague and indeterminate so long as the meaning of the index is not exactly defined. Such a definition cannot be given on empirical grounds only but requires theoretical considerations." [9]

**Important basic concepts: market price and commodity**

In this chapter it will be shown that why the concept of commodity using in the economics and the index theory is insufficient.

In the measurement we use the average market prices at which a commodity is purchased or sold when measuring inflation. We have no other reasonable choice, even though these prices do not match with the equilibrium points of supply and demand, but only converge at them. The choices of individual consumers may even differ significantly from each other, but on the whole the purchased quantity of commodities reflects consumers' judgments about these prices. *William Stanley Jevons* put in 1866: "Such complicated laws as those of the economy cannot be accurately traced in individual cases. Their operation can only be detected in aggregates and by the method of averages."[10]

A commodity is described as follows: "In economics, a commodity is an economic good or service that has full or substantial fungibility"[11] And by another similar interpretation: "A commodity is a basic good used in commerce that is interchangeable with other goods of the same type."[12] These wordings represent a physical, and not an economic definition. For economics, taking into consideration the physical properties of a commodity is important and necessary, but not sufficient. The concept of a commodity must involve a 'substantial' interchangeability with the same product based on its physical properties (fungibility) *and* through the 'universal equivalent' an interchangeability with other commodities (substitutability).[13] We can call this for quantity of 'economic value' or 'utility' of commodity which depends on constantly changing value judgments of consumers.[14]

[9] Ragnar Frisch: Annual Survey of General Economic Theory: the Problem of Index Numbers, Econometrica, 1936/4

[10] https://publicpolicy.pepperdine.edu/academics/research/faculty-research/intellectual-foundations/19th-century-economists/will_brie.htm

[11] See https://en.wikipedia.org/wiki/Commodity

[12] See, Chen, James: Commodity, https://www.investopedia.com/terms/c/commodity.asp

[13] I am aware that the concept of commodity has a narrower and a broader interpretation. I intended my findings to be applicable to any interpretation.

[14] Among several other important practical questions arise for instance, what happens when two or more goods are perfectly substitutable for each other. Should be considered them as independent goods or not?

**Distinguishing the uncompensated and compensated price change**

In this chapter we will go back to original wording of Alfred Marshall on price elasticity of demand. In light of this phrasing, we will interpret the uncompensated and compensated price change.

In the words of Alfred Marshall, "The elasticity (or responsiveness) of demand in a market is great or small, depending on whether the amount demanded increases much or little for a given fall in price; and diminishes much or little for a given rise in price."[15] Similarly, "The Income elasticity of demand is the quantity demanded of a particular product depends not only on its own price (see elasticity of demand) and on the price of other related products (see cross price elasticity of demand), but also on other factors such as income. The purchases of certain commodities may be particularly sensitive to changes in nominal and real income."[16]

Income elasticity is measured as the ratio of the percentage change in quantity demanded to the percentage change in income. Similarly price elasticity is measured as the ratio of the percentage change in quantity demanded to the percentage change in price. That is, if a 10% increase in income causes 25% increase in consumption of a commodity the income elasticity of demand is 25%/10% = 2.5. Suppose that 10% increase in income and 25% increase in demand was accompanied by 12.5 percent decrease in price. The price elasticity of demand is as follows: 25%/-12.5% = -2. We used the same change in quantity of demand to the calculation of both the income elasticity and the price elasticity. Who can tell us exactly to what extent a change in demand is a consequence of an increase in income and to what extent a decrease in price? I suppose that today, no one can do this. Without answering this question, we never will be able to measure inflation exactly.

*Eugen Slutsky* was who pointed out that changes in demand are resulted by an uncompensated and a compensated price change.[17] He used derivation to demonstrate this remarkable discovery. However his method was not suitable for splitting into the

[15] See Alfred Marshall: Principles of Economics. 8th ed. London: Macmillan. Chapter IV. p. 86. Available online: https://www.econlib.org/library/Marshall/marP.html
[16] See, https://stats.oecd.org/glossary/detail.asp?ID=3233
[17] See, Allen, R. G. D.: The Work of Eugen Slutsky, Vol. 18, No. 3 (Jul., 1950), pp. 209-216. (The original article of Slutsky was not available for me.)

uncompensated and the compensated price change from each other in general.[18] The two types of price effect on demand can be distinguished according to how the ratio of price to quantity has changed for a commodity, or more simply, with the price elasticity of demand.

Based on the Marshallian definition of price elasticity of demand uncompensated price change can be interpreted as follows: the amount spent of a commodity changes, but the price relative is equal to the quantity relative. In other words the price elasticity of demand is 1. The case where the price relative is equal to the quantity relative is considered as an uncompensated price change, as in my view the price change was not compensated in such a case.

Compensated price change is simply what has remained beyond uncompensated price change. A bit more detail, the case of uncompensated price change, the compensated price change can be interpreted as follows: the ratio of price to quantity is changing, but the amount spent on a commodity remains unchanged. That is the price relative is reciprocal to the quantity relative, and vice versa. According to this definition, a compensated price change occurs when and only when the price / quantity ratio of the goods changes.

It is important to note that the two types of price changes are complementary each other. In other words, by defining the CPI formula separately for uncompensated and compensated price changes, the product of these gives the change in the general price level.

## ‘Hidden’ properties of ‘ideal’ formula and definition of CPI formula for uncompensated and compensated price changes

In this chapter firstly, based on my practical research work I argue that the ‘ideal’ index formula would give consistent results if only uncompensated price changes occurred. I will then give a simple formula for the uncompensated price changes and after that a formula for compensated price changes. While I also point out the difference between common unit of measurement for uncompensated and compensated price changes.

Using the ‘ideal’ formula in an analysis of consumer price changes I was wander why the direct indices differ from chain indices. When I formed groups from the commodities I experienced that the gaps between the direct and chain indices were the widest in the case of commodities sensitive to the price change and the narrowest in the case of strangely

[18] I note that in the economic literature the pair of concepts of uncompensated and compensated price changes and the pair of concepts of change due to income effect and change due to substitution effect often are using as synonyms. This is, of course, possible if the two pairs of concepts mean the same thing.

behaving goods of which the purchased quantity followed the price increase. I have come to the conclusion that the inconsistency problem is likely to be caused by the inadequate measurement of compensated price changes by this formula. Then, I had no idea yet about the appropriate formulas, but I was fully aware that what I found was most important outcome, 'a place to stand on' which allows for resolving the formula problem.

Following this finding I identified that with variable price and volume data the 'ideal' formula measures consistently if only $p_t^i/q_t^i = p_s^i/q_s^i \;\; i = \overline{1,n}$, or (what is the same) $q_s^i/q_t^i = p_s^i/p_t^i \; i = \overline{1,n}$ exist. This case I have identified as uncompensated price changes, in acronym *UcPCh*. Since the price relatives and the quantity relatives are equal for each product, both the price and the volume indices are equal to each other. As in this special case it is true for each element, $\frac{p_s^i}{p_t^i} = \sqrt{\frac{p_s^i q_s^i}{p_t^i q_t^i}}$ and $\frac{q_s^i}{q_t^i} = \sqrt{\frac{p_s^i q_s^i}{p_t^i q_t^i}}$, that is this partial price index and quantity index are equal to the square root of the so-called value index.[19] That is

$$P_{ts}^{UcPCh} = \sqrt{\frac{\sum_{i=1}^n p_s^i q_s^i}{\sum_{i=1}^n p_t^i q_t^i}} \text{ and } Q_{ts}^{UcPCh} = \sqrt{\frac{\sum_{i=1}^n p_s^i q_s^i}{\sum_{i=1}^n p_t^i q_t^i}}.$$

If this special case would exist in practice, the Bowley-Fisher 'ideal' index formula would provide transitive and identical results to the formulas given above. As it can be seen these partial indices are square roots of value index, where the common unit of measurement, according to I. Fisher's theorem,[20] are the amounts spent on the commodities. However, this definition of the common measures can be applied only to the uncompensated price changes, but won't be to the compensated ones.

Logically, the pair of compensated price changes is complementary to the uncompensated pair. In this case, the price relatives and quantity relatives are reciprocal to each other, that is $p_s^i/p_t^i = q_t^i/q_s^i \;\; i = \overline{1,n}$. So between the price relatives and quantity relatives inverse proportionality exists. This case I identified as compensated price changes *(CPCh)* or change due to substitution effect for general using.

In the case of compensated price changes the relationship of price relatives and quantity relatives is as follows: $\frac{p_s^i}{p_t^i} \times \frac{q_s^i}{q_t^i} = 1 \;\; i = \overline{1,n}$. So using a geometric average is appropriate and only this formula gives consistent results in this special case:

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[19] I called this theorem a symmetry rule.

[20] I. Fisher: "…values (in fact, amounts spent on commodities – F.L.) afford the only common measure for comparing the streams commodities produced, exchanged, or consumed, and afford almost the only basis of weighting which has ever been seriously proposed." Irving Fisher, Idem, p. 45.

$$P_{ts}^{CPCh} = \sqrt[n]{\prod_{i=1}^{n} \frac{p_s^i}{p_t^i}} \text{ and } Q_{ts}^{CPCh} = \sqrt[n]{\prod_{i=1}^{n} \frac{q_s^i}{q_t^i}}.$$

This formula originally was introduced by William Stanley Jevons in 1863.[21] Although Jevons' formula is appropriate, gives consistent results only if compensated price changes have occurred. In this special case, the common unit of measurement presumably is the equi-marginal utility, in accordance with *Hermann Heinrich Gossen's* Second Law.[22] This means that neither Fisher's theorem nor Gossen's Second Law have a general validity with regard to the common unit of measurement. Fisher's theorem can only be applied to price changes without compensation, while Gossen's second law can only be applied to compensated price changes.

The uncompensated and compensated price change reflects two different reactions of the consumers of the market situation. The fact that the two types of price changes have different averaging properties is based on these two different consumer reflections of the income and price change. This has not yet answered the question of how an arbitrary price change can be divided into uncompensated and compensated price changes.

**Why one type of index formula, like the 'ideal' or Jevons' one can never be appropriate?**

In this chapter I illustrate with a simple numerical example why one type of index formula**,** like the 'ideal' one may not be appropriate.

The next table is constructed in such a way that in the horizontal direction (between countries), the direct proportionality (i.e. uncompensated price changes) prevails between the price and quantity relatives. While in the vertical direction (between time periods) the relationships between the price and quantity relatives are reciprocal i.e. compensated price changes exist:

| | *A country* | | | | *B country* | | | |
|---|---|---|---|---|---|---|---|---|
| | $q^1$ | $p^1$ | $q^2$ | $p^2$ | $q^1$ | $p^1$ | $q^2$ | $p^2$ |
| *0 time period* | 80 | 25 | 12.5 | 40 | 40 | 12.5 | 25 | 80 |
| *1 time period* | *20 | 100 | 25 | 20 | 10 | 50 | 50 | 40 |

Using the 'ideal' formula, let's see the comparison *A* country with *B* country in *0* time period:

[21] See: Jevons, William Stanley: Investigations in Currency and Finance, available online at http://www.archive.org/stream/investigationsi01jevogoog#page/n38/mode/2up

[22] Gossen, Hermann Heinrich: The Laws of Human Relations and the Rules of Human Action Derived Therefrom, Cambridge, Mass. : MIT Press, ©1983, p. 14.

$$P_{AB}^{'id'} = \sqrt{\frac{80\times12.5+12.5\times80}{80\times25+12.5\times40}\times\frac{40\times12.5+25\times80}{40\times25+25\times40}} = 100\%$$

and

$$Q_{AB}^{'id'} = \sqrt{\frac{40\times25+25\times40}{80\times25+12.5\times40}\times\frac{40\times12.5+25\times80}{80\times12.5+12.5\times80}} = 100\%$$

This means that both the price level and quantity level are the same in both countries in period *0*. As a direct proportionality prevail between the price and quantity relatives, these results are probably correct.

Thereafter let us see the comparison *A* country with B country in *1* time period:

$$P_{AB}^{'id'} = \sqrt{\frac{20\times50+25\times40}{20\times100+25\times20}\times\frac{10\times50+50\times40}{10\times100+50\times20}} = 100\%$$

and

$$Q_{AB}^{'id'} = \sqrt{\frac{20\times25+25\times40}{80\times25+12.5\times40}\times\frac{20\times100+25\times20}{80\times100+12.5\times20}} = 100\%$$

This means that both the price level and the quantity level are the same in both countries in period *1* too. As direct proportionality prevails between the simple indices, these results are probably correct.

We can conclude that an appropriate formula was used in both case and that both the price level and quantity level are the same in both countries, in both periods. Consequently, the vertical price index of *A* country must be equal to the vertical price of *B* country. Similarly, the vertical quantity index of *A* country must be equal to the vertical quantity index of *B* country.

The vertical indices for the *A* country:

$$P_{01}^{'id'} = \sqrt{\frac{80\times100+12.5\times20}{80\times25+12.5\times40}\times\frac{20\times100+25\times20}{20\times25+25\times40}} = 234.5\%$$

and

$$Q_{01}^{'id'} = \sqrt{\frac{20\times25+25\times40}{80\times25+12.5\times40}\times\frac{20\times100+25\times20}{80\times100+12.5\times20}} = 42.6\%$$

…and for the *B* country the next indices:

$$P_{01}^{'id'} = \sqrt{\frac{40\times50+25\times40}{40\times12.5+25\times80}\times\frac{10\times50+50\times40}{10\times12.5+50\times80}} = 85.3\%$$

and

$$Q_{01}^{'id'} = \sqrt{\frac{10\times12.5+50\times80}{40\times12.5+25\times80}\times\frac{10\times50+50\times40}{40\times50+25\times40}} = 117.3\%$$

The price indices, and similarly, the quantity indices, of the two countries should have been equal. In spite of this, the differences are huge, nearly threefold. In this simple example can be seen, if compensated price changes happened the 'ideal' formula is not appropriate.

Those who blindly believe in the superiority of the ‘ideal’ formula must face this embarrassing contradiction and must face that the 'ideal' formula is not universal.

If we used a suitable formula for a vertical comparison, namely the Jevons one, the price index would have been 141.4% and the quantity index would have been 70.7% in the case of both countries. Since in real life any change within the range of interpretation involves both types of changes, i.e. uncompensated and compensated price changes we need a formula that measures both types of changes at the same time. To do this, we need to divide the price change into uncompensated and compensated price change.

**Separating uncompensated price changes from compensated ones**

In this chapter it will be shown how the price relatives and quantity relatives can be separated into a component of an uncompensated price change and a component of a compensated one in order to be averaged according to their averaging properties.

My theory is based on an axiom, ‘a place to stand on’ namely if the price changes are uncompensated the price and quality indices are equal to each other and are equal with the square roots of the value index. Therefore, if we want to know what happened beyond the uncompensated price changes, as one option we can divide the price and quantity data of current period with the square roots of amount relatives as follows: $p_s^i/\sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}}\ i=\overline{1,n}$ and $q_s^i/\sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}}\ i=\overline{1,n}$. These data will be indicated for $p_{s'}^i$ and $q_{s'}^i$. After that, the price relative and the quantitative relative can be broken out into two parts as follows:

$$\frac{p_s^i}{p_t^i}=\frac{p_s^i}{p_{s'}^i}\times\frac{p_{s'}^i}{p_t^i}\ \ i=\overline{1,n}\ \text{ and } \frac{q_s^i}{q_t^i}=\frac{q_s^i}{q_{s'}^i}\times\frac{q_{s'}^i}{q_t^i}\ \ i=\overline{1,n}.$$

In this case, the first elements of the multiplications (right side of the equalities) represent the uncompensated price change, while the second one the compensated one. Now we can substitute the values of the $p_{s'}^i$ and $q_{s'}^i$ into the above equations (taking the general item):

$$\frac{p_s^i}{p_{s'}^i}=\frac{p_s^i}{p_s^i/\sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}}}=\sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}}\ \text{ and }\ \frac{p_{s'}^i}{p_t^i}=\frac{p_s^i/\sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}}}{p_t^i}=\sqrt{\frac{p_s^i}{p_t^i}/\frac{q_s^i}{q_t^i}}$$

and

$$\frac{q_s^i}{q_{s'}^i} = \frac{q_s^i}{q_s^i \Big/ \sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}}} = \sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}} \text{ and } \frac{q_{s'}^i}{q_t^i} = \frac{q_s^i \Big/ \sqrt{\frac{q_s^i p_s^i}{q_t^i p_t^i}}}{q_t^i} = \sqrt{\frac{q_s^i}{q_t^i} \Big/ \frac{p_s^i}{p_t^i}}$$

We received the following

$$\frac{\boldsymbol{p_s^i}}{\boldsymbol{p_t^i}} = \sqrt{\frac{\boldsymbol{q_s^i p_s^i}}{\boldsymbol{q_t^i p_t^i}}} \sqrt{\frac{\boldsymbol{p_s^i}}{\boldsymbol{p_t^i}} \Big/ \frac{\boldsymbol{q_s^i}}{\boldsymbol{q_t^i}}} \text{ and } \frac{\boldsymbol{q_s^i}}{\boldsymbol{q_t^i}} = \sqrt{\frac{\boldsymbol{q_s^i p_s^i}}{\boldsymbol{q_t^i p_t^i}}} \sqrt{\frac{\boldsymbol{q_s^i}}{\boldsymbol{q_t^i}} \Big/ \frac{\boldsymbol{p_s^i}}{\boldsymbol{p_t^i}}}$$

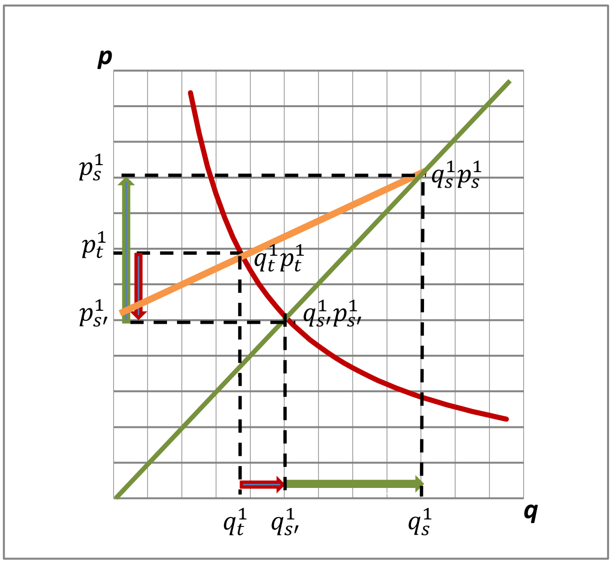
I consider such a breaking down of price and quantity relatives is one of the most important results on the road to solving the measurement problem.

So far the CPI calculation based on averaging of the price relatives and quantity relatives (left sides of equalities). The right sides of the equations show how the price relatives and quantity relatives can be isolated into a component of an uncompensated price change and a component of a compensated one in order to be averaged according to their averaging properties.

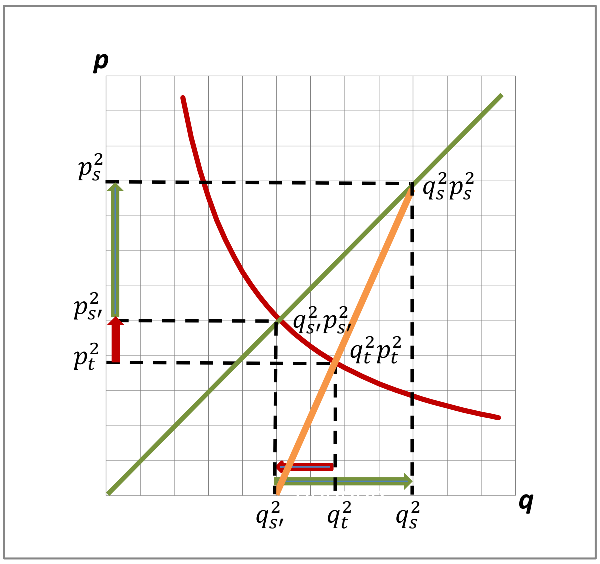
## Graphical representation of the uncompensated and the compensated price change

In this chapter I will explain with help of diagrams what I did in previous chapter.

**1st case: an example for supply-driven change**




**2nd case: an example for demand-driven change**




impacts of uncompensated price change — impacts of compensated price change

This graphical representation clearly shows what interpretation of uncompensated and compensated price changes has got in this study. These diagrams illustrate first, since elasticity curves have been used, how a price and quantity relative needs to break down according to their different averaging properties.

It can be seen that, if we extend the lines that connect the two points of amount spent in the first, supply-driven case, it intersects the Y axis and in the second, demand-driven case, the X axis. (As I mentioned in the footnote No. 2 in my interpretation supply-driven case exists, if $q_s/q_t > p_s/p_t$ and demand-driven case exists, if $q_s/q_t < p_s/p_t$.)

In order to show the sections of uncompensated price changes and the sections of compensated price changes from the $q_t \rightarrow q_s$ and $p_t \rightarrow p_s$ distances, we need to draw straight lines from the origins through the $q_s p_s$ points (green lines). After that, we need to draw equilateral hyperbolas with the coordinate axes as its asymptote through the $q_t p_t$ points (red lines), which never show 'indifferent changes', indeed it is just the opposite. Where the straight lines and the rectangular hyperbolas intersect each other, those points are the inflection ones where a switch takes place from the uncompensated price change to the compensated one.

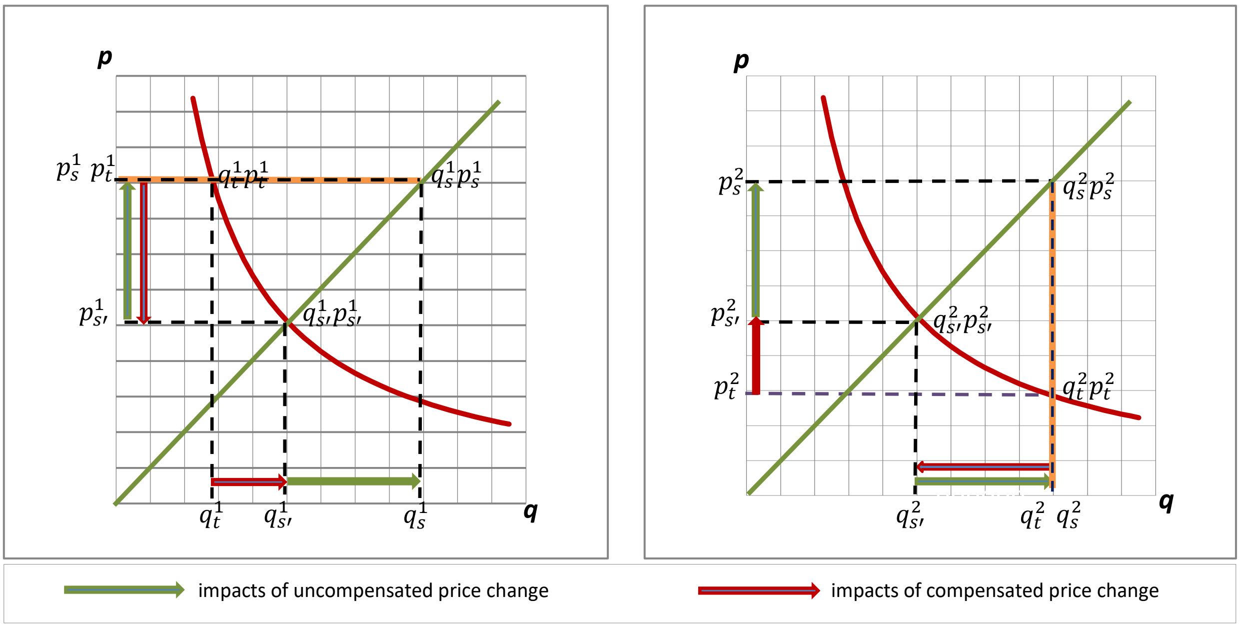
The $q_{s'} \rightarrow q_s$ and the $p_{s'} \rightarrow p_s$ sections (green arrows), where the $q_s/q_{s'} = \sqrt{\frac{q_s p_s}{q_t p_t}}$ and the $p_s/p_{s'} = \sqrt{\frac{q_s p_s}{q_t p_t}}$ represent the uncompensated price changes. The $q_t \rightarrow q_{s'}$ and the $p_t \rightarrow p_{s'}$ sections (red arrows) $q_{s'}/q_t = \sqrt{\frac{q_s}{q_t} / \frac{p_s}{p_t}}$ and $p_{s'}/p_t = \sqrt{\frac{p_s}{p_t} / \frac{q_s}{q_t}}$ show the compensated price changes. The $q_s/q_{s'} = \sqrt{\frac{q_s p_s}{q_t p_t}}$ and $p_s/p_{s'} = \sqrt{\frac{q_s p_s}{q_t p_t}}$ sub-indices (green arrows) are equal to each other. While the $q_{s'}/q_t = \sqrt{\frac{q_s}{q_t} / \frac{p_s}{p_t}}$ and $p_{s'}/p_t = \sqrt{\frac{p_s}{p_t} / \frac{q_s}{q_t}}$ sub-indices (red arrows) are reciprocal to each other.

**Graphical representation if only the quantity or price has changed**

The next two diagrams will show how the change in price or quantity level can be measured if only the quantity or price has changed.

I note that these cases are not unique, as demand can change even if prices remain unchanged, for example due to changes in purchasing power. Or the rent of apartments can change, even if the demand remains unchanged. The method used to measure inflation should be able to handle such situations. If it cannot do so, as the methods provided by index theory, then we can be sure the methods it offers is inadequate. (See the charts in the next page.)

**3rd case: an example if only the quantity changed** **4nd case: an example if only the price changed**




The main massage of these diagrams is as follows: in the 3rd case, if only quantity has changed this alone implies two type changes in price, namely, an uncompensated and a compensated ones not only in quantity (X axis) but in prices (Y axis) too. The rate of uncompensated price change (Y axis – green arrow), which here is $> 1$ and the rate of compensated price change (Y axis – red arrow), which here is $< 1$, are reciprocal to each other. That is, their product is 1. As the two type of price changes have different averaging properties, during the averaging process their joint contribution to change in price level basically depends on whether the amount spent on this item is below average or above it.

The same can be said if we interchange the interchangeable in the 4th diagram.

**Set of developed formulas - compilation of the dual formula**

In the next chapter the formulas developed for the uncompensated and compensated price change can be found. Their multiplication gives the complete formula. This chapter contains formulas to measure whether an economy was demand- or supply-driven in a given period.

I have defined two different formulas, indicating exactly which formula applies to what and also that the product of the two formulas gives the change in price level or the change in the consumption level. Accordingly the CPI and volume index formulas for the uncompensated price changes the partial indicators for measuring uncompensated price changes are as follow:

$$\sqrt{\frac{\sum_{i=1}^{n} q_s^i p_s^i}{\sum_{1=1}^{n} q_t^i p_t^i}} \textbf{ and } \sqrt{\frac{\sum_{i=1}^{n} q_s^i p_s^i}{\sum_{1=1}^{n} q_t^i p_t^i}}$$

As mentioned, the index numbers calculated with these formulas are the same as the index numbers calculated with the 'ideal' formula if only uncompensated price changes have occurred.

As the $\sqrt{\frac{p_s}{p_t}/\frac{q_s}{q_t}}$ and $\sqrt{\frac{q_s}{q_t}/\frac{p_s}{p_t}}$ have the same properties as $\frac{p_s^i}{p_t^i}$ and $\frac{q_s^i}{q_t^i}$ have, i.e. $\sqrt{\frac{p_s}{p_t}/\frac{q_s}{q_t}} \times \sqrt{\frac{q_s}{q_t}/\frac{p_s}{p_t}} = 1$ if we substitute them into the formulas for compensated price change we receive the following formulas: $\sqrt[n]{\prod_{i=1}^{n}\sqrt{\frac{p_s}{p_t}/\frac{q_s}{q_t}}}$ and $\sqrt[n]{\prod_{i=1}^{n}\sqrt{\frac{q_s}{q_t}/\frac{p_s}{p_t}}}$ That is formulas for measuring compensated price changes are as follow:

$$\sqrt[2n]{\prod_{i=1}^{n}\frac{p_s^i}{p_t^i}/\frac{q_s^i}{q_t^i}} \textbf{ and } \sqrt[2n]{\prod_{i=1}^{n}\frac{q_s^i}{q_t^i}/\frac{p_s^i}{p_t^i}}$$

The index numbers calculated with these formulas are the same as the index numbers calculated with the Jevons formula if only compensated price changes have occurred.

The collection of formulas is as follows:

| | Indicators for measuring the prices changes | Indicators for measuring the quantity changes |
|---|---|---|
| Partial indicators for measuring uncompensated price changes | $\sqrt{\frac{\sum_{i=1}^{n} q_s^i p_s^i}{\sum_{1=1}^{n} q_t^i p_t^i}}$ | $\sqrt{\frac{\sum_{i=1}^{n} q_s^i p_s^i}{\sum_{1=1}^{n} q_t^i p_t^i}}$ |
| Partial indicators for measuring compensated price changes | $\sqrt[2n]{\prod_{i=1}^{n}\frac{p_s^i}{p_t^i}/\frac{q_s^i}{q_t^i}}$ | $\sqrt[2n]{\prod_{i=1}^{n}\frac{q_s^i}{q_t^i}/\frac{p_s^i}{p_t^i}}$ |
| Complete indicators for measuring the total changes in price or volume level | $\sqrt{\frac{\sum_{i=1}^{n} q_s^i p_s^i}{\sum_{1=1}^{n} q_t^i p_t^i}} \times \sqrt[2n]{\prod_{i=1}^{n}\frac{p_s^i}{p_t^i}/\frac{q_s^i}{q_t^i}}$ | $\sqrt{\frac{\sum_{i=1}^{n} q_s^i p_s^i}{\sum_{1=1}^{n} q_t^i p_t^i}} \times \sqrt[2n]{\prod_{i=1}^{n}\frac{q_s^i}{q_t^i}/\frac{p_s^i}{p_t^i}}$ |
| Statuses of the indicators the economy is a demand driven | $\sqrt[2n]{\prod_{i=1}^{n}\frac{p_s^i}{p_t^i}/\frac{q_s^i}{q_t^i}} > 1$ or $\sqrt[2n]{\prod_{i=1}^{n}\frac{q_s^i}{q_t^i}/\frac{p_s^i}{p_t^i}} < 1$ | |
| Statuses of the indicators if the economy is a supply driven | $\sqrt[2n]{\prod_{i=1}^{n}\frac{p_s^i}{p_t^i}/\frac{q_s^i}{q_t^i}} < 1$ or $\sqrt[2n]{\prod_{i=1}^{n}\frac{q_s^i}{q_t^i}/\frac{p_s^i}{p_t^i}} > 1$ | |

The formulas in the table, in addition to providing an accurate method of measuring inflation, are a mathematical description of the most important movement, i.e. shift in the equilibrium point of the supply-demand in market economies.

Of the two partial average formulas, one measures the uncompensated price change and the other the compensated one. As the uncompensated and compensated price changes are complements of each other, their multiplication gives the total change in price level or consumption level. Analysing the two types of price changes, among others, can provide an additional opportunity for a better understanding of consumer behaviour of specific groups of households.

**How the indicators introduced work in practice: an example from the time of the financial crisis**

An example presents that the indicator calculated with a conventional formula underestimates the change in price level and overestimates the change in consumption level. And it presents how the change of behaviour of the market actors during the time of the 2008 financial crisis can be measured and how this behaviour changed.

For this purpose I use some food consumption and food prices data for Germany for the years either side of the financial crisis. The selection of the data was random in the sense that I considered all the products for which data could be obtained. (Base data can be found in the Appendix.)

**Impact of the Financial Crisis of 2008 on a Slice of Food Market in Germany (%)**

| | Nomination | 06/05 | 07/06 | 08/07 | 09/08 | 10/09 | 11/10 | 12/11 |
|---|---|---|---|---|---|---|---|---|
| **1.** | $\boldsymbol{P_{ts} \times Q_{ts}}\ (\boldsymbol{V_{ts}})$ | 100.2 | 105.4 | 106.8 | 97.2 | 96.8 | 107.4 | 100.5 |
| **2.** | $\boldsymbol{P_{ts}^{UcPCh}}$ | 100.1 | 102.7 | 103.3 | 98.6 | 98.4 | 103.7 | 100.2 |
| **3.** | $\boldsymbol{P_{ts}^{CPCh}}$ | **102.4** | **102.7** | **103.6** | **97.3** | **99.8** | **102.9** | **100.8** |
| **4.** | $\boldsymbol{P_{ts}^{dual}}$ | 102.5 | 105.4 | 107.0 | 95.9 | 98.1 | 106.7 | 101.1 |
| **5.** | $\boldsymbol{P_{ts}^{\prime ideal\prime}}$ | 101.5 | 103.8 | 107.3 | 95.9 | 96.9 | 106.3 | 102.9 |
| **6.** | $\boldsymbol{P_{ts}^{\prime ideal\prime} / P_{ts}^{dual}}$ | **99.0** | **98.5** | **100.3** | **100.0** | **98.8** | **99.6** | **101.8** |
| **7.** | $\boldsymbol{Q_{ts}^{UcPCh}}$ | 100.1 | 102.7 | 103.3 | 98.6 | 98.4 | 103.7 | 100.2 |
| **8.** | $\boldsymbol{Q_{ts}^{CPCh}}$ | **97.6** | **97.4** | **96.6** | **102.8** | **100.3** | **97.2** | **99.2** |
| **9.** | $\boldsymbol{Q_{ts}^{dual}}$ | 97.8 | 100.0 | 99.8 | 101.4 | 98.6 | 100.7 | 99.4 |
| **10.** | $\boldsymbol{Q_{ts}^{\prime ideal\prime}}$ | 98.8 | 101.6 | 99.5 | 101.4 | 99.9 | 101.1 | 97.6 |
| **11.** | $\boldsymbol{Q_{ts}^{\prime ideal\prime} / Q_{ts}^{dual}}$ | **101.0** | **101.5** | **99.7** | **100.0** | **101.2** | **100.4** | **98.3** |

The impact of the financial crisis on real market developments occurred with a time lag. The first raw shows radical decrease in amount spent on these fourteen products in 2009 and 2010 years. Beside the amount spent on these commodities the prices were those that dropped significantly in the 2009 and 2010 years (see the 4th row). Consumption increased in the crisis years (see the 9th row). The producers increased supply to compensate for the fall in prices and the consumers bought more at lower prices.

The 3rd and the 8th rows show that this slice of the German food market typically is a demand driven. However, this characteristic of it changed in 2009 and did so again slightly in 2010. Namely, it became supply driven. Probably this turn was largely due to the consequences of the financial crisis.

It was sufficient to include the ‘ideal’ index in the table as in this numerical example the gaps between the Laspeyres and the Paasche indices are negligible. With the exception of two years, the ‘ideal’ price index shows lower price growth, while the ‘ideal’ volume index shows a higher quantity growth than the dual indicator. During this period the ‘ideal’ price index underestimated by 0.3 percentage points the price increase while the ‘ideal’ volume index overestimated by 0.3 percentage points the consumption growth to the dual indicators taken as an annual average.

These figures mean nothing less than that the price increase calculated using the traditional method was 14.9 percent over this period, while the exact calculation shows a 17.3 percent increase. Furthermore, according to the conventional method, consumption was essentially stagnant (99.8 percent) over this period. While the exact calculation shows a decrease of 2.3 percent.

* * *

## APPENDEX

### Per capita food consumption and consumer prices in Germany in 2005-2012 years*

| Commodity/ +Year | Unit | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|
| **Wheat flour** | **Kilo** | 67,80 | 64,2 | 63,7 | 62,80 | 66,40 | 70,90 | 70,90 | 69,43 |
| | **Price** | 0,37 | 0,39 | 0,48 | 0,61 | 0,56 | 0,51 | 0,56 | 0,60 |
| **Potatoes** | **Kilo** | 63,00 | 61,1 | 60,7 | 64,30 | 64,50 | 57,00 | 65,20 | 58,70 |
| | **Price** | 0,59 | 0,75 | 0,81 | 0,72 | 0,70 | 0,75 | 0,77 | 0,71 |
| **Sugar** | **Kilo** | 35,90 | 34,2 | 35,3 | 33,60 | 35,20 | 33,70 | 32,00 | 32,10 |
| | **Price** | 0,92 | 0,92 | 0,92 | 0,91 | 0,82 | 0,69 | 0,71 | 0,87 |
| **Milk** | **Litre** | 53,00 | 53,30 | 53,70 | 54,90 | 54,10 | 53,50 | 53,80 | 54,30 |
| | **Price** | 0,54 | 0,54 | 0,61 | 0,66 | 0,53 | 0,58 | 0,61 | 0,58 |
| **Cheese** | **Kilo** | 21,50 | 22,00 | 22,30 | 22,20 | 22,90 | 23,50 | 23,80 | 23,70 |
| | **Price** | 4,07 | 4,19 | 4,62 | 5,50 | 5,03 | 4,32 | 4,67 | 4,76 |
| **Beef and veal** | **Kilo** | 12,10 | 11,9 | 12,7 | 12,30 | 12,50 | 12,80 | 13,10 | 13,00 |
| | **Price** | 7,12 | 7,50 | 7,38 | 7,72 | 7,95 | 7,95 | 8,68 | 9,13 |
| **Pork** | **Kilo** | 54,10 | 54,5 | 55,4 | 54,40 | 54,10 | 54,80 | 54,50 | 52,70 |
| | **Price** | 5,23 | 5,04 | 4,98 | 5,25 | 5,23 | 5,13 | 5,35 | 5,70 |
| **Chicken** | **Kilo** | 17,50 | 16,7 | 17,8 | 18,30 | 18,80 | 18,70 | 19,10 | 18,50 |
| | **Price** | 6,44 | 6,55 | 7,01 | 7,49 | 7,02 | 6,87 | 7,23 | 7,44 |
| **Butter** | **Kilo** | 5,30 | 5,30 | 5,20 | 5,10 | 4,80 | 4,80 | 5,00 | 5,00 |
| | **Price** | 3,20 | 3,08 | 3,72 | 3,24 | 2,92 | 3,88 | 4,44 | 3,52 |
| **Margarine** | **Kilo** | 5,7 | 5,4 | 5,4 | 5,6 | 5,3 | 5,1 | 4,9 | 4,9 |
| | **Price** | 1,48 | 1,52 | 1,54 | 1,84 | 1,84 | 1,70 | 1,80 | 1,92 |
| **Vegetable oil** | **Litre** | 11,3 | 11,3 | 11,3 | 11,2 | 11,3 | 11,2 | 11,2 | 11,1 |
| | **Price** | 0,76 | 0,82 | 0,86 | 1,17 | 1,21 | 0,98 | 1,36 | 1,29 |
| **Eggs** | **10 eggs** | 20,5 | 20,9 | 20,9 | 20,8 | 21,0 | 21,4 | 21,2 | 21,7 |
| | **Price** | 1,31 | 1,27 | 1,26 | 1,46 | 1,48 | 1,41 | 1,41 | 1,27 |
| **Apple** | **Kilo** | 36,50 | 34,30 | 28,90 | 28,00 | 30,00 | 26,60 | 25,90 | 24,70 |
| | **Price** | 1,16 | 1,26 | 1,30 | 1,41 | 1,26 | 1,28 | 1,40 | 1,42 |
| **Banana** | **Kilo** | 10,70 | 10,10 | 10,60 | 10,80 | 10,80 | 10,30 | 10,40 | 10,30 |
| | **Price** | 1,28 | 1,16 | 1,15 | 1,20 | 1,21 | 1,13 | 1,16 | 1,22 |
| **Cauliflower** | **Kilo** | 2,30 | 2,10 | 2,00 | 2,10 | 2,10 | 2,10 | 2,20 | 2,30 |
| | **Price** | 1,02 | 1,15 | 1,12 | 1,05 | 1,08 | 1,17 | 1,09 | 1,11 |

*Milk, Beef and veal, Pork and Apple price for 2005 was estimated.
Source: Federal Statistical Office of Germany, and Agrarmarkt Informations-Gesellschaft mbH (AMI)

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