Behavior-based Price Discrimination in the Domestic and International Mixed duopoly

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Abstract

We investigate mixed markets in which a social welfare-maximizing public firm and a private firm engage in behavior-based price discrimination. We consider two cases: one where the private firm is completely owned by domestic shareholders and one where it is completely owned by foreign shareholders. In the domestic mixed duopoly, BBPD is irrelevant from the viewpoint of social welfare. This is because poaching does not occur. In the international mixed duopoly, BBPD reduces the public firm's market share but improves domestic social welfare. This is because the outflow to foreign shareholders decreases. We also consider domestic and international pure duopoly and find that the presence of public firms reduces welfare loss caused by BBPD.

JEL classification: D43, H42,L13
Keywords : Behavior-based price discrimination, Mixed oligopoly, Foreign firms, Privatization

1 Introduction

Price discrimination is widely observed in many industries such as air travel, supermarket, telecommunication, and web retailing. The price discrimination that firms offer different prices depend on consumers' past purchases is termed Behaviour-based price discrimination (BBPD). Since the seminal works by Villas-Boas (1999) and Fudenberg and Tirole (2000), numerous studies have investigated BBPD. Chen and Pearcy (2010) discuss intertemporal dependence of brand preference. Liu and Shuai (2013, 2016)

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investigate price discrimination in a multi-dimensional model. Esteves and Reggiani (2014) and Zhang et al. (2019) introduce elastic demand. These articles do not include firms fully or partially owned by governments. However, in the real world, they also engage in BBPD.

In the telecommunication industry, many state-owned enterprises (SOEs) compete with domestic or foreign private firms. In Europe, several firms that are partially owned by the governments provide mobile services (e.g. Orange, Telenor, and Telia). In developing countries, there is a tendency in which multi-national firms have large market shares in mobile phone markets with SOEs. Telkomsel, which is owned by the Indonesian government, competes with Ooredoo, which is a company with Qatari capital. In Pakistan, PTCL competes with China mobile. In India, BSNL competes with Vi which is a company with some British capital. In mobile phone markets, consumers can choose between pre-paid plans or post-paid plans. Companies often offer rivals' consumers special discounts when they contract post-paid plans with rivals' consumers. In developing countries, pre-paid plans are the mainstream. However, post-paid plans are expected to increase in the future due to the increase in communication volume and the development of finance.

This paper provides analyses of BBPD in mixed duopoly markets. We follow the brand preference approach based on Hotelling models (Fudenberg and Tirole, 2000). We assume that the public firm maximizes domestic social welfare and the private firm maximizes its profit. In a domestic (international) mixed duopoly, the public firm competes with a private firm which completely owned by domestic (foreign) shareholders. Consumers' preferences are private information. In the first period, firms offer uniform prices. In the second period, firms offer different prices based on the consumers' purchase histories. We investigate the impacts of privatization on social welfare by comparing welfare loss caused by BBPD between mixed duopoly and pure duopoly.

We find that BBPD is irrelevant from the viewpoint of social welfare in the domestic mixed duopoly, contrasting to the result in Fudenberg and Tirole (2000) who show that BBPD is beneficial to consumers and detrimental to firms and social welfare. In the domestic mixed duopoly, BBPD does not reduce domestic social welfare since poaching does not occur in the second period. This is because the public firm makes indifferent consumers locate at the center of the line set to minimizes the sum of disutilities of taste mismatch. We also show that BBPD is beneficial to domestic social welfare in the international mixed duopoly. In the international mixed duopoly, BBPD is not detrimental to domestic social welfare. BBPD increases the private firm's total market share but reduces its profit. This is brought about by the public firm's behavior of lowering its prices to reduce the outflow to foreign shareholders. In both cases, BBPD improves consumer surplus, like Fudenberg and Tirole (2000). We also find that the presence of public firms reduces welfare loss caused by BBPD, although BBPD never improves social welfare in pure duopolies, discussed in Fudenberg and Tirole (2000).

Our paper relates to two strands of literature. Firstly, our paper is related to a strand of literature which deals with BBPD. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), and Choe and Matsushima (2021) show that BBPD is detrimental to firms unless firms are sufficiently asymmetric since it intensifies competition. Carroni (2016), Jing (2017), and Rhee and Thomadsen (2017) introduce asymmetric firms and show that BBPD is beneficial to firms. Our paper introduces asymmetric objective functions and shows that BBPD is detrimental to firms in the domestic and international mixed duopolies.

This paper is also related to strand of literature which deals with the international mixed duopoly. Since Fjell and Pal (1996), many studies discuss international mixed oligopoly (e.g. Pal and White (1998), Matsumura (2003), and Matsumura et al. (2009)). They typically show that the presence of foreign private firms makes the public firm lower its prices and increases consumer surplus. Lyu and Shuai (2017) analyze the international mixed market by using Hotelling models and indicates that the presence of the foreign firm increases transport costs. Matsushima and Matsumura (2003), Heywood and Ye (2009a), Ye and Wu (2015), and Heywood et al. (2021) use spatial price discrimination models to discuss mixed markets. Matsushima and Matsumura (2006) and Heywood and Ye (2009b) analyze spatial price discrimination models of mixed markets with foreign private firms. In spatial price discrimination models, firms offer different prices based on consumers' location. In our model, we assume that firms offer different prices based on the consumers' purchase histories. We find that BBPD increases the sum of disutilities of taste mismatch in the international mixed duopoly.

The remainder of this paper is organized as follows. Section 2, Section 3, and Section 4 describes models. We consider two cases: the domestic mixed duopoly and the international mixed duopoly. Section 5 presents welfares analysis. Section 6 is the conclusion.

2 Preliminary

We extend Fudenberg and Tirole (2000) by considering mixed duopoly markets. Two firms, A and B compete in two periods. Let firm A be a public firm, which maximizes domestic social welfare, and firm B be a private firm, which maximizes its profit. The firms produce homogeneous goods at a constant marginal cost c and simultaneously set their prices in each period.

Consumers uniformly distribute over $[\underline{\theta},\ \overline{\theta}]$ $(\underline{\theta} = -\overline{\theta},\ \underline{\theta} < 0)$. The parameter θ represents how much consumers prefer goods B over goods A. To simplity to analysis, we assume that $c > \overline{\theta}$. Consumers' preferences are constant over time. Consumers buy one unit of product either firm A or firm B in each period. The per-period utilities for a consumer indexed by θ purchasing from A and B at price p are $u_A = v - \frac{\theta}{2} - p$ and $u_B = v + \frac{\theta}{2} - p$, respectively. We assume that v is sufficiently large so that all consumers purchase in equilibrium. Firms and consumers discount their future by the common factor $\delta \in [0,1)$.

3 Domestic mixed duopoly

We firstly discuss a market where a public firm competes with a domestic private firm, which is completely owned by domestic shareholders.

3.1 Uniform pricing

We consider the benchmark case where BBPD is not feasible. The firms can not identify the first-period decisions of consumers or BBPD is illegal. The two-period model reduces to two replications of the static equilibrium. We solve the static model.

Let θ^U denote the consumer who is indifferent between choosing firm A and firm B. Consumers in $[\underline{\theta},\ \theta^U]$ choose firm A and consumers in $[\theta^U,\ \overline{\theta}]$ choose firm B. Firm A and firm B offer p_A and p_B to consumers. Then, θ^U satisfies $v - \frac{\theta^U}{2} - p_A = v + \frac{\theta^U}{2} - p_B$. Solving it for θ^U yields

$$\theta^U = p_B - p_A. \tag{1}$$

Per-period domestic social welfare under uniform pricing is the sum of per-period consumer surplus and per-period profits of firms, and is given by

$$sw^U = (v-c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}(\theta^U)^2, \tag{2}$$

Firm A chooses p_A to maximize (2) and firm B chooses p_B to maximize $\pi_B = (p_B - c)(\overline{\theta} - \theta^U)$.

The first-order conditions yield $p_A^* = p_B^* = c + \overline{\theta},\ \theta^{U*} = 0$. Hence, the total discounted consumer surplus, the total discounted profits, and total discounted social welfare in equilibrium under uniform pricing in the domestic mixed duopoly are

$$\begin{aligned} CS^U &= (1+\delta)[(v-c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - 2(1+\delta)\overline{\theta}^2, && (3) \\ \Pi_A^U &= \Pi_B^U = (1+\delta)\overline{\theta}^2, && (4) \\ SW^U &= (1+\delta)[(v-c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2]. && (5) \end{aligned}$$

3.2 BBPD

We now assume that BBPD is feasible. In the first period, each firm sets a single price simultaneously. In the second period, they offer different prices to consumers who have different purchase histories after they observe the first-period behaviors of consumers. We derive the subgame perfect Nash equilibrium by backward induction.

The second period

Let θ^* denote the consumer who is indifferent between choosing firm A and firm B in the first period. Consumers in $[\underline{\theta}, \theta^*]$ chose firm A and consumers in $[\theta^*, \overline{\theta}]$ chose firm B in the first period. Firm A and firm B offer α and β to its past consumers and offer $\hat{\alpha}$ and $\hat{\beta}$ to the rival's past consumers, respectively, in the second period. Consumer θ on $[\underline{\theta}, \theta^*]$ continues to purchase from firm A if $v - \frac{\theta}{2} - \alpha \geq v + \frac{\theta}{2} - \hat{\beta}$, otherwise, switches to firm B. Consumer θ on $[\theta^*, \overline{\theta}]$ continues to purchase from firm B if $v - \frac{\theta}{2} - \hat{\alpha} \leq v + \frac{\theta}{2} - \beta$, otherwise, switches to firm A. Considering the possibility of corner solutions, we obtain the locations of the indifferent consumers on $[\underline{\theta}, \theta^*]$ and $[\theta^*, \overline{\theta}]$, θ_A and θ_B, respectively.

$$\theta_A = \begin{cases} \hat{\beta} - \alpha, & \text{if} \quad \hat{\beta} - \alpha \leq \theta^*, \\ \theta^*, & \text{if} \quad \hat{\beta} - \alpha \geq \theta^*, \end{cases} \tag{6}$$

$$\theta_B = \begin{cases} \theta^*, & \text{if} \quad \beta - \hat{\alpha} \leq \theta^*, \\ \beta - \hat{\alpha}, & \text{if} \quad \beta - \hat{\alpha} \geq \theta^*. \end{cases} \tag{7}$$

Domestic social welfare in the second period under BBPD is given by

$$sw_2^D = (v-c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 + \frac{1}{2}\theta^{*2} - \frac{1}{2}\theta_A^2 - \frac{1}{2}\theta_B^2, \tag{8}$$

where the superscript "D" stands for price discrimination. Firm A chooses α and $\hat{\alpha}$ to maximize (8) and firm B chooses β and $\hat{\beta}$ to maximize $\pi_{B2}^D = (\beta - c)(\overline{\theta} - \theta_B) + (\hat{\beta} - c)(\theta^* - \theta_A)$.

By taking into account the parametric conditions in (6) and (7), we solve the maximization problems, leading to

$$\alpha = \begin{cases} \hat{\beta}, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}, \quad (9) \\ \hat{\beta} - \theta^*, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (10) \end{cases}$$

$$\hat{\alpha} = \begin{cases} \beta, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (11) \\ \beta - \theta^*, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}, \quad (12) \end{cases}$$

$$\hat{\beta} = \begin{cases} \frac{1}{2}\alpha + \frac{c + \theta^*}{2}, & \text{if} \quad \alpha \geq \mathrm{c} - \theta^*, \quad (13) \\ \alpha + \theta^*, & \text{if} \quad \alpha \leq \mathrm{c} - \theta^*, \quad (14) \end{cases}$$

$$\beta = \begin{cases} \frac{1}{2}\hat{\alpha} + \frac{c + \overline{\theta}}{2}, & \text{if} \quad \hat{\alpha} \leq \mathrm{c} + \overline{\theta} - 2\theta^*, \quad (15) \\ \hat{\alpha} + \theta^*, & \text{if} \quad \hat{\alpha} \geq \mathrm{c} + \overline{\theta} - 2\theta^*. \quad (16) \end{cases}$$

These reaction functions can be depicted in Figure 1.[1] Figure 1 shows that there is a unique equilibrium represented by E_A in (a) and (b) and E_B in (c), respectively.[2] However, there are multiple equilibria in (d). We assume that a social welfare-maximizing firm puts a little more value on the consumer surplus than on the producer surplus. Firm A lowers its price as much as possible and then equilibrium is determined at E_B in (d). Thus, equilibrium prices in the second period are

$$\hat{\alpha} = \beta = c + \overline{\theta}, \ \ \alpha = c - \theta^*, \ \ \hat{\beta} = c, \qquad \text{if} \quad \underline{\theta} \le \theta^* \le 0, \tag{17}$$
$$\alpha = \hat{\beta} = c + \theta^*, \ \ \hat{\alpha} = c + \overline{\theta} - 2\theta^*, \ \ \beta = c + \overline{\theta} - \theta^*, \qquad \text{if} \quad 0 \le \theta^* \le \overline{\theta}. \tag{18}$$

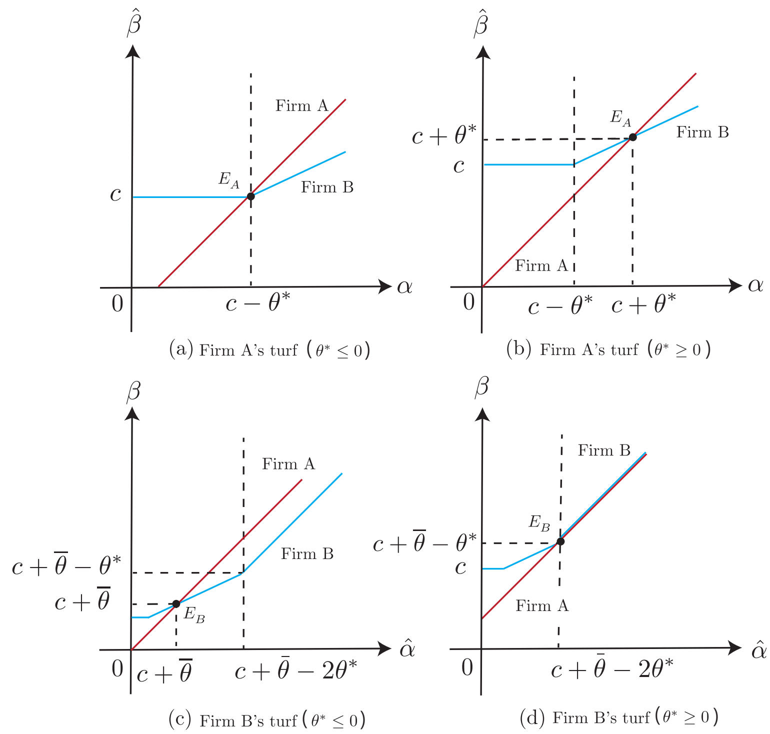

Figure 1: The second period's equilibrium in the domestic mixed duopoly

Therefore, we have

$$sw_2^D = (v - c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2, \quad \text{for all} \quad \underline{\theta} \le \theta^* \le \overline{\theta}, \tag{19}$$

[1]Since $\underline{\theta} \le \theta^* \le \overline{\theta}$, $min\{c + \overline{\theta} - 2\theta^*\} = c - \overline{\theta} > 0$. Hence, $(c + \overline{\theta} - 2\theta^*)$ is positive for all $\underline{\theta} \le \theta^* \le \overline{\theta}$.

[2]Following Fudenberg and Tirole (2000), we assume that a profit-maximizing firm does not charge prices below the marginal cost.

$$\pi_{B2}^D = \begin{cases} \overline{\theta}^2, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (20) \\ \theta^{*2} + (\overline{\theta} - \theta^*)^2, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \quad (21) \end{cases}$$

The first period
We obtain the indifferent consumers in the first period. In the case of $\underline{\theta} \leq \theta^* \leq 0$, the consumer, θ^*, is indifferent between choosing firm A in the first period at price a and then choosing firm A in the second period at price α, or choosing firm B in the first period at price b and then choosing firm A at price $\hat{\alpha}$. Thus, θ^* satisfies $v - \frac{\theta^*}{2} - a + \delta(v - \frac{\theta^*}{2} - \alpha) = v + \frac{\theta^*}{2} - b + \delta(v - \frac{\theta^*}{2} - \hat{\alpha})$. Substituting the second-period equilibrium prices and solving it for θ^* yields

$$\theta^* = \frac{b - a + \delta\overline{\theta}}{1 - \delta}, \quad \text{if} \quad \underline{\theta} \leq \theta^* \leq 0. \tag{22}$$

In the same way, in the case of $0 \leq \theta^* \leq \overline{\theta}$, the consumer, θ^*, is indifferent between choosing firm A in the first period at price a and then choosing firm B in the second period at price $\hat{\beta}$, or choosing firm B in the first period at price b and then choosing firm B at price β. Thus, θ^* satisfies $v - \frac{\theta^*}{2} - a + \delta(v + \frac{\theta^*}{2} - \hat{\beta}) = v + \frac{\theta^*}{2} - b + \delta(v + \frac{\theta^*}{2} - \beta)$. Substituting the second-period equilibrium prices and solving it for θ^* yields

$$\theta^* = \frac{b - a + \delta\overline{\theta}}{1 + 2\delta}, \quad \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \tag{23}$$

Firm A chooses a to maximize $SW^D = sw_1^D + \delta sw_2^D$ and firm B chooses b to maximize $\Pi_B^D = \pi_{B1}^D + \delta\pi_{B2}^D$. Their objective functions can be obtained as

$$SW^D = \left[(v - c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}\theta^{*2}\right] + \delta\left[(v - c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2\right], \tag{24}$$

and

$$\Pi_B^D = \begin{cases} (b - c)\left(\dfrac{(1 - 2\delta)\overline{\theta} - b + a}{1 - \delta}\right) + \delta\overline{\theta}^2, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (25) \\ \dfrac{(b + 2\delta a - (1 + 2\delta)c - 2\delta^2\overline{\theta})((1 + \delta)\overline{\theta} - b + a)}{(1 + 2\delta)^2} + \delta\overline{\theta}^2, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}, \quad (26) \end{cases}$$

where θ^* in (24) is given by (22) if $\underline{\theta} \leq \theta^* \leq 0$, and is given by (23) if $0 \leq \theta^* \leq \overline{\theta}$.

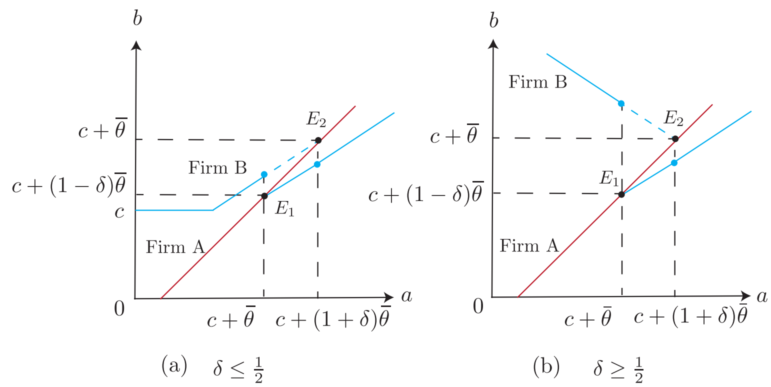

Figure 2: The first period's equilibrium in the domestic mixed duopoly

The differentiation of the above objective functions yields

$$a = b + \delta\overline{\theta}, \qquad \text{for all} \quad \underline{\theta} \leq \theta^* \leq \overline{\theta}, \tag{27}$$

$$b = \begin{cases} \dfrac{1}{2}a + \dfrac{c+(1-2\delta)\overline{\theta}}{2}, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (28) \\ \dfrac{1-2\delta}{2}a + \dfrac{(1+2\delta)c+(1+\delta+2\delta^2)\overline{\theta}}{2}, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \quad (29) \end{cases}$$

These reaction functions can be depicted in Figure 2. The private firm's reaction function has two local optimal solutions for each a in the range $c+\overline{\theta} \leq a \leq c+(1+\delta)\overline{\theta}$. In the range, given firm A's price, firm B set b according to either (28) or (29) and earn the following profit, respectively.

$$\Pi_B^D = \begin{cases} \dfrac{1}{4(1-\delta)}\left[a-c+(1-2\delta)\overline{\theta}\right]^2 + \delta\overline{\theta}^2, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (30) \\ \dfrac{1}{4}\left[a-c+(1-\delta)\overline{\theta}\right]^2 + \delta\overline{\theta}^2, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \quad (31) \end{cases}$$

These are obtained by substitution of (28) into (25) and (29) into (26), respectively. We find that (30) is larger than (31) if $a < c+(\delta-\sqrt{1-\delta})\overline{\theta},\ c+(\delta+\sqrt{1-\delta})\overline{\theta} < a$. Since $c+(\delta+\sqrt{1-\delta})\overline{\theta} < c+\overline{\theta}$, firm B earn a larger profit according to (28) in the range of $c+\overline{\theta} \leq a \leq c+(1+\delta)\overline{\theta}$. Therefore, equilibrium is given by E_1 in Figure 2.

Proposition 1 *Equilibrium prices in the domestic mixed duopoly are given by*

$$a = \hat{\alpha} = \beta = c+\overline{\theta},\ b = c+(1-\delta)\overline{\theta},\ \alpha = \hat{\beta} = c,$$

satisfies the following inequalities

$$a = \hat{\alpha} = p_A^* > \alpha,\ p_B^* = \beta \geq b > \hat{\beta},\ \theta^* = \theta_A = \theta_B = 0.$$

Proposition 1 implies that poaching does not occur in the domestic mixed duopoly.

Total discounted consumer surplus, total discounted profits, and total discounted domestic social welfare in equilibrium under BBPD in the domestic mixed duopoly are

$$CS^D = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta})+\frac{1}{4}\overline{\theta}^2+\frac{1}{4}\underline{\theta}^2]-2\overline{\theta}^2, \tag{32}$$

$$\Pi_A^D = \Pi_B^D = \overline{\theta}^2, \tag{33}$$

$$SW^D = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta})+\frac{1}{4}\overline{\theta}^2+\frac{1}{4}\underline{\theta}^2]. \tag{34}$$

Proposition 2 *Comparing the outcome under uniform pricing,* (3)*,* (4)*, and* (5)*, we have*

$$CS^U \leq CS^D, \Pi_i^U \geq \Pi_i^D, SW^U = SW^D,\ (i = A, B).$$

Thus, BBPD does not affect domestic social welfare since poaching does not occur in the domestic mixed duopoly.

BBPD is beneficial to consumers, detrimental to firms, and neutral to domestic social welfare in the domestic mixed duopoly. Fudenberg and Tirole (2000) consider two symmetric profit-maximizing firms and show that BBPD is beneficial to consumers and detrimental to firms and social welfare. Our result shows that BBPD does not harm domestic social welfare, unlike Fudenberg and Tirole (2000). Poaching allows the reallocation of consumers to the firm they like less and reduces the second-period social welfare in their model. However, Proposition 1 shows that poaching does not occur in our model. This is because the social welfare-maximizing public firm minimizes the sum of disutilities of taste mismatch.

4 International mixed duopoly

We secondly discuss a market where a public firm competes with a foreign private firm, which is completely owned by foreign shareholders.

4.1 Uniform pricing

We consider the case where BBPD is not feasible. Since foreign private firm profits flow out to foreign owners, per-period domestic social welfare under uniform pricing in the international mixed duopoly is the sum of per-period consumer surplus and the per-period profit of firm A, and is given by

$$sw^{Uf} = v(\overline{\theta}-\underline{\theta})+\frac{1}{4}\overline{\theta}^2+\frac{1}{4}\underline{\theta}^2-\frac{1}{2}(\tilde{\theta}^U)^2-c(\tilde{\theta}^U-\underline{\theta})-p_B^f(\overline{\theta}-\tilde{\theta}^U). \tag{35}$$

where the superscript "f" stands for the international mixed duopoly and $\tilde{\theta}^U$ is given by $p_B^f - p_A^f$. Solving as in the previous section, we obtain $p_A^{f*} = c$, $p_B^{f*} = c + \frac{\overline{\theta}}{2}$, $\tilde{\theta}^{U*} = \frac{\overline{\theta}}{2}$. Thus, we have

$$
\begin{aligned}
CS^{Uf} &= SW^{Uf} = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - (1+\delta)\frac{3}{8}\overline{\theta}^2, && (36)\\
\Pi_A^{Uf} &= 0, && (37)\\
\Pi_B^{Uf} &= (1+\delta)\frac{1}{4}\overline{\theta}^2. && (38)
\end{aligned}
$$

4.2 BBPD

We now assume that BBPD is feasible.

The second period

Let θ^{*f} denote the first-period indifferent consumer in the international mixed duopoly. Firm A and firm B offer α^f and β^f to its past consumers and offer $\hat{\alpha}^f$ and $\hat{\beta}^f$ to the rival's past consumers, respectively, in the second period. As in the previous section, we obtain the locations of the indifferent consumers on $[\underline{\theta}, \theta^{*f}]$ and $[\theta^{*f}, \overline{\theta}]$, θ_A^f and θ_B^f, respectively.

$$
\theta_A^f = \begin{cases} \hat{\beta}^f - \alpha^f, & \text{if} \quad \hat{\beta}^f - \alpha^f \leq \theta^{*f}, \\ \theta^{*f}, & \text{if} \quad \hat{\beta}^f - \alpha^f \geq \theta^{*f}, \end{cases} \qquad (39)
$$

$$
\theta_B^f = \begin{cases} \theta^{*f}, & \text{if} \quad \beta^f - \hat{\alpha}^f \leq \theta^{*f}, \\ \beta^f - \hat{\alpha}^f, & \text{if} \quad \beta^f - \hat{\alpha}^f \geq \theta^{*f}. \end{cases} \qquad (40)
$$

In the case of the international mixed duopoly, domestic social welfare in the second period under BBPD is given by

$$
\begin{aligned}
sw_2^{Df} = {} & v(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 + \frac{1}{2}(\theta^{*f})^2 - \frac{1}{2}(\theta_A^f)^2 - \frac{1}{2}(\theta_B^f)^2 \\
& - c(\theta_A^f - \underline{\theta}) - \hat{\beta}^f(\theta^{*f} - \theta_A^f) - c(\theta_B^f - \theta^{*f}) - \beta^f(\overline{\theta} - \theta_B^f). \qquad (41)
\end{aligned}
$$

Firm A chooses α^f and $\hat{\alpha}^f$ to maximize (41) and firm B chooses β^f and $\hat{\beta}^f$ to maximize $\pi_{B2}^{Df} = (\beta^f - c)(\overline{\theta} - \theta_B^f) + (\hat{\beta}^f - c)(\theta^{*f} - \theta_A^f)$. By taking into account the parametric conditions in (39) and (40), we solve the public firm's maximization problems, leading to

$$
\alpha^f = \begin{cases} c, & \text{if} \quad \hat{\beta}^f \leq c + \theta^{*f}, \qquad (42) \\ \hat{\beta}^f - \theta^{*f}, & \text{if} \quad \hat{\beta}^f \geq c + \theta^{*f}, \qquad (43) \end{cases}
$$

and

$$\hat{\alpha}^f = \begin{cases} c, & \text{if} \quad \beta^f \geq c + \theta^{*f}, \quad (44) \\ \beta^f - \theta^{*f}, & \text{if} \quad \beta^f \leq c + \theta^{*f}. \quad (45) \end{cases}$$

The private firm's maximization problem is the same as in (13), (14), (15), and, (16).

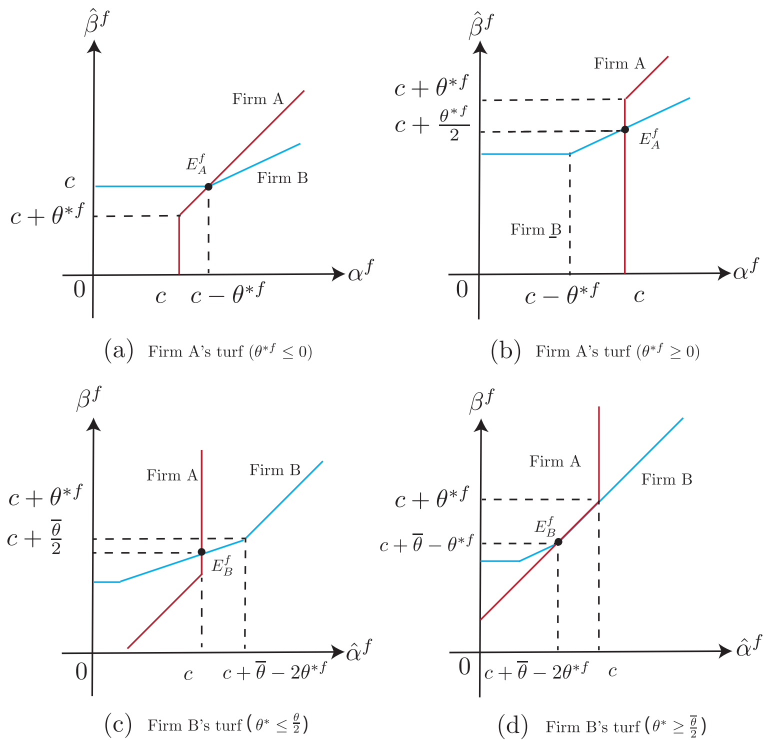

Figure 3: The second period's equilibrium in the international mixed duopoly

The reaction functions can be depicted as Figure 3. Proceeding in the same way, equilibrium prices in the second period are

$$\alpha^f = c - \theta^{*f}, \; \hat{\beta}^f = c, \; \hat{\alpha}^f = c, \; \beta^f = c + \frac{\overline{\theta}}{2}, \qquad \text{if} \quad \underline{\theta} \leq \theta^{*f} \leq 0, \quad (46)$$

$$\alpha^f = c, \; \hat{\beta}^f = c + \frac{\theta^{*f}}{2}, \; \hat{\alpha}^f = c, \; \beta^f = c + \frac{\overline{\theta}}{2}, \qquad \text{if} \quad 0 \leq \theta^{*f} \leq \tfrac{\overline{\theta}}{2}, \quad (47)$$

$$\alpha^f = c, \; \hat{\beta}^f = c + \frac{\theta^{*f}}{2}, \; \hat{\alpha}^f = c + \overline{\theta} - 2\theta^{*f}, \; \beta^f = c + \overline{\theta} - \theta^{*f}, \qquad \text{if} \quad \tfrac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}. \quad (48)$$

Therefore, we have

$$sw_2^{Df} = (v-c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{3}{8}(\theta^{*f})^2 - (\overline{\theta} - \theta^{*f})^2, \quad (49)$$

$$\pi_{B2}^{Df} = (\overline{\theta} - \theta^{*f})^2 + \frac{1}{4}(\theta^{*f})^2, \quad (50)$$

if $\frac{\overline{\theta}}{2} \le \theta^{*f} \le \overline{\theta}$. In this case, firm A does not poach firm B's consumer but firm B poach firm A's consumer since $\theta_A^f = \frac{1}{2}\theta^{*f}$ and $\theta_B^f = \theta^{*f}$.

We show that only the case of $\frac{\overline{\theta}}{2} \le \theta^{*f} \le \overline{\theta}$ is possible in equilibrium in Appendix.

The first period

In the case of $\frac{\overline{\theta}}{2} \le \theta^{*f} \le \overline{\theta}$, the consumer, θ^{*f}, is indifferent between choosing firm A in the first period at price, a^f, and then choosing firm B in the second period at price, $\hat{\beta}^f$, or choosing firm B in the first period at price, b^f, and then choosing firm B at price, β^f. Thus, θ^{*f} satisfies $v - \frac{\theta^{*f}}{2} - a^f + \delta(v + \frac{\theta^{*f}}{2} - \hat{\beta}^f) = v + \frac{\theta^{*f}}{2} - b^f + \delta(v + \frac{\theta^{*f}}{2} - \beta^f)$. Substituting the second period equilibrium prices and solving it for θ^* yields

$$\theta^{*f} = \frac{2(b^f - a^f + \delta\overline{\theta})}{2+3\delta}, \quad \text{if} \quad \frac{\overline{\theta}}{2} \le \theta^{*f} \le \overline{\theta}. \tag{51}$$

From (49) and (50), objective functions of firm A and firm B are given by

$$\begin{aligned} SW^{Df} = & \big[v(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}(\theta^{*f})^2 - c(\theta^{*f} - \underline{\theta}) - b^f(\overline{\theta} - \theta^{*f})\big] \\ & + \delta\big[(v-c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{3}{8}(\theta^{*f})^2 - (\overline{\theta} - \theta^{*f})^2\big], \end{aligned} \tag{52}$$

and

$$\Pi_B^{Df} = (b^f - c)(\overline{\theta} - \theta^{*f}) + \delta[(\overline{\theta} - \theta^{*f})^2 + \frac{1}{4}(\theta^{*f})^2]. \tag{53}$$

The differentiation of the above objective functions yields the following reaction functions.

$$2(2+3\delta)c - (4+\delta)\delta\overline{\theta} + 5\delta b^f - (4+11\delta)a^f = 0, \tag{54}$$

$$2(2+3\delta)c + (4+\delta^2)\overline{\theta} + 4(1-\delta)a^f - 2(4+\delta)b^f = 0. \tag{55}$$

Solving (54) and (55) gives equilibrium prices in the first period and substituting these prices into (48), we can obtain equilibrium prices.

Proposition 3 *Equilibrium prices in the international mixed duopoly are given by*

$$a^f = c - \frac{\delta(6-\delta)}{2(7\delta+8)}\overline{\theta}, \quad b^f = c + \frac{5\delta^2 + 2\delta + 8}{2(7\delta+8)}\overline{\theta}, \quad \theta^{*f} = \frac{2(3\delta+2)}{7\delta+8}\overline{\theta},$$

$$\alpha^f = c, \quad \hat{\beta}^f = c + \frac{3\delta+2}{7\delta+8}\overline{\theta}, \quad \hat{\alpha}^f = c - \frac{5\delta}{7\delta+8}\overline{\theta}, \quad \beta^f = c + \frac{\delta+4}{7\delta+8}\overline{\theta}.$$

Thus, we have

$$p_A^f = \alpha^f \ge a^f \ge \hat{\alpha}^f, \;\; p_B^f \ge b^f \ge \beta^f > \hat{\beta}^f. \;\; \theta_A^f < \theta^{*f}, \;\; \theta_B^f = \theta^{*f}.$$

We can check that θ^{*f} satisfies $\frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}$ and $\theta_A^f = \frac{1}{2}\theta^{*f}$.

Proposition 3 implies that BBPD decreases equilibrium prices and only the private firm poaches its rival's consumers. Furthermore, Proposition 3 indicates that BBPD increases the private firm's total market share. Its total market share under uniform pricing is $2(\frac{\overline{\theta}}{2})$ and that under BBPD is $\frac{5\delta+10}{7\delta+8}\overline{\theta}$. It is apparent that the latter is larger than the former if $\delta \in [0, 1)$. Firm A captures more than three-quarters of the market by charging a below-marginal-cost price in the first period under BBPD. This makes poaching by the private firm less costly in the second period since consumers in firm A's turf have higher relative preferences for the private firm's goods as the public firm's first-period market share is larger.

Total discounted consumer surplus, total discounted profits, and total discounted domestic social welfare in equilibrium under BBPD in the international mixed duopoly are

$$CS^{Df} = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - \frac{47\delta^3 + 44\delta^2 + 36\delta + 48}{2(7\delta+8)^2}\overline{\theta}^2, \quad (56)$$

$$\Pi_A^{Df} = -\frac{\delta(6-\delta)(13\delta+12)}{2(7\delta+8)^2}\overline{\theta}^2, \quad \Pi_B^{Df} = \frac{25\delta^3 + 62\delta^2 + 56\delta + 32}{2(7\delta+8)^2}\overline{\theta}^2, \quad (57)$$

$$SW^{Df} = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - \frac{17\delta^3 + 55\delta^2 + 54\delta + 24}{(7\delta+8)^2}\overline{\theta}^2. \quad (58)$$

Proposition 4 *Comparing the outcome under uniform pricing,* (36)*,* (37)*, and* (38) *yields*

$$CS^{Uf} \leq CS^{Df},\ \Pi_i^{Uf} \geq \Pi_i^{Df},\ SW^{Uf} \leq SW^{Df},\ \ (i = A, B).$$

Thus, BBPD improves domestic social welfare although it increases the private firm's total market share in the international duopoly.

BBPD is beneficial to consumers and domestic social welfare but detrimental to the firms in the international mixed duopoly. We have previously mentioned that Fudenberg and Tirole (2000) show that BBPD is beneficial to consumers and detrimental to firms and social welfare. However, our results show that BBPD increases domestic social welfare in the international mixed duopoly.

Total discounted domestic social welfare in the international mixed duopoly (SW^{Df}) can be defined as that in the domestic mixed duopoly (SW^D) minus the total discounted sum of disutilities of taste mismatch and total discounted private firm's profit. BBPD increases the total discounted sum of disutilities of taste mismatch. The sum of disutilities of taste mismatch is increased by the public firm's below-marginal-cost pricing in the first period. It is decreased by the private firm's poaching in the second period. On the other hand, BBPD decreases the private firm's profit. BBPD increases the private firm's market share but decreases its equilibrium prices as shown in Proposition 3. Therefore, BBPD increases total discounted domestic social welfare in the international mixed duopoly.

5 Welfare

In this section, we discuss welfare analysis. We investigate the impacts of privatization on social welfare.

Suppose that the public firm is privatized in the domestic mixed duopoly. There are two profit-maximizing firms completely owned by domestic shareholders. Domestic social welfare under uniform pricing and under BBPD in an domestic pure duopoly can be obtained as follows:

$$SW^{Up} = (1+\delta)[(v-c) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2], \quad (59)$$

$$SW^{Dp} = (1+\delta)[(v-c) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - \frac{1}{9}\delta\overline{\theta}^2, \quad (60)$$

where "p" stands for the pure duopoly. It is obvious that $SW^{Up} \geq SW^{Dp}$, and we can see that BBPD lowers domestic social welfare in the domestic pure duopoly. On the other hand, Proposition 2 shows that $SW^U = SW^D$.

Suppose that the public firm is privatized in the international mixed duopoly. There are a profit-maximizing firm completely owned by domestic shareholders and that completely owned by foreign shareholders. Domestic social welfare under uniform pricing and under BBPD in an international pure duopoly can be obtained as follows:

$$SW^{Upf} = SW^{Dpf} = (1+\delta)[(v-c) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - (1+\delta)\overline{\theta}^2. \quad (61)$$

We can see that BBPD is neutral to domestic social welfare in the domestic international duopoly. Note that equilibrium prices and differential consumers in those cases are consistent with Fudenberg and Tirole (2000). On the other hand, Proposition 4 shows that $SW^{Uf} \leq SW^{Df}$.

Therefore, we have the following proposition.

Proposition 5 *The presence of public firms reduces welfare loss caused by BBPD.*

The public firm prevents poaching in the domestic mixed duopoly and lowers equilibrium prices in the international mixed duopoly. As shown in Fjell and Pal (1996), the public firm is more aggressive in pricing when it competes with the foreign private firm.

6 Conclusion

We extend Fudenberg and Tirole (2000) by considering mixed duopoly to investigate mixed markets in which the social welfare-maximizing public firm and private firm engage in behavior-based price discrimination. We find that BBPD is irrelevant from the viewpoint of social welfare when the public firm competes with the domestic private firm. The public firm minimizes the sum of disutilities of taste mismatch and then both firms do not poach rivals' consumers. On the other hand, if the public firm privatized, BBPD reduces domestic social welfare since both firms poach rivals' consumers. We also find that BBPD improves domestic social welfare when the public firm competes with the foreign private firm. BBPD reduces the outflow to foreign shareholders although it increases the foreign private firm's market share. If the public firm is privatized, BBPD is irrelevant from the viewpoint of social welfare.

Lin and Matsumura (2012), Cato and Matsumura (2012), and Haraguchi and Matsumura (2014) investigate the effect of partial privatization, developed by Matsumura (1998) on social welfare. Matsumura et al. (2009) and Cato and Matsumura (2012) introduces the stockholding ratios of foreign investors in private firms. Introducing partial privatization and a private firm owned by both domestic shareholders and foreign shareholders into this model remains for future research.

Appendix

A The case of $\underline{\theta} \leq \theta^{*f} \leq 0$

In the case of $\underline{\theta} \leq \theta^{*f} \leq 0$, the first-period indifferent consumer can be derived as

$$\theta^{*f} = \frac{b-a}{1-\delta}. \tag{62}$$

The objective functions of firm A and firm B are

$$\begin{aligned} SW^{Df} &= \left[v(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}\theta^{*f2} - c(\theta^{*f}-\underline{\theta}) - b^f(\overline{\theta}-\theta^{*f})\right] \qquad (63)\\ &\quad +\delta\left[(v-c)(\overline{\theta}-\underline{\theta}) - \frac{1}{8}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2\right], \\ \Pi_B^{Df} &= (b^f-c)(\overline{\theta}-\theta^{*f}) + \delta\frac{\overline{\theta}^2}{4}. \qquad (64) \end{aligned}$$

First-order conditions are

$$(1-\delta)c + \delta b - a = 0, \tag{65}$$
$$c + (1-\delta)\overline{\theta} + a - 2b = 0. \tag{66}$$

Solving these equations yields $a = c + \frac{\delta(1-\delta)}{2-\delta}\overline{\theta}$ and $b = c + \frac{1-\delta}{2-\delta}\overline{\theta}$. Hence, we obtain $\theta^{*f} = \frac{1-\delta}{2-\delta}\overline{\theta}$, which violates $\underline{\theta} \leq \theta^{*f} \leq 0$.

B The case of $0 \leq \theta^{*f} \leq \frac{\overline{\theta}}{2}$

In the case of $0 \leq \theta^{*f} \leq \frac{\overline{\theta}}{2}$, the first-period indifferent consumer can be derived as

$$\theta^{*f} = \frac{2(b-a)}{2-\delta}. \tag{67}$$

The objective functions of firm A and firm B are

$$\begin{aligned} SW^{Df} &= \big[v(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}\theta^{*f2} - c(\theta^{*f}-\underline{\theta}) - b^f(\overline{\theta}-\theta^{*f})\big] \quad (68) \\ &\quad +\delta\big[(v-c)(\overline{\theta}-\underline{\theta}) - \frac{1}{8}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 + \frac{1}{8}\theta^{*f2}\big], \\ \Pi_B^{Df} &= (b^f - c)(\overline{\theta}-\theta^{*f}) + \delta[\frac{\theta^{*f2}}{4} + \frac{\overline{\theta}^2}{4}]. \quad (69) \end{aligned}$$

First-order conditions are

$$2(2-\delta)c + \delta b - (4-\delta)a = 0, \tag{70}$$

$$(2-\delta)^2\overline{\theta} + 2(2-\delta)c + 4(1-\delta)a - 2(4-3\delta)b = 0. \tag{71}$$

Solving these equations yields $a = c + \frac{\delta(2-\delta)}{2(8-5\delta)}\overline{\theta}$ and $b = c + \frac{(2-\delta)(4-\delta)}{2(8-5\delta)}\overline{\theta}$. Hence, we obtain $\theta^{*f} = \frac{2(2-\delta)}{8-5\delta}\overline{\theta}$, which violates $0 \leq \theta^{*f} \leq \frac{\overline{\theta}}{2}$.

References

[1] Carroni, E. (2016) Competitive customer poaching with asymmetric firms. *International Journal of Industrial Organization* , 48, pp. 173-206.

[2] Cato, S., Matsumura, T. (2012) Long-run effects of foreign penetration on privatization policies. *Journal of Institutional and Theoretical Economics*, 168 (3), pp. 444-454.

[3] Chen, Y. (1997) Paying customers to switch. *Journal of Economics and Management Strategy*, 6 (4), pp. 877-897.

[4] Chen, Y. (2008) Dynamic price discrimination with asymmertic firms. *Journal of Industrial Economics* , 56 (4), pp. 729-751.

[5] Chen, Y., Pearcy, J. (2010) Dynamic pricing: When to entice brand switching and when to reward consumer loyalty. *RAND Journal of Economics*, 41 (4) , pp. 674-685.

[6] Choe, C., Matsushima, N. (2021) Behavior-Based Price Discrimination and Product Choice. *Review of Industrial Organization*, 58 (2), pp. 263-273.

[7] Chung, H.S. (2016) Behavior-Based Price Discrimination with Experience Goods. *Manchester School* , 84 (5), pp. 675-695.

[8] Esteves, R.-B., Reggiani, C. (2014) Elasticity of demand and behaviour-based price discrimination. *International Journal of Industrial Organization* , 32 (1), pp. 46-56.

[9] Fjell, K., Pal, D. (1996) A mixed oligopoly in the presence of foreign private firms. *Canadian Journal of Economics* , 29 (3), pp. 737-743.

[10] Fudenberg, D., Tirole, J. (2000) Customer poaching and brand switching. *RAND Journal of Economics* , 31 (4), pp. 634-657.

[11] Garella, P.G., Laussel, D., Resende, J. (2021) Behavior based price personalization under vertical product differentiation. *International Journal of Industrial Organization* , 76, art. no. 102717, .

[12] Haraguchi, J., Matsumura, T. (2014) Price versus quantity in a mixed duopoly with foreign penetration. *Research in Economics*, 68 (4), pp. 338-353.

[13] Heywood, J.S., Wang, S., Ye, G. (2021) Partial Privatization Upstream with Spatial Price Discrimination Downstream. *Review of Industrial Organization*, 59 (1), pp. 57-78.

[14] Heywood, J.S., Ye, G. (2009) Mixed ologopoly, sequential entry and spatial price diecrimination. *Economic Inquiry*, 47 (3), pp. 589-597.

[15] Heywood, J.S., Ye, G. (2009) Mixed oligopoly and spatial price discrimination with foreign firms. *Regional Science and Urban Economics* , 39 (5), pp. 592-601.

[16] Jing, B. (2017) Behavior-based pricing, production efficiency, and quality differentiation. *Management Science*, 63 (7), pp. 2365-2376.

[17] Lin, M.H., Matsumura, T. (2012) Presence of foreign investors in privatized firms and privatization policy. *Journal of Economics/ Zeitschrift fur Nationalokonomie*, 107 (1), pp. 71-80.

[18] Liu, Q., Shuai, J. (2016) Price discrimination with varying qualities of information. *B.E. Journal of Economic Analysis and Policy* , 16 (2), pp. 1093-1121.

[19] Lyu, Y., Shuai, J. (2017) Mixed duopoly with foreign firm and subcontracting. *International Review of Economics and Finance*, 49, pp. 58-68.

[20] Matsushima, N., Matsumura, T. (2003) Mixed oligopoly and spatial agglomeration [Oligopole mixte et agglomération spatiale]. *Canadian Journal of Economics* , 36 (1), pp. 62-87.

[21] Matsushima, N., Matsumura, T. (2006) Mixed oligopoly, foreign firms, and location choice. *Regional Science and Urban Economics*, 36 (6), pp. 753-772.

[22] Matsumura, T. (1998) Partial privatization in mixed duopoly. *Journal of Public Economics*, 70 (3), pp. 473-483.

[23] Matsumura, T. (2003) Stackelberg mixed duopoly with a foreign competitor. *Bulletin of Economic Research* , 55 (3), pp. 275-287.

[24] Matsumura, T., Matsushima, N., Ishibashi, I. (2009) Privatization and entries of foreign enterprises in a differentiated industry. *Journal of Economics/ Zeitschrift fur Nationalokonomie* , 98 (3), pp. 203-219.

[25] Pal, D., White, M.D. (1998) Mixed oligopoly, privatization, and strategic trade policy. *Southern Economic Journal* , 65 (2), pp. 264-281.

[26] Rhee, K.-E., Thomadsen, R. (2017) Behavior-based pricing in vertically differentiated industries. *Management Science*, 63 (8), pp. 2729-2740.

[27] Villas-Boas, J.M. (1999) Dynamic competition with customer recognition. *RAND Journal of Economics*, 30 (4), pp. 604-631.

[28] Ye, G., Wu, W. (2015) Privatization and merger in a mixed oligopoly with spatial price discrimination. *Annals of Regional Science*, 54 (2), pp. 561-576.

[29] Zhang, T., Huo, Y., Zhang, X., Shuai, J. (2019) Endogenous third-degree price discrimination in Hotelling model with elastic demand. *Journal of Economics/ Zeitschrift fur Nationalokonomie*, 127 (2), pp. 125-145.