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Abstract

There are two major rationality theories in social sciences, such as in economics and psychology, namely, economic rationality and bounded rationality. In terms of theoretical physics, these rationality theories are about accelerated states. This paper proposes a new theory, called ordinary rationality, which refers to ground states. Ordinary rationality theory is a principled theory that includes eight principles. Ordinary rationality also shares three meta-properties with Higgs fields, and its function is modeled by the Higgs mechanism in the standard model of particle physics. The present paper also discusses the relation between the notion of mass in physics and the notion of mass in social sciences: namely, it addresses the issue about P-mass vs. S-mass.

1. Moving from Trivial to Spontaneous Symmetry Breaking

In the western philosophy of law, the notion of the ordinary man is essential due to the jury practice in the western justice tradition, though how to define the notion of the ordinary man is a long-term controversy. Current mainstream microeconomics treats ordinary men as Newtonian dots. Based on the so-called large number law, individual minds are canceled out with one another, and thus they are averaged statistically. Within behavioral economics, ordinary men seem proven to be bounded [4], biased and heuristic [3], illusory, or irrational. The above framework has two layers. The first layer is a normative theory, and the second layer attaches some additional psychological principles and empirical evidence. This research paradigm is called trivial symmetry breaking.

The rationality issue is essential to theorizing economic thought. Modern economics has well-established modeling instruments and analytical technologies, such as equilibrium and margin analyses, within the framework of Newtonian mechanics. We observe that ordinary rationality shares three properties and corresponding functions with the Higgs field in the standard model of particle physics. Previous work [8][9][11] has established the gauge symmetry
groups $U(1)$ for market dynamics, $SU(3)$ for sub-economic dynamics, and $SU(2)$ for economic externality (e.g., policies) dynamics. This paper aims to outline the spontaneous symmetry breaking framework by applying the Higgs mechanism approach to model ordinary rationality function.

The rest of this paper is arranged as follows. Section 2 introduces the notion of ordinary rationality, which is characterized by eight principles. Section 3 introduces three meta-properties shared by ordinary rationality and the Higgs field in the standard model of particle physics. Section 4 discusses the relation between physical mass $P$-mass and social mass $S$-mass. A complete example of the Higgs mechanism is given in the Appendix.

2. Ordinary Rationality

Ordinary people are rational. This commonly shared ordinary rationality is characterized by eight principles: high selectivity, subjective certainty (acting), taking a null action, sunk cost, hesitation, emotion, better life, and face. These principles are explored below.

(i) Principle of high selectivity. Ordinary people are highly selective about what information to acquire or to ignore, and what business to take care of or to not take care of. They are not bounded. They are selective, and being selective is rational, particularly in the information overloading era.

(ii) Principle of subjectivity. This is the concept originally given by Wittgenstein. As he explained, doubting everything is not a doubt. People can only doubt what they have believed in. Although this world is full of uncertainties, as decision theorists used to say, an ordinary man has established many subjective certainties. For the majority of everyday life, ordinary men are just acting routinely, rather than frequently taking decisive actions that would make themselves feel crazy and nervous.

(iii) Principle of taking null actions. People ordinarily do not like making decisions because it is costly. Modern decision theory views preference not as a purely mental state, but as taking actions. When one receives two competitive offers, one will often hold both for a while because one has earned them as one’s wealth. People ordinarily accept one of the two competitive offers, often when the deadline gets closer because that simultaneously means the declined offer becomes the opportunity cost. The most common decision people ordinarily make is to take a null action. In a situation with two competing candidates, when both are your close friends lobbying hard for your vote, you might find a reason to not vote to minimize the opportunity cost of losing a close friend. Additionally, we all know that after a customer hesitates in a store for a while, when he or she is asked if any help is needed, the most common answer would be, “don’t worry about me, I will be fine.” The customer is taking a null action.

Note that taking a null action is an action. We denote routine acting as $\phi$ and denote taking null action as $\{\phi\}$. They are different, but the difference cannot be directly observed from the behavior perspective. This can be seen as the economic version of Einstein’s principle of equivalence in the general theory of relativity.
(iv) Principle of sunk cost. Margin analysis is the main technology used to study efficiency in economics. Consider investing in a unit of a scarce resource, it is efficient if the marginal gain (calculated by some differential operations or derivatives) is greater than the marginal cost. Thus, margin analysis is only about the near future or the present. Hence, current textbooks in economics teach us to never look back at what happened before. This is called the sunk cost. Nevertheless, our history is part of who we are, and it will ordinarily have influences on what we do or how we behave on the market at the present or in the near future. Taking sunk cost seriously makes perfectly rational sense.

(v) Principle of hesitating-ness. In market dynamics, hesitation (spin) is the intrinsic property of a demand or a supply, which carries a market charge. In sub-economic dynamics, hesitation is the intrinsic property of impulses. For ordinary rationality, hesitation is treated as an inertial habit at a general level.

(vi) Principle of emotion. By common sense, ordinary people are often emotional in the market and everyday life.

(vii) Principle of a better life. This principle stands by itself but needs to be spelled out. The underlying assumption of macroeconomics is that the economy will go up in the long run. The underlying assumption for microeconomics is that people ordinarily want to improve their lives. The present work concerns the rationality theory in economics. Any rationality theory that favors a worse life would go beyond the scope of ordinary rationality in economic mechanics.

(viii) Principle of face (faith). Ordinary people take face seriously and wish to be respected. This mental routine is ordinary and it is the bottom line for ordinary rationality.

3. Properties Shared with Higgs Field

As Nobel Laurate F. Wilczek pointed out, Newtonian physics pre-assumed a zeroth law: Matter is measured by mass, and the mass is not created nor annihilated. However, Einstein told us that energy is more fundamental to matter because \( E = mc^2 \). The Newtonian vacuum is empty, but in modern theoretic physics, vacuums are ground states with the minimum level of energy by fluctuation. Ordinary rationality shares exactly three general properties with Higgs field given below.

(i) They are ground states with the minimum level of energy, which are with non-zero expected value.

(ii) They are inertial systems, with spin 0. Higgs field is a scalar function, which is value oriented and serves as the gauge freedom in a wave equation. In this sense, Ordinary rationality is a simple, one-dimensional value function.

(iii) They are degenerate states, which have spectrums, rather than singles, of eigenvalues for the observation.
Ordinary rationality is a degenerate state, and similarly to commonly shared cognition, provides routines while it depends how ordinary individuals take these routines in their own ways. In addition, the Higgs mechanism involves creating and then vanishing the so-called Goldstone field, which in our context means that emotion is characterized by the Goldstone mode.

In everyday life, we do most things routinely without making decisions. We follow these routines based on the subject certainties established from our experiences. We get up in the morning, take a shower, eat some breakfast, and then go to work. We are trying not to make decisions often, so we can avoid paying a high opportunity cost. We like keeping everyday life normal, and we dislike letting life drive us crazy or nervous. This is the base of our ordinary daily life called ground state. This kind of normal life is often overlooked by rationality theories. In terms of Newtonian physics, the “ground state” of our normal life is treated as an empty vacuum. Nevertheless, we all know that keeping our life ordinary is not so easy. It demands certain effort and energy, of which we try to keep at a minimum level. That means, mathematically speaking, that the expectation value is non-zero.

In our daily life, we keep being ordinary, and that is essentially rational. For most parts of our daily life, we are just acting, instead of taking deliberate actions. When we have to make decisions, oftentimes we take a null action. We keep working because we intend to secure our life. In most of our life, it is similar to an inertial system. Even when we feel emotional, oftentimes we naturally let emotions go and in physics term, try not to accelerate them. Even when we feel hesitant, we try not to let it drive us crazy; from a meta-cognition viewpoint, we intend not to be bothered by too much by second-order thinking. In terms of physics, the inertial systems spin zero.

As ordinary men, we all share ordinarily rationality. On one hand, no matter who you are, be it president, professor, student, or parent, we all share something in common which is ordinary rationality. Everyone has an ordinary nature which can be personified as “ordinary man”. On the other hand, each one of us is special. Ordinary rationality lives with individual differences which can be personified as “ordinary men”. Thus, ordinary rationality is a degenerate state. In terms of gauge field theory, the ordinary man stands for the global level, while the ordinary men stand for the local level.

From the above analyses, ordinary rationality can be reasonably characterized by the Higgs field. But, what is the point for doing this? In physics, all the gauge particles are originally treated as massless. There are no mass terms in its langrangian formulas. The Higgs field is responsible for interacting with gauge particles, such that the gauge particles become massive. There is a popular metaphor which describes the Higgs field like a swimming pool. When one stands on the land, you do not feel that your clothes are heavy. But, if one jumps into the swimming pool, you would feel the weight of clothes. Ordinary rationality is also like a swimming pool. The degree of results for an event in our life needs to be measured by using ordinary rationality as the ground state. The bigger the difference, the more massive the result. In the society, how successful a new policy is going to be depends on how ordinary people take it. On the market, how successful a new product is going to be actually depends on how ordinary
customers accept it and buy into it. The Higgs particle is sometimes called God’s particle in popular books. This reminds us of the saying that customers are the God of the market.

4. P-mass vs. S-mass

Here, P-mass stands for physical mass and S-mass stands for social mass. We are going to discuss the similarity between the two. First, we briefly discuss the so-called Higgs mechanism in particle physics. By the standard model of particle physics, the Higgs mechanism is responsible for spontaneous symmetry breaking. The Higgs field is a complex scalar field, which can be written as,

\[ \Phi = a + [h(x) + ig(x)] \]

where \( a \) is responsible for global spontaneous symmetry breaking, and \([h(x) + ig(x)]\) is responsible for local spontaneous symmetry breaking. The following is a brief description on how the Higgs mechanism works: First, treat \( h(x) \) as the Higgs field, which is a real scalar, and \( g(x) \) is called Goldstone field. Second, \([h(x) + ig(x)]\) has an exponential form \( \rho e^{i\theta(x)} \), where \( \rho \) is called Higgs field, and \( \theta \) is called Goldstone field. By gauge field theory, at the local level, \( \theta(x) \) is the phase function of the internal phase space for a wave function, \( \Phi \). In order to keep gauge transformation conformal from one state to another state, it is necessary to introduce what is called the gauge field, defined by

\[ A_\mu = A'_\mu - \frac{1}{e} \partial_\mu \theta(x). \]

There are two key steps in Higgs mechanism. The first one is to introduce the notion of free gauge field, defined by

\[ B_\mu = A'_\mu - \frac{1}{q} \partial_\mu. \]

In the popular saying, this step is described as “the goldstone particle is eaten by the gauge field.” The second key step is to define \( \rho = (v + \chi) \), where \( \chi \) becomes the true Higgs field. Then, define

\[ v = \sqrt{\frac{u^2}{\lambda}}, \quad M = ve \]

where \( u \) and \( \lambda \) are potential parameters in the original massless Langrangian. Finally, it can derive a term \( \frac{1}{2} M^2 B^2 \) in the resulting Langrangian. By the popular saying, it means that after eating the Goldstone particle, the free gauge field acquired the mass \( M^2 \). Let us call \( M^2 \) the P-mass. For the detailed step-by-step derivation for this example of the Higgs mechanism, see the Appendix.
What has been overlooked is the relation between Goldstone field and potential parameters. This leaves some mystery about the Higgs mechanism. The present author has not seen any documented discussion on this issue. Here, I only want to point out that the notion of free gauge is actually treating the gauge field as a geometric matter, which involves the so-called Barry phase in gauge field theory. Thus, a better understanding of the Higgs mechanism should take the nonintegrable phase factor into consideration. The integral formalism approach to gauge field theory was originally proposed by C.N. Yang [7]. A more complete technical discussion on the relation between Goldstone field and potential parameters would go beyond the scope of this paper. Below, I would like to provide a parallel story from the perspective of ordinary rationality.

Ordinary people become emotional from time to time. Let us characterize emotions as in Goldstone mode. Emotions are private matters, which do not cause any mass in public. In this sense, we say, emotions are massless. But emotions could cumulatively build up, and eventually trigger certain actions. Actions are public matters. Consequently, these actions may result in some trouble for society. In such situations, people used to say, “you have left a lot of mass.” This is what we mean by S-mass. Accordingly, the society would have to do some job in order to solve the problem. This job is called the social gauge field, and this job is massive.

Appendix

Below we provide an example to show how the Higgs mechanism works. The derivation is partly taken from Aitchison [1] and Yang [10]. This example is about the Higgs field itself, which is straightforward, and for reader’s convenience, is spelled out step-by-step without any omission. During the derivation, the key formulas are numbered by (H1), definitions are numbered by (Dj), and step formulas are not numbered. Notations and definitions are listed below.

\[ \varphi = \rho e^{i\theta}, \quad \varphi^\dagger = \rho e^{-i\theta} \] (D1), (D2)

\[ D_\mu \varphi = (\partial_\mu - ieA_\mu)\varphi, \quad (D_\mu \varphi)^\dagger = (\partial_\mu + ieA_\mu)\varphi \] (D3), (D4)

\[ \rho = \frac{1}{\sqrt{2}}(v + \chi), \quad v = \sqrt{\frac{\mu^2}{\lambda}} \] (D5), (D6)

\[ B_\mu \equiv A_\mu - \frac{1}{e} \partial_\mu v, \quad M = ve \] (D7), (D8)

Consider a massless Langrangian

\[ \mathcal{L} = -\frac{1}{4} F_{\mu
u} F^{\mu\nu} + (D_\mu \varphi)^\dagger D_\mu \varphi + \mu^2 \varphi^\dagger \varphi - \lambda(\varphi^\dagger \varphi)^2 \] (H1)

Since \[ \varphi^\dagger \varphi = \rho e^{-i\theta} \rho e^{i\theta} = \rho^2 e^0 = \rho^2 \]

\[ D_\mu \varphi = (\partial_\mu - ieA_\mu)\rho e^{i\theta} = \partial_\mu (\rho e^{i\theta}) - ipeA_\mu e^{i\theta} = e^{i\theta} \partial_\mu \rho + \rho \partial_\mu (e^{i\theta}) - ipeA_\mu e^{i\theta} \]

\[ = e^{i\theta} \partial_\mu \rho + e^{i\theta} i\rho \partial_\mu \theta - ipeA_\mu e^{i\theta} = e^{i\theta} [\partial_\mu \rho + i\rho (\partial_\mu \theta - eA_\mu)] \]
\((D_\mu \varphi)^\dagger D_\mu \varphi = e^{-i\theta} e^{i\theta} [\partial_\mu \rho + i \rho (\partial_\mu \theta - eA_\mu)] [\partial_\mu \rho - i \rho (\partial_\mu \theta - eA_\mu)]
\)
\[
= [\partial_\mu \rho + i \rho (\partial_\mu \theta - eA_\mu)] [\partial_\mu \rho - i \rho (\partial_\mu \theta - eA_\mu)] \]
\[
= (\partial_\mu \rho)^2 + \rho^2 (\partial_\mu \theta - eA_\mu)^2
\]
\[
\phi^\dagger \phi = \mu^2 \rho^2
\]
\[
\lambda (\phi^\dagger \phi)^2 = \lambda (\rho^2)^2 = \lambda \rho^4
\]
\[
(H1) \quad \Rightarrow \quad L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \rho^2 (\partial_\mu \theta - eA_\mu)^2 + (\partial_\mu \rho)^2 + \mu^2 \rho^2 - \lambda \rho^4 \quad (H2)
\]
Since \((\partial_\mu \rho)^2 = [\partial_\mu \frac{1}{\sqrt{2}} (v + \chi)]^2 = \frac{1}{2} [\partial_\mu (v + \chi)]^2 = \frac{1}{2} (\partial_\mu v + \partial_\mu \chi)^2
\]
\[
= \frac{1}{2} (\partial_\mu \sqrt{\frac{\mu^2}{\lambda}} + \partial_\mu \chi)^2 = \frac{1}{2} (0 + \partial_\mu \chi)^2 = \frac{1}{2} (\partial_\mu \chi)^2
\]
\[
\rho^2 (\partial_\mu \theta - eA_\mu)^2 = \rho^2 [e \left( A_\mu - \frac{1}{e} \partial_\mu \rho \right)]^2 = \rho^2 e^2 B^2
\]
\[
\rho^2 = \frac{1}{\sqrt{2}} (v + \chi)^2 = \frac{1}{2} (v + \chi)^2 = \frac{1}{2} v^2 + v \chi + \frac{1}{2} \chi^2
\]
\[
\rho^2 e^2 B^2 = \left( \frac{1}{2} v^2 + v \chi + \frac{1}{2} \chi^2 \right) e^2 B^2 = \frac{1}{2} v^2 e^2 B^2 + e^2 v \chi B^2 + \frac{1}{2} e^2 \chi^2 B^2
\]
\[
= \frac{1}{2} M^2 B^2 + e^2 v \chi B^2 + \frac{1}{2} e^2 \chi^2 B^2,
\]
\[
(H2) \quad \Rightarrow \quad L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} M^2 B^2 + e^2 v \chi B^2 + \frac{1}{2} e^2 \chi^2 B^2 + \frac{1}{2} (\partial_\mu \chi)^2 + \mu^2 \rho^2 - \lambda \rho^4 \quad (H3)
\]
Since \(\mu^2 \rho^2 = \mu^2 \frac{(\chi + v)^2}{\sqrt{2}} = \frac{1}{2} \mu^2 (\chi + v)^2
\]
\[
= \frac{1}{2} \mu^2 (\chi^2 + v^2 + 2 \chi v) = \frac{1}{2} \mu^2 \chi^2 + \frac{1}{2} \mu^2 v^2 + \mu^2 \chi v
\]
\[
= \frac{1}{2} \mu^2 \chi^2 + \frac{1}{2 \lambda} \mu^4 + \mu^2 \frac{\mu^2}{\sqrt{\lambda}} \chi = \frac{1}{2} \mu^2 \chi^2 + \frac{1}{2 \lambda} \mu^4 + \mu^3 \chi \frac{1}{\sqrt{\lambda}}
\]
\[
\lambda \rho^4 = -\lambda \left( \frac{v + \chi}{\sqrt{2}} \right)^4 = -\frac{\lambda}{4} [(v + \chi)^4] = -\frac{\lambda}{4} [(v + \chi)^2]^2
\]
\[
= -\frac{\lambda}{4} [(\chi^2 + v^2 + 2 \chi v)^2] = -\frac{\lambda}{4} [(\chi^2 + v^2)^2 + 2 \chi v]^2
\]
\[
= -\frac{\lambda}{4} [(\chi^2 + v^2)^2 + 4 \chi v (\chi^2 + v^2) + 4 \chi^2 v^2]
\]
\[
= -\frac{\lambda}{4} (\chi^4 + v^4 + 2 \chi^2 v^2 + 4 \chi v^3 + 4 \chi^3 v^2 + 4 \chi^2 v^2)
\]
\[
= -\frac{\mu^4}{4 \lambda} - \frac{\lambda}{4} \chi^4 - \frac{1}{2} \mu^2 \chi^2 - \mu^3 \chi \frac{1}{\sqrt{\lambda}} - \sqrt{\lambda} \mu \chi^3 - \mu^2 \chi^2
\]
\[
\mu^2 \rho^2 - \lambda \rho^4 = \frac{1}{2} \mu^2 \chi^2 + \frac{1}{2 \lambda} \mu^4 + \mu^3 \chi \frac{1}{\sqrt{\lambda}} - \frac{\mu^4}{4 \lambda} - \frac{\lambda}{4} \chi^4 - \frac{1}{2} \mu^2 \chi^2 - \mu^3 \chi \frac{1}{\sqrt{\lambda}} - \sqrt{\lambda} \mu \chi^3 - \mu^2 \chi^2
\]
\[
\frac{\mu^4}{4\lambda} - \frac{\lambda}{4} \chi^4 - \sqrt{\lambda} \mu \chi^3 - \mu^2 \chi^2
\]

(H3) \Rightarrow \quad \mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} M^2 B^2 + e^2 v \chi B^2 + \frac{1}{2} e^2 \chi^2 B^2

\quad + \frac{1}{2} (\partial_\mu \chi)^2 + \frac{\mu^4}{4\lambda} - \frac{\lambda}{4} \chi^4 - \sqrt{\lambda} \mu \chi^3 - \mu^2 \chi^2 \quad (H4)

Notice the second term on the right side of the equation, the free gauge field \( B^2 \) is with a mass term \( M^2 \). H4 is a massive Lagrangian.

References


