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Abstract

This study develops a two-country new Keynesian (NK) model that incorporates deep habits in consumption and investigates the macroeconomic dynamics under the optimal coordinated monetary policy. We show that in response to the structural shocks, the central bank changes the interest rate significantly in the two-country open economy model compared with the closed economy where the central bank is reluctant to move interest rates. When a deep consumption habit exists, the international central bank can exploit the terms of trade externalities. Habit formation might boost the expenditure switching effect, which differentiate the aggressiveness of the central bank between closed and open economies with deep habit. Moreover, we showed that the deviations from the law of one price, or the goods-specific real exchange rate, generated endogenously by the deep habit are significantly related to the degree of home bias. In particular, the deviations fully disappeared when there is no home bias.

JEL codes: E52; E58

Keywords: Optimal monetary policy; Deep habit; Policy coordination; Commitment;

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1 Introduction

Incorporating habit formation in household consumption into macroeconomic theoretical models has been increasingly important. In particular, in medium-scale dynamic stochastic general equilibrium (DSGE) models, habit formation in household consumption is often employed to reproduce the hump-shaped dynamics of endogenous variables in response to structural shocks (Christiano, Eichenbaum and Evans, 2005, Smets and Wouters, 2007).

Recently, Ravn, Schmitt-Grohé and Uribe (2006) extended the concept of habit formation to "deep habits," in a form that inherits the empirically desirable hump-shaped dynamics.[1] In particular, traditional (or "superficial") habits are formed for aggregated goods, whereas deep habits are formed for goods-by-goods.[2] Deep habits provide new insights by generating additional externalities in models.[3] For example, originally Ravn et al. (2006) highlighted that because deep habits change the demand structure of households, price setting decisions of final-goods firms and markups of the goods price over its cost could also be altered, thus bringing crowding-in effects of fiscal policy[4] (Ravn et al., 2006, Zubairy, 2014).

Compared to the contributions of deep habit to research focused on fiscal policy, a few previous studies have considered the role of deep habit in optimal monetary policy. In particular, in focusing on open economy NK models, no model has incorporated deep habits into optimal monetary policy, to the best of the author's knowledge.

This present study develops a two-country NK model with deep habits in household consumption. Specifically we extended Leith, Moldovan and Rossi (2009)'s closed economy NK model with deep habit to a two-country version (e.g., (Corsetti, Dedola and Leduc, 2010);

[1] See, for example, Kormilitsina and Zubairy (2018) and Zubairy (2014).

[2] Cantore, Levine and Melina (2014) used the Bayesian estimation to compare the performance of superficial and deep habits in terms of data fitting in a DSGE model.

[3] Our paper considers external (Smets and Wouters, 2007) rather than internal habit formation (Fuhrer, 2000, Christiano et al., 2005). In the former, households cannot internalize the externalities of their utility on other households' utility because they care about other households' consumption rather than their own past consumption. ("catching up with the Joneses" effect) See Ravn et al. (2006) and Leith, Moldovan and Rossi (2012) in detail.

[4] Ravn et al. (2006) highlighted that the dynamics of the markup generated by the deep habit are quite different from the markup derived from sticky prices in the standard new Keynesian (NK) model. Our model avoids confusion between the two by formulating them distinctly, as we will demonstrate later.

(Ravn, Schmitt-Grohe and Uribe, 2007); (Jacob and Uusküla, 2019)), while omitting the aspects of fiscal policy for simplicity. We then study the central bank's behavior in solving the Ramsey problem, which maximizes both countries' economic welfare in a coordinated fashion.

By extending the closed model to a two-country model, we should also consider the terms of trade externality.[5] In other words, the central bank's optimal response is required to find a balance between consumption externalities due to deep habit, terms of trade externalities, and the cost of nominal rigidities. Consequently, the simulations in this study provide new insights that are not available in either the model with deep habit in a closed economy or that without deep habit in an open economy.

Our main findings are as follows. When a deep habit exists, unlike in the standard NK model, the central bank faces a trade-off between the output gap and inflation stability even in response to a productivity shock. In a closed economy model, previous studies have shown that when the central bank conducts the optimal commitment policy, it is reluctant to move the interest rate in the closed economy model significantly. Meanwhile, we show that the central bank changes the interest rate significantly in the two-country open economy model. The key to this finding lies in the asymmetric changes in domestic and foreign markups generated by the deep habit and the resulting changes in relative demand. These, combined with the central bank's behavior and the externalities of the terms of trade that are unique to the open economy model, lead to markedly different results from those of a closed economy. When a deep consumption habit exists, the international central bank can exploit the externalities of the terms of trade. Habit formation might boost the expenditure switching effect, which differentiates the aggressiveness of the central bank between closed and open economies with deep habits. Also, we showed that the deviations from the law of one price (LOP), or the goods-specific real exchange rate, generated by the deep habit are significantly related to the degree of home bias. In particular, the deviations fully disappeared in the absence of home bias.

Here, we briefly review the role of habit formation in macroeconomic modeling to specify the scope of our research. First, habit formation has been used as a device to generate real rigidity in many fields. For instance, Abel (1990) incorporated the "catching with the Joneses effect" in the utility function into the asset pricing model to examine the equity premium

[5] See Clarida, Galí and Gertler (2002) and Pappa (2004).

puzzle. Regarding the "deep" habit, its similarities to the customer market model (Bils, 1989) were pointed out by previous studies, because deep habits alter demand behavior at the goods-by-goods level, which influence pricing at the individual firm level as the customer market.[6]

Incorporating deep habit into the NK model has drawn attention to how deep habit produces countercyclical markup. In other words, under the deep habit, firms may lower prices and markups to increase expected future demand. Thus, "*price may magnify, rather than stabilize, demand movements*" (Bils, 1989). The influence of countercyclical markups on optimal monetary policy is significant: as discussed by Leith et al. (2012), under a deep habit, the central bank cannot lower interest rates sufficiently even when they should (e.g., due to positive productivity shocks). This is because lowering the interest rate will cause firms to lower their markups, leading to undesirable overconsumption. This dilemma generates the trade-off between the stability of inflation and the output gap, which implies a deviation from the "divine coincidence." In the context of optimal policy, Givens (2016) explored the role of deep habit in the welfare gains from commitment relative to discretion and showed that deep habits weaken the stabilization trade-offs facing a discretionary planner.

Some of these properties from countercyclical markup may be carried over in the two-country model; however, which properties are inherited and to what extent are not always clear. Thus, this is one of our research questions. In the two-country model, deep habit is noteworthy because it affects international price markups. Specifically, the deep habit can endogenously generate deviations from the LOP(Ravn et al., 2007). It can also explain the incomplete pass-through of exchange rates to international prices (Jacob and Uusküla, 2019).

We develop the two-country model based on Corsetti et al. (2010)[7], incorporating deep habit in consumption similarly to Ravn et al. (2007), Jacob and Uusküla (2019). Our two-country model reproduces the deviation from the LOP as in Ravn et al. (2007), and we investigate the optimal commitment policy following Leith et al. (2009, 2012).

The remainder of the paper is organized as follows. Section 2 describes the model structure. Section 3 describes the central bank's optimal monetary policy. Section 4 demonstrates the impulse responses to the structural shocks and provides economic intuition about the results.

[6] Indeed, Hong (2019) embedded customer capital due to deep habits into a standard model of firm dynamics with entry and exit.

[7] To be precise, our model directly refers to the model demonstrated by Bodenstein, Guerrieri and LaBriola (2019), a simplified version of Corsetti et al. (2010) and Benigno and Benigno (2006).

Finally, Section 5 briefly concludes our research and the results.

2 The Model

We develop the two-country model based on Corsetti et al. (2010), incorporating deep habit in consumption as Ravn et al. (2007), Jacob and Uusküla (2019). As aforementioned, our two-country model reproduces the LOP deviation Ravn et al. (2007).

The two countries are symmetrical and of the same size. The present study's model is based on the models of Benigno and Benigno (2006), Corsetti et al. (2010), and Bodenstein et al. (2019). We incorporate a deep habit into the simple nonlinear NK model with nominal rigidities, following Leith, Moldovan and Rossi (2015), Leith et al. (2009) and Jacob and Uusküla (2019).

Note that most NK models that deal with real inertia due to deep habit and nominal rigidity adopt Rotemberg (1982)-type adjustment cost as price stickiness for convenience of aggregation. In terms of two major types of price stickiness, that is, Rotemberg-type and Calvo (1983)-type, the same Phillips curve can be obtained up to a first-order approximation (Woodford, 2003a). However, Lombardo and Vestin (2008) showed that the two pricing assumptions could yield different social inflation costs when the steady-state is inefficient. Considering this difference, we choose the combination of deep habit and Calvo pricing by following Leith et al. (2009). Specifically, we allocate these two rigidities separately: intermediate goods firms face Calvo-type nominal rigidities, whereas final-goods firms' demand functions are affected by households' deep habit. In such a way, as suggested by Leith et al. (2009), the model can be built with Calvo pricing and deep habit without losing its desirable aggregate properties.

The model structure of the foreign country is symmetrical to that of the home country. Unless otherwise noted, we denote foreign variables with an asterisk.

2.1 Households

Households derive utility from the consumption of both home and foreign goods, and they form habits at the level of individual goods rather than the aggregate of goods. When habits are "deep," consumers form inseparable preferences at the individual goods level over time, thereby generating the habit-persistence at a goods-by-goods level (Jacob and Uusküla, 2019).

A representative household k maximize lifetime utility for an infinite period of time.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(X_t^k\right)^{1-\sigma}}{1-\sigma} - \chi \frac{\left(N_t^k\right)^{1+\upsilon}}{1+\upsilon} \right] \tag{1}$$

where X is habit-adjusted aggregate consumption of household, β is the discount factor, σ denotes the inverse of the intertemporal elasticities of habit-adjusted consumption, υ is the inverse of the intertemporal elasticities of work, and χ is the relative weight on disutility from time spent working. Moreover, X is a CES composite of habit-adjusted consumption of domestic goods X_D and foreign goods X_M with the elasticity of substitution of η:

$$X_t = \left(\omega^{\frac{1}{\eta}} X_{D,t}^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} X_{M,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

where ω is a degree of home bias in consumption. X_D and X_M are habit-adjusted domestic consumption and imported consumption, respectively. A CES aggregates of the variety of goods $i \in [0,1]$ with elasticity of substitution of ϵ:

$$X_{D,t} = \left[\int_0^1 \left(C_{D,t}(i) - \theta_D S_{D,t-1}(i)\right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

$$X_{M,t} = \left[\int_0^1 \left(C_{M,t}(i) - \theta_M S_{M,t-1}(i)\right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

where S_D is the stock of consumption habits, consisting of the stock at the end of the last period and the habits that will be newly formed in the current period.

$$S_{D,t}(i) = \varrho_D S_{D,t-1}(i) + (1-\varrho_D) C_{D,t}(i) \tag{5}$$

$$S_{M,t}(i) = \varrho_M S_{M,t-1}(i) + (1-\varrho_M) C_{M,t}(i) \tag{6}$$

Note that if $\varrho = 0$, $S_{D,t}(i) = C_{D,t}(i)$ where $C_D(i)$ is the average consumption of domestic goods independent of household k. In this case, in each period, household k refers to the average household consumption in the previous period with respect to good i and allocates a portion of it to habit formation. Note that habitual consumption does not yield utility by definition; it is habit-adjusted consumption that yields utility.

By the term of the stock of habit consumption, demand functions are characterized independent of the goods price. The demand function for each good is described as follows:

$$C_{D,t}(i) = \left(\frac{P_{D,t}(i)}{P_{D,t}} \right)^{-\epsilon} X_{D,t} + \theta_D S_{D,t-1}(i) \tag{7}$$

$$C_{M,t}(i) = \left(\frac{P_{M,t}(i)}{P_{M,t}} \right)^{-\epsilon} X_{M,t} + \theta_M S_{M,t-1}(i) \tag{8}$$

where $P_{D,t}(i)$ and $P_{M,t}(i)$ are price of domestic goods i and price of imported goods i respectively. In addition,

$$P_{D,t} = \left(\int_0^1 P_{D,t}^{1-\epsilon}(i) di \right)^{\frac{1}{1-\epsilon}} \tag{9}$$

$$P_{M,t} = \left(\int_0^1 P_{M,t}^{1-\epsilon}(i) di \right)^{\frac{1}{1-\epsilon}}. \tag{10}$$

We express the budget constraint for household k as follows:

$$\int_0^1 \left[P_{D,t}(i) C_{D,t}^k(i) + P_{M,t}(i) C_{M,t}^k(i) \right] di + E_t \left\{ Q_{t,t+1} D_{t+1}^k \right\} \leq D_t^k + W_t N_t^k + \Phi_t$$

where W_t is the nominal wage, D_t is the nominal payoff on the portfolio of assets, $Q_{t,t+1}$ is the one-period stochastic discount factor for nominal payoffs relevant to the domestic household. Following Clarida et al. (2002), we assume households have access to a complete set of contingent claims, traded internationally. Φ_t is dividend (or profits) from the firms owned by households.

By using Equation (7)–(10), we obtain

$$P_{D,t} X_{D,t}^k + P_{M,t} X_{M,t}^k + v_t^k + E_t \left\{ Q_{t,t+1} D_{t+1}^k \right\} \leq D_t^k + W_t N_t^k + \Phi_t \tag{11}$$

and

$$v_t = \theta_D \int_0^1 P_{D,t}(i) S_{D,t-1}(i) di + \theta_M \int_0^1 P_{M,t}(i) S_{M,t-1}(i) di \tag{12}$$

where v is the expenditure on the stock of habitual consumption.

Households maximize lifetime utility (1) subject to (12). Meanwhile, the following is the first-order condition for habit-adjusted consumptions:

$$\left(\frac{\omega}{1-\omega} \right)^{1/\eta} \left(\frac{X_{D,t}}{X_{M,t}} \right)^{-1/\eta} = \frac{P_{D,t}}{P_{M,t}} \tag{13}$$

$$(X_t)^{\sigma} \left[\omega^{1/\eta} \left(\frac{X_{D,t}}{X_t} \right)^{-1/\eta} \right]^{-1} \chi (N_t)^{\upsilon} = \frac{W_t}{P_{D,t}} \tag{14}$$

$$\beta E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma + 1/\eta} \left(\frac{X_{D,t+1}}{X_{D,t}} \right)^{-1/\eta} \left(\frac{P_{D,t}}{P_{D,t+1}} \right) \right] R_t = 1 \tag{15}$$

where $R_t \equiv 1/E_t\{Q_{t,t+1}\}$ denotes the risk-free gross nominal interest rate between periods t and $t+1$. The superscript k is dropped in the optimal conditions due to the assumption of homogeneity. Note that the form of consumption Euler equation is equivalent to the one under traditional (or superficial) habit formation.

2.2 Final goods Producers

Final-goods firms produce final goods by CES-aggregating intermediate goods. We assume that domestic final-goods firms purchase the intermediate goods produced only in their home country. The final-goods firms face monopolistic competition and determine the optimal price and markups conditional on the demand function.

Note that the demand function is affected by the deep habit, as shown in Equation (7). Unlike the standard NK model, the model with deep habits has demand function, including the term independent of the current price. The demand function depends on the past (external average, not self) demand. Consequently, the current pricing behavior affects current and future expected demands. This is called the intertemporal effect of a deep habit (Ravn et al., 2006).

Firm i produces final goods $Y(i)$ by CES-aggregating intermediate goods $Y(i,j)$.

$$Y_t(i) = \left(\int_0^1 Y_t(i,j)^{\frac{\xi-1}{\xi}} dj \right)^{\frac{\xi}{\xi-1}} \tag{16}$$

Then, final goods are shipped to home and the foreign country.

$$Y_t(i) = Y_t^D(i) + Y_t^{EX}(i) \tag{17}$$

where $Y_t^D(i)$ is domestic output for domestic consumers and $Y_t^{EX}(i)$ is an exported output. Thus, the market clearing condition of domestic output are

$$Y_t^D(i) = C_{D,t}(i) \tag{18}$$

$$Y_t^{EX}(i) = C^*_{M,t}(i). \tag{19}$$

Profit maximization for final-goods firms

Standard NK models assume that the final-goods firms face perfect competition. Under the perfect competition, price and marginal cost are equal, and price markup does not occur due to firms' optimal problems. Thus, the profit of the final-goods firm reaches zero. However, this is not the case when a household has deep habit-formation for consumption. The demand function has a constant term that is independent of its price, which makes room for non-zero profit (price markup to marginal cost). We define the profit of a final-goods firm as follows:

$$\Phi_t(i) \equiv \Phi_{D,t}(i) + \Phi^*_{M,t}(i) \tag{20}$$

$$\Phi_{D,t}(i) \equiv \left(P_{D,t}(i) - P^m_{D,t}(i)\right) C_{D,t}(i) \tag{21}$$

$$\Phi^*_{M,t}(i) \equiv \left(\mathcal{E}_t P^*_{M,t}(i) - P^m_{D,t}(i)\right) C^*_{M,t}(i) \tag{22}$$

where $\Phi(i)$ is the total profit of the final good firm i, $\Phi_D(i)$ is the profit from domestic sales, and $\Phi^*_M(i)$ is the profit from sales to foreign households.

When households have deep habits, demand for goods may differ between domestic and foreign countries, and firms would naturally differentiate prices in each market. Ravn et al. (2007) named this as "pricing to habits," showing that deviations from the LOP occur endogenously.[8] [9]

Final-goods produced for domestic consumption

Final-goods firms maximize the discounted present value of future domestic sales profits:

$$\max_{\{P_{D,t}(i), C_{D,t}(i)\}} E_t \sum_{s=0}^{\infty} Q_{t,t+S} \Phi_{D,t+s}(i) \tag{23}$$

subject to the demand function with deep-habit consumption (7) and intertemporal substitution of consumption (15)

$$C_{D,t+s}(i) = \left(\frac{P_{D,t+s}(i)}{P_{D,t+s}}\right)^{-\epsilon} X_{D,t+s} + \theta_D C_{D,t+s-1}(i) \tag{24}$$

$$Q_{t,t+s} = \beta^s \left(\frac{X_{t+s}}{X_t}\right)^{-\sigma+1/\eta} \left(\frac{X_{D,t+s}}{X_{D,t}}\right)^{-1/\eta} \left(\frac{P_{D,t}}{P_{D,t+s}}\right) \tag{25}$$

Substituting (21) into (23), we derive the first-order conditions of this problem:

$$\Lambda_{D,t}(i) = \left(P_{D,t}(i) - P^m_t(i)\right) + \theta_D E_t \left[Q_{t,t+1} \Lambda_{D,t+1}(i)\right] \tag{26}$$

$$C_{D,t}(i) = \Lambda_{D,t}(i) \left[\epsilon \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\epsilon-1} (P_{D,t})^{-1} X_{D,t}\right] \tag{27}$$

where $\Lambda_{t,D}(i)$ is the Lagrange multiplier.

[8] Monacelli (2005) related incomplete exchange rate pass-through on import prices to LOP gap in a small-open NK model. In addition to deep habit, several other candidates for real rigidities lead to deviations from the LOP, such as local distribution costs (Corsetti and Dedola, 2005). See Jacob and Uusküla (2019).

[9] Our model can be regarded as a type of model employing local currency pricing (LCP). For LCP, see Engel (2011) and Corsetti et al. (2010). Fujiwara and Wang (2017) extended Engel's model to focus on non-cooperative policy games under LCP in a two-country DSGE model.

Final-goods shipped to a foreign country

Similarly, we set up the profit maximization problem for the exported goods toward foreign country, $Y_t^{EX} = C_{M,t}^*$,

$$\max_{\{\mathcal{E}_t P_{M,t}^*(i), C_{M,t}^*(i)\}} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \left(\mathcal{E}_t P_{M,t+s}^*(i) - P_{t+s}^m(i) \right) C_{M,t+s}^*(i) \tag{28}$$

subject to the demand function

$$C_{M,t+s}^*(i) = \left(\frac{P_{M,t+s}^*(i)}{P_{M,t+s}^*} \right)^{-\epsilon} X_{M,t+s}^* + \theta_M^* C_{M,t+s-1}^*(i) \tag{29}$$

and the discount factor (25).

Note that domestic firms choose the final-goods price in terms of home currency $\mathcal{E}_t P_M^*(i)$, whereas foreign households determine their consumption behavior in terms of foreign currencies. Moreover, foreign households may have different consumption habits from those in the home country, even for the identical goods.

The following are the first-order conditions of this problem:

$$\Lambda_{M,t}^*(i) = \left(\mathcal{E}_t P_{M,t}^*(i) - P_t^m(i) \right) + \theta_M^* E_t \left[Q_{t,t+1} \Lambda_{M,t+1}^*(i) \right] \tag{30}$$

$$C_{M,t}^*(i) = \Lambda_{M,t}^*(i) \left[\epsilon \left(\frac{P_{M,t}^*(i)}{P_{M,t}^*} \right)^{-\epsilon-1} \left(\mathcal{E}_t P_{M,t}^* \right)^{-1} X_{M,t}^* \right] \tag{31}$$

2.3 Intermediate goods producers

The structure of intermediate firms is similar to that of Leith et al. (2009). The intermediate goods firm derives the marginal cost function by solving the cost minimization problem and determining the optimal price through the solution of the profit maximization problem. The following is the production function of the firm j that produces the intermediate good $Y(i,j)$:

$$Y_t(i,j) = A_t N_t(i,j). \tag{32}$$

From the cost minimization problem for intermediate goods firms, we derive $MC_t = W_t/A_t$, which is common across firms.

Under monopolistic competition, intermediate goods firms face the following demand function:

$$Y_t(i,j) = \left(\frac{P_t^m(i,j)}{P_t^m(i)} \right)^{-\xi} Y_t(i) \tag{33}$$

and

$$P_t^m(i) \equiv \left(\int_0^1 (P_t^m)^{1-\xi}(i,j)dj \right)^{\frac{1}{1-\xi}}. \tag{34}$$

where $P_t^m(i,j)$ is the price of the intermediate good j that is put into the final good i, whereas $P_t^m(i)$ is the average price index of the intermediate goods in terms of the final good i. We omit the subscript D or M from the intermediate price $P_t^m(i,j)$ because we assume that intermediate goods $Y_t(i,j)$ are not traded internationally.

Intermediate goods firm maximizes the discounted sum of profit in the face of Calvo (1983)-type nominal rigidity. The nominal profit of intermediate goods firm is defined as $\Phi_t^m(i,j) \equiv (P_t^m(i,j) - MC_t)Y_t(i,j)$. Meanwhile, the following is the profit maximization problem:

$$\max_{P_t^{mo}(i,j)} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s}(P_t^{mo}(i,j) - MC_{t+s})Y_{t+s}(i,j)$$

subject to the demand function for the intermediate goods (33)

$$Y_{t+s}(i,j) = \left(\frac{P_t^{mo}(i,j)}{P_{t+s}^m(i)} \right)^{-\xi} Y_{t+s}(i) \tag{35}$$

and the discount factor (25), where α is the degree of price stickiness and $P_t^{mo}(i,j)$ is optimal price of intermediate goods j for final goods i.

The first-order condition is as follows:

$$\frac{P_t^{mo}(i,j)}{P_{D,t}} = \left(\frac{\zeta_t}{\zeta_t - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s X_{t+s}^{-\sigma+1/\eta} X_{D,t+s}^{-1/\eta} mc_{t+s} \left(P_{t+s}^m(i) \right)^{\xi} Y_{t+s}(i)}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s X_{t+s}^{-\sigma+1\eta} X_{D,t+s}^{-1/\eta} \left(\frac{P_{D,t+s}}{P_{D,t}} \right)^{-1} \left(P_{t+s}^m(i) \right)^{\xi} Y_{t+s}(i)} \tag{36}$$

where $mc_t \equiv MC_t/P_{D,t}$ is the real marginal cost in terms of domestic prices and $\zeta_t/(\zeta_t - 1)$ is the desired markup, $\ln \zeta_t = (1-\rho_m)\ln \epsilon + \rho_m \zeta_{t-1} + \varepsilon_t^m$. Equation (36) can be rewritten using the auxiliary variables $K_{1,t}, K_{2,t}$ as follows:

$$\frac{P_t^{mo}(i,j)}{P_{D,t}} = \left(\frac{\zeta_t}{\zeta_t - 1} \right) \frac{K_{1,t}}{K_{2,t}} \tag{37}$$

where

$$K_{1,t} \equiv E_t \sum_{s=0}^{\infty} (\alpha\beta)^s X_{t+s}^{-\sigma+1/\eta} X_{D,t+s}^{-1/\eta} mc_{t+s} \left(\frac{P_{t+s}^m(i)}{P_{D,t}} \right)^{\xi} Y_{t+s}(i)$$

$$\Rightarrow K_{1,t} = X_t^{-\sigma+1/\eta} X_{D,t}^{-1/\eta} mc_t \left(\frac{P_{D,t}}{P_t^m} \right)^{-\xi} Y_t(i) + \alpha\beta E_t \left[K_{1,t+1} (\Pi_{D,t+1})^{\xi} \right] \tag{38}$$

and

$$K_{2,t} \equiv E_t \sum_{s=0}^{\infty} (\alpha\beta)^s X_{t+s}^{-\sigma+1/\eta} X_{D,t+s}^{-1/\eta} \left(\frac{P_{D,t+s}}{P_{D,t}}\right)^{-1} \left(\frac{P_{t+s}^m(i)}{P_{D,t}}\right)^{\xi} Y_{t+s}(i)$$

$$\Rightarrow K_{2,t} = X_t^{-\sigma+1\eta} X_{D,t}^{-1/\eta} \left(\frac{P_{D,t}}{P_t^m}\right)^{-\xi} Y_t(i) + \alpha\beta E_t \left[K_{2,t+1}(\Pi_{D,t+1})^{\xi-1}\right]. \quad (39)$$

We can drop the subscript i from its price level; hence, $P_t^m = P_t^m(i)$, assuming symmetric intermediate goods.

The following is the distribution of the prices of international goods firms:

$$(P_t^m)^{1-\xi} = \alpha(P_{t-1}^m)^{1-\xi} + (1-\alpha)(P_t^{mo}(j))^{1-\xi} \quad (40)$$

2.4 Terms of trade and the goods-specific real exchange rate

We define the home country's terms of trade as the relative prices of foreign imported goods:

$$TOT_t \equiv \frac{P_{M,t}}{P_{D,t}} \quad (41)$$

When the LOP does not hold, we define the degree of deviation as the LOP gap. For the final goods produced in the home country, Y, we define the gap between domestic price and the foreign price in terms of home currency, or the "goods-specific real exchange rate," as:

$$\psi_t \equiv \frac{\mathcal{E}_t P_{M,t}^*}{P_{D,t}} \quad (42)$$

Similarly, for a foreign good Y^*, the deviation between the price in the foreign and home countries in terms of foreign currency is defined as follows.

$$\psi_t^* \equiv \frac{P_{M,t}/\mathcal{E}_t}{P_{D,t}^*} \quad (43)$$

2.5 Consumer price index, aggregate consumption, and real exchange rate

When incorporating deep habits into the model, we can no longer define the consumer price index (CPI) in a typical manner. We define the CPI following Ravn et al. (2007)[10]

$$P_t \equiv \gamma_p P_{D,t} + (1-\gamma_p) P_{M,t} \quad (44)$$

[10] This definition of the CPI is reminiscent of the fixed base year method.

where $\gamma_p = \frac{P_D}{P_D + P_M}$ is the relative weight of home goods prices at the steady-state. Following this definition, we also defined total consumption as follows:

$$C_t \equiv \frac{P_{D,t}C_{D,t} + P_{M,t}C_{M,t}}{P_t} = \frac{P_{D,t}C_{D,t} + P_{M,t}C_{M,t}}{\gamma_p P_{D,t} + (1-\gamma_p)P_{M,t}} \tag{45}$$

$$= \frac{C_{D,t} + TOT_t C_{M,t}}{\gamma_p + (1-\gamma_p)TOT_t} \tag{46}$$

Using the foreign CPI defined similarly, we define the aggregate level of real exchange rate as follows:

$$q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{\gamma_p^* TOT_t/\psi_t^* + (1-\gamma_p^*)\psi_t}{\gamma_p + (1-\gamma_p)TOT_t} \tag{47}$$

From Equation (47), the aggregate real exchange rate obviously depends on the goods-specific real exchange rates and the terms of trade. In addition, the fixed weights γ_p and γ_p^* govern the extent to which changes in the goods-specific real exchange rate affect the aggregate exchange rate.

2.6 Equilibrium

2.6.1 Aggregate output

Aggregating the intermediate goods set for j, with production function (32) and the demand function (33), we can express the intermediate goods as follows:

$$Y_t(i)\Delta_t(i) = A_t N_t(i) \tag{48}$$

where $\Delta_t(i) \equiv \int_0^1 \left(\frac{P_t^m(i,j)}{P_t^m(i)}\right)^{-\xi} dj$. Following Leith et al. (2015), we can drop the i subscript, with final goods-producing sectors being symmetric:

$$Y_t = \frac{A_t}{\Delta_t} N_t \tag{49}$$

$$\Delta_t = \int_0^1 \left(\frac{P_t^m(j)}{P_t^m}\right)^{-\xi} dj. \tag{50}$$

2.6.2 Perfect financial market

We assume a perfect financial market, which implies the following:

$$Q_{t,t+1} = \beta \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \left(\frac{\omega^{1/\eta}(X_{D,t+1}/X_{t+1})^{-1/\eta}}{\omega^{1/\eta}(X_{D,t}/X_t)^{-1/\eta}}\right) \left(\frac{P_{D,t}}{P_{D,t+1}}\right)$$

$$\mathcal{E}_t Q_{t,t+1}^* = \beta \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \left(\frac{(1-\omega)^{1/\eta}(X_{M,t+1}^*/X_{t+1}^*)^{-1/\eta}}{(1-\omega)^{1/\eta}(X_{M,t}^*/X_t^*)^{-1/\eta}}\right) \left(\frac{\mathcal{E}_t P_{M,t}^*}{\mathcal{E}_{t+1} P_{M,t+1}^*}\right)$$

A perfect capital market implies that both government bonds' prices are equal in the same currency denomination, $Q_{t,t+1} = \mathcal{E}_t Q^*_{t,t+1}$. By simplifying these equations, we derive an international risk sharing condition in terms of habit-adjusted consumption associated with the goods-specific real exchange rate.

$$k_0 \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X_t^*}{X_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{M,t}}{X_{D,t}}\right)^{-\eta} = \psi_t \tag{51}$$

where $k_0 \equiv \psi_0 \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X_0^*}{X_0}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{M,0}}{X_{D,0}}\right)^{-\eta}$ which is normalized to 1 as an assumption.

2.6.3 Aggregate Profits

The aggregate nominal profits in a home country can be expressed as follows:

$$\begin{aligned}\Phi_t &\equiv \int_0^1 \int_0^1 \Phi_t^m(i,j)djdi + \int_0^1 \Phi_{D,t}(i)di + \int_0^1 \Phi^*_{M,t}(i)di \\ &= P_{D,t}Y_t^D + \mathcal{E}_t P^*_{M,t} Y_t^{EX} - W_t N_t. \end{aligned} \tag{52}$$

We derive Equation (52) in Appendix A. Using Equation (52), we expressed the aggregate resource constraint as follows:

$$P_{D,t}C_{D,t} + \mathcal{E}_t P^*_{M,t} C^*_{M,t} = P_{D,t}Y_t^D + \mathcal{E}_t P^*_{M,t} Y_t^{EX} = \Phi_t + W_t N_t \tag{53}$$

3 Optimal monetary policy

This section considers the optimal cooperative monetary policy under commitment with instrument variables as $\pi_{D,t}$ and $\pi^*_{D,t}$. Consider the following Lagrangian:

$$L = \max_{\boldsymbol{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[U(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) + \boldsymbol{\lambda}_t f(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t)\right] \tag{54}$$

where $\boldsymbol{y}_t$ is a vector of endogenous variables and $\boldsymbol{u}_t$ is a vector of exogenous variables, $\boldsymbol{\lambda}_t$ is a vector of Lagrange multipliers.

$$U(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) = (1-\gamma)\left[\frac{X_t^{1-\sigma}}{1-\sigma} - \chi\frac{N_t^{1+\upsilon}}{1+\upsilon}\right] + \gamma\left[\frac{(X_t^*)^{1-\sigma}}{1-\sigma} - \chi\frac{(N_t^*)^{1+\upsilon}}{1+\upsilon}\right] \tag{55}$$

is the policymaker's objective function and $f(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) = 0$ are the constraints provided by the equilibrium path of the private economy (Appendix B). We compute the first-order conditions of this problem using Dynare. The Ramsey policy is computed by approximating the equilibrium system around the perturbation point where the Lagrange multipliers are at their steady state (Appendix C). Consequently, the optimal decision rules are computed around this steady state of the endogenous variables and the Lagrange multipliers. More details are available at Dynare's website.[11]

To compare performance with a policy that is different from the optimal policy (i.e., Taylor rule), we consider the following rule (Bodenstein et al., 2019):

$$R_t = R \left(\frac{R_{t-1}}{R} \right)^{\phi_R} \pi_{D,t}^{\phi_\pi (1-\phi_R)} \tag{56}$$

$$R_t^* = R^* \left(\frac{R_{t-1}^*}{R^*} \right)^{\phi_R} (\pi_{D,t}^*)^{\phi_\pi (1-\phi_R)} \tag{57}$$

which will be replaced by the first-order conditions of the policymakers.

4 Quantitative analysis

4.1 Calibration

This section describes the parameters used in the simulations. Table 1 summarizes calibrations of the parameters. We set the discount rate β to 0.99. The relative risk aversion σ (or the inverse of the habit-adjusted intertemporal substitution of consumption) is set to 2.0. The elasticity of substitution between home and foreign goods η is set to 2.0. These two values have some implications in an open economy NK model; Clarida et al. (2002) and Pappa (2004) showed that a shock occurring in one country has a positive spillover effect on the other country when $\sigma\eta > 1$ in their models. This feature of the spillover effect is carried over to our model.

Following Ravn et al. (2007), we assume that the degree of habits is equal among countries and goods.[12] Specifically, we set $\theta_D = \theta_M^* = \theta_M = \theta_D^* = \theta = 0.4$. This value might be lower than the one in previous studies. According to the brief survey of Leith et al. (2012, 2015), macro-based estimates of deep habits vary from relatively low value of 0.53 as in Ravn, Schmitt-Grohé and Uribe (2012) to extremely high values of 0.95–0.97 as in Ravn et al. (2006) and Lubik and Teo (2014). Meanwhile, micro-based estimates have much lower values of 0.29–0.5 (Ravina, 2005).

[11] https://www.dynare.org/

[12] Following Ravn et al. (2006), we impose symmetry assumptions $\theta_D = \theta_D^*, \theta_M = \theta_M^*$. Moreover, further assumptions $\theta_D = \theta_M$ are imposed to make the steady state system more tractable.

Note that as much as empirical plausibility, the determinacy problems should be considered. Models with deep habits are prone to indeterminacy due to the countercyclical markup behavior (Zubairy, 2014, Ravn, Schmitt-Grohe, Uribe and Uuskula, 2010, Jacob and Uusküla, 2019). Moreover, in our two-country model with sticky prices, the interaction caused by deep habits might be more reinforced. To avoid the problem, we keep the degree of habit somewhat lower: we use $\theta_i = 0.4$ as a benchmark and compare the results by varying the value from 0 (case with no deep habit) to 0.6.

Moreover, we set the degree of home bias ω to 0.85, in line with Bodenstein et al. (2019). For most of the remaining parameters, we followed Leith et al. (2015, 2009). We set the elasticity of substitution for both final goods ϵ and intermediate goods ξ to 11, the degree of price stickiness α to 0.75, and the Frisch labor supply elasticity (Inverse of the intertemporal elasticity of work) υ to 4.0. Furthermore, we set the AR(1) coefficients for technology and markup shocks to $\rho_a = 0.9$ and $\rho_\psi = 0.9625$, respectively.

The relative weight of the disutility of labor χ is set to 1, the coefficient of the lagged nominal interest rate in the Taylor rule ϕ_R to 0.6, and the coefficient of inflation ϕ_π to 1.5. The home country weight in the coordinated Ramsey policy is set to $(1-\gamma)$ =0.5, and the domestic price weight to CPI in the steady-state γ_p is set to 0.5 for both home and foreign countries.

4.2 Impulse responses

In this section, we observe the impulse responses of the foreign and the home country in the case of exogenous shocks that occurred in the foreign country. As in other open models, foreign shocks lead to spillovers to the home country, which are affected by parameters specific to open models (e.g., the degree of economic openness). In addition, the nature of economic dynamics also depends significantly on the degree of deep habit; hence, the optimal policy response also varies. In the following, we observe the responses to productivity and markup shocks, respectively.

Foreign productivity shock

Figure 1 shows the impulse response to a productivity shock that occurred in a foreign country. This figure compares impulse responses by varying the degree of deep habit from the case with no habit formation ($\theta = 0$) to the strong case ($\theta = 0.6$).

[Figure 1 around here.]

First, we review the effects of foreign productivity shock on the foreign country itself. Basically, the effects of the shock are in line with the results of previous studies in a closed economy model: a positive productivity shock raises potential output and puts downward pressure on inflation. Considering optimal monetary policy, the central bank reduces these fluctuations by lowering interest rates. Thus, in the absence of deep habit formation in consumption, the central bank can fully control the fluctuations in inflation and the output gap associated with changes in productivity shocks. The solid blue line in Figure 1 (no deep habit) shows that the PPI inflation rate in a foreign country is fully stabilized to the productivity shocks. Although not shown in the figure, the output gap in the foreign country is also stable because the demand is stimulated as much as potential output growth, resulting in increased production and no change in the output gap.

When the degree of deep habit is large, households are likely to over-consume and the resulting increase in output deviates from the efficiency level, because firms are willing to lower the markups over the goods' prices strategically to increase expected future demand and profits, which in called the countercyclical response of markups. In other words, firms' pricing behavior amplifies rather than stabilizes fluctuations in demand, which is consistent with the empirical evidence.[13] From figure 1, we can see that with deep habits, the habit-adjusted consumption is smaller than that in the absence of habit formation, even though the output is larger than that in the absence of habit formation. This difference expresses excessive consumption.

Although the earlier discussion is consistent with a closed economy case, for example, Leith et al. (2015), the degree of deep habit and the magnitude of decreases in nominal interest rate show a sharp contrast in closed and open economies. The central bank is reluctant to lower interest rates in a closed economy because it does not want to lower markups and increase over-consumption. However, in an open economy, the nominal interest rate is lowered in proportion to the degree of deep habit. This may result from internationally cooperative central banks' incentives to increase global welfare by exploiting terms of trade and the real exchange rate externalities, which could outweigh the negative effects of excessive consumption by foreign households.

Consequently, the central bank faces a trade-off between stabilizing the output gap and

[13] See for example Rotemberg and Woodford (1999).

stabilizing inflation and eventually, the central bank may initially allow deflation. As aforementioned, deep habit generates the endogenous trade-off even in a productivity shock, which implies the deviation from the "devine coincidence."[14] The greater the degree of habit formation, the stronger this effect will be in contrast to the closed economy case.

Turning to an open economy aspect, the change in markups caused by productivity shocks and the resulting change in the demand function results in different pricing of the same good at home and a foreign country, that is, deviation from the LOP, which appreciates the goods-specific real exchange rate for foreign-produced goods. The higher the degree of deep habit, the greater the change in the real exchange rate and this change will spill over to the home country, as we can see in the figure.[15]

Why would combining habit formation externalities with terms of trade externalities lead to improved economic welfare? The intuition is as follows: foreign productivity shocks increase foreign output, which worsens the terms of trade in a foreign country, with or without deep habits. In addition, in the case of deep habits, the increase in foreign output causes habit formation and increase in demand, which deviates from the LOP, causing the real exchange rate to fluctuate, resulting in further deterioration of the terms of trade. As a result, in the home country, imported consumption increases, whereas consumption of domestically produced goods decreases (not shown in the figure). In other words, habit-adjusted consumption increases, whereas domestic production decreases, reducing the negative utility of labor and thus contributes to the improvement of overall economic welfare.

When there is a deep habit in consumption, the international central bank can exploit the externalities of the terms of trade. Habit formation might boost the expenditure switching effect. This difference contrasts the aggressiveness of the central bank between closed and open economies with deep habit.

[14] Givens (2016) introduced other common ways of overcoming divine coincidence, aside from deep habit, such as putting a supply shock on the Phillips curve (Clarida, Gali and Gertler, 1999) and adding an interest rate stabilization term to the central bank's objective function (Amato and Laubach, 1999, Woodford, 2003b). Moreover, Ravenna and Walsh (2006) showed that the endogenous cost-push shock can be generated by introducing a cost channel for monetary policy via borrowing constraint, resulting in an endogenous cost-push shock to the NK Phillips curve.

[15] Although not shown in the figure, the real exchange rate q_t shows the same impulse response as the goods-specific real exchange rate evaluated in terms of home goods, ψ, as long as $\gamma_p + \gamma_p^* = 1$ is satisfied.

The degree of home bias

To highlight the impact specific to an open economy, we examine the impulse response to productivity shocks by varying the home bias in consumption, while keeping the degree of deep habit at the baseline level of 0.4 in Figure 2. The green lines with "x" in Figures 1 and 2 are the baseline case ($\theta_D = 0.40$, $\omega = 0.85$) and are identical each other.

[Figure 2 around here.]

When the degree of home bias is changed from the baseline, the impulse response within the foreign country to a foreign productivity shock does not change significantly, whereas the spillover to the home country changes due to the impact on the terms of trade. This can be explained by the fact that home households are willing to consume more foreign goods when the home bias is small. The smaller the home bias, the more foreign output is imported in the home country through changes in the terms of trade, which leads to the lower the domestic home production and the labor to be required.

Interestingly, when home bias is nonexistent, the goods-specific real exchange rate changes are completely offset. The absence of a home bias in consumption implies that changes in the demand occur equally in the home and the foreign country, in response to changes in relative prices: the LOP holds. However, when there is a strong home bias, the price of a foreign good in a foreign country falls in response to positive productivity shock, whereas the demand and the price for the foreign good in a home country remains almost unchanged. Thus, the greater the home bias, the larger the change in the good-specific real exchange rate and the terms of trade. This finding is somewhat novel: Ravn et al. (2007) also indicated that structural shocks endogenously change goods-specific real exchange rate but the relationship between home bias and deep habit in consumption has not been discussed.

Commitment versus Taylor rule based simple monetary policy

To clarify the role of the central bank in the earlier discussion, we show impulse responses to a productivity shock that occurred in a foreign country under commitment, compared to the rule-based simple monetary policy (Taylor rule) in Figure 3. The solid blue line is the commitment case and the red line with a circle is the Taylor rule, with $\theta = 0.4$ and $\omega = 0.85$. Other parameters follow the benchmark calibration.

[Figure 3 around here.]

The figure shows that the responses under the Taylor rule do not differ so much from those under the commitment case. One of the features to be mentioned is that the foreign markup is significantly lower in the commitment case than under the Taylor rule. Figure 1 shows that markups do not respond to productivity shocks when there is no deep habit in consumption, which also holds under the Taylor rule. As discussed earlier, the markup differences between with and without deep habit illustrate how an internationally coordinated monetary policy authority exploits the deep habit of consumption. By lowering the price markup more than in the simple rule case, the commitment policy seeks to bring the positive spillover effects of foreign shocks to the home country. In other words, the central bank is trying to increase import consumption through habit formation and exploiting trade externality. Consequently, habit-adjusted consumption in the home country responds more strongly than under the Taylor rule.

As shown in Figure 2, the effect becomes more evident as the home bias in consumption becomes small. In a coordinated monetary policy, the externalities of the terms of trade and the deep habit of consumption are combined to maximize worldwide social welfare. The results contrast previous studies on habit formation and optimal monetary policy in closed economies.

Foreign markup shock

Leith et al. (2015) showed that time-varying markups generated under deep habits play an important role in the optimal policy response to cost-push shocks. Ravn et al. (2007) showed that markups cause endogenous changes in goods-specific real exchange rates. Considering these two, we show the impulse response of foreign intermediate goods firms to a fall in the desired markup (i.e., markup shock) in Figure 4.

[Figure 4 around here.]

A reduction in the desired markup leads to a fall in inflation and an increase in output. In other words, the markup shock works as a negative cost-push shock. The central bank faces a trade-off between the output gap and inflation stability in response to the markup shock, even in the absence of a deep habit.

As in a productivity shock, the markup does not move in the absence of deep habit in consumption. However, when there is a deep habit, the markups fall in response to the degree of deep habit. The greater the degree of deep habit, the greater the fall in the goods-specific real exchange rate and terms of trade. In response to the markup changes, the central bank cuts interest rates more and tries to reduce the negative utility of labor in a home country by encouraging imports rather than consumption of domestically produced goods.

This result is also in contrast to previous studies: in a closed economy, interest rates are temporarily raised to eliminate overconsumption through deep habit formation caused by lowering markups. Consequently, the markup initially rises and then falls in a closed economy case, whereas the two-country model has no such temporary markup increase. The central bank considers the world as a whole, not just the economic welfare of a foreign country, even if the shock occurs in a foreign country. In this model framework, the benefits of spillovers are likely to offset the disadvantages of overconsumption.

Figure 5 confirms the earlier discussion by changing the home bias. As in the case of productivity shocks, the central bank also tries to exploit spillovers to the home country in response to a markup shock in a foreign country. The smaller the home bias, the more the home country would import foreign goods, and the central bank tries to take this opportunity to further reduce foreign firms' markups.

[Figure 5 around here.]

Finally, we examine how the impulse response to a markup shock differs between optimal commitment and rule-based simple monetary policies (Figure 6). The calibration of the parameters is the same as for the productivity shock case (Figure 3). Again, as in the previous discussion, the markups of foreign firms are much lower under the commitment policy, which highlights the role of optimal monetary policy.

[Figure 6 around here.]

5 Concluding remarks

Our paper shows how the dependence between the deep habits in consumption and optimal monetary policy in the closed NK model may differ when extended to a two-country NK model. We show that in response to the structural shocks, the central bank changes the interest rate significantly in the two-country open economy model, contrasted with a closed economy, where the central bank is reluctant to move interest rates. When there is a deep habit in consumption, the international central bank can exploit the externalities of the terms of trade. Habit formation might boost the expenditure switching effect, which differentiate the aggressiveness of the central bank between closed and open economies with deep habit. Also, we showed that the deviations from the LOP, or the goods-specific real exchange rate, generated by the deep habit are significantly related to the degree of home bias. In particular, the deviations are fully disappeared when there is no home bias.

Our study considered an optimally coordinated central bank; however, the discretionary policy remains to be considered. Investigating the gain in commitment, as in Givens (2016), is one possible avenue for future research. Another future work is investigating the gains of international coordination by comparing the result of uncoordinated Nash equilibrium.[16]

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Appendix

A Aggregate Profits

Following Leith et al. (2009), we express the aggregate profits of intermediate goods firms as follows:

$$
\begin{aligned}
&\int_0^1 \int_0^1 \Phi_t^m(i,j)djdi \\
&= \int_0^1 \int_0^1 (P_t^m(i,j) - MC_t^n)Y_t(i,j)djdi \\
&= \int_0^1 \int_0^1 \left(P_{D,t}(i,j) - \frac{W_t}{A_t}\right)\left(\frac{P_t^m(i,j)}{P_t^m(i)}\right)^{-\xi}(Y_{D,t}(i) + Y_{EX,t}(i))djdi \\
&= \int_0^1 \int_0^1 \left[\frac{P_t^m(i,j)^{1-\xi}}{P_t^m(i)^{-\xi}}(Y_{D,t}(i) + Y_{EX,t}(i)) - \frac{W_t}{A_t}A_tN_t(i,j)\right]djdi \\
&= \int_0^1 (P_t^m(i))^{\xi}(Y_{D,t}(i) + Y_{EX,t}(i))\left(\int_0^1 (P_t^m)^{1-\xi}(i,j)dj\right)di - W_t\int_0^1\int_0^1 N_t(i,j)djdi \\
&= \int_0^1 (P_t^m(i))^{\xi}(Y_{D,t}(i) + Y_{EX,t}(i))(P_t(i)^m)^{1-\xi}di - W_tN_t \\
&= \int_0^1 (P_t^m(i))(Y_{D,t}(i) + Y_{EX,t}(i))di - W_tN_t
\end{aligned}
$$

Aggregate profits of final goods firms are:

$$
\begin{aligned}
&\int_0^1 \Phi_{D,t}(i)di + \int_0^1 \Phi_{M,t}^*(i)di \\
&= \int_0^1 (P_{D,t}(i) - P_t^m(i))Y_{D,t}(i)di + \int_0^1 (\mathcal{E}_tP_{M,t}^*(i) - P_t^m(i))Y_{EX,t}(i)di \\
&= \int_0^1 (P_{D,t}(i)Y_{D,t}(i) + \mathcal{E}_tP_{M,t}^*(i)Y_{EX,t}(i))di - \int_0^1 P_t^m(i)(Y_{D,t}(i) + Y_{EX,t}(i))di.
\end{aligned}
$$

Aggregate profits in a home country is thus as follows:

$$
\begin{aligned}
\Phi_t \equiv &\int_0^1\int_0^1 \Phi_t^m(i,j)djdi + \int_0^1 \Phi_{D,t}(i)di + \int_0^1 \Phi_{M,t}^*(i)di \\
&= \int_0^1 (P_t^m(i))(Y_{D,t}(i) + Y_{EX,t}(i))di - W_tN_t \\
&+ \int_0^1 (P_{D,t}(i)Y_{D,t}(i) + \mathcal{E}_tP_{M,t}^*(i)Y_{EX,t}(i))di \\
&- \int_0^1 P_t^m(i)(Y_{D,t}(i) + Y_{EX,t}(i))di \\
&= \int_0^1 (P_{D,t}(i)Y_{D,t}(i) + \mathcal{E}_tP_{M,t}^*(i)Y_{EX,t}(i))di - W_tN_t \\
&= P_{D,t}Y_{D,t} + \mathcal{E}_tP_{M,t}^*Y_{EX,t} - W_tN_t
\end{aligned}
$$

The first and second terms on the right-hand side of the last equation can be derived by combining the CES aggregation properties of production function and its prices with the final goods firm's cost minimization problem.

B System of Equilibrium Conditions

Complete-market condition,

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}}\left(\frac{X_t^*}{X_t}\right)^{-\sigma+1/\eta}\left(\frac{X_{M,t}^*}{X_{D,t}}\right)^{-\eta}=\psi_t \tag{B.1}$$

Habit adjusted aggregate consumption

$$X_t=\left(\omega^{\frac{1}{\eta}}X_{D,t}^{\frac{\eta-1}{\eta}}+(1-\omega)^{\frac{1}{\eta}}X_{M,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{B.2}$$

$$X_t^*=\left(\omega^{\frac{1}{\eta}}X_{D,t}^{*\frac{\eta-1}{\eta}}+(1-\omega)^{\frac{1}{\eta}}X_{M,t}^{*\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{B.3}$$

From habit adjusted consumption (7), assuming that goods i are symmetric for households, we can write:

$$X_{D,t}=C_{D,t}-\theta_D S_{D,t-1} \tag{B.4}$$

$$X_{M,t}=C_{M,t}-\theta_M S_{M,t-1} \tag{B.5}$$

$$X_{D,t}^*=C_{D,t}^*-\theta_D^* S_{D,t-1}^* \tag{B.6}$$

$$X_{M,t}^*=C_{M,t}^*-\theta_M^* S_{M,t-1}^* \tag{B.7}$$

The stocks of habit consumption,

$$S_{H,t}=\varrho_H S_{H,t-1}+(1-\varrho_H)\,C_{H,t} \tag{B.8}$$

$$S_{F,t}=\varrho_F S_{F,t-1}+(1-\varrho_F)\,C_{F,t} \tag{B.9}$$

$$S_{H,t}^*=\varrho_H^* S_{H,t-1}^*+(1-\varrho_H^*)\,C_{H,t}^* \tag{B.10}$$

$$S_{F,t}^*=\varrho_F^* S_{F,t-1}^*+(1-\varrho_F^*)\,C_{F,t}^* \tag{B.11}$$

First order conditions

$$\left(\frac{\omega}{1-\omega}\right)^{1/\eta}\left(\frac{X_{D,t}}{X_{M,t}}\right)^{-1/\eta}=\frac{P_{D,t}}{P_{M,t}}=TOT_t^{-1} \tag{B.12}$$

$$\omega^{-1/\eta}\chi(X_t)^{\sigma-1/\eta}X_{D,t}^{1/\eta}N_t^{\upsilon}=\frac{W_t}{P_{D,t}}=w_t \tag{B.13}$$

where $w_t \equiv W_t/P_{D,t}$.

Euler equation of consumption

$$1 = \beta E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t+1}}{X_{D,t}} \right)^{-1/\eta} \left(\frac{P_{D,t}}{P_{D,t+1}} \right) \right] R_t \tag{B.14}$$

$$1 = \beta E_t \left[\left(\frac{X^*_{t+1}}{X^*_t} \right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t+1}}{X^*_{D,t}} \right)^{-1/\eta} \left(\frac{P^*_{D,t}}{P^*_{D,t+1}} \right) \right] R^*_t \tag{B.15}$$

the distribution of intermediate goods prices

$$(P^m_t)^{1-\xi} = \alpha (P^m_{t-1})^{1-\xi} + (1-\alpha)(P^{mo}_t(j))^{1-\xi}$$

Therefore,

$$1 = \alpha \left(\frac{1}{\Pi^m_t} \right)^{1-\xi} + (1-\alpha) \left(\frac{P^{mo}_t}{P^m_t} \right)^{1-\xi} \tag{B.16}$$

The evolution of price dispersion

$$\Delta^m_t = \alpha (\Pi^m_t)^{\xi} \Delta^m_{t-1} + (1-\alpha) \left(\frac{P^{mo}_t}{P^m_t} \right)^{-\xi} \tag{B.17}$$

Optimal intermediate price

$$\frac{P^{mo}_t}{P_{D,t}} = \left(\frac{\zeta_t}{\zeta_t - 1} \right) \frac{K_{1,t}}{K_{2,t}} \tag{B.18}$$

$$\frac{P^{mo*}_t}{P^*_{D,t}} = \left(\frac{\zeta^*_t}{\zeta^*_t - 1} \right) \frac{K^*_{1,t}}{K^*_{2,t}} \tag{B.19}$$

where

$$K_{1,t} = X_t^{-\sigma+1/\eta} X_{D,t}^{-1/\eta} mc_t \left(\frac{P_{D,t}}{P^m_t} \right)^{-\xi} Y_t + \alpha\beta E_t \left[K_{1,t+1} (\Pi_{D,t+1})^{\xi} \right] \tag{B.20}$$

$$K^*_{1,t} = (X^*_t)^{-\sigma+1/\eta} (X^*_{D,t})^{-1/\eta} mc^*_t \left(\frac{P^*_{D,t}}{P^{m*}_t} \right)^{-\xi} Y^*_t + \alpha\beta E_t \left[K^*_{1,t+1} (\Pi^*_{D,t+1})^{\xi} \right] \tag{B.21}$$

and

$$K_{2,t} = X_t^{-\sigma+1/\eta} X_{D,t}^{-1/\eta} \left(\frac{P_{D,t}}{P^m_t} \right)^{-\xi} Y_t + \alpha\beta E_t \left[K_{2,t+1} (\Pi_{D,t+1})^{\xi-1} \right] \tag{B.22}$$

$$K^*_{2,t} = (X^*_t)^{-\sigma+1/\eta} (X^*_{D,t})^{-1/\eta} \left(\frac{P^*_{D,t}}{P^{m*}_t} \right)^{-\xi} Y^*_t + \alpha\beta E_t \left[K^*_{2,t+1} (\Pi^*_{D,t+1})^{\xi-1} \right] \tag{B.23}$$

Where $\Pi_{D,t} = P_{D,t}/P_{D,t-1}$.

Note that when intermediate goods firms can decide its price, subscript j can be removed from its optimal price since all firms behave the same manner, $P^{mo}_t(j) = P^{mo}_t$ for all j.

Aggregate production function

$$Y_t = Y_{D,t} + Y_{EX,t} = \frac{A_t}{\Delta_t} N_t \tag{B.24}$$

$$Y_t^* = Y_{D,t}^* + Y_{EX,t}^* = \frac{A_t^*}{\Delta_t^*} N_t^* \tag{B.25}$$

Aggregate resource constraint are derived from home and foreign households' budget constraints,

$$P_{D,t}Y_{D,t} + \mathcal{E}_t P_{M,t}^* Y_{EX,t} = P_{D,t}C_{D,t} + \mathcal{E}_t P_{M,t}^* C_{M,t}^*$$

$$\Rightarrow Y_{D,t} + \psi_t Y_{EX,t} = C_{D,t} + \psi_t C_{M,t}^* \tag{B.26}$$

$$P_{D,t}^* Y_{D,t}^* + P_{M,t} Y_{EX,t}/\mathcal{E}_t = P_{D,t}^* C_{D,t}^* + P_{M,t} C_{M,t}/\mathcal{E}_t$$

$$\Rightarrow Y_{D,t}^* + \psi_t^* Y_{EX,t}^* = C_{D,t}^* + \psi_t^* C_{M,t} \tag{B.27}$$

Behavior of deep habit formation

$$\Lambda_{D,t}(i) = (P_{D,t}(i) - P_t^m(i)) + \theta_D E_t \left[Q_{t,t+1} \Lambda_{D,t+1}(i) \right]$$

$$Q_{t,t+1} = \beta \left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{P_{D,t}}{P_{t+1}} \right)$$

Dropping i and eliminate $Q_{t,t+1}$ from the first equation,

$$\Lambda_{D,t} = (P_{D,t} - P_t^m) + \beta\theta_D E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{P_{D,t}}{P_{D,t+1}} \right) \Lambda_{D,t+1} \right]$$

$$\Rightarrow \lambda_{D,t} = \left(1 - \frac{P_t^m}{P_{D,t}} \right) + \beta\theta_D E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \lambda_{D,t+1} \right] \tag{B.28}$$

where $\lambda_{D,t} \equiv \Lambda_{D,t}/P_{D,t}$ is Lagrange multiplier in real term. And,

$$C_{D,t} = \epsilon \lambda_{D,t} X_{D,t} \tag{B.29}$$

Similarly,

$$\Lambda_{M,t}^* = (\mathcal{E}_t P_{M,t}^* - P_t^m) + \beta\theta_M^* E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{P_{D,t}}{P_{D,t+1}} \right) \Lambda_{M,t+1}^* \right]$$

Dividing both sides by $\mathcal{E}_t P_{M,t}^*$ and using $\psi_t = \mathcal{E}_t P_{M,t}^*/P_{D,t}$,

$$\lambda_{M,t}^* = \left(1 - \frac{P_t^m}{\psi_t P_{D,t}} \right) + \beta\theta_M^* E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{\psi_{t+1}}{\psi_t} \right) \lambda_{M,t+1}^* \right] \tag{B.30}$$

where $\lambda^*_{M,t} = \Lambda^*_{M,t}/(\mathcal{E}_t P^*_{M,t})$. Also,

$$C^*_{M,t} = \epsilon\lambda^*_{M,t} X^*_{M,t} \tag{B.31}$$

The demand for goods produced in a foreign country, taking into account the deep habit, can also be derived as follows:

$$\Lambda^*_{D,t}(i) = (P^*_{D,t}(i) - P^{m*}_t(i)) + \theta^*_D E_t\left[Q^*_{t,t+1}\Lambda^*_{D,t+1}(i)\right]$$
$$Q^*_{t,t+1} = \beta\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta}\left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta}\left(\frac{P^*_{D,t}}{P^*_{t+1}}\right)$$

Dropping i and eliminate $Q^*_{t,t+1}$ from the first equation,

$$\Lambda^*_{D,t} = (P^*_{D,t} - P^{m*}_t) + \beta\theta^*_D E_t\left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta}\left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta}\left(\frac{P^*_{D,t}}{P^*_{D,t+1}}\right)\Lambda^*_{D,t+1}\right]$$
$$\Rightarrow \lambda^*_{D,t} = \left(1 - \frac{P^{m*}_t}{P^*_{D,t}}\right) + \beta\theta^*_D E_t\left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta}\left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta}\lambda^*_{D,t+1}\right] \tag{B.32}$$

where $\lambda^*_{D,t} \equiv \Lambda^*_{D,t}/P^*_{D,t}$ is Lagrange multiplier in real term. And,

$$C^*_{D,t} = \epsilon\Lambda^*_{D,t}(P^*_{D,t})^{-1}X^*_{D,t} \Rightarrow C^*_{D,t} = \epsilon\lambda^*_{D,t}X^*_{D,t} \tag{B.33}$$

Similarly,

$$\Lambda_{M,t} = (P_{M,t}/\mathcal{E}_t - P^{m*}_t) + \beta\theta_M E_t\left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta}\left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta}\left(\frac{P^*_{D,t}}{P^*_{D,t+1}}\right)\Lambda_{M,t+1}\right]$$

Dividing both sides by $P_{M,t}/\mathcal{E}_t$ and using $\psi^*_t = \frac{P_{M,t}/\mathcal{E}_t}{P^*_{D,t}}$,

$$\frac{\Lambda_{M,t}}{P_{M,t}/\mathcal{E}_t} = \left(1 - \frac{P^{m*}_t}{\psi^*_t P^*_{D,t}}\right) + \beta\theta_M E_t\left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta}\left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta}\left(\frac{\psi^*_{t+1}}{\psi^*_t}\right)\left(\frac{\Lambda^*_{M,t+1}}{\mathcal{E}_{t+1}P^*_{M,t+1}}\right)\right]$$
$$\Rightarrow \lambda_{M,t} = \left(1 - \frac{P^{m*}_t}{\psi^*_t P^*_{D,t}}\right) + \beta\theta_M E_t\left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta}\left(\frac{X^*_{D,t}}{X_{D,t+1}}\right)^{1/\eta}\left(\frac{\psi^*_{t+1}}{\psi^*_t}\right)\lambda_{M,t+1}\right] \tag{B.34}$$

where $\lambda_{M,t} = \Lambda_{M,t}/(P_{M,t}/\mathcal{E}_t)$. Also,

$$C_{M,t} = \epsilon\Lambda_{M,t}\frac{1}{P_{M,t}/\mathcal{E}_t}X_{M,t}$$
$$\Rightarrow C_{M,t} = \epsilon\lambda_{M,t}X_{M,t} \tag{B.35}$$

Inflation, relative prices and real variables

Terms of trade

$$TOT_t = \frac{P_{M,t}}{P_{D,t}} \tag{B.36}$$

$$TOT_t^* = \frac{P_{M,t}^*}{P_{D,t}^*} \tag{B.37}$$

Goods-specific real exchange rate

$$\psi_t = \frac{\mathcal{E}_t P_{M,t}^*}{P_{D,t}} \tag{B.38}$$

$$\psi_t^* = \frac{P_{M,t}/\mathcal{E}_t}{P_{D,t}^*} \tag{B.39}$$

Terms of trades and goods-specific real exchange rates are combined as follows:

$$\psi_t \psi_t^* = \frac{\mathcal{E}_t P_{M,t}^* P_{M,t}/\mathcal{E}_t}{P_{D,t} P_{D,t}^*} = TOT_t \cdot TOT_t^* \tag{B.40}$$

optimal relative intermediate goods price

$$p_t^{mo} \equiv \frac{P_t^{mo}}{P_{D,t}} \tag{B.41}$$

$$p_t^{mo*} \equiv \frac{P_t^{mo*}}{P_{D,t}^{m*}} \tag{B.42}$$

Price markup of final goods firms

$$\mu_t \equiv \frac{P_{D,t}}{P_t^m} \tag{B.43}$$

$$\mu_t^* \equiv \frac{P_{D,t}^*}{P_t^{m*}} \tag{B.44}$$

We can express the relative optimal price to average intermediate goods price as follows:

$$\frac{P_t^{mo}}{P_t^m} = \frac{P_t^{mo}}{P_{D,t}} \frac{P_{D,t}}{P_t^m} = p_t^{mo} \mu_t \tag{B.45}$$

$$\frac{P_t^{mo*}}{P_t^{m*}} = p_t^{mo*} \mu_t^* \tag{B.46}$$

Intermediate goods price inflation

$$\Pi_t^m = \frac{P_t^m}{P_{t-1}^m} \tag{B.47}$$

$$\Pi_t^{m*} = \frac{P_t^{m*}}{P_{t-1}^{m*}} \tag{B.48}$$

Domestic Price inflation

$$\Pi_{D,t} \equiv \frac{P_{D,t}}{P_{D,t-1}} = \frac{P_{D,t}}{P_t^m}\frac{P_{t-1}^m}{P_{D,t-1}}\frac{P_t^m}{P_{t-1}^m} = \frac{\mu_t}{\mu_{t-1}}\Pi_t^m \tag{B.49}$$

$$\Pi_{D,t}^* = \frac{\mu_t^*}{\mu_{t-1}^*}\Pi_t^{m*} \tag{B.50}$$

Real wage

$$w_t = \frac{W_t}{P_{D,t}} \tag{B.51}$$

$$w_t^* = \frac{W_t^*}{P_{D,t}^*} \tag{B.52}$$

Real marginal cost

$$mc_t = \frac{MC_t}{P_{D,t}} = \frac{w_t}{A_t} \tag{B.53}$$

$$mc_t^* = \frac{w_t^*}{A_t^*} \tag{B.54}$$

C Steady state

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X^*}{X}\right)^{-\sigma+1/\eta} \left(\frac{X_M^*}{X_D}\right)^{-\eta} = \psi \tag{C.1}$$

$$X = \left(\omega^{\frac{1}{\eta}} X_D^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} X_M^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{C.2}$$

$$X^* = \left(\omega^{\frac{1}{\eta}} X_D^{*\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} X_M^{*\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{C.3}$$

$$X_D = C_D - \theta_D S_D \tag{C.4}$$

$$X_M = C_M - \theta_M S_M \tag{C.5}$$

$$X_D^* = C_D^* - \theta_D^* S_D^* \tag{C.6}$$

$$X_M^* = C_M^* - \theta_M^* S_M^* \tag{C.7}$$

$$S_D = C_D \tag{C.8}$$

$$S_M = C_M \tag{C.9}$$

$$S_D^* = C_D^* \tag{C.10}$$

$$S_M^* = C_M^* \tag{C.11}$$

$$\left(\frac{\omega}{1-\omega}\right)^{1/\eta}\left(\frac{X_D}{X_M}\right)^{-1/\eta} = TOT^{-1} \tag{C.12}$$

$$\left(\frac{\omega}{1-\omega}\right)^{1/\eta}\left(\frac{X_D^*}{X_M^*}\right)^{-1/\eta} = TOT(\psi\psi^*)^{-1} \tag{C.13}$$

$$\omega^{-1/\eta}\chi(X)^{\sigma-1/\eta}(X_D)^{1/\eta}(N)^{\upsilon} = w \tag{C.14}$$

$$\omega^{-1/\eta}\chi(X^*)^{\sigma-1/\eta}(X_D^*)^{1/\eta}(N^*)^{\upsilon} = w^* \tag{C.15}$$

$$1 = \beta\Pi_D^{-1}R \tag{C.16}$$

$$1 = \beta\Pi_D^{*-1}R^* \tag{C.17}$$

$$1 = \alpha(\Pi^m)^{-1+\xi} + (1-\alpha)(p^{mo}\mu)^{1-\xi} \tag{C.18}$$

$$1 = \alpha(\Pi^{m*})^{-1+\xi} + (1-\alpha)(p^{mo*}\mu^*)^{1-\xi} \tag{C.19}$$

$$\Delta^m = \frac{(1-\alpha)(p^{mo}\mu)^{-\xi}}{1-\alpha(\Pi^m)^{\xi}} \tag{C.20}$$

$$\Delta^{m*} = \frac{(1-\alpha)(p^{mo*}\mu^*)^{-\xi}}{1-\alpha(\Pi^{m*})^{\xi}} \tag{C.21}$$

$$p^{mo} = \left(\frac{\zeta}{\zeta-1}\right)\frac{K_1}{K_2}$$

$$p^{mo*} = \left(\frac{\zeta^*}{\zeta^*-1}\right)\frac{K_1^*}{K_2^*}$$

$$K_1 = \frac{(X)^{-\sigma+1/\eta}(X_D)^{-1/\eta}mc(\mu)^{-\xi}Y}{1-\alpha\beta(\Pi_D)^{\xi}} \tag{C.22}$$

$$K_1^* = \frac{(X^*)^{-\sigma+1/\eta}(X_D^*)^{-1/\eta}mc^*(\mu^*)^{-\xi}Y^*}{1-\alpha\beta(\Pi_D^*)^{\xi}} \tag{C.23}$$

$$K_2 = \frac{(X)^{-\sigma+1/\eta}(X_D)^{-1/\eta}\mu^{-\xi}Y}{1-\alpha\beta(\Pi_D)^{\xi-1}} \tag{C.24}$$

$$K_2^* = \frac{(X^*)^{-\sigma+1/\eta}(X_D^*)^{-1/\eta}(\mu^*)^{-\xi}Y^*}{1-\alpha\beta(\Pi_D^*)^{\xi-1}} \tag{C.25}$$

$$\Rightarrow (p^{mo})^{-1} = \left(\frac{\zeta}{\zeta-1}\right)\frac{1-\alpha\beta(\Pi_D)^{\xi-1}}{1-\alpha\beta(\Pi_D)^{\xi}}mc \tag{C.26}$$

$$(p^{mo*})^{-1} = \left(\frac{\zeta^*}{\zeta^*-1}\right)\frac{1-\alpha\beta(\Pi^*_D)^{\xi-1}}{1-\alpha\beta(\Pi^*_D)^{\xi}}mc^* \tag{C.27}$$

$$Y = \frac{A}{\Delta^m}N \tag{C.28}$$

$$Y^* = \frac{A^*}{\Delta^{m*}}N^* \tag{C.29}$$

$$mc = \frac{w}{A} \tag{C.30}$$

$$mc^* = \frac{w^*}{A^*} \tag{C.31}$$

$$Y_D + \psi Y_{EX} = C_D + \psi_t C^*_M \tag{C.32}$$

$$Y^*_D + \psi^*_t Y^*_{EX} = C^*_D + \psi^* C_M \tag{C.33}$$

$$\mu = [1-(1-\beta\theta_D)\lambda_D]^{-1} \tag{C.34}$$

$$\psi\mu = [1-(1-\beta\theta^*_M)\lambda^*_M]^{-1} \tag{C.35}$$

$$\mu^* = [1-(1-\beta\theta^*_D)\lambda^*_D]^{-1} \tag{C.36}$$

$$\psi^*\mu^* = [1-(1-\beta\theta_M)\lambda_M]^{-1} \tag{C.37}$$

$$C_D = \epsilon\lambda_D X_D \tag{C.38}$$

$$C^*_M = \epsilon\lambda^*_M X^*_M \tag{C.39}$$

$$C^*_D = \epsilon\lambda^*_D X^*_D \tag{C.40}$$

$$C_M = \epsilon\lambda_M X_M \tag{C.41}$$

$$A = 1 \tag{C.42}$$

$$A^* = 1 \tag{C.43}$$

$$\zeta = \varepsilon^m \tag{C.44}$$

$$\zeta^* = \varepsilon^{m*} \tag{C.45}$$

$$\Pi^m = \Pi_D \tag{C.46}$$

$$\Pi^{*m} = \Pi^*_D \tag{C.47}$$

For the sake of tractability, we will make some simplifying and symmetry assumptions. Eliminating λ, C in both countries

$$\Rightarrow \mu = \left[1 - \frac{1-\beta\theta_D}{(1-\theta_D)\epsilon}\right]^{-1}$$
$$\psi\mu = \left[1 - \frac{1-\beta\theta_M^*}{(1-\theta_M^*)\epsilon}\right]^{-1}$$
$$\mu^* = \left[1 - \frac{(1-\beta\theta_D^*)}{(1-\theta_D^*)\epsilon}\right]^{-1}$$
$$\psi^*\mu^* = \left[1 - \frac{(1-\beta\theta_M)}{(1-\theta_M)\epsilon}\right]^{-1}$$

Symmetry assumptions, $\theta_D = \theta_D^*, \theta_M = \theta_M^*$.

$$\Rightarrow \mu = \mu^*$$
$$\psi = \psi^*$$

Further assumptions, $\theta_D = \theta_M$. $\psi = \psi^* = 1$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X_M^*}{X_D}\right)^{-\eta} = 1$$

Finally, assuming $\Pi^m = \Pi_D = 1$, prices at steady state are determined and the deterministic steady state system equations are uniquely determined analytically.

Parameter	Value	Description
β	0.99	Subjective discount factor
σ	2.0	Inverse of the intertemporal elasticities of habit-adjusted consumption
ω	0.85	Degree of home bias in consumption
ϵ	11	Elasticity of substitution across final goods
ξ	11	Elasticity of substitution across intermediate goods
η	2.0	Elasticity of substitution between home and foreign goods
θ_i, θ_i^*	0.4	Degree of habit persistence ($i = D, M$)
ϱ_i, ϱ_i^*	0.0	Persistence of habit stock ($i = D, M$)
υ	4.0	Inverse of the intertemporal elasticities of work
χ	1	Relative weight on disutility from time spent working
α	0.75	Degree of price stickiness
ϕ_R	0.6	Coefficient on lagged nominal interest rate of Taylor rule
ϕ_π	1.5	Coefficient on inflation of Taylor rule
ρ_ψ	0.9625	Persistence of markup shock
ρ_a	0.9	Persistence of technology shock
γ	0.5	Weight on home country in Ramsey policy
γ_p, γ_P^*	0.5	Steady state level of relative weight of domestic prices in the CPI

Table 1: structural parameter values used in simulations

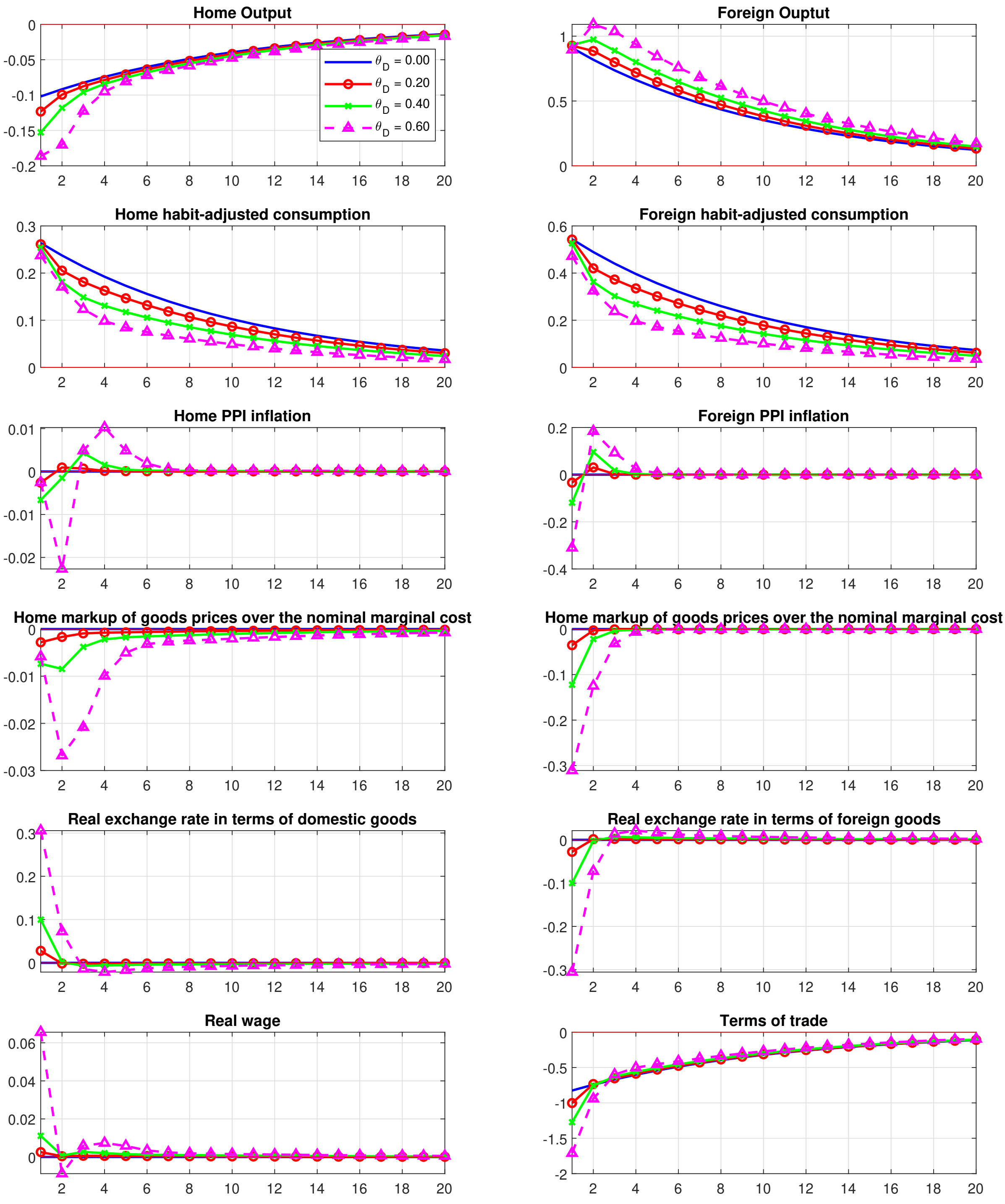

Figure 1: Impulse responses of a foreign productivity shock

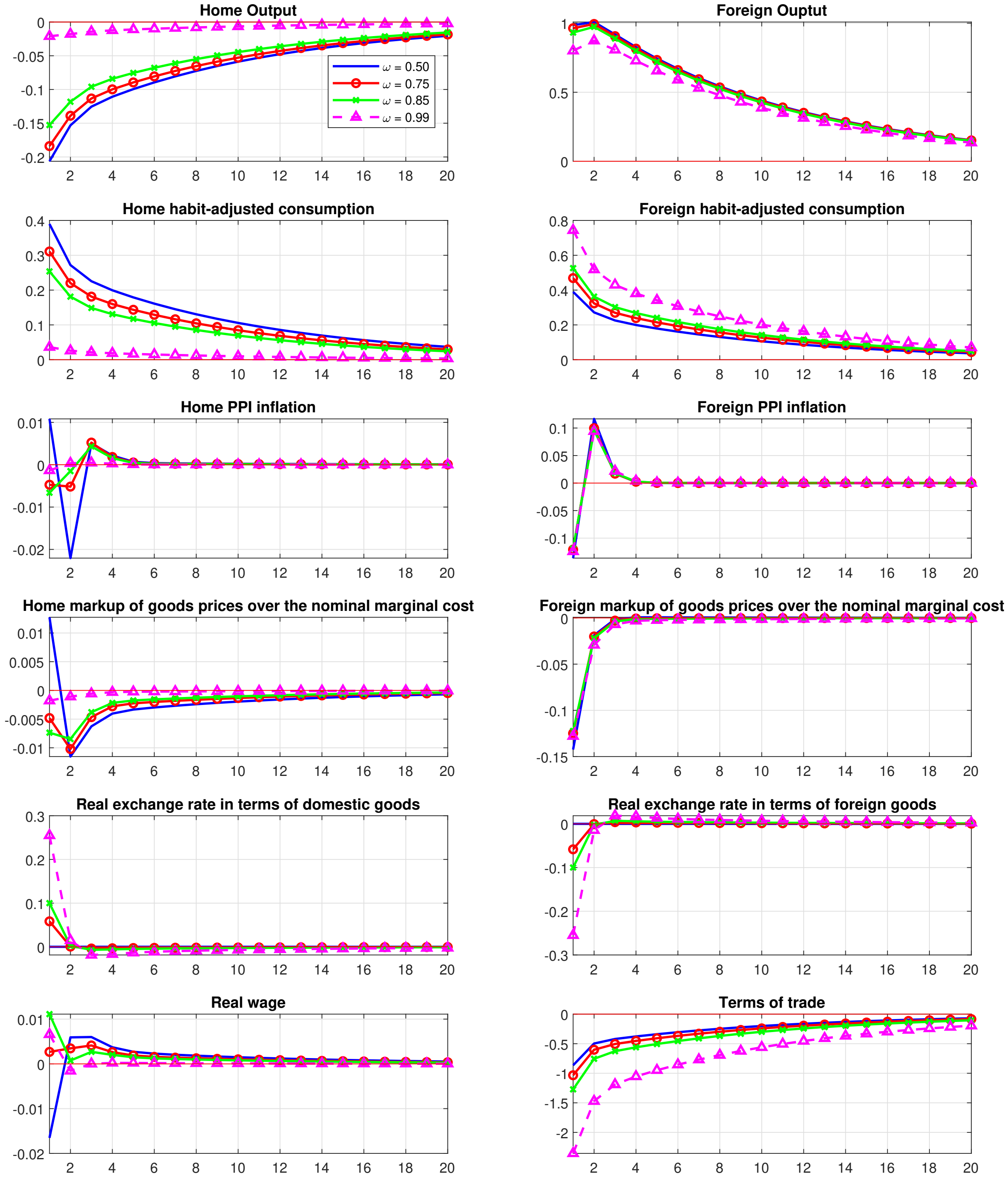

Figure 2: Impulse responses of a foreign productivity shock with respect to changing the degree of home bias

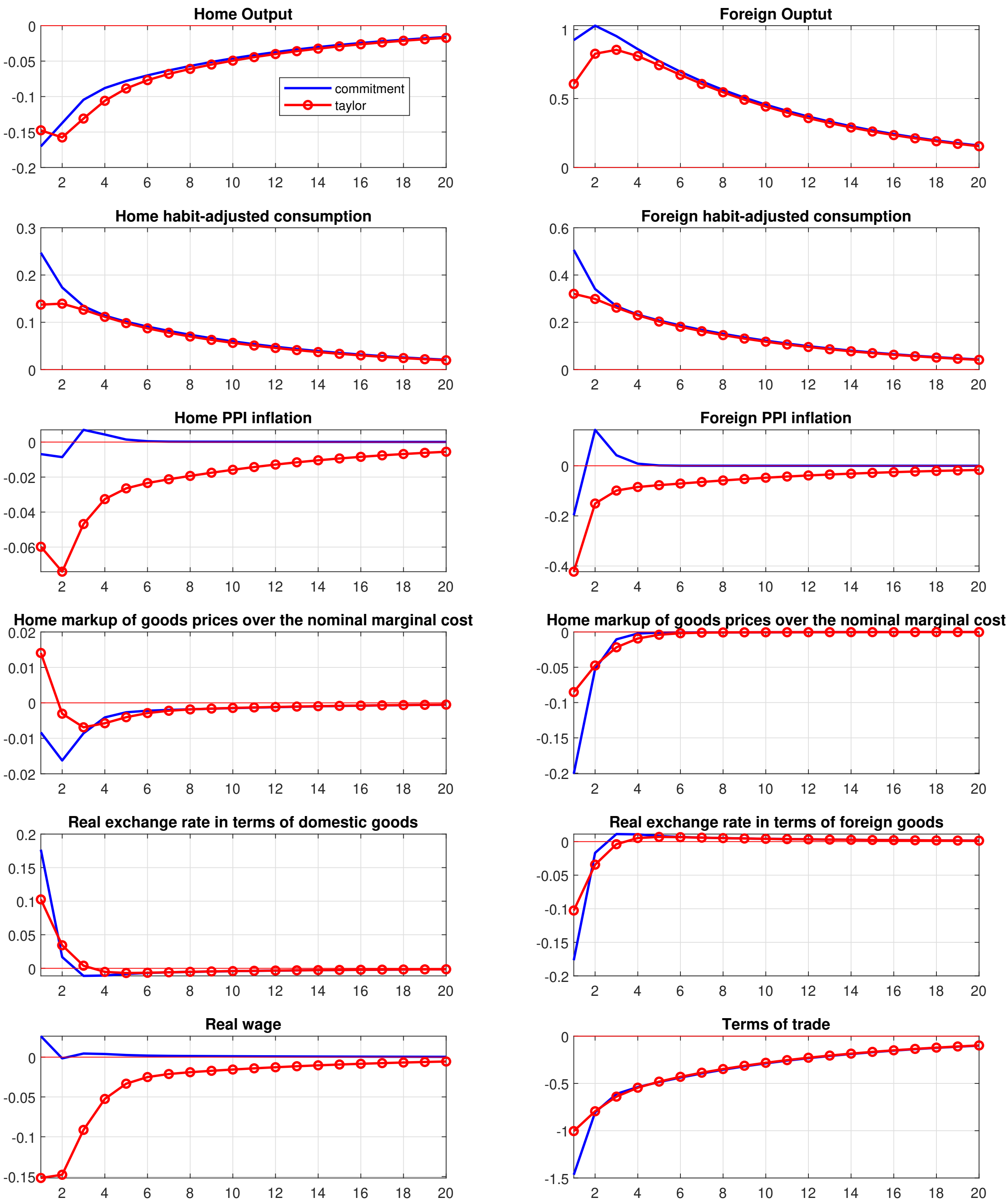

Figure 3: Impulse responses of a foreign productivity shock: Commitment v.s. Taylor rule

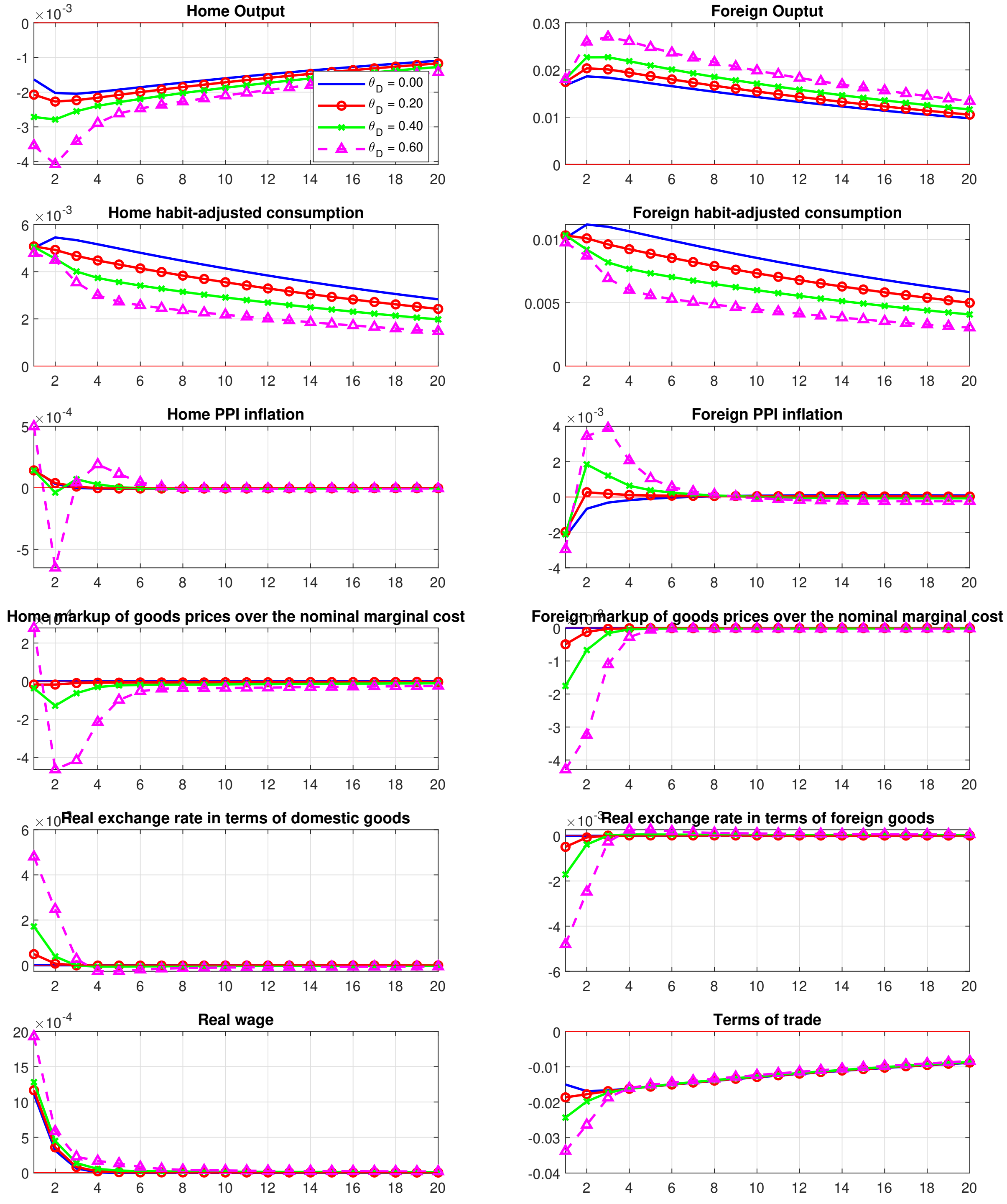

Figure 4: Impulse responses of a foreign markup shock

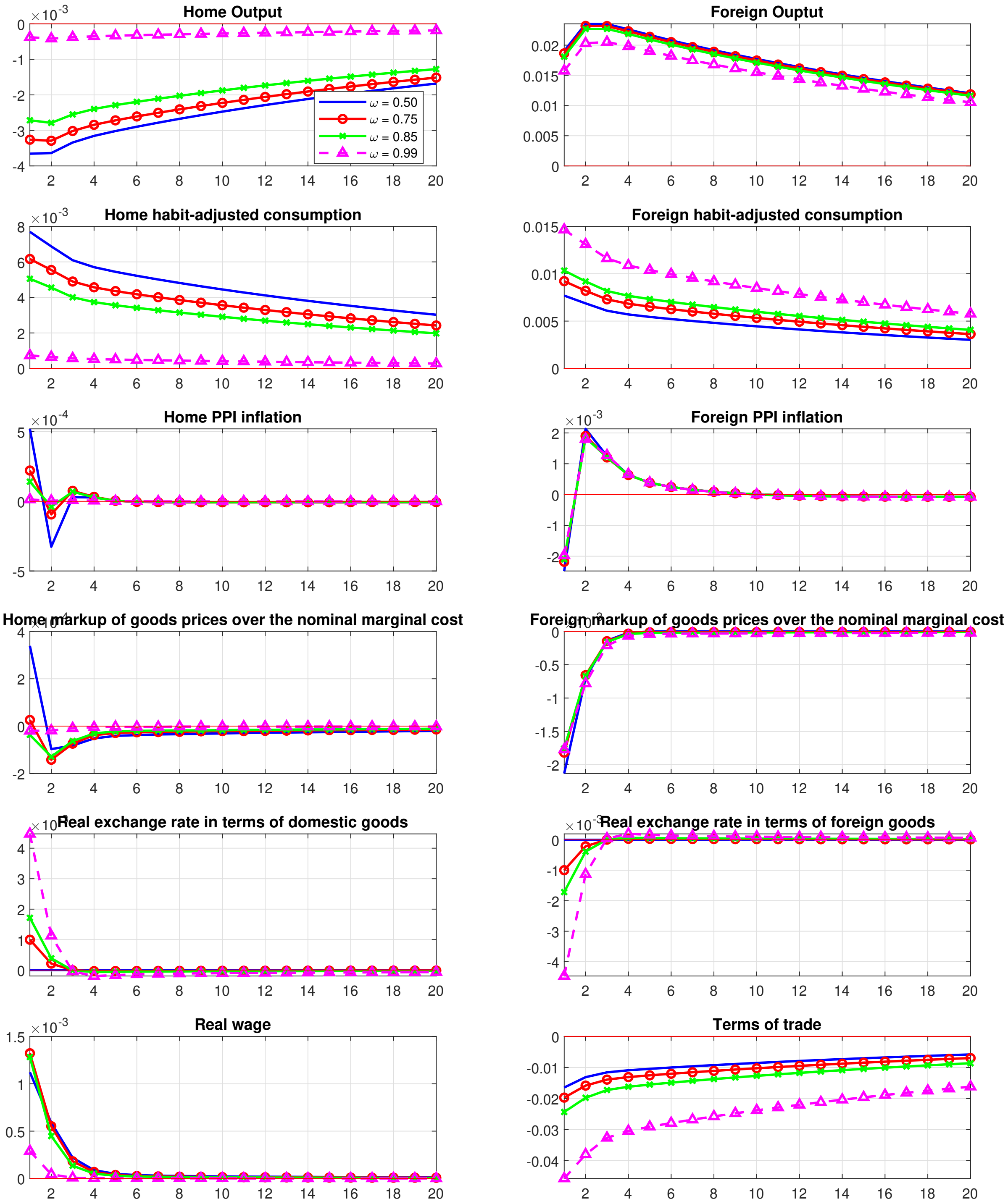

Figure 5: Impulse responses of a foreign markup shock with respect to changing the degree of home bias

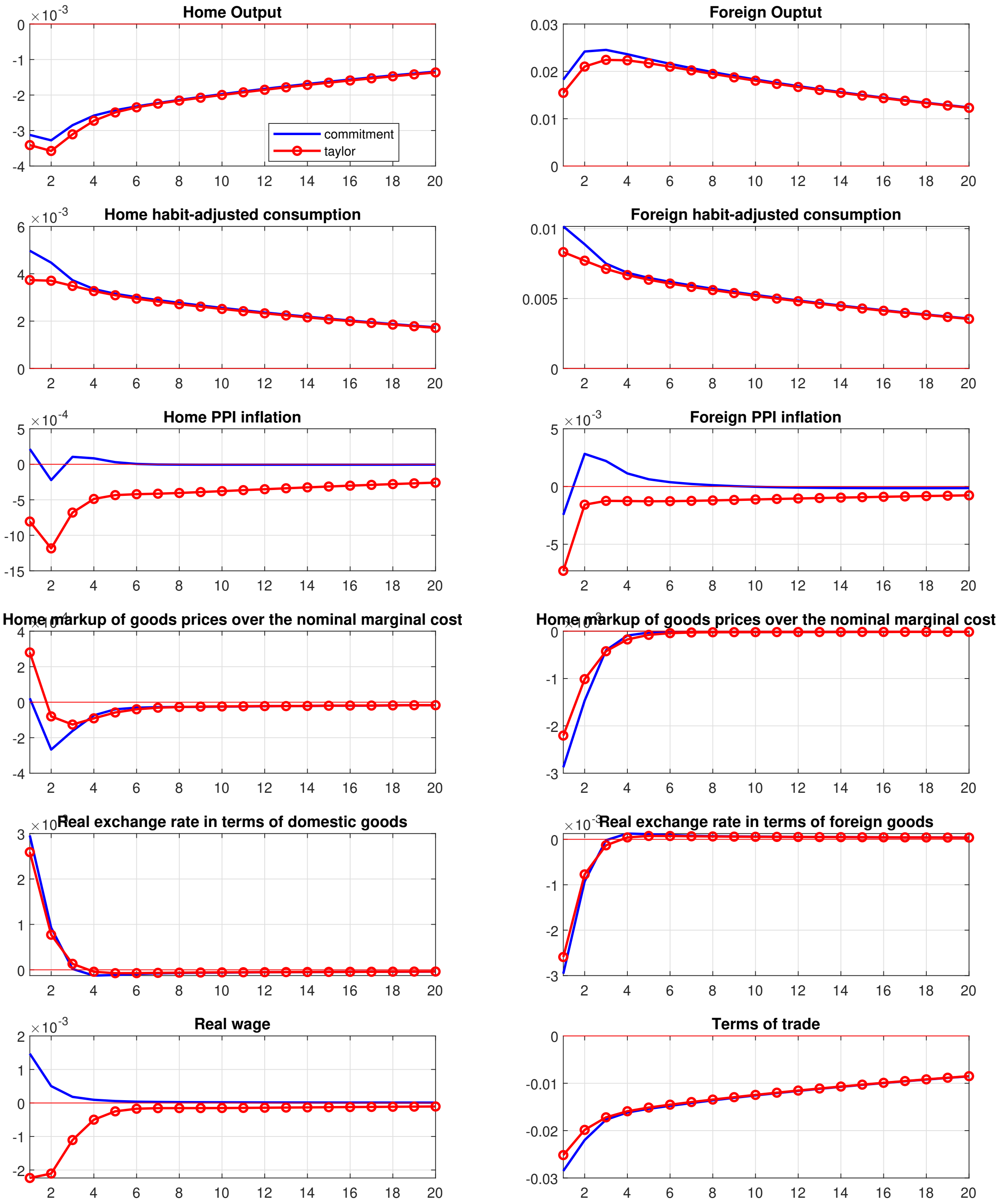

Figure 6: Impulse responses of a foreign markup shock: Commitment v.s. Taylor rule