Comparative Analysis of Market Efficiency and Volatility of Energy Prices Before and During COVID-19 Pandemic Periods

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COMPARATIVE ANALYSIS OF MARKET EFFICIENCY AND VOLATILITY OF ENERGY PRICES BEFORE AND DURING COVID-19 PANDEMIC PERIODS

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Abstract

The Covid-19 pandemic has affected energy demand and pricing globally due to different lockdown measures embarked on by governments in different economies. As a result, prices of oil and petroleum products dropped drastically at the peak of the pandemic period. The present paper, therefore, investigates the effect of the pandemic on energy markets and compared the levels of market efficiency, volatility, and volatility persistence. Two 5-monthly daily data windows are considered, each for the period before and during the pandemic, and an updated nonlinear fractional integration approach in time series analysis is employed. Having considered prices of Crude oil, Gasoline, Diesel, Heating oil, Kerosene, and Propane from US markets, we find that energy markets are less efficient during the Covid-19 pandemic period, even though with higher volatility but with lesser volatility persistence compared to the period before the pandemic. Thus, volatility shocks last for a shorter period during the 5-month pandemic period than in the 5-month period that precedes the pandemic. It is hoped that the findings of this work will be of interest to oil marketers and administrators in the international oil markets.

Keywords: Energy price; Covid-19 pandemic; Efficient market; Volatility persistence; Fractional integration
JEL Classification: C22; Q41; Q48

1. Introduction

Fama (1965; 1970) efficient market theory is often used to explain market participation in stocks and other asset price markets. This does not exclude energy markets where energy sources such as crude oil and petroleum products are traded on daily basis. The entire crude oil market is a complex, turbulent, and opaque international financial market as noted in Norouzi and Fani (2020). Thus, the level of efficiency of such an energy market will be useful in evaluating energy investment in a bid to develop the energy market further. The efficiency/inefficiency in the energy market will render useful information to energy market players. As defined in Fama (1965) and as adapted to the case of the energy market, efficiency posits that own past information in the energy market is expected to predict the future dynamics of market prices. As quoted directly, the standard definition of an efficient market states that: "In an efficient market, at any point in time, the actual price of a security will be a good estimate of its intrinsic value". A more refined definition by Fama (1970) on the Efficient Market Hypothesis (EMH) is that prices of assets have complete past information, and in the event of the arrival of new information, these prices re-adjust for the assets to be re-valued rightly. Thus, returns of asset prices at the market are expected to be predictable for EMH to hold (see Lim and Brooks, 2011), and in this case, investors cannot make abnormal returns. Asset prices follow a random walk movement since returns are deemed to be unpredictable.

Quite several papers have investigated market efficiency in energy pricing markets using crude oil and petroleum products such as gasoline, diesel, kerosene, heating oil, and natural gas as sources of energy. Table 1 gives a cursory look at this literature.

PUT TABLE 1 AROUND HERE

On the volatility of energy prices, we have numerous pieces of literature as documented in Table 2. Efficiency and volatility of asset pricing are inseparable, as efficiency is a function of price differences, that is, returns. This could also be price movement as it follows a random walk. Market volatility is a function of variation from price returns, and while volatility persistence tells us about the time such volatility lasts when triggered by external shocks, and in an efficient market, the time needed for the effect of price shocks on volatility to fizzle out is very short. Market volatility is often proxied by absolute or squared returns in daily frequency series.

PUT TABLE 2 AROUND HERE

The analysis approach considered in the present paper is novel and scarcely applied in the analysis of energy pricing, as detailed in the literature. This is the fractional integration (unit root) method in time series econometrics. It is a more general method to classical unit root testing of Box et al. (2015) as it allows for fractional unit root testing in time series other than unit root 0 as in autoregressive moving average (ARMA) and its integrated ARMA version with unit roots 1,2, etc. These roots are too restrictive for any time series observations since in a real sense these may not be an exact integer number. Fractional integration is determined by parameter d which lies in the stationary interval as in $-0.5 < d < 0.5$, and nonstationary interval as in $0.5 \leq d < 3$. Most time series are integrated of order 1 as noted in Box et al. (2015), while others are nonstationarity of the higher order of d (see Shittu and Yaya, 2011; Yaya and Gil-Alana, 2020). A subset of fractional integration is a long-range dependency which is more general, where $0 < d < 1$ and $0 < d < 0.5$ is the case of stationary mean reversion, while $0.5 < d < 1$ is the case of nonstationary mean reversion. Mean reversion refers to the tendency of an asset time series to return to its original trend path after being induced by external shocks (Fama and French, 1992), while in the case of non-mean reversion ($d \geq 1$), the effects of shocks on assets series persist indefinitely. In the case of market efficiency of energy pricing, after being induced by the Covid-19 health crisis as a source of shocks, if the energy price series tends to drift away from its mean level, it is said to be persistent and non-mean reverting.

On the other way round, if after energy price series are induced by Covid-19 or any other external shocks, the series reverts to their mean level, they are said to be mean-reverting.

The present paper, therefore, investigates the level of market efficiency and volatility persistence of energy pricing before and during the Covid-19 pandemic, using a 5-month daily data window in each case, by employing fractional integration test. Market efficiency of energy price series, in this case, means that prices series are I(d = 1) as in the case of random walk, which further implies that the first difference series of price series (i.e. the log-returns) are I(d = 0). Evidence of market inefficiency, thus means that I(d < 1) which is the case of long-range dependency of the series. The fractional integration approach is as well applied to absolute returns used as volatility persistence proxy. Due to the nonlinear nature of the time series, our analysis is based on an updated method in Gil-Alana and Yaya (2020) that allows for smooth nonlinear time trends based on Fourier functions. The classical fractional integration method is the Robinson (1994) linear model which is limited to the linear time trend context. The estimation of the model is based on the Lagrange Multiplier (LM) principle with the Whittle function.

Fractional integration framework with linear and nonlinear regimes in the dynamics of oil and energy prices has been considered in recent papers. The persistence of prices and returns of crude oil at the WTI market across bull and bear phases were considered in Gil-Alana et al. (2016) in which the bear market of oil persisted longer than the bull market. The author further stated the implication this had for market efficiency. Olubusoye and Yaya (2016) also considered persistence, asymmetry, and jumps in oil and petroleum products' returns and found crude oil to persist relatively differently from petroleum products prices. Other papers such as Gil-Alana, Yaya, and Awe (2017) used the fractional integration approach to investigate movement between oil and gold (Yaya, Tumala and Udomboso, 2016; Gil-Alana, Yaya and Awe (2017), natural gas pricing (Yaya, Gil-Alana and Carcel, 2015), oil price and US dollar exchange rate returns (Yaya et al., 2017). These are very recent papers adopting a persistence approach in modeling oil and energy prices in the market, whereas the applied methodology in the present paper is an updated approach. Nonlinearity check is prominent in oil and energy modeling as noted in Kapetanios, Snell, and Shin (2003), therefore the property is recommended to be tested in the analysis of the time series. It is hoped that the investigation of efficiency and volatility persistence in energy pricing markets will interest readers.

The rest of the paper is structured as follows: Section 2 presents and describes the datasets, giving some initial pretests. Section 3 presents the statistical methodology via a fractional integration framework. Section 4 presents the main findings and section 5 renders the concluding remarks.

2. Data presentation

Daily spot prices of six oil and petroleum products, namely Crude oil (West Texas Intermediate market), Gasoline, Heating oil, Diesel (New York Harbour market), Kerosene (US Gulf Coast market), and Propane (Mont Belvieu Texas market) were analyzed. The datasets were retrieved from the website of the US Energy Information and Administration (EIA) at https://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm. The energy prices are quoted in US dollars per barrel while the natural gas (Propane) is quoted in US dollar per Million Btu. The time series spans from 1 October 2019 to 17 August 2020.

Following WHO (2020), the Covid-19 pandemic declaration was on 11 March 2020, and this date serves as a break date for our time series. We then have a subsample before and during the pandemic. In Figure 1, we present plots of each energy series with a vertical line dividing the series into two subsamples A and B. In the plots, energy price drops are observed, commencing around January 2020 and this was further triggered by the Covid-19 pandemic in February-March, 2020, with oil and petroleum prices hitting their lowest prices in March-April,

2020. The transformed log-returns series of these energy prices showed obvious price fluctuations due to volatility during the pandemic phase in subsample B in all the six plots. This gingered further probing into the market efficiency level and volatility at energy markets before and during the pandemic.

PUT FIGURE 1 AROUND HERE

As part of the data exploration, descriptive measurements are computed for energy prices, their log-returns, and absolute returns used as volatility proxy, for the two subsamples A and B. Results are presented for price level and log-returns in Table 3. Energy prices were found to be higher before the pandemic, as WTI oil traded at an average price of $48.89 per barrel as against $34.40 per barrel during the pandemic. Oil recorded a maximum price of $63.27 since October 2019, with a minimum price of $14.10 in the first 5 months since October 2020. Minimum daily prices reported for all energy series are a result of a sharp decline in prices of oil and petroleum products before the WHO announcement of Covid-19 as a pandemic. The results of JB (Jarque-Bera) test statistics imply that the dynamics of oil and petroleum products' pricing are asymmetric and possibly heteroscedastic since neither follow Gaussian distribution. During the pandemic (i.e. Subsample B), energy prices are lower except for propane with an average price of $0.46 per million Btu. This is expected since the plot of Propane pricing in Figure 1 has indicates that Propane gas has almost recovered completely from price drops. The maximum price before the pandemic was $0.56 per Million Btu as against $0.53 per Million Btu during the pandemic. In other petroleum products, there is still a wide margin in price differences before and during the pandemic. Since these energy sources are gaining back their price levels quickly, log-returns for these prices are positive indicating an increase in prices. By looking at estimates of standard deviations of returns, we found higher variation for prices of WTI oil, Heating oil, Diesel, and Kerosene during the pandemic implying the possibility of higher price volatility during the episode.

PUT TABLE 3 AROUND HERE

These prices differences serve as indications to investigate market price dynamics and behaviour of market participants for the possibility of abnormal gain or not. In what follows, we describe the fractional integration methodology used to investigate market efficiency and volatility persistence between the two trading phases.

3. Fractional integration framework

Long-range dependence and long memory are often used to describe time series persistence, often estimated under fractional integration setup. A time series has long memory if its spectral density function has a pole at frequency null, and its autocorrelation function ρ_{y_t} decays exponentially slowly as in,

$$\rho_{y_t}(k) = ck^{2d-1} \text{ as } k \to \infty \quad (1)$$

for fractional persistence parameter, d in the interval, $0 < d < 0.5$, with k being the time lag and constant, c. Thus, the autocorrelation sums to infinity, i.e. $\sum_{k=-\infty}^{\infty} \rho_{y_t}(k) = \infty$ (see Granger, 1980; Granger and Joyeux, 1980, Marinucci and Robinson (1999).

As applied in this paper, the general fractional integration approach considers the model,

$$y_t = \rho(L; d)x_t + u_t, \qquad t = 1,2, \dots, N \quad (2)$$

where y_t is the observed energy price series of size N, $\rho(L; d)x_t$ is the backward shift operation, with lag L, $(L'x_t = x_{t-1})$. The standard I(d) model is,

$$\rho(L; d) = (1 - L)^d \quad (3)$$

where d is the fractional persistence parameter. By extending (2) above to include a nonlinear deterministic function that captures both nonlinear cycle and persistence in the time series, we have the model,

$$y_t = \phi(t) + \rho(L;d)x_t + u_t, \qquad t = 1,2,\dots,N \tag{4}$$

where $\rho(L;d)$ is of the form in (3) and $\phi(t)$ is the Fourier function, capturing multiple structural breaks in the energy series as nonlinear cycles and smooth breaks. This model is proposed in Gil-Alana and Yaya (2020):

$$\phi(t) = \alpha + \beta t + \sum_{k=1}^{n} \lambda_k \sin(2\pi kt/T) + \sum_{k=1}^{n} \gamma_k \cos(2\pi kt/T); \qquad n \le T/2; \quad t = 1,2,\dots,N \tag{5}$$

where α and β are the intercept and linear, t trend slope, respectively; λ_k and γ_k are parameters driving the amplitude and displacement of the Fourier form which induces the nonlinearity, respectively; Fourier form expansion is determined by frequency n and k is a particular frequency number set as equal to 1, 2, …. n, and N is the number of observations. Thus, λ_k and γ_k (for all k) determine nonlinearity in the entire process and the significance of one or both parameter implies nonlinearity in the energy series, while for $\lambda_k = \gamma_k = 0$, the process is linear in time, and this becomes the fractional integration unit root test model of Robinson (1994) and Dolado et al. (2002).

By combining (3) with (5) and with further transformation, we have,

$$y_t^* = \alpha 1_t^* + \beta t_t^* + \sum_{k=1}^{n} \lambda_k \sin_{k,t}^* + \sum_{k=1}^{n} \gamma_k \cos_{k,t}^* + u_t, \qquad t = 1,2,\dots, \tag{6}$$

where $\sin_{k,t}^* = (1-L)^{d_o} \sin(2\pi k t/T)$, $\cos_{k,t}^* = (1-L)^{d_0} \cos(2\pi j_k t/T)$, $y_t^* = (1-L)^{d_o} y_t$: $1_t^* = (1-L)^{d_o} 1_t$: and $t_t^* = (1-L)^{d_o} t_t$, . The model in (7) then becomes linear in parameters and the coefficients $\hat{\theta}^T = \left(\hat{\alpha}, \hat{\beta}, \hat{\lambda}_1, \dots, \hat{\lambda}_n, \hat{\gamma}_1, \dots, \hat{\gamma}_n\right)^T$ are easily estimated using the Ordinary Least Squares (OLS) method. The model residuals are then obtained as,

$$\hat{u}_t = y_t - \hat{\alpha} 1_t^* + \hat{\beta} t_t^* + \sum_{k=1}^{n} \hat{\lambda}_k \sin_{k,t}^* + \sum_{k=1}^{n} \hat{\gamma}_k \cos_{k,t}^*, \qquad t = 1, 2, \dots, \tag{7}$$

and log-likelihood estimates are obtained. Thus, model diagnostic criteria such as the Sum of Squares Residuals (SSR) and Akaike information criterion (AIC) are obtained. The major problem is how to obtain the optimal *k* value. A grid search experiment is set up where *k* takes values from 0 to 5 with increments of 0.1. Thus, with each *k* value, the model in (6) is estimated and information criteria are obtained. The optimal *k* value is such that gives minimum SSE and AIC among 50 sets of *k* values (see Gil-Alana and Yaya, 2020).

4. Empirical Results

From (7), by assuming the linear fractional integration framework of Robinson (1994) such that that the nonlinear Fourier form parameters λ_k and γ_k are insignificantly different from 0, then we have resulted in Tables 4-7 for price energy levels, log-returns and absolute squared returns, respectively. Both price levels and returns inform market efficiency since the random walk hypothesis is expected to be unpredictable in the returns, and prices are assumed to be I(1) processes. Robinson (1994) considers testing $d = d_o$ for any real h-vector d_o value such that the I(d = 1) hypothesis is rejected if the lower bound of the confidence interval (CI) of obtained d value is above 1. Also, mean reversion in the stationary range ($0.5 < d < 1$) is decisively rejected once the upper bound of CI(d) is less than 1. Robinson's (1994) approach relies on three regression models, of no regressors, intercept only, and a linear trend as in the Augmented Dickey-Fuller (ADF) unit root set up (see Dickey and Fuller, 1979). This makes the approach robust to modeling intercept and linear trends. For the three deterministic terms, selected models are based on the significance of intercept and linear trend parameters. These results are in bold in the reporting tables. In Table 4 Subsample A, that is, the subsample of energy prices before the Covid-19 pandemic, none of the intercept and linear trends are significant in the deterministic terms, thus a model with no regressors is selected as observed in bold. Here, the I(1) hypothesis of persistence estimate d is unrejected in the case of WTI Oil, Heating oil, Diesel, Kerosene, and Propane prices, while the I(1) hypothesis is rejected in favour of I(d > 1) in the case of Gasoline prices. By looking at the results in the second subsample (B), that is during the Covid-19 period, in the selected results in bold, we found lower persistence of energy prices compared to corresponding values in Subsample A. Thus, price persistence is lowered during the Covid-19 pandemic. The fact that persistence levels during the Covid-19 pandemic deviate further away from d = 1 as in Subsample A means that energy markets are less efficient during this phase. The market efficiency is more pronounced in the case of WTI Oil as the persistence estimate is found to be about 0.29 during the pandemic as against 0.96 before the pandemic. So this triggers more fear in the crude oil market as the mean reversion tendency is high.

PUT TABLE 4 AROUND HERE

By further probing into the market efficiency level of energy markets based on log-returns, we present the results in Table 5. By looking at the results based on the selected deterministic terms in bold, we found more deviation from the I(d = 0) hypothesis in a negative direction in the results of Subsample A compared to the results for Subsample A?. Thus, energy markets are more efficient before Covid-19 pandemic compared to the current pandemic period that we are in. Based on the results in Tables 4 and 5, market participants in oil and petroleum products' trading are likely to make an excessive profit during this period, which is a blessing in disguise.

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In Table 6, the results of volatility persistence are presented. Recall in the descriptive measurements that the Covid-19 period induces more price volatility compared to the period before the Covid-19 pandemic. We only found exceptions in the case of Gasoline and Propane. The results of fractional persistence on the persistence of such volatility imply that volatility shocks during the Covid-19 pandemic will last for a shorter period compared to the period before the pandemic. This is based on smaller persistence *d* values during the pandemic compared to those estimates found before the pandemic. The WTI oil price indicates anti persistence value, thus persistence level cannot be ascertained, and this is the only exception.

PUT TABLE 6 AROUND HERE

Further probe into the results of fractional persistence on the price level for market efficiency and absolute returns for volatility is based on a nonlinear approach proposed in Gil-Alana and Yaya (2020). The results obtained are presented in Tables 7 and 8 for price level and absolute returns series, respectively. For the energy price series in Table 7, nonlinearity is pronounced as Fourier function parameters λ_k and γ_k are significant except in the case of Heating oil during pandemic subsample. Note, optimal *k* values in each case were determined by a grid search among *k* values from 0 to 5 with a step increase of 0.1. The optimal *k* values are as well reported in the results table.[1] Time series persistence is found to be higher during the pandemic (Subsample B) compared to the period before the pandemic (Subsample A), as the fractional d value for WTI Oil was 0.7122 before the pandemic and this is about 0.1100 implying that the WTI Oil market is less efficient during this period compared to the earlier period before the pandemic. Thus, the results also support evidence that market players are likely to make abnormal returns during the pandemic. Volatility persistence results in Table 8 indicate the antipersistence of absolute returns, implying the inability to judge the persistence of volatility based on the nonlinear fractional integration framework.

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5. Concluding remarks

The present paper investigates market efficiency and volatility persistence in energy pricing markets, by using price series of Crude oil, Gasoline, Diesel, Heating oil, Kerosene, and

[1] Optimal values of k are determined with corresponding Log likelihood, SSE and AIC estimates. These estimates are not reported in this paper but are available on request.

Propane. In the analysis, the pricing of these energy sources in the period before, and during the Covid-19 pandemic are compared for market efficiency and volatility persistence, using a 5-month daily data window, each, for the period before and during the pandemic. The analysis is based on a fractional integration approach in time series econometrics which allows the persistence parameter to be estimated on the price level series as well as on the returns and absolute returns used for volatility proxy. We also consider nonlinearity in the modeling of persistence, thus, an updated framework by Gil-Alana and Yaya (2020) that allows for a smooth nonlinear time trend with Fourier functions is employed.

We found that energy markets were less efficient during the Covid-19 pandemic, even though with higher volatility but with lesser volatility persistence compared to the period before the pandemic. Thus, volatility shocks lasted for a shorter period during the 5-month pandemic period than in the 5-month that preceded the pandemic. Thus, energy marketers were not prone to arbitrage profit-making during the Covid-19 pandemic period.

The findings of this paper have some implications. First, it shows that pandemics are likely to have an adverse impact on energy markets with its consequences on the prices of oil and petroleum products. For instance, higher volatility discourages investment in the market because existing and prospective investors become apprehensive for the fear of losing their capital or funds. Besides, declining investment forces market players (such as firms) to reduce employment of factors of production including labour leading to falling incomes and aggregate demand, all of which re-enforces the uncertainty in the business environment. Thus, policymakers are advised to devise a mechanism to reduce the negative effect of Covid-19 and future pandemics on the energy market. To achieve this, there is a need for international collaboration and cooperation to develop the global healthcare system via increased and sustained funding of healthcare facilities particularly in poor countries with dilapidated infrastructures. This will not only help in reducing the Covid-19 pandemic but also prepare countries to tackle future health crises. Besides, there is a need for a continuous awareness campaign to enlighten people all over the world to embrace the ‘new normal’ and non-pharmaceutical measures to check the spread of the Covid-19 virus.

Our analysis is subject to criticism but we have employed an up-to-date persistence approach based on nonlinearity. Meanwhile, due to the limitation of the short trading period for the time series of price movements of those energy markets, we recommend persistence analysis based on neural network nonlinearity as in Yaya et al. (2021) for testing for energy market efficiency and their volatility persistence.

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The data that support the findings of this study are openly available in US Energy Information and Administration (EIA) at https://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm

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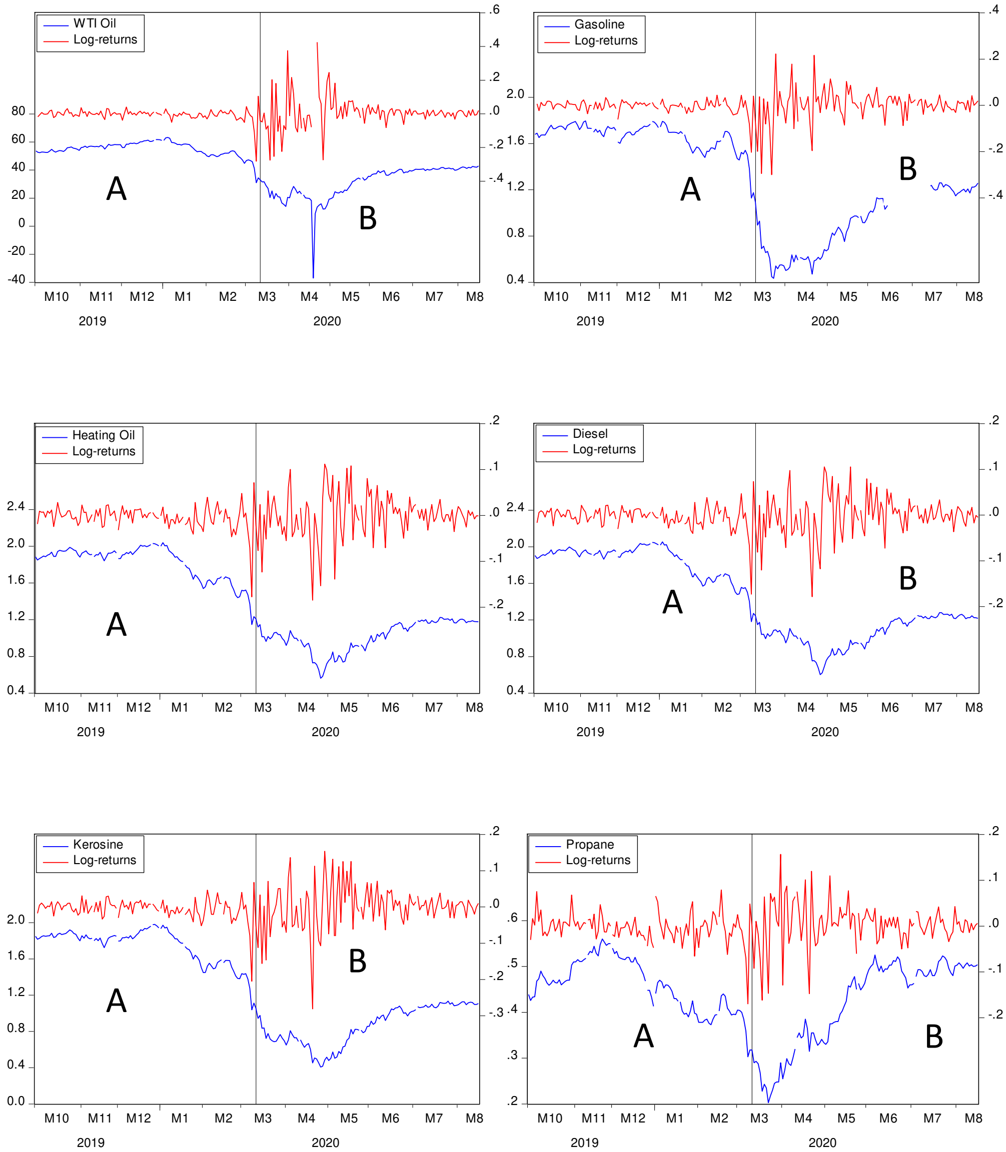

Figure 1: Plots of Oil and Petroleum Products pricing

Table 1: A Cursory Review of Literature on Market Efficiency of energy pricing

Authors	Objectives	Methodology	Data Structure	Findings
Lean et al (2010)	Market efficiency of oil spot and futures prices	Mean-variance and stochastic dominance approaches	From 1989 to 2008	No arbitrage returns at oil spot and futures prices while spot and futures oil markets rationally efficient
Zhang (2013)	Weak form efficiency of world-main crude oil markets.	Generalized spectral method	World crude oil price data from January 2 to 20 May 1987.	Highest level of market efficiency is found for Brent and WTI oil, while anti-synchronization is also found in the two markets
Mensi et al. (2014)	Time-varying levels of weak-form efficiency and the presence of structural breaks in crude oil markets	Hurst and Shannon entropy methods.	Crude oil benchmarks over the period from January 1990 to September 2012	The Hurst exponent produced more market efficient estimate than when Shannon entropy is used
Sharma (2017)	Explored how high-frequency data generating trading strategies using information flow direction between Indian and US crude oil future markets.	Vector error correction	Daily data from 2013 to 2015	Close relationship between the US and Indian market prices, while US markets are more efficient than Indian markets.
Qiao et al (2019)	Market efficiency of oil stocks under various types of oil price changes	Quantile regression and interval-valued factor pricing models	March 2018 to 2nd February 2019	Oil stocks tend to be overpriced due to negative shocks
Ghazzani and Ebrahimi (2019)	Adaptive market hypothesis to the	Automatic portmanteau and	Daily returns of crude oil from 2003 to 2018	Brent and WTI oil have high-efficiency levels.

	efficient market hypothesis	generalized spectral tests		
Kuruppuarachchi et al. (2019)	Proposed novel futures market efficiency index in investigating efficiencies of four major energy commodities	Heteroscedastic prices and time-varying risk premiums	1990-2016	significant delayed, contemporaneous, and potential information spillovers among term premiums of the energy commodities
Tiwari et al. (2019)	Measuring oil price efficiency using long memory dependencies	ARFIMA model	Monthly data from 1990 to 2017.	Time variation in the efficiency of oil returns
Arshad et al. (2020)	Investigating weak-form of market efficiency in crude oil markets during different economic cycles over multi-scales.		Crude oil prices from 1996 to 2018	Brent crude oil prices are weak-form efficient
Apergis and Gangopadhyay (2020)	Studying oil price for substitution effect on pollution in Vietnam		Data from 1982 to 2015	asymmetric interlinkages
Lim and Lee (2020)	Proposing a two-stage method of portfolio theory and panel data analysis in analyzing oil refining industry	Markowitz portfolio optimization	2005 to 2016	the negative effect of Crude oil production and energy of the OECD on the efficiency of the oil refinery industry.

	efficiency of OECD countries.			
David et al. (2020)	Proposing detrended fluctuation analysis, Hurst exponent, and fractal dimension in investigating the dynamic behaviour of ethanol and gasoline prices in Brazil.	Detrended fluctuation analysis	Weekly prices from January 2011 to December 2016.	Subtle path toward market efficiency for ethanol with clear moving toward better efficiency for the gasoline market in Brazil
Yang et al. (2020)	Investigating pricing of crude oil features in Shanghai International Exchange (INE)	Cointegration and Granger causality	Data on crude oil spot prices from 26th March 2018 to 2nd February 2019	The evidence of Granger causality is mixed but supports the efficiency of the Shanghai International Exchange (INE) in the Asia-pacific region

Compiled by the Authors

Table 2: A Cursory Review of Literature on Volatility Persistence of Energy pricing

Authors	Objectives	Methodology	Data Structure	Findings
Narayan and Narayan (2007)	The volatility of crude oil price	GARCH model	Daily data for the period 1991 to 2006.	Inconsistent evidence of asymmetry and persistence of shocks with the possibility that oil prices would change over short periods
Golpe, et al. (2012)	Persistence in the consumption of natural gas	State-space model	Quarterly data from January 1973 to March 2010.	Positive policy shocks in natural gas consumption having permanent effects.
Ozdemir, et al. (2013)	Persistence in Brent crude oil spot and futures prices	ARIMA model	Monthly data between October 1993 and December 2011	Brent crude oil spot and futures are persistent
Charles and Darne (2013)	Impact of outliers and structural changes on the volatility persistence of crude oil markets.	GARCH-type models	Daily data from 2nd January 1985 and 17th June 2011	With outliers in the volatility modeling improving the understanding of volatility in crude oil markets
Bastianin et al (2016)	Effects of oil prices shocks on the stock market volatility of the G7 countries	Structural VAR model	Daily data for G7 countries between February 1973, and January 2015.	The irresponsibility of stock market volatility to oil supply shocks while demand shocks impact significantly on the stock market volatility of G7 countries.
Olubusoye and Yaya (2016)	Persistence and volatility of oil and petroleum products' prices	Fractional integration and Generalized Autoregressive (GAS) model variants	Different daily sample sizes, far back 1986 to 2014	Crude oil persist relatively differently from petroleum products prices
Gil-Alana et al. (2016)	Persistence in price and volatility of WTI oil price and also identify bull and bears market prices	Fractional integration	Monthly data between September 1859 and July 2015.	Oil price was found to be non-stationary and volatility exhibits long-memory while the degree of persistence increases when market phases are identified.

Ewig and Malik (2017)	Endogenously determined structural breaks within asymmetric GARCH models in reducing reduce volatility persistence in oil	Asymmetric GARCH model	Crude oil prices from 1st January 2000 to 31st December 2015	Good and bad news has significantly more impact on volatility if structural breaks are incorporated
Andriosopoulos et al. (2017)	in the extent to which the financially troubled EU markets affected energy prices during financial crises.	GARCH model	March 2004 and March 2014	Significant contagion effects like energy volatility during the EU financial crisis.
Chun, et al. (2019)	Performance of crude oil hedge portfolios in five periods	Stochastic Volatility (SV), GARCH, and diagonal BEKK model were used	Daily closing prices of the UK between 6/23/1988 and 9/29/2017.	Hedging strategies based on the SV model outperformed both the GARCH and BEKK models in terms of variance reduction.
Mokni and Youssef (2019)	Persistence between crude oil prices and GGC countries stock markets	Copula functions	Daily data from 2010 to 2017.	Strong persistence of the upper tail compared to the lower tail while the persistence of dependence is affected by the oil crises and not affected by the asymmetric variation of oil prices.
Okorie and Lin (2020)	Volatility connectedness and hedging strategy possibilities between both 10 cryptocurrency and crude oil markets	Adopted MGARCH-DCC and VAR-MGARCH-BEKK approaches	Daily data from 29/04/2013 to 17/09/2019	Significant volatility spillovers in both hedging and market possibilities.
Maitra et al. (2020)	Volatility spillover and connectedness between liner	Dynamic conditional equicorrelations	Daily data from 3rd January 2000 to 14th January 2019.	The volatility co-movement between liner shipping and oil companies 'stock returns increased during the global financial crisis and European debt crisis.

	shipping and oil markets	and spillover index was utilized		
Sarwar et al. (2020)	Volatility spillover of oil and stock market returns.	BEKK-GARCH model	Multi-frequency data, over the period 1st July 1997 to 31st December 2015.	Spillover between oil and stock markets.
Zavadska. et al. (2020)	Volatility patterns in Brent crude oil spot and futures prices during four major crises as it affected oil markets.	GARCH type models	The data sample spans from 7th December 1988 to 31st December 2013.	Higher level of volatility during crises that was directly associated with oil supply/demand disruptions and higher volatility persistence during financial crises

Compiled by the Authors

Table 3: Descriptive measurements

Subsample A: Before the Covid-19 pandemic										
	Prices					Log-returns				
	Mean	**Max.**	**Min.**	**Std. dev.**	**JB**	**Mean**	**Max.**	**Min.**	**Std. Dev.**	**JB**
WTI Oil	49.89	63.27	14.10	12.72	52.82***	-0.0063	0.3747	-0.2814	0.0705	631.91***
Gasoline	1.49	1.80	0.43	0.41	63.37***	-0.0080	0.2222	-0.2999	0.0582	782.20***
Heating oil	1.68	2.04	0.92	0.34	24.16***	-0.0051	0.1001	-0.1774	0.0310	432.36***
Diesel	1.71	2.05	0.95	0.34	24.83***	-0.0050	0.0976	-0.1728	0.0306	397.50***
Kerosene	1.58	1.98	0.65	0.41	32.02***	-0.0076	0.1364	-0.2047	0.0399	292.57***
Propane	0.43	0.56	0.20	0.09	11.37***	-0.0024	0.1565	-0.1696	0.0438	93.56***
Subsample B: During the Covid-19 pandemic										
Prices	**Prices**					**Log-returns**				
	Mean	**Max.**	**Min.**	**Std. dev.**	**JB**	**Mean**	**Max.**	**Min.**	**Std. Dev.**	**JB**
WTI Oil	34.40	42.89	12.17	8.83	19.70***	0.0158	0.4258	-0.2730	0.0735	688.46***
Gasoline	1.02	1.26	0.47	0.23	11.84***	0.0082	0.2168	-0.1960	0.0496	108.80***
Heating oil	1.02	1.23	0.56	0.18	8.99***	0.0020	0.1119	-0.1846	0.0500	35.33***
Diesel	1.06	1.28	0.60	0.19	8.61***	0.0022	0.1054	-0.1783	0.0442	37.75***
Kerosene	0.89	1.13	0.41	0.23	10.99***	0.0051	0.1535	-0.2810	0.0599	117.42***
Propane	0.46	0.53	0.32	0.06	14.78***	0.0043	0.1195	-0.1473	0.0352	81.31***

*** denotes the significance of the Jarque-Bera (JB) normality test at a 5% level.

Table 4: Robinson (1994) Fractional integration results on Energy Prices

Subsample A: Before the Covid-19 pandemic			
	No regressors	**An intercept**	**A linear time trend**
WTI Oil	**[0.9574 (1.1072) 1.2570]**	[0.9555 (1.1054) 1.2552]	[0.9316 (1.0860) 1.2404]
Gasoline	**[1.0008 (1.1719) 1.3429]**	[1.0008 (1.1720) 1.3431]	[0.9710 (1.1492) 1.3274]
Heating oil	**[0.9497 (1.0971) 1.2445]**	[0.9543 (1.1065) 1.2586]	[0.9030 (1.0676) 1.2322]
Diesel	**[0.9419 (1.0881) 1.2343]**	[0.9458 (1.0962) 1.2465]	[0.8941 (1.0566) 1.2191]
Kerosene	**[0.9703 (1.1126) 1.2548]**	[0.9782 (1.1274) 1.2766]	[0.9281 (1.0888) 1.2495]
Propane	**[0.9457 (1.1143) 1.2830]**	[0.9457 (1.1143) 1.2830]	[0.9225 (1.0992) 1.2759]
Subsample B: During the Covid-19 pandemic			
	No regressors	**An intercept**	**A linear time trend**
WTI Oil	[0.4311 (0.5564) 0.6817]	[0.4324 (0.5564) 0.6805]	**[0.2897 (0.4360) 0.5823]**
Gasoline	[0.8427 (0.9787) 1.1147]	[0.8141 (0.9453) 1.0764]	**[0.6780 (0.8362) 0.9943]**
Heating oil	**[0.7998 (0.9540) 1.1082]**	[0.8025 (0.9576) 1.1127]	[0.7845 (0.9447) 1.1049]
Diesel	**[0.8216 (0.9738) 1.1260]**	[0.8236 (0.9762) 1.1288]	[0.8051 (0.9631) 1.1211]
Kerosene	**[0.8283 (0.9608) 1.0932]**	[0.8249 (0.9560) 1.0871]	[0.7785 (0.9202) 1.0619]
Propane	[0.8872 (1.0007) 1.1142]	[0.8567 (0.9899) 1.1231]	**[0.7492 (0.9064) 1.0635]**

In bold are selected estimates of d based on significant parameters of the deterministic function.Estimates of d are in the curved bracket, enclosed within the squared brackets for lower and upper confidence bands for d.

Table 5: Fractional integration results on Log-returns series

Subsample A: Before the Covid-19 pandemic			
	No regressors	**An intercept**	**A linear time trend**
WTI Oil	[-0.1137 (0.0351) 0.1839]	[-0.1137 (0.0356) 0.1848]	**[-0.2403 (-0.0736) 0.0930]**
Gasoline	[-0.0295 (0.1422) 0.3140]	[-0.0283 (0.1427) 0.3137]	**[-0.0950 (0.0846) 0.2643]**
Heating oil	[-0.1183 (0.0412) 0.2007]	[-0.1181 (0.0415) 0.2010]	**[-0.2593 (-0.0765) 0.1062]**
Diesel	[0.1292 (0.2863)0.4435]	[-0.1284 (0.0289) 0.1862]	**[-0.2669 (-0.0876) 0.0918]**
Kerosene	[-0.0837 (0.0737) 0.2311]	[-0.0831 (0.0743) 0.2316]	**[-0.1919 (-0.0185) 0.1549]**
Propane	[-0.0824 (0.0895) 0.2615]	[-0.0824 (0.0895) 0.2614]	**[-0.1722 (0.0164) 0.2051]**
Subsample B: During the Covid-19 pandemic			
	No regressors	**An intercept**	**A linear time trend**
WTI Oil	**[-0.6844 (-0.2913) 0.1019]**	[-0.5861 (-0.2327) 0.1207]	[-0.6844 (-0.2913) 0.1019]
Gasoline	**[-0.2326 (-0.0857) 0.0612]**	[-0.2249 (-0.0791) 0.0667]	[-0.2326 (-0.0857) 0.0612]
Heating oil	**[-0.1892 (-0.0169) 0.1553]**	[-0.1620 (0.0036) 0.1693]	[-0.1892 (-0.0169) 0.1550]
Diesel	**[-0.1572 (0.0119) 0.1810]**	[-0.1347 (0.0289) 0.1925]	[-0.1572 (0.0119) 0.1810]
Kerosene	**[-0.2000 (-0.0572) 0.0856]**	[-0.1727 (-0.0342) 0.1044]	[-0.2000 (-0.0572) 0.0856]
Propane	**[-0.2750 (-0.1172) 0.0406]**	[-0.2546 (-0.1018) 0.0510]	[-0.2750 (-0.1172) 0.0406]

In bold are selected estimates of d based on significant parameters of the deterministic function. Estimates of d are in the curved bracket, enclosed within the squared brackets for lower and upper confidence bands for d.

Table 6: Fractional integration results on Absolute returns series

Subsample A: Before the Covid-19 pandemic			
	No regressors	**An intercept**	**A linear time trend**
WTI Oil	**[0.2669 (0.4387) 0.6106]**	[0.2655 (0.4375) 0.6096]	[0.2133 (0.3919) 0.5706]
Gasoline	[0.1660 (0.3187) 0.4715]	[0.165 (0.3178) 0.4701]	**[0.0916 (0.2522) 0.4127]**
Heating oil	[0.2001 (0.3647) 0.5294]	[0.1992 (0.3639) 0.5286]	**[0.1047 (0.2847) 0.4646]**
Diesel	[0.2145 (0.3805) 0.5466]	[0.2135 (0.3797) 0.5460]	**[0.1189 (0.3006) 0.4824]**
Kerosene	[0.2303 (0.3922) 0.5541]	[0.2293 (0.3913) 0.5534]	**[0.1470 (0.3218) 0.4966]**
Propane	[-0.0105 (0.1606) 0.3318]	[-0.0092 (0.1615) 0.3323]	**[-0.0973 (0.0937) 0.2847]**
Subsample B: During the Covid-19 pandemic			
	No regressors	**An intercept**	**A linear time trend**
WTI Oil	[-0.2761 (0.0665) 0.4091]	[-0.2847 (0.0636) 0.4119]	**[-0.8728 (-0.5192) -0.1657]**
Gasoline	[0.0477 (0.1840) 0.3203]	[0.0490 (0.1876) 0.3262]	**[-0.2874 (-0.1054) 0.0766]**
Heating oil	[0.0928 (0.2171) 0.3413]	[0.0928 (0.2174) 0.3421]	**[-0.0213 (0.1146) 0.2505]**
Diesel	[0.0723 (0.1973) 0.3224]	[0.0723 (0.1979) 0.3235]	**[-0.0806 (0.0608) 0.2022]**
Kerosene	[0.1178 (0.2418) 0.3659]	[0.1183 (0.2435) 0.3686]	**[-0.0629 (0.0811) 0.2251]**
Propane	[0.1425 (0.2842) 0.4258]	[0.1438 (0.2879) 0.4320]	**[-0.1028 (0.0716) 0.2460]**

In bold are selected estimates of d based on significant parameters of the deterministic function. Estimates of d are in the curved bracket, enclosed within the squared brackets for lower and upper confidence bands for d.

Table 7: Gil-Alana and Yaya (2020) Fractional integration results on Energy Prices

Subsample A: Before the Covid-19 pandemic						
	k_{opt}	d	α	β	λ_k	γ_k
WTI Oil	0.1	**[0.7122(0.8951)1.0780]**	**467.10 (1.72)**	**-14.48 (-3.61)**	**2648.74 (3.78)**	**-474.12 (-1.78)**
Gasoline	0.3	**[0.8723(1.0609)1.2495]**	2.51 (1.17)	**-0.07 (-1.68)**	**3.43 (2.08)**	-2.80 (-1.21)
Heating oil	0.1	**[0.7476(0.9363)1.1250]**	1.82 (0.12)	-0.21 (-0.83)	40.06 (0.87)	-2.06 (-0.13)
Diesel	0.8	**[0.7582(0.9456)1.1330]**	0.32 (0.49)	**-0.008 (-1.97)**	0.04 (0.32)	-0.26 (0.11)
Kerosene	0.4	**[0.7823(0.9640)1.1457]**	0.88 (0.48)	**-0.029 (-1.72)**	**1.11 (3.00)**	-0.95 (-1.05)
Propane	0.3	**[0.7168(0.9005)1.0842]**	52.96 (0.95)	**-1.63 (-2.01)**	**96.81 (3.05)**	-59.51 (-1.26)
Subsample B: During the Covid-19 pandemic						
	k_{opt}	d	α	β	λ_k	γ_k
WTI Oil	1.6	**[0.1100 (0.2782) 0.4464]**	**-15.84 (-5.30)**	**0.30 (6.89)**	-1.89 (-1.11)	**6.12 (3.94)**
Gasoline	1.3	**[0.4927 (0.6818) 0.8709]**	**-0.45 (-4.21)**	**0.008 (5.74)**	**-0.13 (-3.45)**	0.04 (1.09)
Heating oil	1.3	**[0.6416 (0.8278) 1.0140]**	-0.25 (-1.22)	0.003 (1.53)	-0.02 (-0.43)	**0.14 (2.80)**
Diesel	1.4	**[0.6651 (0.8466) 1.0281]**	-0.32 (-1.32)	**0.004 (1.81)**	0.008 (0.15)	**0.14 (2.93)**
Kerosene	0.1	**[0.6428 (0.8021) 0.9614]**	**38.70 (3.44)**	**-0.62 (-3.28)**	**103.95 (3.27)**	**-38.62 (-3.47)**
Propane	1.0	**[0.6187 (0.8012) 0.9837]**	**-0.17 (-2.59)**	**0.002 (3.46)**	-0.02 (-0.77)	**-0.05 (-2.53)**

Note, in the first column are the k_{opt} values i.e. optimal value of frequency k that gives minimum SSE and AIC values. In the second column of the results table, estimates of d are in a curved bracket, enclosed within the squared brackets for lower and upper confidence bands for d. In the third to sixth columns are parameters of the nonlinear fractional integration with t statistics in parentheses.In bold indicates significant estimates at a 5% level. The critical values of the t-tests at this level of significance are based on a one-sided hypothesis with t = 1.64.

Table 8: Gil-Alana and Yaya (2020) Fractional integration results on Absolute returns

Subsample A: Before the Covid-19 pandemic						
	k_{opt}	**d**	**α**	**β**	**$λ_k$**	**$γ_k$**
WTI Oil	0.1	**[-0.0287(0.1716)0.3719]**	**-3.51 (-12.1)**	**0.07 (28.5)**	**13.0 (29.0)**	**3.49 (12.4)**
Gasoline	0.1	[-0.1847 (0.0003) 0.1853]	**-2.81 (-19.9)**	**0.06 (281.0)**	**-10.71 (193.0)**	**2.79 (20.4)**
Heating oil	0.1	[-0.1093 (0.0932) 0.2957]	**-1.71 (-10.8)**	**0.04 (92.4)**	**-6.99 (-81.5)**	**1.70 (11.0)**
Diesel	0.1	[-0.0885 (0.1149) 0.3183]	**-1.68 (-10.4)**	**0.04 (79.3)**	**-6.85 (-72.2)**	**1.67 (10.6)**
Kerosene	0.1	[-0.0407 (0.1526) 0.3459]	**-2.55 (12.0)**	**0.06 (50.1)**	**-9.86 (-49.5)**	**2.54 (12.3)**
Propane	0.1	[-0.1460 (0.0475) 0.2410]	0.19 (1.01)	**0.003 (4.68)**	**-0.51 (-4.82)**	-0.19 (-1.03)
Subsample B: During the Covid-19 pandemic						
	k_{opt}	**d**	**α**	**β**	**$λ_k$**	**$γ_k$**
WTI Oil	0.1	**[-1.0783 (-0.5924) -0.1065]**	**-1.87 (-7.84)**	**0.11 (6.37)**	**-1.14 (-4.91)**	**1.76 (7.88)**
Gasoline	0.1	**[-0.4434 (-0.2276) -0.0118]**	**0.05 (6.82)**	**-0.0008 (-6.26)**	-0.002 (-0.36)	**0.01 (2.76)**
Heating oil	0.1	[-0.2766 (-0.1043) 0.0680]	0.006 (0.92)	-9.9E-05 (-0.86)	**0.02 (3.96)**	**-0.01 (-3.94)**
Diesel	0.1	[-0.3015 (-0.1282) 0.0451]	0.007 (1.41)	-0.0001 (-1.26)	**0.02 (4.26)**	**-0.007 (-2.75)**
Kerosene	0.1	[-0.2337 (-0.0620) 0.1097]	0.01 (1.57)	-0.0003 (-1.54)	**0.02 (3.35)**	-0.006 (-1.32)
Propane	0.1	[-0.2592 (-0.0542) 0.1508]	**0.03 (4.62)**	**-0.0006 (-4.46)**	-0.0005 (-0.09)	**0.10 (2.83)**

Note, in the first column are the k_{opt} values i.e. optimal value of frequency k that gives minimum SSE and AIC values. In the second column of the results table, estimates of d are in a curved bracket, enclosed within the squared brackets for lower and upper confidence bands for d. In the third to sixth columns are parameters of the nonlinear fractional integration with t statistics in parentheses. In bold indicates significant estimates at a 5% level. The critical values of the t-tests at this level of significance are based on a one-sided hypothesis with t = 1.64.