Gold and Silver prices, their stocks and market fear gauges: Testing fractional cointegration using a robust approach

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Abstract

The present paper investigates the long-run relationships between daily prices, stocks and fear gauges of gold and silver by employing an updated fractional cointegrating framework, that is, the Fractional Cointegrating Vector Autoregression (FCVAR). The initial unit root tests results indicate that the series are I(d)s with values of d around 1 in all cases, and these are homogenous in the paired cointegrating series. Evidence of cointegration is found in the three pairs (prices, stocks and market gauge indices), while these cointegrations are only time-varying in the case of market gauge indices for the commodities. The fact that cointegration exists in prices and stocks of gold and silver implies the possibility that gold and silver prices and stocks can interchangeably be used to access the performances of the commodity markets, with the recommendation that the two commodities are not to be traded in the same portfolio.

Keywords: Fractional cointegration; FCVAR; Gold; Silver; Mean reversion; Market fear gauges
JEL Classification: C22; C32

1. Introduction

As history has it, gold and silver have always been considered as precious metals, and they are often seen as replacements to each other in order to minimize similar types of risk in portfolios (Mani and Vuyyuri 2005; Ciner, 2001; Escribano and Granger, 1998). Comparable to gold, silver also serves many purposes and at times, it serves as an inflationary hedge. In the opinion of metal traders, silver is considered to have higher volatility compared to gold. However, gold and silver possess distinctive features and serve different purposes, and their markets need to be separated.

Gold and silver are the two most valuable commodities that are traded globally by investors. The physical price of gold is the amount of US dollars that is needed to purchase an ounce of gold, and similarly to silver. As at the time of reporting, an ounce cost of gold is the price of 72 ounces of silver, thus gold is relatively expensive compared to silver. Silver prices change more in bull and bear period compared to gold as the price of silver is easily manipulated for industrial demand. Instead of holding these physical commodities due to market pricing and substitute competitions, investors also consider gold-like assets. Gold mining stocks are examples of gold-like assets (Dar, Bhanja and Paul, 2019). These assets behave like other stocks which are affected by company management strategy, extraction costs and reserves, debt, and macroeconomic policies. Thus, gold mining stocks are expected to behave like gold, that is, a commodity or a share or both. Market fear index computed for any financial series measures uncertainty at a particular time in the market, with values around 0 implying much stable market values, and values close to 100 imply quite unstable or crisis-induced market values. Market fear index is computed by the Chicago Board of Exchange (CBOE) for both gold and silver stocks. The CBOE rely on Exchange Traded Fund (ETF) values for stocks.[1] Thus, price level, stock performances and fear index levels (volatility velocity) are parameters often used to assess the performances of gold and silver at commodity markets.

Prominent among ETFs that track gold mining stocks are VanEcK Vectors Gold Miners ETF (GDX), VanEcK Vectors Junior Gold Mine (GDXJ), iShares Gold Trust (IAU), SPDR Gold Trust (GLD) and Aberdeen Standard Physical Gold (SGOL), while silver stock

[1] These are various gold and silver miners' stock indices. Some are ETF which tracks stocks indices of the respective commodity.

markets are iShares Silver Trust (SLV), ProShares Ultra Silver (AGQ), iShares MSCI Global Silver Mine (SLVP), among others.[2] When we mention the stock market, its importance is inescapable, as it is not possible to have a world without a stock market. The market contributes prominently to the economic development of any country and can have a negative impact when not properly monitored. Recently, as reported by VanEcK Vectors Gold Miners ETF, a total return of 61.1% was posted by gold compared to the total return of the S&P500 of 8.9% in the last 12 months. Also, Gold stocks have outperformed the broader market, that is, the FTSE Gold Russell 1000 total return of 19.7% as of December 2020. Global X Silver Miners ETF reflected that silver posted a total return of 40.6% compared to the S&P 500's total return of 5.8%, thereby significantly outperformed the S&P 500 index in the last 12 months. These reports indicate that gold and silver stocks have outperformed the broader commodity market dramatically, with gold having higher performance.

A good number of researches have been conducted, studying the movement and relationship between gold and silver stocks and their prices. Michael & Swanson (1981) studied the efficiency in the gold and silver market and examined the relationship between the two markets. The study could not find a correlation of any macroeconomic variable with price movements of gold and silver. The dynamic nature of the performance of silver and gold prices could not allow the application of any traditional model and hence the study failed to ascertain the effectiveness of gold and silver markets. In a study conducted by Ciner (2001) on silver and gold prices on the Tokyo Commodity Exchange (TOCOM), the study adopted Johansen's (1991) maximum likelihood cointegration analysis. The study found that the cointegration of the silver and gold market has disappeared as far back in the 1990s. In the study, a conclusion was drawn that over time, the long term connection between silver and gold prices has dematerialized. The two markets should therefore be separately

[2] The VanEcK Vectors Gold Miners ETF seeks to track price performance of NYSEÂ® Arca Gold Miners Index. The fund invests at least 80% of its investment on stocks from gold mining companies.

approached and not be regarded as a replacement of each other to eliminate certain risks types. Christian et al. (2015) in their study used Residual Augmented Least Squares (RALS) test to study the cointegration of gold and silver prices. The RALS test takes into account the dynamism in price which could be a result of the financial crisis. The result of the RALS test shows more evidence of cointegration in prices of Gold and Silver than based on the results obtained from the standard Dickey-Fuller test. In a study carried out by Zhu et al. (2016), the authors examined the quantile behaviour of cointegration between gold and silver prices by adopting the Quantile Autoregressive Distributed Lag (QARDL) model. The study found evidence of cointegration and suggested that the existence of cointegration is a result of the tail-quantiles that falls outside the interquartile range and revealed time-varying cointegrating coefficients that might not be found in the result of traditional analysis. The study further revealed that, in the short-run dynamics, the contemporaneous change in gold price overtook that of the silver price. Another study by Karsten (2018) employed the quantile cointegration model to study the long-run relationship between gold and silver prices. The study applied a state-dependent and a time-varying cointegrating vector. The result of the study could not show evidence of constant cointegration between gold and silver, but a nonlinear long-run relationship exists between the two variables. The study further indicates that there is an asymmetry in the relationship between the variable.

Gil-Alana, Yaya, and Awe (2017) in their study employed fractional integration and cointegration method to examine the co-movements of gold and oil prices. At first, they employed the standard unit root and cointegration tests. The results show that the two series are individually integrated of orders one, however, cointegrated. They later applied the fractional cointegration methods and found evidence of fractional cointegration relationship between the two variables, with the long-run relationship having an order of integration less than 0.5. Bibhuti et al. (2019) in their study examined the dynamic causal relationship between the returns of silver and gold in the Indian market. They used monthly data that spans the period, June 1991 to June 2018. They, at first employed a rolling window bootstrap approach to study the causal relationship between the variables and then employed wavelet-based time-varying and a non-linear Granger-causality test to examine the causality between variables. The study shows evidence that there exist significant positive effects and time-varying negative causality running from gold to silver, that is, a unidirectional causality from gold to silver.

By using gold and silver mining stocks or ETFs, Naylor, Wongchoti, and Gianotti (2011) investigated whether abnormal returns are available through gold and silver ETFs. Using the Capital Asset Pricing Model (CAPM), the study showed that abnormal returns could not be achieved through the Standard and Poor's Depository Receipt (SPDR) Gold Shares (GLD) fund. Emmrich and McGroarty (2013) investigated the diversification of gold ETF benefits, the authors found that ETFs reduce portfolio volatility more than bullion, but noted that the sample period for ETFs was shorter. Ivanov (2013) investigated the effect of gold and silver ETFs on price discovery in their respective futures markets. The author argued that the foundation of the ETFs has led to a reduction in the importance of the future of the ETFs, which are now leading price discoveries for both markets. Dar, Bhanja and Paul (2019) investigated correlations of gold mining stocks on gold and equity prices in the US and the UK and found strong positive relationship existing up to some time points in the datasets while weak relationship exists in another period. The authors, in Paul, Bhanja and Dar (2019) further investigated the co-movement among those assets using convectional Maximal Overlap Discrete Wavelet Transform correlation matrix with partial wavelet coherence. The results showed high coherence of gold price and gold mining stocks, while the gold price and gold mining stocks are weakly coherent.

The CBOE Gold and Silver ETF volatility indices have been used in some papers to study their usefulness and interrelationships. Jubinski and Lipton (2013) examined the interrelationship between implied and contemporaneous volatility and gold, silver, and oil future returns. The study found a statistically significant positive relationship between gold and silver future returns and implied volatility, but not contemporaneous. The findings of the study support the view of gold as a safe haven and silver as a pure commodity. Yu et al., (2019) investigated the appropriateness of the CBOE gold and silver volatility indices in forecasting the realised volatility (RV) of gold futures volatility in China. The authors employed the Heterogeneous Autoregressive (HAR) and Ridge regression models. The models were used with the China gold futures volatility indices and the CBOE gold and silver volatility indices. The study showed that the models with CBOE gold and silver volatility indices show significant predictive performance than the models with their counterpart. Korhan and Negar (2015) investigated the long-run relationship between gold price, oil price, oil price volatility index and gold price volatility index using the Autoregressive Distributed Lag (ARDL) cointegration approach. The study found a long-run equilibrium among all the four variables considered. The study further revealed that the variables have a long-run impact on S&P500 stock market price index, however, the gold price has the highest impact on the stock price in short-run and long-run. Boscaljon and Clark (2013) examined the degree of market uncertainty as measured by the CBOE volatility index (VIX). The results of the study revealed that huge increases in the VIX index result in a positive unusual return on assets in the gold and silver ore companies and the SPDR Gold Shares (GLD) exchange of traded funds (ETF). The performance of common stocks in the gold and silver ore industries and the GLD ETFs are examined about the level of market uncertainty. Market uncertainty is increased by 10 per cent, 25 per cent and 50 per cent in the VIX index compared to its 75-day moving average.

With the fact that gold and silver are closed substitute, the present study seeks to investigate the market performance of the two in cointegrating analyses. Specifically, we investigate gold and silver daily prices, their stocks and their market fears (volatility indices) using the updated fractional cointegrating framework. This is the fractional cointegrating vector autoregressive (FCVAR) model of Johansen and Nielsen (2012). The results obtained are interesting since classical unit root tests could have wrongly judged the order of the paired time series, and the tests are limited in their applicability. Whereas, the overall cointegration orders of the FCVAR are found to be long-range dependent (LRD) as against I(0) series in the classical definition of cointegration. The LRD means that the effects of the shocks in the long run equilibria in the three cointegrating results are temporal.

The findings in the paper are quite interesting since this is the first paper that studied the three market performance indicators of gold and silver using fractional integration and cointegration. The results will clear the air with readers, researchers and investors as par each variable when gold and silver performances are being discussed, particularly their prices and stocks. Also, testing such relationships has implications for portfolio management and its construction since it hardly pays investors to include assets of similar pricing relationships in the same portfolio.

The rest of the paper is structured as follows: Section 2 presents the fractional cointegration framework applied in the paper. Section 3 presents the data and the empirical results, while section 4 concludes the paper.

2. Fractional Cointegrating framework

We begin the procedure by estimating fractional differencing parameters in the series individually. This was carried out by adopting the semiparametric log-periodogram regression method by Geweke-Porter-Hudak (GPH) (Robinson, 1995a), and the Gaussian semi-parametric Local Whittle (LW) estimation method by Robinson (1995b). Robinson

(1995b), in his work, utilized the LW estimator proposed by Künsch (1987) which approximates the Maximum Likelihood Estimate (MLE) in the frequency domain.

The GPH method assumes that the spectrum of a time series process is of the form,

$$f(\lambda) = \left|1 - e^{-i\lambda}\right|^{-2d} f^{*}(\lambda), \tag{1}$$

where $f^{*}(\lambda)$ is expected to be the spectral density function that corresponds to the short-run components, Autoregressive Moving Average (ARMA) of the process. The estimated parameter *d,* in the frequency domain, was computed using the spectral density function at low frequencies represented by *m* Fourier frequencies, $\lambda_j = 2\pi j/n$, $j = 1, \ldots, m < n/s$. With the use of the periodogram,

$$I(\lambda) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} e^{it\lambda}\left(y_t - \bar{y}\right)^2\right|, \tag{2}$$

to approximate the spectral density function,

$$f(\lambda) = \left\{4\sin^2(\lambda/2)\right\} f_0(\lambda) \tag{3}$$

The estimate of parameter *d* is based on the GPH estimator,

$$\hat{d} = \frac{-\sum_{j=1}^{m}\left(y_j - \bar{y}\right)\log\left\{I\left(\lambda_j\right)\right\}}{2\sum_{j=1}^{m}\left(y_j - \bar{y}\right)^2} \tag{4}$$

As an approximation to the MLE in the frequency domain for large n, the LW (Gaussian Semi-parametric) estimator is given as

$$\hat{d} = \arg\min\left(\log\left[m^{-1}\sum_{j=1}^{m}\left\{\frac{I\left(\lambda_j\right)}{\lambda_j^{-2d}}\right\}\right] - 2dm^{-1}\sum_{j=1}^{m}\log\left(\lambda_j\right)\right) \tag{5}$$

For $d \in (-0.5, 0.5)$, the estimator is consistent and this consistency depends on the bandwidth, satisfying $\frac{1}{m}+\frac{m}{n}\to 0$ as $n\to\infty$. Velasco (1999) further proved the consistency of the estimator when $d\in(-0.5,1)$ and asymptotic normal for $d\in(-0.5,0.75)$.

Having estimated the fractional integrating parameters in the individual series, performing the test of homogeneity of the orders of integration and Hausman-type test of no cointegration is necessary and of high importance for justification sake. The null hypothesis of homogeneity (equality) of the fractional-order test is given as:

$$H_0 : d_x = d_y, \qquad (6)$$

where d_x and d_y are the orders of integration of the two series, respectively (see Robinson and Yajima, 2002). The following test statistic is used,

$$\hat{T}_{xy} = \frac{m^{1/2}\left(\hat{d}_x - \hat{d}_y\right)}{\left\{\frac{1}{2}\left[1-\hat{G}_{xy}^2\big/\left(\hat{G}_{xx}\hat{G}_{xy}\right)\right]\right\}^{1/2} + h(T)}, \qquad (7)$$

where $h(T)>0$ and $\hat{G}_{xy}$ is the (xy)th element of $\hat{\Lambda}(\lambda_j)^{-1} I(\lambda_j)\hat{\Lambda}(\lambda_j)$ with $\hat{\Lambda}(\lambda_j) = diag\left\{e^{i\pi\hat{d}_x/2}\lambda^{-\hat{d}_x}, e^{i\pi\hat{d}_y/2}\lambda^{-\hat{d}_y}\right\}$.

Thus, the equality of fractional integration parameters *d* implies that gold and silver (prices, their stocks or their volatility velocity /fear indices) share common stochastic trend if there is a joint memory parameter d_0 for the paired series such that $d_0 < \min\left(d_{gold}, d_{silver}\right)$. Fractional integration generalizes the unit cointegration of Engle and Granger (1987) which assumes that d is restrictively set at unity (i.e. d = 1) for each cointegrating series, and $d_0 = 0$. Details of other definitions of fractional cointegration and estimation methods are found in Robinson (2008). Based on this theory, a more general approach to fractional cointegration is the fractional cointegrating vector autoregressive (FCVAR) model of Johansen (2008) and

Johansen and Nielsen (2012). The model specification is derived from the cointegrating VAR (CVAR) model of Johansen (2005),

$$\Delta y_t = \alpha\beta' L y_t + \sum_{i=1}^{k} \Gamma_i \Delta L^i y_t + \varepsilon_t \tag{8}$$

where $\Pi = \alpha\beta'$ is the long-run equilibrium. By replacing Δ and L in (8) by their fractional versions Δ^b and $L_b = 1 - \Delta^b$, respectively, and using $y_t = \Delta^{d-b} x_t$,

$$\Delta^d x_t = \alpha\beta' L_b \Delta^{d-b} x_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b{}^i y_t + \varepsilon_t \tag{9}$$

is obtained where ε_t is a matrix of i.i.d. $N(0, \Omega)$. In the FCVAR model, the parameters Γ_i measure the short-run behaviour of the multivariate variables, y_t. Parameters β are the cointegrating relations in the system, measuring the long-run equilibria, while α measures the speed of adjustment towards the equilibrium for each of the multiple variables.

Finally, the FCVAR above is linear whereas cointegration is assumed to be constant over time, t. We then consider testing constant cointegration against time-varying (TV) cointegration following Bierens and Martins (2010). The authors applied the TV vector error correction model in which the cointegrating relations are nonlinear smooth processes. Chebyshev polynomials in time, $\left[P_{i,t}(t)\right]$ are used (Cuestas and Gil-Alana, 2016). The TV vector error correction model is given as,

$$\Delta y_t = \alpha \left[\sum_{i=0}^{m} \xi_i P_{i,n}(t) \right]' y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t \tag{11}$$

where $y_t \in R^k$, $\varepsilon_t \approx i.i.d. N_k(0, \Omega)$ and n is the number of observations, $P_{0,n}(t) = 1$, $P_{i,n}(t) = \sqrt{2}\cos\left[i\pi(t - 0.5)/n\right]$ for $t = 1, 2, ..., n,$ and $i = 1, 2, 3, ...$ Thus, the null of constant cointegration, as in Johansen (1995) with $\Pi_t' = \Pi = \alpha\beta'$ is tested against the alternative of

$\Pi_t' = \Pi = \alpha\beta_t = \left[\sum_{i=0}^{m} \xi_i P_{i,n}(t)\right]'$, where α and β are $k \times r$ matrices of rank r. In the case of bivariate cointegration, r is set at 1.

3 Data and Empirical Results

The data used in the paper are daily prices, stock indices and market fear gauges (volatility velocity) of gold and silver. Gold and silver prices (GCF and SIF) are London bullion prices at the close of the trading day. Gold and silver stock indices are the VanEck Vectors Gold Miners ETF (GDX) and iShares Silver Trust (SLV) indices, respectively; and these were retrieved from Yahoofinance website, https://yahoofinance.com. The volatility indices for gold and silver stocks are the market fear gauges of the two commodities, computed by the Chicago Board Options Exchange (CBOE) and are labelled as GVZCLS and VXSLVCLS, respectively. These were retrieved from the Federal Reserve Bank of St Louis Economic Database (FRED) at the website https://fred.stlouisfed.org. The time-series span from 04 May 2011 to 31 July 2020, based on data availability for all the series.

Plots of each paired series are given in Figures 1-3. In Figure 1, gold price (GCF) and silver price (SIF) are plotted; it is found that silver prices decreased slowly over the period since 2011. This is glaring between 2013 and around 2014. The price of gold rises from the first quarter of 2013 till the date of reporting due to the increase in the safe-haven appeal for gold as a result of the global decline in economic growth, and interruption in the economic activities which have driven investors away from risky assets. From the second quarter of 2012 to the second quarter of 2013, we observe a sharp decline in the prices of gold and silver. From February 2016 to mid-August 2016, we observe a gradual increase in the price of both gold and silver after which there is a gradual decrease till late December 2016 reaching $1056 per ounce in December 2016. Prices of gold rebound back astronomically since then. The Covid-19 period, around March-April 2020 caused another sharp decline, however, silver prices quickly rebound back. Prices of these two commodities show likely co-movements to a certain extent. By looking at the performance of stocks of these two commodities (GDX and SLV), from the start of the data sample in 2011, there is a sharp decrease in stock markets performance and this stabilized in late 2015 (see Figure 2). Gold stocks gained momentum in 2016 for about 8 months and the prices decreased and later stabilized. Both stock indices experienced a sharp decline in March-April 2020 Covid-19 pandemic period and stock markets have since then recorded higher performance. By looking at their volatility velocity/fear gauge indices, those long spikes in Figure 3 for gold volatility index (GVZCLS) and (VXSLVCLS) coincide with the period of market turbulence, where commodity prices reduced sharply due to market uncertainty. The variations in Gold volatility velocity index are likely to be higher than that of silver as it is observed in Figure 3. This is due to market demand and supply of gold during the turbulence period since 2011.

PUT FIGURES 1-3

We present the descriptive statistics in Table 1. Gold prices have a mean value of \$1383.59, a median value of \$1309.2, a minimum and a maximum value of \$1050.8 and \$1998.0, respectively. The skewness value (0.70) for gold prices indicates that the price movement over time is positively skewed with a platykurtic distribution (kurtosis = 2.33). Silver prices have a mean value of 20.63, a value very lower (difference of 1362.96) compared to that of gold, the median value of 17.52, a minimum and maximum value of 1050.8 and 1998.0, respectively. The skewness and kurtosis values are 1.44 and 4.07, respectively, indicating a positively skewed and a leptokurtic distribution. Gold and silver stocks indices have mean values of 29.20 and 19.79, respectively. There is a little difference (difference < 10) between the mean values of gold and silver stock indices; minimum and maximum values for gold stock indices are 12.47 and 66.63, respectively, while the minimum and maximum values for silver-stock indices are 11.21 and 47.26, respectively. The stock indices for both gold and silver have a positively skewed and a leptokurtic distribution (skewness for gold and silver stocks indices are 1.25 and 1.43, respectively; kurtosis for gold and silver stock indices are 3.24 and 4.02, respectively). Gold and silver volatility indices have mean values of 16.84 and 29.14, respectively; Minimum and maximum values for gold volatility indices are 8.88 and 48.98, respectively, while the minimum and maximum values for silver volatility indices are 14.89 and 100.66, respectively. This is an indication that the silver markets are more volatile compared to the gold market. The result of the volatility indices indicates a positively skewed distribution for gold and silver volatility indices, both having a sharper peaked (leptokurtic) distribution.

PUT TABLE 1

Table 2 presents the results of the classical unit root tests, the ADF (Augmented Dickey-Fuller) and PP (Phillip Peron) tests, conducted on the gold and silver prices, stock indices, and volatility indices, respectively. The tests were carried out using three regression cases: i) with no intercept and trend, ii) with intercept only, and iii) with intercept and trend. The results reflect no rejections of the unit root hypothesis in the gold and silver prices, and gold and silver stock indices. For the case of gold and silver volatility velocity indices, the decision of unit root is inconclusive as intercept and intercept with trend models indicates no unit root in the paired series based on ADF and PP tests. Noting that unit root tests (ADF and PP) are insensitive to fractional unit roots, thus there is the need for a robust unit root test that judged accurately the unit root order of the time series since unit root testing is important in modelling, forecasting and policymaking (Box, Jenkins and Reinsel, 2008).

PUT TABLE 2

Table 3 presents the results of the fractional unit root (i.e. fractional integration) on the time series, using both Whittle semi-parametric and log-periodogram (GPH) approaches.

The results are computed for three periodogram points m = $j^{0.6}$, m = $j^{0.7}$ and m = $j^{0.8}$. Fractional integration estimates, *d*s are computed fairly around 1 in all cases across the three periodogram points for the six time series. By comparing the series pairwise i.e. GCF/SIF, GDX/SLV and GVZCLS/VXSLVCLS, the corresponding fractional integration orders for corresponding periodogram points are fairly similar. Thus, we conducted the homogeneity of fractional order test since equality of unit root, as recommended in Engle and Granger (1987) is part of the cointegration procedure. The results of the test, discussed earlier in the methodology are given in Table 4. The test results indicated no significant differences in the paired fractional orders since test statistics are all less than 1.96 two-tail t-tests.

PUT TABLE 3

PUT TABLE 4

With the motivation, we applied a more general and newly proposed fractional cointegration method, which tests for fractional cointegration and as well estimated fractional cointegration model. In the bivariate setting applied in this paper, we have the results of the rank test for the FCVAR model in Table 5 Panel a. The test computes the log-likelihood and Likelihood Ratio (LR) statistics for rank 0, 1 and 2. The results show that cointegrating rank of 0 against 1 is rejected based on the LR statistic 12.827, and rank 1 cannot be rejected further against rank 2. Thus, fractional cointegration exists between the three paired variables (gold and silver prices, their stocks and their volatilities). In Table 5 Panel b, we present the FCVAR model results. The fractional integration estimate $\hat{d}$ is the joint estimate for paired series. For the three cointegrating pairs GCF/SIF, GDX/SIF and GVZCLS/VXSLVCLS, these are 0.980, 1.003 and 1.016, respectively, while the cointegrating order $\hat{b}$ is not less than 0.5 in each case implying strong cointegration.[3]. Thus estimates $\hat{d}$ - $\hat{b}$, i.e. $\hat{d}_0$ are values that are less than corresponding values of fractional integration reported in Table 3 and these are in long-range dependency range, $0 < \hat{d} - \hat{b} < 1$ unlike as in the classical cointegration where $\hat{d} - \hat{b} = 0$.

PUT TABLE 5

Having established fractional cointegration, the onus is to check for robustness since Johansen's cointegration tests are based on linearity. Thus, we checked for constant cointegration against time-varying cointegration using Bierens and Martins (2010) earlier described. The results, as presented in Table 6 showed no rejection of constant cointegration against-time varying cointegration in gold and silver prices (GCF/SIF) and gold and silver stocks (GDX/SLV) relationship while constant cointegration is rejected against time-varying cointegration for gold and silver fear gauges (GVZCLS/VXSLVCLS).

PUT TABLE 6

Our paper is the first along the line of thoughts using fractional cointegration to study the dynamics of prices, stocks behaviour and market fear (volatility velocity) of gold and silver. Previous works by authors such as Christian et al. (2015), Zhu et al. (2016) and Bibhuti et al. (2019) did obtain cointegration evidence between gold and silver but they are different in the methodological approaches. On gold and silver mining stocks and their market fear gauges, works are few on their inter-relationships, ours is still the first putting the three gold and silver performance measurements in a unified analysis.

4. Conclusions

[3] Nielsen and Popiel (2018) noted $0 < b < 0.5$ as the weak cointegration case while strong cointegration is when $0.5 < b < d$.

The paper investigated long-run relationships between gold and silver prices, their stocks, as well as their market, fear indices. Gold and silver prices are the daily closing prices at London bullion prices, while Gold and silver stock indices analyzed are the VanEck Vectors Gold Miners ETF (GDX) and iShares Silver Trust (SLV) indices, respectively. The volatility indices for the commodity stocks are the Chicago Board Options Exchange (CBOE) market fear gauges of the two commodities. These three market indicators for gold and silver are often used to weigh market performances of the commodities, noting that silver is a close substitute to gold. Having considered historical datasets from 04 May 2011 to 31 July 2020, we considered updated fractional cointegration framework between the two commodities by checking if paired prices of the commodities, their stocks and their fear indices are cointegrated. First, fractional integration test indicated the plausibility of series being I(d) with values of d around 1 in all cases and homogeneity test that gold and silver have similar persistence order. Second, by testing and estimating FCVAR, the rank test showed evidence of fractional cointegration in the paired series for prices, stocks and fear indices. Third, detected cointegrations for prices and stocks of gold and silver are constant, while cointegration for their fear indices is time-varying.

The findings imply that prices of gold and silver determine stocks market behaviour of the commodities. Our findings have implications for portfolio managers in the sense that it does not often pay investors to include assets of similar pricing relationships in the same portfolio.

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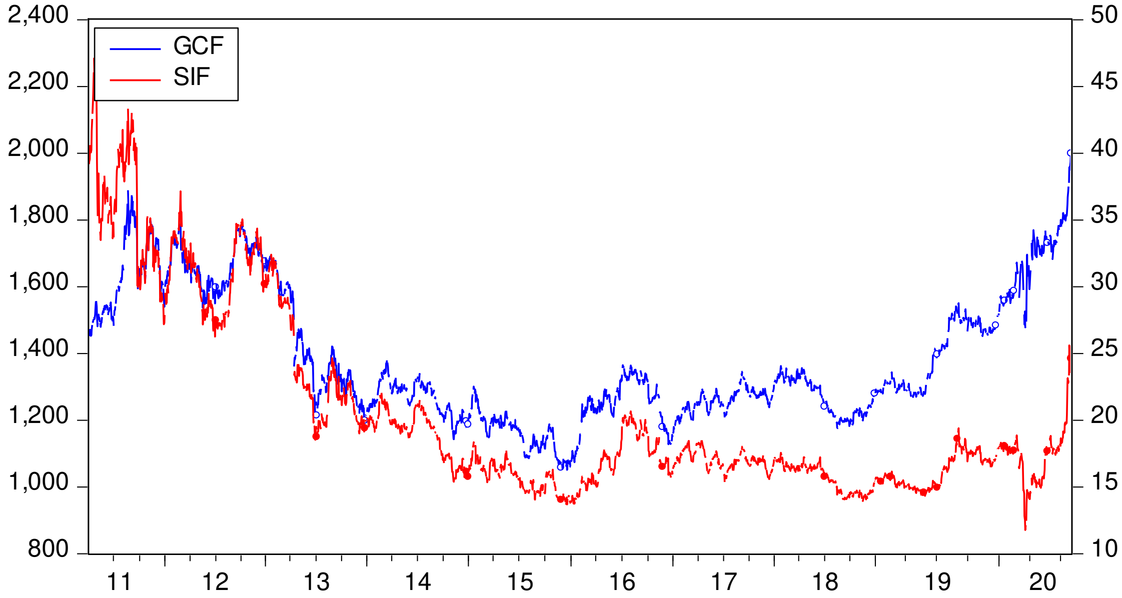

Figure 1: Co-movement of Gold and Silver prices

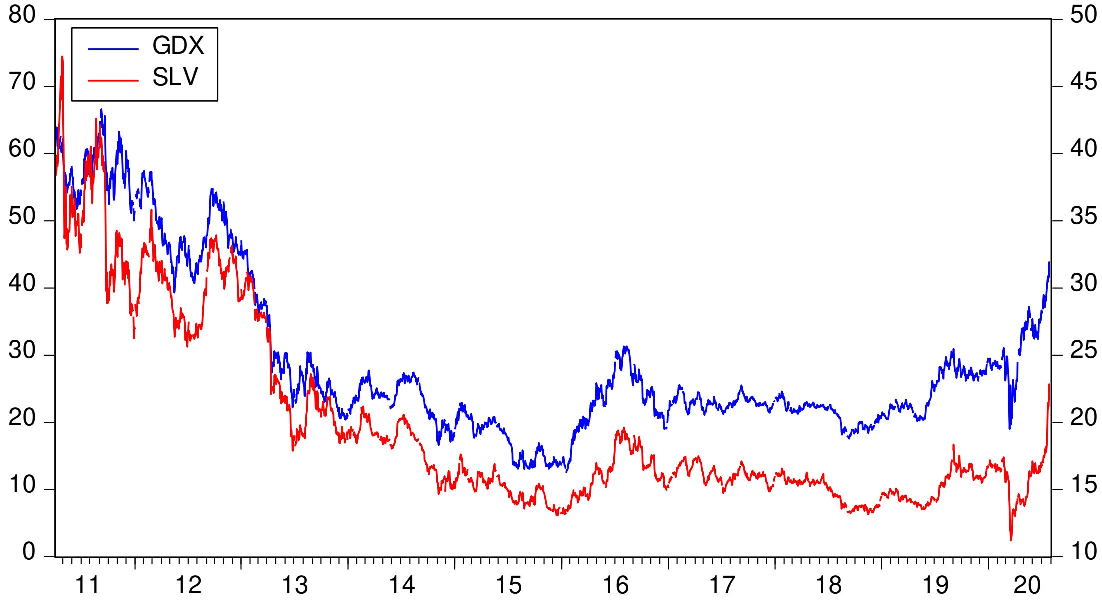

Figure 2: Co-movement of Gold and Silver stock indices

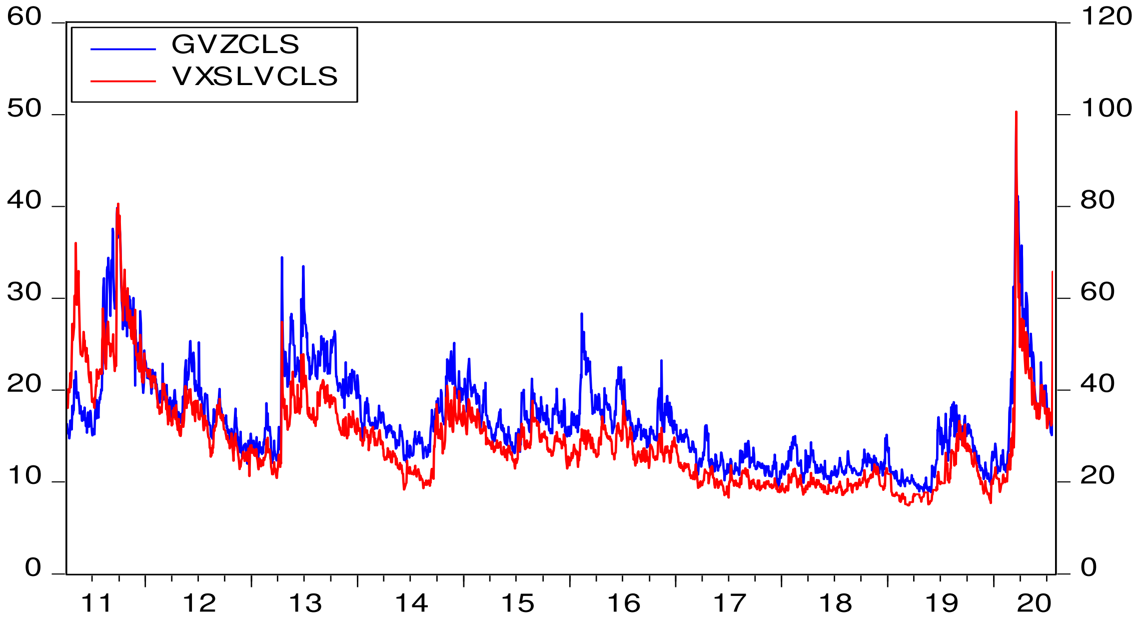

Figure 3: Co-movement of Gold and Silver Fear gauge/Volatility indices

Table 1: Descriptive statistics

	GCF	SIF	GDX	SLV	GVZCLS	VXSLVCLS
Mean	1383.59	20.63	29.20	19.79	16.84	29.14
Median	1309.20	17.52	24.05	16.54	15.99	27.22
Maximum	1998.00	48.58	66.63	47.26	48.98	100.66
Minimum	1050.80	11.77	12.47	11.21	8.88	14.89
Std. Dev.	198.67	6.98	12.60	6.98	5.28	10.49
Skewness	0.70	1.44	1.25	1.43	1.37	1.61
Kurtosis	2.33	4.07	3.42	4.02	5.92	7.04
Jarque-Bera	233.52	925.61	632.26	896.62	1621.69	2704.79
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2: Unit root tests

Ticker	None	Intercept only	Intercept with trend
ADF test			
GCF	0.6897 [0]	-0.5107 [0]	-0.1039 [0]
SIF	-1.4990 [1]	-2.7112 [1]	-2.2548 [1]
GDX	-1.4827 [0]	-2.4578 [0]	-1.3345 [0]
SLV	-1.5678 [0]	-2.6687 [0]	-2.1443 [0]
GVZCLS	-1.1014 [3]	-5.0809 [0]***	-5.5738 [0]***
VXSLVCLS	-1.3274 [1]	-4.5342 [1]***	-4.9589 [1]***
PP test			
GCF	0.7358 [13]	-0.4189 [12]	0.0633 [14]
SIF	-1.4995 [8]	-2.6781 [8]	-2.3314 [9]
GDX	-1.4974 [3]	-2.4443 [4]	-1.2733 [2]
SLV	-1.5405 [11]	-2.6987 [11]	-2.3271 [12]
GVZCLS	-0.9277 [33]	-4.4575 [20]***	-4.9808 [18]***
VXSLVCLS	-0.6987 [36]	-3.5745 [28]***	-3.9000 [26]***

*** denotes significant of unit root test at 5% level. In squared brackets are optimal lag numbers for augmentation selected based on minimum information criteria in the case of ADF test, and Newey-West bandwidth number is selected based on Bartlett kernel spectral estimation method.

Table 3: Fractional integration estimates based on Whittle semi-parametric and Log-periodogram regression

Ticker	m.	Whittle Semi-parametric	Log-Periodogram
GCF	$T^{0.6}$	0.9785	0.9623
	$T^{0.7}$	0.9405	0.9439
	$T^{0.8}$	0.8831	0.8836
SIF	$T^{0.6}$	0.8868	0.8985
	$T^{0.7}$	0.9814	1.0282
	$T^{0.8}$	0.9720	0.9953
GDX	$T^{0.6}$	0.9384	0.9896
	$T^{0.7}$	0.9558	0.9691
	$T^{0.8}$	0.9662	0.9913
SLV	$T^{0.6}$	0.8909	0.9083
	$T^{0.7}$	0.9870	1.0407
	$T^{0.8}$	1.0006	1.0198
GVZCLS	$T^{0.6}$	0.9037	0.8756
	$T^{0.7}$	0.8667	0.8681
	$T^{0.8}$	0.8530	0.8815
VXSLVCLS	$T^{0.6}$	0.8540	0.8716
	$T^{0.7}$	0.7981	0.7928
	$T^{0.8}$	0.8437	0.8796

Note: total sample T is 2434 and the three periodogram points, $T^{0.6}$, $T^{0.7}$ and $T^{0.8}$ are 107, 233 and 508, respectively

Table 4: Test of Homogeneity of fractional integration orders

Ticker	Test statistic		
	m = $T^{0.6}$	**m = $T^{0.7}$**	**m = $T^{0.8}$**
GCF/SIF	1.0550	1.5541	0.2636
GDX/SLV	0.6572	1.2548	1.6716
GVZCLS/VXSLVCLS	1.4087	1.5333	0.8523

Critical value at 5% is 1.96

Table 5: FCVAR estimation results

a. **Rank test results based on the FCVAR**

Variables	Rank	Log-lik	LR stat.
GCF/SIF	0	-12738.12	12.827
	1	**-12731.77**	**0.128**
	2	-12731.71	----
GDX/SLV	0	-5225.73	25.996
	1	**-5213.83**	**2.201**
	2	-5212.73	----
GVZCLS/VXSLVCLS	0	-8117.94	105.477
	1	**-8083.76**	**37.113**
	2	-8065.20	----

Note, max k value was set at 5 and optimal k value was selected based on minimum information criteria. In bold denotes point of further rejection of rank 0 against none rejection of rank 1 in the likelihood ratio (LR) test statistic. Thus, fractional cointegration exists and it is only one relationship in the case of bivariate.

b. **FCVAR model**

Variables	$\hat{d}$	$\hat{b}$	$\hat{\beta}$
GCF/SIF	0.980 (0.012)	0.837 (0.108)	[-277.869, 1.000, -72.081]
GDX/SLV	1.003 (0.020)	0.500 (0.142)	[-7.919, 1.000, -1.150]
GVZCLS/VXSLVCLS	1.016 (0.108)	0.560 (0.063)	[6.824, 1.000, -2.545]

FCVAR model diagnostic tests were carried out at 12 using Q statistic and LM tests. The models were found to be adequate. These result are available on request. In parentheses are the standard errors of the corresponding estimates.

Table 6: Test of constant cointegration against time-varying cointegration

Ticker	Test statistic		
	j = 1	**j = 2**	**j = 3**
GCF/SIF	11.3696	11.6661	13.8029
GDX/SLV	10.3908	8.0793	8.1652
GVZCLS/VXSLVCLS	41.4148***	39.4032***	37.2561***

Wald test critical value at 5% is 18.4 for periodogram points j =1,2,3.