Equilibrium in the Insurance Industry: Price and Probability of Insolvency

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EQUILIBRIUM IN THE INSURANCE INDUSTRY:

PRICE AND PROBABILITY OF INSOLVENCY

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INTRODUCTION

The standard and well-known result from the economic insurance literature is that a risk-averse consumer will purchase full insurance when insurance prices are actuarially fair [Arrow, 1963;Mossin, 1968; Smith, 1968].[1] Some exceptions to this rule have been shown [Cook and Graham, 1977; Doherty and Schlesinger, 1983, 1990; Mayers and Smith, 1983].[2] Risk neutrality on the part of the insurer coupled with perfect competition implies that insurance prices will be actuarially fair. The risk neutral insurance firm is indifferent regarding risk as long as the premium equals the expected loss from the risk.

There is always some possibility of insolvency for the insurer unless the excess capital held is larger than the total possible loss that could be experienced. Nevertheless, the assumption of risk neutrality on the part of the insurer is a rational assumption and is justified with a simple explanation concerning profit maximization and Bertrand competition between two insurers. That explanation is given in the first part of this essay.

The secondary goal of this essay is to show an insurance market equilibrium defined by an insurance product price and a probability of insolvency for the insurer(s).

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[1] Proof:
Suppose P represents the probability of no accident and (1-P) the probability of an accident, Suppose a risk averse consumer who has initial wealth of W and can purchase a percentage of insurance coverage α, where $0 \leq \alpha \leq 1$ which costs a total of αx and returns x units of consumption/ wealth in the event of a claim/ accident. Also suppose that the severity of a loss is Ω. The von Neumann-Morgenstern expected utility function for the consumer is then $EU = (1-p)U(W-\alpha x) + pU(W-\Omega + x(1-\alpha))$. The consumer maximizes this expected utility function (no budget constraint) by choosing an amount of insurance coverage, x, to purchase. The first order condition of the EU function is then $(1-p)U'(W-\alpha x)\alpha + p(1-\alpha)U'(W-\Omega + x(1-\alpha))$, rearranging terms we get the equation $\frac{(1-p)U'(W-\alpha x)}{pU'(W-\Omega + x(1-\alpha))} = \frac{(1-\alpha)}{\alpha}$, and we note that actuarially fair insurance means that prices are proportional to risk, which implies that $(1-p)/p = (1-\alpha)/\alpha$ which implies that $U'(W-\alpha x) = U'(W-\Omega + x(1-\alpha))$ which implies that $W-\alpha x = W-\Omega + x(1-\alpha)$ which says that the consumer purchases full insurance because $\Omega = x$.

[2] Cook and Graham (1977) showed that a rational insurance purchaser "will typically not fully insure an irreplaceable commodity;" Doherty and Schlesinger (1983) showed that full insurance depends on the correlation between insurable and uninsurable risk; Doherty and Schlesinger (1990) show that rational insurance purchasing when there is some probability of contract non-performance (default) leads to less than full insurance coverage; Mayers and Smith (1983) demonstrate "that when the payoffs of the policy are correlated with the payoffs to the individual's other assets, the demand for insurance contracts is generally not a separable portfolio decision."

THE MODEL

While insurers compete on price to reach a level at or around the actuarially fair price, the insureds themselves are assumed to be concerned with financial quality as previous research has already modeled the insurance market [Doherty and Schlesinger, 1990; Cummins and Mahul, 2003; Rees, et al, 1999; Cummins and Lamm-Tennant, 1994; Cummins and Danzon, 1997; Gron, 1994; Winter 1988, 91a, 94; Cagle and Harrington, 1995]. It is assumed that insureds are concerned with the probability of insolvency of their insurer; this would seem to be a rational assumption. As the probability of insolvency increases, the probability of not receiving indemnity for a loss increases, making the value of the insurance to the insured conditional upon the probability of insolvency for the insurer. A representative insured is taken for the entire insurance market. Two symmetric insurance firms competing on price are introduced in this part of the essay.

This essay outlines a basic model where the equilibrium is defined by a price and a probability of insolvency. The probability of insolvency represents the financial quality of the insurer; the probability of contract non-fulfillment on the part of the insurer. Previous research has been done on the rational purchase of insurance coverage when there is a positive probability of contract non-performance and the impact that financial quality has on the demand for insurance.[3]

MOTIVATION FOR THE ASSUMPTION OF RISK-NEUTRALITY OF AN INSURER

Consider the standard Rothschild-Stiglitz view of the world where two states of the world are modeled, W^1 and W^2, without an insured loss and with an insured loss respectively as shown in Figure 1.1. Consider an initial endowment E and consider a high and a low risk insured. Suppose the entire endowment could be lost in the event that the second state of the world is reached without insurance. Consider the case of a pooling equilibrium in Figure 1.1 represented by contract P which cannot exist as

[3] See, for example, Doherty and Schlesinger, 1990; Cummins and Mahul, 2003; Tapiero and Jacque, 1987; also see Cagle and Harrington, 1995; and Cummins and Danzon, 1997.

shown by Rothschild and Stiglitz. Any insurer offering contract γ will be able to make a profit by attracting the low-risk insureds and not the high-risk insureds and so the pooling equilibrium cannot exist because the existence of the γ contract alongside a pooling equilibrium would contradict the profit maximization portion of the definition of equilibrium. Any insurer offering a contract of γ would make a profit, but would lose the low-risk market to another insurer who could offer a contract closer to α^L, the contract along the fair-odds, actuarially fair insurance line for the low-risk insured.[4]

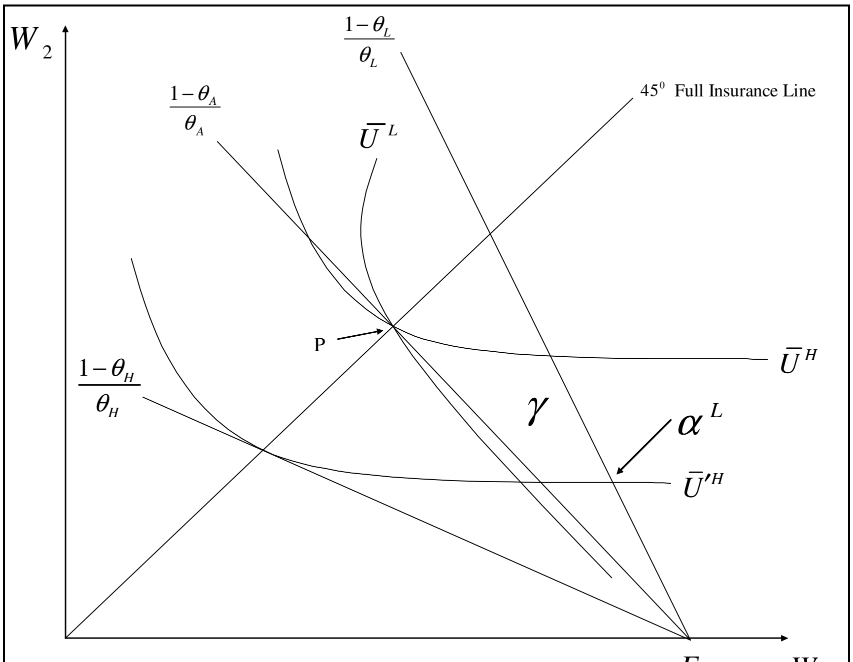

Figure 1.1

<the upward sloping indifference curve in Figure 1.1 needs to be fixed>

As a condition of profit maximization, the insurer will necessarily take on any risk that pays a premium greater than the actuarially fair expected loss. This competitive behavior implies that insurers

[4] It is left to the reader to prove it a necessary and sufficient condition that a risk-neutral insurer prices insurance at an actuarially fair insurance price when under Bertrand competition.

will price insurance along the actuarially fair insurance line where the low-risk insured is offered contract α^L, the separating contract for the low-risk insurance market.

FINANCIAL QUALITY

The assumptions necessary for the equilibrium in this essay rely upon the financial quality hypothesis that insurers are willing to pay more for an insurance policy which is backed by an insurer with superior financial quality. As long as an insurer receives a price which is greater than or equal to the expected loss, the insurer is profit maximizing. And yet, it is straightforward that an insurer who charges a price which is strictly greater than the expected loss has a lower probability of insolvency than an insurer receiving a price exactly equal to the expected loss.[5] They are both profit maximizing, but the former insurer is receiving larger profits while also having a lower probability of insolvency. The regulator fulfills the role of verifying the accuracy of the financial quality/ solvency of the insurers.

Doherty and Schlesinger (1990) note that the actuarially fair insurance price for the insurance product which includes some probability of contract nonperformance (insolvency) is actually lower than the actuarially fair insurance price for the insurance product underwritten by an insurer which is perfectly solvent. Although the willingness to pay more for an insurance product with a lower probability of insolvency for the insurer holds true in accordance with the results of Doherty and Schlesinger, it should be noted that insurance in the case of the property and casualty insurance industry includes some social risk which necessarily implies that insurance will not be actuarially fair.[6]

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[5] An insurer who earns greater underwriting profit has a lower probability of insolvency. Note that as the price, P, increases, the probability of insolvency, Ψ decreases. The probability of insolvency function is introduced in equation (1.8) and in extended form is written as: $\Psi = \exp\left\{ \frac{-2K \left(\sum_{i=1}^{n} Y_i \left(P_i - E[\Omega_i] - \beta \right) - rK \right)}{Y^2 \operatorname{var}\left(E[\Omega\ ] \right)} \right\}$

[6] Social risk exists whenever wealth in two different states of the world differ from each other. If the wealth is the same in all states of the world (claim or not insurance claim) then full insurance is possible. In the case of property and casualty insurance, consider the case of a home insurance claim or an automobile insurance claim. If a home or

The insurer who is able to fake financial quality without holding excess costly capital reserves is able to raise insurance prices in response to the utility maximizing behavior of insurance consumers who wish to be insured by a financially solvent insurer. This argument is useful in motivating a rationale for the existence of a regulator whose purpose is to audit insurers to determine their true probability of insolvency.

The simplifying assumption is made that this is a costless process and that the regulator attains perfect information regarding the probability of insolvency and passes that information on to the consumers who then maximize their utility.

From this point onwards, the Bertrand price competition is left behind and a profit maximizing monopolist is assumed. This change allows for a simplified representation of the calculation of an equilibrium which involves a price and a probability of insolvency.

There are three states of the world for consumers, $W_1 = E - P$ is wealth for the consumer in state one when the insured has no loss but has paid a premium P for insurance. $W_2 = E - P - \Omega + \alpha\Omega$ is wealth for the consumer in state two when a premium was paid, a loss occurs, the insurer is solvent, and the claim is paid; the insured recovers $\alpha\Omega$ of the loss. $W_3 = E - P - \Omega$ is wealth for the consumer when

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an automobile is damaged, this represents the destruction of some wealth for society, even though the insured is indemnified. The existence of social risk implies that insurance cannot be actuarially fair:

Proof:
Suppose the total wealth before an accident is not equal to the total wealth after an accident, $W_1 > W_2$, then the consumer will necessarily not be able to fully insure and $\Omega \neq x$. The expected utility function for the insurance consumer is $EU = (1-p)U(W_1 - \alpha x) + pU(W_1 - \Omega + x(1-\alpha))$ and the first order condition rearranged is $\frac{(1-p)U'(W_1 - \alpha x)}{pU'(W_1 - \Omega + x(1-\alpha))} = \frac{(1-\alpha)}{\alpha}$ but $(W_1 - \alpha x) \neq (W_1 - \Omega + x(1-\alpha))$ because $W_1 > W_2$. So $\frac{U'(W_1 - \alpha x)}{U'(W_1 - \Omega + x(1-\alpha))} < 0$ which implies that $\frac{(1-p)}{p} > \frac{(1-\alpha)}{\alpha}$ which proves that with social risk, prices are higher than the otherwise actuarially fair price.

a premium is paid, a loss occurs and the insurer is insolvent so that the insured does not recover anything from the insurer.

The consumer's objective is to maximize expected utility by choosing a level of insurance coverage α. Equation (1.1) shows the consumers objective function.

$$EU(\alpha) = (1-\phi)U(W_1) + \phi(1-\Psi)U(W_2) + \phi\Psi U(W_3) \quad (1.1)^7$$

The first order condition for the maximization of (1.1) is $EU' = 0$ as in,

$$(1-\phi)U'(W_1) + \phi(1-\Psi)U'(W_2) + \phi\Psi U'(W_3) = 0 \quad (1.2)$$

Note that for every level of insurance coverage α there is a probability that the insured will not receive indemnity. Although there may be an upper limit on the total amount that the insurer would ever have to pay out in claims, if the cost of holding capital is strictly positive, the insured will never be able to fully insure. This is a condition which could imply actuarially unfair insurance in addition to the social-risk condition noted earlier. Note that if the probability of insolvency is zero, the consumer's utility maximization problem reduces to the standard one. Let α^* be the solution to the consumer's utility maximization problem and then consider the consumer's indirect utility function, equation (1.3).

$$V^* = (1-\phi)U(W_1) + \phi(1-\Psi)U(W_2) + \phi\Psi U(W_3) \quad (1.3)$$

The consumer's demand for insurance coverage is then a function of ϕ, Ψ, Ω, P, E as is the indirect utility function. For simplicity, consider the case where the endowment, probability of loss, and expected loss are held constant so that the demand for insurance and the indirect utility are only a function of Ψ, P. For any given level of indirect utility V^*, there is a tradeoff that the consumer is then willing to make between the probability of insolvency and the price, Ψ, P so that $\Psi(P)$ and $P(\Psi)$. The

[7] This is based on a standard von Neumann-Morgenstern expected utility equation with three potential states of the world.

relationship is assumed to be one-to-one; for any level of indirect utility, there is a unique Ψ for any given P and there is a unique P for any given Ψ.

The insurance firm then optimizes profit by recognizing this demand for insurance. The profit maximizing function of the insurer is shown in equation (1.4) where the expected profit, $E[\pi]$ is a function of quantity Y, prices P, expected losses $E[\Omega]$, constant marginal transaction cost β the cost of capital r, and capital itself K. The insurer maximizes profit by choosing price and capital. The cost of capital r is the difference between the opportunity cost of capital and the return on investment from investing the capital. It is assumed that $r > 0$.

$$\underset{P,K}{Max}: E[\pi] = Y(P - E[\Omega] - \beta) - rK \qquad (1.4)$$ [8]

As a function of prices and probability of insolvency, the required capital can be found; equation (1.4) can then be rewritten as it is in equation (1.5) where capital is a function of price and probability of insolvency. Note that demand, Y, can also be written as a function of prices and probability of insolvency by substituting the consumer's demand function into the monopolist's profit maximization function. The relationship between capital, prices, and the probability of insolvency are shown in equation (1.6) and further in the appendix to this essay.

$$\underset{P,\Psi}{Max}: E[\pi] = \left[Y\left(P,\Psi\right)\right](P - E[\Omega] - \beta) - r\left[K(P,\Psi)\right] \qquad (1.5)$$

Insolvency risk is given an explicit value in this essay based on an actuarial theory regarding the probability of insolvency.[9] The probability of insolvency can be modeled across infinite or finite time. The probability of insolvency is a function of the capital held at the start of time, the expected profit, and

[8] It is assumed that the Y risks are independent from each other.

[9] For a more detailed explanation of the probability of insolvency/ ruin, the reader is directed towards either (Kaas, et al, 2001) or (Booth, et al, 1999) and the appendix to this essay.

the variance of that expected profit. Equation (1.6) shows that the probability of insolvency for an insurer Ψ, is a function of capital, K, expected profit, $E[\pi]$, and the variance of the expected profit, $\operatorname{var}(E[\pi])$.

$$\Psi = \exp\left\{\frac{-2K\left(E[\pi]\right)}{\operatorname{var}\left(E[\pi]\right)}\right\} \tag{1.6}$$ [10]

The first order condition for the profit maximization problem of equation (1.5) is shown in equation (1.7).

$$\frac{\partial E[\pi]}{\partial P} = \frac{\partial E[\pi]}{\partial \Psi} = 0 \tag{1.7}$$

Note that if demand was not a function of the probability of insolvency, the first order condition would revert to the standard monopolist's profit maximization problem where the first order condition would be $[Y'](P - \Omega - \beta) = 0$.

CONCLUSION

From equation (1.7) comes the equilibrium price and probability of insolvency P^*, Ψ^*. Although the probability of insolvency is not utilized by the Canadian *Office of the Superintendent of Financial*

[10]The probability of insolvency for the firm is a function of capital held at the beginning of the period of time under consideration, expected profit, and the variance of expected profit. The capital of the insurer increases continuously and decreases stepwise; premiums accumulate and (relatively large) claims are paid when they occur. It is assumed that the distribution of potential losses for each insurance policy is an exponential distribution and that the aggregation of the policies represents the overall distribution of potential losses for the insurer as a whole. The probability of insolvency is then the probability of having negative capital. More precisely, for a finite time model, the probability of insolvency is the probability of having more claims in a period than all premiums received in that period plus the capital held at the start of the period. The probability of insolvency function can be expanded by using equation (1.4), the explicit expected profit function of the insurer $E[\pi] = Y(P - E[\Omega] - \beta) - rK$, the probability of insolvency function then becomes: $\Psi = \exp\left\{\frac{-2K\left(Y\left(P - E[\Omega] - \beta\right) - rK\right)}{Y^2 \operatorname{var}\left(E[\Omega]\right)}\right\}$

Institutions (OSFI), OSFI does use a minimum capital test (MCT) to ensure some measure of financial quality on the part of insurers.[11]

[11] The *MCT For Federally Regulated Property and Casualty Insurance Companies* requires capital to be held for Policy Liabilities. The regulation requires the insurer to hold onto capital which is calculated as a "margin" (percentage) of the potential liabilities. The regulation states on page 18 that "The margins establish a balance between the recognition of varying risks associated with different classes of insurance and the administrative necessity to minimize the test's complexity." Future research could be done on a cost-benefit analysis of the extra administrative cost that a more complex solvency regulation might involve compared to the benefit in terms of efficiency improvements for the industry.

APPENDIX

This appendix provides additional information and analysis of the probability of insolvency function. It shows the numerous ways in which an insurer can improve financial quality. It shows the first and second partial derivatives of the probability of insolvency function with respect to the variables that determine it. If an insurer wishes to improve financial quality, numerous approaches can be taken besides simply raising prices and/ or holding more capital; operation and distribution structures can be altered and different lines of business with less variance can be underwritten.

The probability of insolvency for the firm is a function of capital held at the beginning of the period of time under consideration, expected profit, and the variance of expected profit. Equation (1.8) shows that the probability of insolvency for an insurer, Ψ , is a function of capital, K, expected profit, $E[\pi]$, and the variance of the expected profit, $\operatorname{var}(E[\pi])$.The capital of the insurer increases continuously and decreases stepwise; premiums accumulate and claims are paid when they occur. It is assumed that the distribution of potential losses for each automobile insurance policy is an exponential distribution and that the aggregation of the policies represents the overall distribution of potential losses for the insurer as a whole. The probability of insolvency is then the probability of having negative capital. More precisely, for a finite time model, the probability of insolvency is the probability of having more claims in a period than all premiums received in that period plus the capital held at the start of the period.

$$\Psi = \exp\left\{\frac{-2K\left(E[\pi]\right)}{\operatorname{var}\left(E[\pi]\right)}\right\} \tag{1.8}$$

The probability of insolvency function can be expanded by using equation (1.4), the expected profit function of the insurer $E[\pi] = Y(P - E[\Omega] - \beta) - rK$, the probability of insolvency function is then shown in expanded form in equation (1.9).

$$\Psi = \exp\left\{\frac{-2K\ \left(Y\ \left(P - E[\Omega] - \beta\right) - rK\right)}{Y^2\ \operatorname{var}\left(E[\Omega]\right)}\right\} \tag{1.9}$$

As the capital reserves at the start of the time period, K, increase, the probability of insolvency decreases as shown in the model. As equation (1.10) shows, an increase in capital leads to a decrease in the probability of insolvency at an increasing rate as the cost of capital, r, is held constant at zero. It could be assumed that the more stable an insurer becomes, the more closely $r \to 0$, where r is considered here the cost of capital and approaches zero because the cost of capital is the cost of borrowing minus the revenue from investing the capital which is merely held to decrease insolvency risk. Assuming that expected loss, quantity, marginal costs, the cost of capital, and capital are all held constant, there exists a relationship where capital is a function of prices and a given level of probability of insolvency, $K(P,\Psi)$.

$$\begin{aligned}\left.\frac{\partial \Psi}{\partial K}\right|_{r=0} &= \frac{4Kr - 2YP + 2Y\beta + 2YE[\Omega]}{Y^2 VarE[\Omega]}\cdot \Psi < 0,\\ \left.\frac{\partial^2 \Psi}{\partial K^2}\right|_{r=0} &= \frac{4Kr - 2YP + 2Y\beta + 2YE[\Omega]}{Y^2 VarE[\Omega]}\cdot \frac{\partial \Psi}{\partial K} + \frac{4r}{Y^2 VarE[\Omega]}\cdot \Psi > 0\end{aligned} \tag{1.10}$$

Note that capital can be written as a function of prices and the probability of insolvency as in equation (1.11) when the other variables are held constant and the quantity is also written as a function of prices and probability of insolvency as shown in equation (1.17).

$$K = \frac{Var\left(E[\Omega]\right)\ln\Psi}{-2Y^{-1}\left(P - E[\Omega_i] - \beta\right) - rY^{-2}} \tag{1.11}$$

As the expected profit increases, the probability of insolvency also decreases. Profitability in this case should be related to the difference between the premium collected and the money paid out in actual damages resulting from claims. The cost of settling the claims and the fixed costs of running the insurance business should themselves be aggregated with the expected payoffs. As equation (1.12) shows, an increase in expected profit reduces the probability of insolvency but at a decreasing rate.

$$\begin{aligned} \frac{\partial\Psi}{\partial\left[E(\pi)\right]} &= \frac{-2K}{\left[VarE(\pi)\right]}\cdot\Psi < 0 \\ \frac{\partial^2\Psi}{\partial\left[E(\pi)\right]^2} &= \frac{-2K}{\left[VarE(\pi)\right]}\cdot\Psi < 0 \end{aligned} \tag{1.12}$$

Contrary to capital and expected profits, an increase in the variance of the expected profit, $V\text{ar}\left(E[\pi]\right)$, which is itself a function of the variance of expected losses, $V\text{ar}\left(E[\Omega\ ]\right)$, and the total quantity of policies, Y, leads to an increase in the probability of insolvency but at a decreasing rate as shown by equation (1.13).

$$\begin{aligned} \frac{\partial\Psi}{\partial\left[Var(E[\pi])\right]} &= \frac{2K\left(E[\pi]\right)}{\left[Var(E[\pi])\right]^2}\cdot\Psi > 0 \\ \frac{\partial^2\Psi}{\partial\left[Var(E[\pi])\right]^2} &= \frac{-8K^2\left[E(\pi)\right]^2}{\left[Var(E[\pi])\right]^5}\cdot\Psi < 0 \end{aligned} \tag{1.13}$$

As noted above, the variance of expected profit can actually be rewritten as a function of total policies underwritten and the variance of expected losses, $\text{var}\left(E[\pi]\right) = Y^2\ \text{var}\left(E[\Omega\ ]\right)$. If we break down the impact that these individual variables have on the probability of insolvency, we see stronger results for the impact that expected losses have on the probability of insolvency and opposite results for the impact that quantity has on the probability of insolvency. Equation (1.14) shows that an increase in the variance of expected losses has an increasing effect on the probability of insolvency but at a decreasing rate.

$$\begin{aligned} \frac{\partial\Psi}{\partial\left[Var(E[\Omega])\right]} &= \frac{2KE(\pi)}{Y^2\left[Var(E[\Omega])\right]^2}\cdot\Psi > 0 \\ \frac{\partial^2\Psi}{\partial\left[Var(E[\Omega])\right]^2} &= \frac{-8K^2\left[E[\pi]\right]^2}{Y^4 Var(E[\Omega])}\cdot\Psi < 0 \end{aligned} \tag{1.14}$$

Note, however, by equation (1.15) that the variance of expected losses has a greater impact on increasing the probability of insolvency than the variance of expected profit. This is because the variance of expected profit is also a function of quantity of policies, Y, which has a negative impact on the probability of insolvency as shown in equation (1.16). It is assumed that the policies, Y, are independent from each other.

$$\frac{\partial \Psi}{\partial Var(E[\Omega])} > \frac{\partial \Psi}{\partial Var(E[\pi])} \tag{1.15}$$

Equation (1.16) confirms the assumption that an increase in quantity of insured risks which are similar but independent from each other reduces solvency risk. It reduces solvency risk, but it does so at a decreasing rate when the cost of capital, r, is held constant. Assuming that expected loss, capital, marginal costs, the cost of capital, and capital are all held constant, there exists a relationship where quantity of insurance is a function of prices and a given level of probability of insolvency, $Y(P,\Psi)$.

$$\begin{aligned} \frac{\partial \Psi}{\partial Y} &= \frac{2KYP - 2KY\beta - 2KYE[\Omega] - 4rK^2}{Y^3 VarE[\Omega]} \cdot \Psi < 0 \\ \left.\frac{\partial^2 \Psi}{\partial Y^2}\right|_{r=0} &= \frac{2KYP - 2KY\beta - 2KYE[\Omega] - 4rK^2}{Y^3 VarE[\Omega]} \cdot \frac{\partial \Psi}{\partial Y} + \frac{-2KP - 2K\beta - 2KE[\Omega]}{3Y^2 VarE[\Omega]} \cdot \Psi < 0 \end{aligned} \tag{1.16}$$

Note that quantity can be written as a function of prices and the probability of insolvency as in equation (1.17) when the other variables are held constant.

$$Y(P,\Psi) = \frac{-2K\left(P - E[\Omega] - \beta\right)}{\ln \Psi \operatorname{var}\left(E[\Omega]\right)} + \frac{P - E[\Omega] - \beta}{rK} \tag{1.17}$$

To further elaborate and extend our model, we can consider the fact that an insurer has some sort of cost structure, β, which we assume to be the marginal cost of transactions (underwriting/ adjusting/ administration) per policy. Equation (1.18) confirms the known fact that an increase/ decrease in marginal costs leads to an increase/ decrease in the probability of insolvency.

$$\begin{aligned} \frac{\partial \Psi}{\partial \beta} &= \frac{2KY}{Y^2\left[Var(E[\Omega])\right]} \cdot \Psi > 0 \\ \frac{\partial^2 \Psi}{\partial \beta^2} &= \frac{2KY}{Y^2\left[Var(E[\Omega])\right]} \cdot \Psi > 0 \end{aligned} \tag{1.18}$$

The most challenging part to model is the cost of capital. If the cost of capital was zero, insurers would hold an infinite quantity of capital to ensure that they never went bankrupt[12]. The cost of capital does not have to be the same for each borrower. The cost of capital issue is also complicated because the

[12] Note that the cost of capital is modeled here as the difference between the cost of borrowing money and the return from investing it. It is assumed that insurers have some cost of capital. This can be because of the requirement of regulators that capital be invested in "safe" investments or because of the transaction costs of borrowing and investing money.

capital is simply held to ensure against bankruptcy and so the money can be re-invested by the insurer. This adds another dimension to the model because regulators often place restrictions on where the capital can be invested. Equation (1.19) shows the impact that the cost of capital, r, has on the probability of insolvency. As the cost of capital increases, the probability of insolvency increases at an increasing rate.

$$\begin{aligned} \frac{\partial \Psi}{\partial r} &= \frac{2K^2}{Y^2\left[Var(E[\Omega])\right]} \cdot \Psi > 0 \\ \frac{\partial^2 \Psi}{\partial r^2} &= \frac{2K^2}{Y^2\left[Var(E[\Omega])\right]} \cdot \Psi > 0 \end{aligned} \tag{1.19}$$

Lastly, equation (1.20) shows the obvious relationship between prices and the probability of insolvency where an increase in the price of insurance, P, leads to an increase in the probability of insolvency but at a decreasing rate.

$$\begin{aligned} \frac{\partial \Psi}{\partial P} &= \frac{-2KY}{Y^2\left[Var(E[\Omega])\right]} \cdot \Psi < 0 \\ \frac{\partial^2 \Psi}{\partial P^2} &= \frac{-2KY}{Y^2\left[Var(E[\Omega])\right]} \cdot \Psi < 0 \end{aligned} \tag{1.20}$$

If an insurer prices insurance actuarially fair and is able to borrow money at an effective cost of capital rate of zero, r=0, then the insurer borrows an infinite amount of capital, will never go bankrupt, and will break even in the long run[13]. We've noted however that consumers are willing to make their insurance purchases based on the insolvency level of an insurer in addition to price: the financial quality hypothesis.

[13] This follows straightforward from the fact that the expected losses are best estimates and that they equal actual losses in the long run.

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