Black Scholes Model

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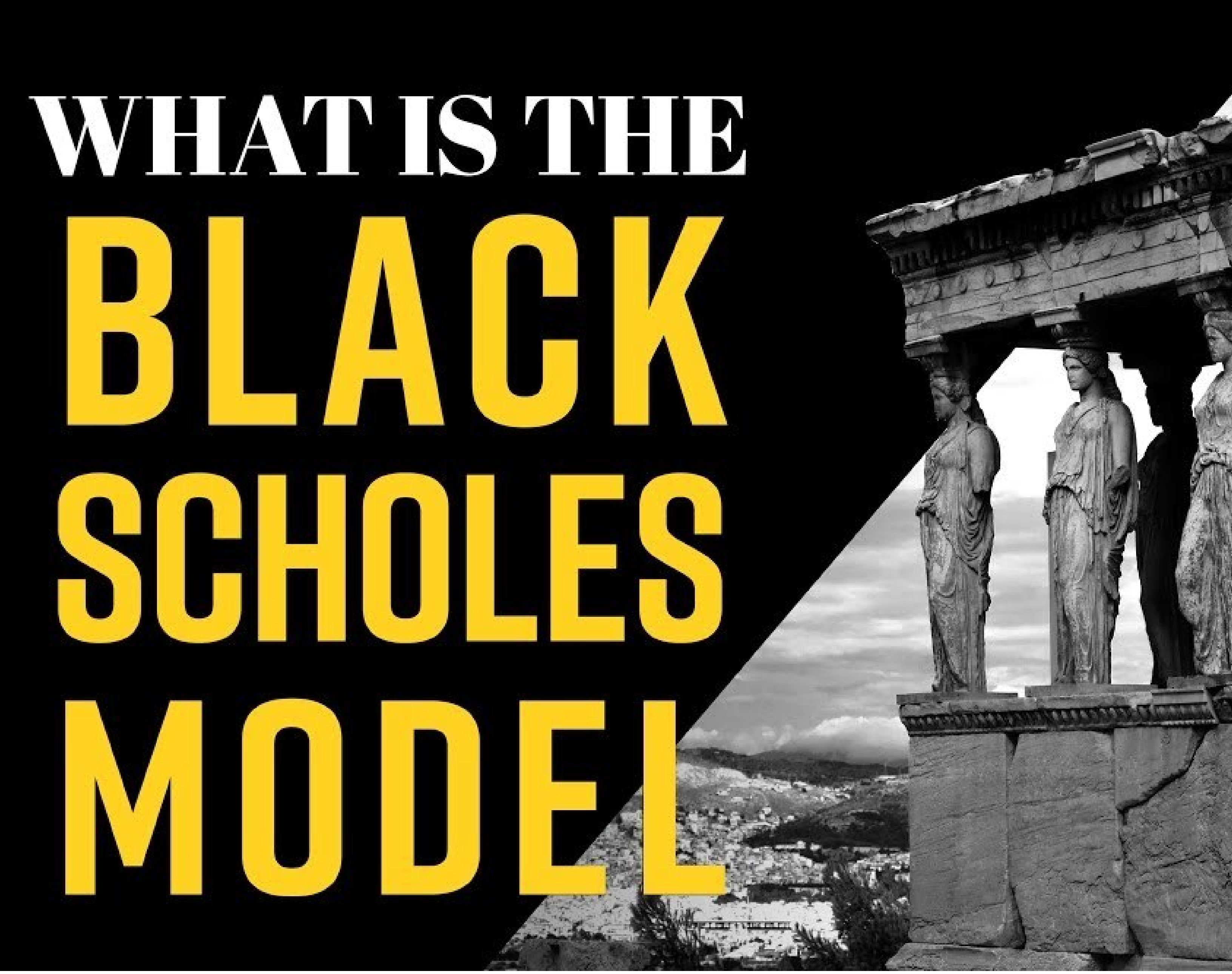
WHAT IS THE
BLACK
SCHOLES
MODEL

Black-Scholes is a pricing model applied as the reference in the derivation of fair price—or the theoretical value for a call or a put option. A call is defined as the decision to buy actual stock at a set price, defined as the strike price; and by a scheduled expiration date. A put option is defined as the opportunity contract providing the owner the right but not the obligation, to sell an exact amount of underlying security at a stated price within a specific time frame.

The call or put option in the Black Scholes model is based on six variables: strike price and underlying stock price, time and type of option, volatility and risk-free rate. The application of the model assumes that these stock or securities recognise its corresponding custom derivatives held to expiration. It is sufficient to state that the Black-Scholes treats a call option as an informal agreement defined as a forward contract with expectation to deliver stock at a contractual price, otherwise indicative in the strike price.

Typically the Black-Scholes model is utilised to price European options (y p) that represents investment options in a selection of financial assets earning risk-free interest rates. In strictness, the model presents the option price as a function of stock price volatility: High volatility is tantamount a high premium price on the option.

$$C = SN\,(d_1) - Ke^{-r(1/365)}Nd_2$$

The variables include:
C = price of a call option. P = price of a put option. S = price of the underlying asset. X = strike price of the option. r = rate of interest. t = time to expiration. s = volatility of the underlying. N represents a standard normal distribution with mean = 0 and standard deviation = 1

$$d_1 = \frac{In\left(\frac{SP}{STR}\right) + \left(\frac{r}{365} + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{In\left(\frac{SP}{SPR}\right) + \left(\frac{r}{365} - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

On the equation r(t/365) recognises the right to buy at the discount on the price of the stock at present utilising a risk free interest rate. The assumption is that r = 0.

The standard deviation of the daily volatility of stock adjusted for time is defined by $\sigma\sqrt{t}$ where the distribution into a standard normal distribution with a standard deviation of 1.

d_1 calculates the cumulative probability to this standard normal point and σ normalises the numerator as the number of standard deviations.

μ is zero, therefore $\frac{\sigma^2}{2}$ is the log-normal zero mean adjustment.

$In\left(\frac{SP}{SPR}\right)$ is the absolute log growth difference between the strike price and the stock price

VOLATILITY MATTERS

In the Black Scholes model, only volatility matters, the *mu* or drift is not important. Drift is supplanted by the risk-free ratios utilising a mathematical construct called risk-neutral probability pricing. That is in the context of a typical stock market, the considerable speculation derives to exploit arbitrage opportunities. Whereas the Black Scholes model establishes proper pricing of options such to eradicate any opportunity for arbitrage. Arbitrage is the simultaneous acquisition and auction of financial assets to profit from an imbalance in the price. Trade profits make up the price differences of identical financial instruments put down on different markets.

Other pricing formulae stated in a series of studies find trading volume strategies to forecast price momentum in terms of magnitude and persistence (DYL, E.A., Yukse, H. Z and Zaynutdinova, G.R., 2019).The theoretical construct detailed in the articles *Momentum strategies*, which looks into the context of earnings drift (CHAN, L., Jegadeesh, N. and Lakonishok, J., 1996); and *The long-run negative drift of post-listing stock return*s that finds after seasoned equity offerings (DHARAN, B.G, and Ikenberry, D., 1995). A contrasting interpretation still based on the same fact put down in the article *The new issues puzzle* (LOUGHRAN, T., and Ritter,J., 1995).

Yet a number of authors suggest for the substitution of the volatility parameter by a volatility function, evidently consistent with maturity biases and moneyness. An example of which is the Deterministic Volatility Function that has no indications of time instability. The deterministic approach performs poorly because a change in volatility is inevitable through time (DUMAS, B., J. Fleming and R. Whaley, 1998).

In the dimension of arbitrage, the book *Investor Intelligencef from Isider Trading* infers gains to have mimicked the large trades of insiders (SEYHUN, H. Nejat, 1997), presenting captured information on insider trading over the past 21 years. The same principle is elaborated further in the article *Conflict of interest and the credibility of underwriter analyst recommendations* (MICHAELY, R., and K. Womack, 1999);while other documents confirm drift after earnings for up to 12 months after the initial upsruge (BERNARD, V. L. and Thomas, J.K., 1990).

GROWTH OPTIMAL PORTFOLIOS VS BLACK SCHOLES

Fundamentally these models rely on the same theoretical foundations and assumptions of the geometric movements of stock price behaviour and risk-neutral valuation. One key advantage of the Black-Scholes model is speed in terms of calculating a very large volume of option prices within a quick span of time. Nevertheless it cannot be used for the accurate valuation of price options with non-British exercises as it strictly calculates the option price at expiration; whereas other growth optimal portfolios recognise the possibility of early exercise of an option. The study on the momentum effects in China concludes there is considerable gain in time-variation momentum strategies (YANG, Y., Gebka B. and R. Hudson, 2019), reiterrated in the article *Enhancing momentum investment strategy using leverage* (FORNER, C., Muradoglu, Y. and S. Sivaprasad, 2018). On contary, a document which examines short-term momentum effect among other variants of portfolio return behavior, concludes that trading volume-based momentum investment strategies should not be used at all (EJAZ, A. and Polak, P., 2018).

Traditional portfolio management emphasises explicitly the determined trade-off between risk and return. The optimal growth portfolio explores the geometric mean that seeks to optimise long-term growth ratios of a portfolio, as an alternate to the traditional Sharpe ratio maximization. Given these, volatility plays a significant role where lower return portfolios can result in higher expected terminal gains when volatility is substantially low.

The work of Sharpe ascertains the notion of the tangency portfolio which determines the optimal excess return per unit of risk. Relevant empirical research concludes the Sharpe ratio to provide nearly three times more return when compared with a portfolio of equal shares of ten stocks (BILIR, H., 2016), and a study in similar vein by Koc Universityconcludes stock gains increase after elevated levels of volume and variability (GÖKÇEN, U and T. Post, 2018).

While growth optimal portfolios are incompatible with short-term requirements, it is thought that the Black Scholes model is apt for short term investments. That is, the geometric Brownian motion captures short time modelling inaccurately because the parameters drift and volatility are constant.

ASPECTS OF REAL PRICE

A number of studies suggest the Black Scholes Models does not particularly describe every aspect of real asset price data, when in fact other models typically do better. A financial simulation study in the appropriateness of financial models in forecasting stock prices on the Ghana, finds the Geometric Brownian Motion model superior (DAMPTEY, I., 2017); and the same as concluded in an earlier empirical research on the validity of the geometric Brownian motion assumption (MARATHE, R., and Ryan, S., 2005). To improve on this flaw, an inquiry into option pricing models specifically examined strike price biases situated in the Black-Scholes model, thought to have skewed implied volatilities in market options indexes (HESTON, S. and Nandi, 2000).

Nonetheless it is interesting to note these characteristics of skewedness and kurtosis result in price performance improvements (BACKUS, D., S. Foresi, K. Li and L. Wu, 1997). Volatilities obtained between different models are relatively the same with percentage variation between 17 and 30 and correlation coefficient of 0.8951 (CORRADO, C. and T. Miller, 1996). The recognised alterations on skewedness give better results when compared to kurtosis adjustment (CAPELLE-BLANCARD, G., E. Jurczenko and B. Maillet, 2001).

BENCHMARKING BLACK SCHOLES MODEL

Apart from financial vehicle, the Black Scholes model has been benchmarked in other industries.

The role of option contracts in the context of the supply chain management benchmarked on the Black Scholes model to embed real price analysis in instances that the demand curve sloped downward. The Black Scholes model recognised the call option provision for retailer right to reorder or return goods at a stationary price. The research concludes that the introduction of options is not exactly a zero-sum game. Option contracts raised the wholesale price, while the volatility of the retail price lessened. Conditions stem from the manufacturer preferences, at the same time the retailer is also better off. Nevertheless, should the uncertainty turn sufficiently high, the introduction of option contracts modify equilibrium prices to the disadvantage of the retailer (BENAROCH, M., Shah, S., & Jeffery, M., 2006).

Another study looks into the potential consequences in the valuation of IT investments by application of nested variation of the Black-Scholes model. Using custom-tailored options as the baseline, severe and unpredictable conditions are determined (BENAROCH, M., & Kauffman, R., 1999).

A study benchmarked on the Black Scholes Model as a comparative measure for Asian options finds at volatility Black-Scholes formula overestimates Asian call options with maturity in 30 days. Black-Scholes model is mainly utilised for standard European options which recognises the asset price upon expiration and on the average is higher than the Asian option settlement price on positive drift. The analysis entailed simulations and bias analysis to derive different volatility schemes to arrive at the conclusion (WIKLUND, E, 2012).

A study presents a real option–based contract model by benchmarking the Black Scholes model in risk sharing for the privatisation of underground infrastructures. In the circumstance of the United States shortage in government funding, about 100 water and sewer systems underwent privatisation. Users have taken on considerable risk even when water and sewer systems churn stable revenue, as compared to a number of infrastructures. The research entails simulations by way of the Black Scholes model as the reliable tool for real options evaluation in the assessment and prediction of values (PARK, T., Kim, B., and H. Kim, 2013).

The Black Scholes model is benchmarked in the simulation of market-based option pricing approaches thought as the risk-neutral valuation method, in the delivery of of Public Private Partnerships for highway projects around the world. Studies find the Black Scholes model as an advanced theory for real option techniques when compared to conventional economic analysis. This approach recognises the risk of underestimating future traffic demand and institutionalised the traffic revenue cap or TRC options on the economic risk profile (ASHURI, B., Kashani, H., Molenaar, K. R., Lee, S., & Lu, J, 2012).

BLACK SCHOLES MODEL A HACKNEYED NOTION

While the Black Scholes model is broadly applied across industries, its sole classic contribution is the recognition of fact: Using risk premium when valuing an option is not necessary because the risk premium is embedded in the stock price. It is to note that way back 1600 options were actively traded in Netherlands in some fashion characterised as an expertise in option pricing and hedging assimilating the heuristic method with distinct resemblance of the put call parity (DE LA VEGA, J., 1688). Amsterdam grain dealers took up options and forward contract on Mesopotamian clay tablets that are traced back to 1750 BC; while active option markets located in Paris, London and New York have been proven to exist in the late 1800s and the early 1900s. Into the 1870s markets developed a well enough sophistication to price for tail events (KAIRYS, J. and Valerio, N., 1997).

Active option arbitrage trading events can be situated to ten German treatises on options authored in the late 1800s (NELSON, S. A., 1904); wherein the origins of Dynamic Hedging using a heuristic method developed at about the turn of the twentieth century (TALEB, N., 1997). To tell the truth, a manuscript deriving a number of option pricing formulas published in 1908, described a risk-neutral option valuation, arbitrage principles for put-call parity and forward price (BRONZIN, V., 1908). The same concept explaining the put-call parity authored in 1910 (DEUTSCH, H, 1910) and in 1961 a descriptive on paper of converter functions for the movement of puts into calls or calls into puts (REINACH, A. M., 1961).

Characteristics of static market-neutral delta hedging were published as early as 1902 (HIGGINS, L. R., 1902), which was followed through with discussion on market neutral delta hedging for at-the money options in 1937 (GANN, W. D., 1937). The idea of jumps and fat tails are seen to have taken shape in 1927 (MILLS, F, 1927). Recognition of option pricing formulas first written in 1900 (BACHELIER, L., 1900) adopted lognormal to replace normal distributed asset price in 1962 (SPRENKLE, C., 1961). Finally the lognormal asset price authored in 1964, practically identical to the Black Scholes 1973 formula (BONESS, A., 1964).

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