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Forecasting Inflation and Output Growth with Credit-Card-Augmented Divisia Monetary Aggregates

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This paper investigates the performance of the Credit-Card-Augmented Divisia monetary aggregates in forecasting U.S. inflation and output growth at the 12-month horizon. We compute recursive and rolling out-of-sample forecasts using an Autoregressive Distributed Lag (ADL) model based on Divisia monetary aggregates. We use the three available versions of those monetary aggregate indices, including the original Divisia aggregates, the credit card-augmented Divisia, and the credit-card-augmented Divisia inside money aggregates. The source of each is the Center for Financial Stability (CFS). We find that the smallest Root Mean Square Forecast Errors (RMSFE) are attained with the credit-card-augmented Divisia indices used as the forecast indicators. We also consider Bayesian vector autoregression (BVAR) for forecasting annual inflation and output growth.

Keyword: Divisia, Credit-Card-Augmented Divisia, Monetary Aggregates, Forecasting, Bayesian vector autoregression, Inflation, Output Growth.

JEL Codes: C32, C53, E31, E47, E51

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1. Introduction

Credit card transactions have never been included in central bank measures of the money supply, since accounting conventions do not permit adding liabilities, such as credit card balances, to assets, such as money. But as credit card transactions have increased, the need to measure the contributions of credit cards to liquidity has grown. Economic aggregation theory permits aggregating over service flows, such as monetary services and credit card transaction volumes, regardless of whether the source of the services are assets or liabilities. The result, as maintained by the Center for Financial Stability (CFS) in New York City, is called the credit-card-augmented Divisia monetary aggregates.

Simple sum measures, which are not based on economic aggregation or index number theory, impute the same weight to each monetary asset in the aggregate, despite the fact that the services of different monetary assets are not perfect substitutes. Unlike the simple sum measure, Divisia monetary aggregates are directly derived from economic theory and impute user cost prices to the marginal utilities of component assets. The resulting component growth rate weights in measuring the growth rate of the Divisia index are the component expenditure shares with user cost pricing. As displayed in Figure 1 and Table 1, the share of cash and checks as payment instruments has been declining. The role of credit cards in transactions can no longer be ignored.

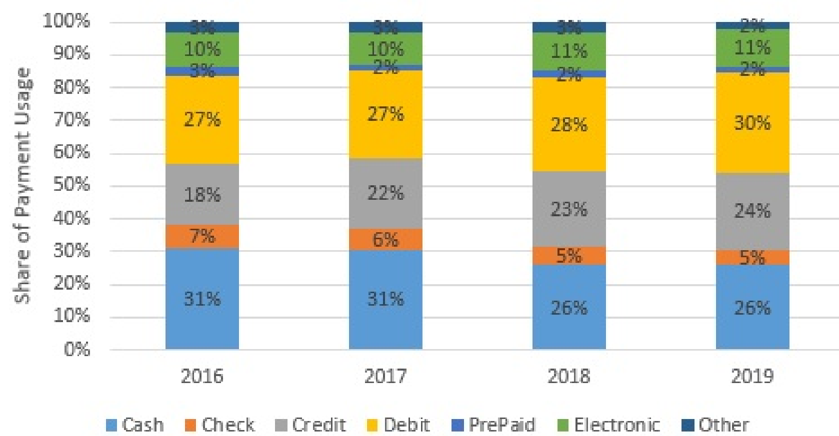

FIGURE 1. U.S. SHARES OF PAYMENT INSTRUMENTS
SOURCE: THE 2019 FEDERAL RESERVE PAYMENT STUDY.

Barnett and Su (2016), extending the well-known Divisia monetary aggregates originated by Barnett (1980), generalized the theory to include credit card transaction services on the demand side. They derive the theory needed to measure the joint services of credit cards and money. Their results are based on index number theory, which measures service flows, and are derived from economic aggregation theory. Barnett and Su (2018) derived the corresponding supply side theory needed to measure the production of the joint services of credit cards and inside money. The resulting supply side index is needed in estimation of the output supply function of banks and to measure value-added in banking. Financial firms' monetary production theory is thereby augmented to include credit card transaction services.

Table 1: Noncash Payment

	2012	2015	2018	2012-15	2015-18
	Value ($ trillions)	Value ($ trillions)	Value ($ trillions)	% change in value	% change in value
Total	78.01	86.78	97.04	3.6	3.8
Credit Cards	2.55	3.05	3.98	**6.2**	**9.3**
Debit Cards	2.10	2.47	3.10	5.5	7.8

Source: The 2019 Federal Reserve Payments Study, https://www.federalreserve.gov/paymentsystems/2019-December-The-Federal-Reserve-Payments-Study.htm

Using the conventional Divisia monetary aggregates, Barnett, Offenbacher, and Spindt (1984) explored the forecast properties of the well-known, linear, fixed-coefficient money demand functions. Barnett, Chauvet, and Leiva-Leon (2016) investigate the performance of univariate and multivariate, linear and nonlinear econometric models in nowcasting nominal GDP growth in real-time with conventional Divisia monetary aggregates includes as possible indictors. Their model, containing information on real economic activity, inflation, interest rates, and Divisia monetary aggregates, is found to be the most accurate in producing real-time nowcasts of nominal GDP growth. Barnett, Chauvet, Leiva-Leon, and Su (2016) consider the use of credit-card-augmented Divisia monetary aggregates as indicators in nowcasting nominal GDP growth. Nowcasting is the prediction of present values, not yet directly measured, such as monthly GDP, which is directly measured only quarterly. The purpose of our paper is to evaluate the performance of credit-card-augmented Divisia monetary aggregates as indicators in predicting future economic variables, rather than current variables. We use the methods of forecasting rather than nowcasting to investigate prediction.

Schunk (2001) compares the forecasting performance of the conventional Divisia monetary aggregates versus the simple sum monetary aggregates and shows that forecasts of U.S. real GDP with a four-variable vector autoregression are most accurate, when including Divisia monetary aggregates.[1] Ellington (2018) assesses the relative empirical benefits of Divisia monetary aggregates by fitting time-varying coefficient VAR models. That paper finds that there is a strong link between Divisia money and economic activity over the business cycle, but the relationship is substantially less prominent with simple sum aggregates. That paper also finds that out of sample forecasts of economic activity from models using Divisia aggregates surpass those using simple sum measures.

We investigate credit-card-augmented Divisia monetary aggregates as predictors of inflation and output growth. We compare the results of forecasting inflation and output growth using these three monetary aggregates: the original Divisia monetary aggregates, the demand-side credit-card-augmented Divisia monetary aggregates, and the supply-side credit-card-augmented Divisia inside money aggregates. All three types of Divisia monetary aggregates are available from the Center for Financial Stability (CFS).

Stock and Watson (1999) investigated forecasts of U.S. inflation at the 12-month horizon, and examined the conventional unemployment rate Phillips curve in a simulated out-of-sample framework. They found that forecasting inflation by the Phillips curve was more accurate than with other macroeconomic variables as indicators, such as interest rates, money, and commodity prices. Stock and Watson (2003) forecasted output and inflation using asset prices and other leading indicators for seven OECD countries. They calculated out-of-sample mean square forecast errors. Rossi and Sekhposyan (2010) empirically analyzed the ability of various economic models in predicting both future industrial production growth and inflation.

[1] The four variables in Schunk (2001) are real GDP, GDP deflator, nominal 6-month treasury bill rate, and monetary aggregates.

We also consider Bayesian vector autoregression. Chin and Li (2019) explored BVAR and evaluated the performance in individual and combination forecasts. They studied whether the inclusion of prior economic and/or non-economic information can improve the forecasting performance of VAR and found that using prior information produces more accurate forecasts.

Our paper is organized as follows. Section 2 discusses the regular Divisia monetary aggregates, the credit-card-augmented Divisia, and credit-card-augmented Divisia inside money aggregates. Section 3 explains the out-of-sample method used in this paper for forecasting inflation and output growth. Section 4 describes the data used for forecasting, and the results of the model are investigated in Section 5. Section 6 explores forecasting by Bayesian vector autoregression. We conclude in Section 7.

2. Divisia Monetary Aggregates

Divisia monetary aggregate growth rates are the share weighted average of component growth rates, with user cost pricing used in computing the expenditure shares. This paper considers three types of Divisia monetary aggregates: the original Divisia, the credit-card-augmented Divisia, and the credit card-augmented Divisia inside money aggregates. We define each in this section.

2.1. Original Divisia Monetary Aggregates

The original Divisia monetary aggregates measure demand-side monetary services, using the economic aggregation and index number theory developed by Barnett (1980). Barnett, Offenbacher, and Spindt (1984) describe the details of the production and use of those aggregates and provide initial empirical results. Barnett (1980) proposes the use of either the Divisia or Fisher ideal index for monetary quantity aggregation with user cost pricing of components. The difference between the two indexes is negligible, being less than the roundoff error in the components, but the Divisia index is easier to explain and interpret than the Fisher ideal index. The resulting quantity index numbers, by either formula, are elements of Diewert's class of superlative quantity index numbers. Barnett's resulting monetary aggregates are strictly preferable to the simple sum monetary aggregates, since the component monetary assets are not perfect substitutes. Relative to aggregation and index number theory, simple sum aggregation over imperfect substitutes is inadmissible.

Barnett proved that the real user cost (equivalent rental price) of monetary asset *i* is

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t}, \tag{1}$$

where r_{it} is the own rate of return on asset i during period t, and R_t is the risk-free rate of return on pure capital, a completely illiquid asset (benchmark rate), during period t.

The Divisia index in growth rate form in continuous time is

$$d \log M(\mathbf{m}_t) = \sum_i s_{it} d \log m_{it}, \tag{2}$$

where $s_{it} = \pi_{it} m_{it} / \boldsymbol{\pi}_t' \boldsymbol{m}_t$ is the expenditure share on monetary asset i and m_{it} is real balances of monetary asset i during period t.

2.2. Credit Card-Augmented Divisia and Inside Money Divisia

Credit-card-augmented Divisia money is the measure of demand-side monetary services, including credit card transaction volumes. Barnett and Su (2016) and Barnett, Chauvet, Leiva-Leon, and Su (2016) introduced the credit card services augmented Divisia monetary aggregates. Conventional simple sum monetary aggregates have never been augmented to include credit card balances, since accounting conventions do not allow adding liabilities to assets. Credit-card-augmented Divisia monetary aggregates are derived from the consumer decision with credit card transaction volumes along with monetary balances entered into the utility function, reflecting the fact that money and credit card transaction volumes provide services, such as liquidity and transactions services. Economic aggregation and index number theory measure service flow, independently of whether from assets or liabilities.

The real user cost of credit card services, derived in Barnett and Su (2016), is:

$$\tilde{\pi}_{jt} = \frac{e_{jt} - R_t}{1 + R_t} \ , \tag{3}$$

where R_t is the risk-free rate of return on a completely illiquid benchmark asset (pure capital) during period t, and e_{jt} is the interest rate on credit card type *j*. There are two categories of consumers using credit cards. Some consumers pay interest to the credit card issuing banks, and the others do not pay interest. The "representative consumer" pays the interest rate e_{jt}, averaged over both categories of consumers, including those who maintain rotating balances, and thereby pay interest on contemporaneous credit card transactions, and those consumers who pay off such credit card transactions before the end of the period, and thereby do not pay explicit interest on credit card transactions. The CFS considers four credit card types, j, including Visa, MasterCard, Discover, and American Express.

The aggregation-theoretic exact approach defines the credit-card-augmented structural aggregator function, M, to be the utility function, v,

$$M_t = M\left(\boldsymbol{m}_t, \boldsymbol{c}_t\right) = v\left(\boldsymbol{m}_t, \boldsymbol{c}_t\right), \tag{4}$$

nested as a weakly separable category utility function within the full utility function, which also contains consumer goods. The growth rate of the Divisia index tracks the growth rate of the exact aggregate $M\left(\boldsymbol{m}_t, \boldsymbol{c}_t\right)$, where $\boldsymbol{m}_t$ is the vector of monetary asset quantities held by the representative consumer during period *t*, and $\boldsymbol{c}_t$ is the vector of the consumer's four credit card transaction volumes during period *t*.

The credit card quantities to include in the augmented Divisia index formula are the monthly credit card transactions volumes, not the credit card balances. Including credit card balances would produce double counting, since they include carried forward rotating balances used for transactions in prior periods. The growth rate of the credit-card-augmented Divisia index is

$$d\ \log M\left(\boldsymbol{m}_t, \boldsymbol{c}_t\right) = \sum_i s_{it} d\ \log m_{it} + \sum_j \tilde{s}_{jt} d\ \log c_{jt}\,, \tag{5}$$

where $s_{it} = \pi_{it} m_{it} / \left(\boldsymbol{\pi}'_t \boldsymbol{m}_t + \tilde{\boldsymbol{\pi}}'_t \boldsymbol{c}_t \right)$ is the expenditure share of monetary asset i in the total service of monetary assets and credit cards, while $\tilde{s}_{jt} = \tilde{\pi}_{jt} c_{jt} / \left(\boldsymbol{\pi}'_t \boldsymbol{m}_t + \tilde{\boldsymbol{\pi}}'_t \boldsymbol{c}_t \right)$ is the expenditure share of the services of credit card type j in the total services of monetary assets and credit cards.

Barnett and Su (2018) employ a production model of financial firms and investigate supply-side aggregation, when financial firms produce both monetary services and credit card transactions services. Inside money is a supply-side concept. Barnett and Su derive the conditions under which a joint supply-side aggregate over monetary and credit card transactions services exists. The existence condition for a supply-side inside-money aggregator function is blockwise weak separability of outputs within financial firm technology, while for a demand-side aggregator function the existence condition is blockwise weak separability within consumer tastes.

The user cost formula for supplied monetary assets differs from the demand-side user cost formula, when a regulatory wedge results from the existence of required reserve, imposing an implicit tax on banks. Until recently, reserve requirements existed and produced an especially large implicit tax during the late 1970s, when interest rates on bank loans were high. Barnett (1987) and Liu, Dery, and Serletis (2019) show that the user cost of supplied monetary assets subject to reserve requirements is

$$\pi_{it} = \frac{\left(1 - k_i\right) R_t - r_{it}}{1 + R_t}, \tag{6}$$

where k_i is the required reserve ratio on monetary asset i, r_{it} is the interest rate paid by the bank on those deposits, and R_t is the bank's rate of return on invested loans. The implicit tax is the foregone interest on uninvested required reserves. The credit-card-augmented Divisia inside money index is calculated from equation (5), using the monetary asset user cost from equation (6).

3. Out-of-Sample Forecasting Method

This section provides our forecasting models. This paper forecasts inflation and real GDP growth at the h-period horizon and focuses on the forecasting model used in Stock and Watson (1999, 2003) and Rossi and Sekhposyan (2009). We test the estimated model's ability to forecast in out-of-sample data. That is, some data are dropped from the estimation sample to see how the model forecasts in the region of the data omitted from the estimation. The model is defined as

$$Y^{h}_{t+h} = \beta_0 + \beta_1(L)X_t + \beta_2(L)Y_t + e^{h}_{t+h} \,, \tag{7}$$

where Y^{h}_{t+h} is either the h-period inflation at time t, defined by $Y^{h}_{t+h} = (1200/h)\ln(CPI_{t+h}/CPI_t) - 1200\ln(CPI_t/CPI_{t-1})$ or the cumulative growth of output over the h-periods at an annual percentage rate, defined by $Y^{h}_{t+h} = (1200/h)\ln(Q_{t+h}/Q_t)$, where X_t is a possible explanatory variable. Output during period *t* is Q_t, and the consumer price index during period t is CPI_t. The variable Y_t on the right hand side of (7) is either the first difference of the period t rate of inflation, $Y_t = 1200\ln(CPI_t/CPI_{t-1}) - 1200\ln(CPI_{t-1}/CPI_{t-2})$, or the period t output growth, $Y_t = 1200\ln(Q_t/Q_{t-1})$, while e^{h}_{t+h} is an error term. The lag polynomials in the lag operator L are $\beta_1(L)$ and $\beta_2(L)$.

We use Autoregressive Distributed Lag (ADL) time series regression in accordance with equation (7). In particular, we estimate the following model

$$Y^{h}_{t+h} = \alpha + \sum_{i=0}^{p}\beta_i Y_{t-i} + \sum_{j=0}^{r}\gamma_j D_{t-j} + e_{t+h} \,, \tag{8}$$

where Y^{h}_{t+h} is the same as in equation (7) and Y_{t-i} for *i* = 0, . . . , p is either p lags of the change in monthly inflation or p lags of the monthly output growth, and D_{t-j}, *i* = 0, . . . , r, is r lags of the change in log difference of Divisia monetary aggregates at time t. We set *h* = 12 to model one year ahead inflation change and output growth.

We estimate equation (8) using in-sample data and recursively forecast inflation and output growth in the out-of-sample period. We forecast using only data before the forecast period in the parameter estimation. The model is estimated by two different methods for out-of-sample forecasting. Figures 2 and 3 show these two forecasting methods. By the first method, we use recursive regression, setting the forecasting window three months ahead to forecast the observation at $t = 4$. We sequentially expand the forecasting window by one month until we have a forecast for all out-of-sample observations. For example, in 2006:09 we consider the forecast of the 12-month inflation rate and output growth from 2006:09 to 2007:09. Then moving forward one month, all the models are re-estimated using all prior data through 2006:10, and the forecast of inflation and output growth is computed from 2006:10 to 2007:10. The data period used in this paper is from 2006:07 to 2020:07, as is discussed in section 4. By using this method recursively, the period over which out-of-sample forecasts are computed is 2007:12 through 2020:07.

The second method to estimate the model is rolling regression using fixed windows of data to re-estimate the model over the periods. We fix the forecasting window at 60, 84, and 120 months. By using those three fixed windows, the periods of out-of-sample forecasts are 2012:09 through 2020:07, 2014:09 through 2020:07, and 2017:09 through 2020:07, respectively.

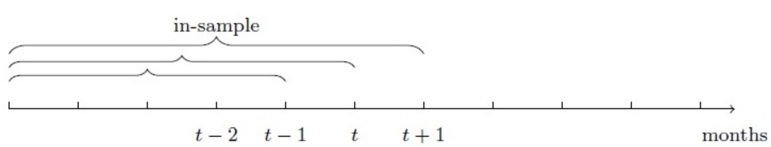

FIGURE 2 RECURSIVE REGRESSION (EXPANDING FORECASTING WINDOW)

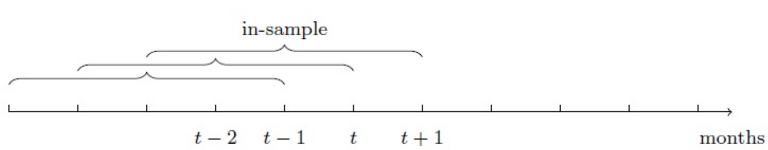

FIGURE 3 ROLLING REGRESSION (FIXED FORECASTING WINDOW)

We calculate the forecast error and the root mean square forecast error (RMSFE) for each estimation to measure forecast performance. The forecast error is defined to be

$$\text{Forecast Error} = Y_{t+1} - \hat{Y}_{t+1|t}, \tag{9}$$

where Y_{t+1} is the value from the actual data not included in the estimation of coefficients in the regression and $\hat{Y}_{t+1|t}$ is a forecast of Y_{t+1} based on the data in the past with the coefficients estimated using data through period t. The RMSFE of the recursive regression is

$$RMSFE_{recursive} = \sqrt{\frac{1}{T-3}\sum_{j=3}^{T} E\left[\left(Y_{j+1} - \hat{Y}_{j+1|j}\right)^2\right]}\ , \tag{10}$$

and the RMSFE of the rolling regression is

$$RMSFE_{rolling} = \sqrt{\frac{1}{T-N}\sum_{j=N}^{T} E\left[\left(Y_{j+1} - \hat{Y}_{j+1|j}\right)^2\right]}\ , \tag{11}$$

where N is the size of the forecasting window, set to be 64, 84, or 120, respectively. To measure the spread of the forecast error distribution and magnitude of a typical forecasting error, we compute RMSFE for all measures of aggregate monetary services. Comparing RMSFE, we can determine whether credit-card-augmented Divisia monetary aggregates perform better as indicators in forecasting than the original Divisia monetary aggregates.

4. Data

To evaluate the credit-card-augmented Divisia monetary aggregates as indicators in forecasting, we forecast inflation and output growth using three different Divisia monetary aggregates: (1) the original Divisia monetary aggregates, (2) the credit-card-augmented Divisia monetary aggregates, and (3) the credit-card-augmented Divisia inside money aggregates.

The three types of Divisia monetary aggregates are available on a monthly basis from the Center for Financial Stability (CFS). The CFS provides Divisia monetary aggregates to the public from 1967:01 along with credit-card-augmented Divisia monetary services and credit-card-augmented Divisia inside-money services from 2006:07. The CFS original Divisia data are level normalized to equal 100 in 1967:01. The credit-card-augmented Divisia money and credit card-augmented Divisia inside money data are level normalized to equal 100 in 2006:7.

Inflation is measured using the seasonally adjusted Consumer Price Index (CPI) with base year of 1984, as reported monthly by the Bureau of Labor Statistics (BLS). For output growth, we use monthly real GDP, as provided by IHS Markit.[2] The period of our data is 2006:07-2020:07 for all variables, since the credit-card-augmented Divisia data are provided from 2006.7. Table A.1 in the Appendix contains a description of our data.

Stock and Watson (1999) include M1, M2, M3, and L (now M4) as explanatory variables. Rossi and Sekhposyan (2010), who include M2 and M3 as indicators in forecasting US output growth and inflation, use an identical model to Stock and Watson (2003), but with different data. We use the M1, MZM, M2, and ALL clusterings of components as narrow money measures and the M3, M4-, and M4 component groupings in the broad money measures for all three Divisia indices. The groupings are as defined by the CFS.

M1 includes currency, demand deposits, travelers checks, and other checkable deposits. M2 includes the components of M1 along with savings deposits, money market accounts, retail money market mutual funds, and small time deposits. MZM stands for Money Zero Maturity. It includes the components of M1 along with the components of M2, other than time deposits, and also money market funds. ALL adds institutional money market funds to the components of M2. M3 is a broad concept of the money, including the components of M2 along with large time deposits, institutional money market funds, and term Eurodollars.[3] M4-

[2] See https://ihsmarkit.com/products/us-monthly-gdp-index.html. First, IHS Markit derives a raw index from various monthly data, mostly source data of the official quarterly GDP of the Bureau of Economic Analysis (BEA). Then they calculate a monthly residual that reconciles the raw index with official GDP at the quarterly frequency. While nowcasting methods provide alternative measures of monthly GDP, the IHS Market measure is the closest to a direct measure. The BEA quarterly GDP data are currently the only available fully directly measured GDP data.

[3] See Barnett, Fisher, and Serletis (1992) and Barnett, Liu, and Mattson (2013).

adds commercial paper to the components of M3, while M4 adds Treasury Bills to the components of M4-. Table A.2 in the Appendix fully defines the component groupings.

5. Empirical Analysis

5.1. Stationarity

Before forecasting, we conduct a unit root test to check whether all variables are stationary. We use the Augmented Dickey-Fuller (ADF) test, which allows for higher order autoregressive processes, from the following equation:

$$\Delta y_t = \mu + \delta t + \phi y_{t-1} + \sum_{i=0}^{k} \theta_i \Delta y_{t-k} + \varepsilon_t . \quad (12)$$

The null hypothesis is that variables are non-stationary, so that $\phi = 1$. We reject the null hypothesis at the 5% significance level, if a *p*-value is less than 0.05. Then the data have no unit root and are stationary. The test statistic for this test is

$$\tau = \frac{\phi - 1}{SE(\phi)}, \quad (13)$$

where $SE(\phi)$ is the standard error of ϕ in equation (12). In the Appendix, Table A.2 shows the results of the Augmented Dickey-Fuller test. All variables are found to be stationary in the forecasting model.

5.2. Bayesian Information Criterion (BIC)

Before we can forecast inflation and output growth, we must choose the number of lags, p and r, in equation (8). We determine the number of lags by the Bayesian Information Criterion (BIC). BIC is a criterion for model selection among a finite set of models and widely used for model identification in time series and linear regression.

BIC is defined as

$$BIC = k \ln(n) - 2\ln(L), \quad (14)$$

where k is the number of parameters in the model, n is the number of observations, and L is the maximized value of the model's likelihood function. Using that criterion, we find $p = 0$ and $r = 0$ in the model, which thereby is ADL(0,0) for both inflation and output growth. Then the right-hand side of the equation (8) has Y_t and D_t with no lags. The model which includes only the information immediately before time t is best at 12-month-ahead forecasting of both inflation and output growth.

5.3. Forecasting Results

We forecast annual US inflation and output growth using the original Divisia, the credit-card-augmented Divisia, and the credit-card-augmented Divisia inside money aggregates, as provided by the Center for Financial Stability (CFS) at M1, M2, MZM, ALL, M3, M4-, and M4 levels of aggregation. We compute Root Mean Square Forecast Error (RMSFE) to evaluate the forecasts. We now provide the results of the forecasts.

5.3.1. Inflation Forecasting

Using 3 months in-sample data at first and expanding the forecasting window by 1 month for recursive regression, we estimate equation (8) by multivariate linear regression in samples of 155 observations (T=155). We then recursively forecast the change in inflation and output growth in the out-of-sample periods. Hence, our first 12-months-ahead out-of-sample forecast is computed at 2006:12. The data set starts at 2006:07. We lose the first two observations, because we generate new variables by taking differences and using lags. Also, since the first regression uses 3 months of observations, forecasts of those 3 months will not be calculated. Table 2 shows the results of RMSFE in the case of the recursive regression.

Table 2: Forecasting Inflation (recursive regression): Root Mean Square Forecast Error

		Full sample	Great Recession	After Great Recession
M1	Divisia	3.8607	5.0661	3.7047
	CA-Divisia	3.8169	5.2434	3.6140
	CA-Divisia Inside	3.6567	4.9477	3.4773
M2	Divisia	3.8330	5.4783	3.5741
	CA-Divisia	3.8070	5.5062	3.5367
	CA-Divisia Inside	3.7834	5.4839	3.5119
MZM	Divisia	3.7828	5.3862	3.5310
	CA-Divisia	3.7598	5.4196	3.4960
	CA-Divisia Inside	3.7304	5.3902	3.4652
ALL	Divisia	3.8243	5.4984	3.5538
	CA-Divisia	3.8065	5.5207	3.5284
	CA-Divisia Inside	3.7853	5.5034	3.5050
M3	Divisia	3.6903	5.8382	3.3143
	CA-Divisia	3.6902	5.8328	3.3172
	CA-Divisia Inside	3.6920	5.8495	3.3160
M4-	Divisia	3.6491	5.6831	3.2991
	CA-Divisia	3.6482	5.6806	3.3001
	CA-Divisia Inside	3.6492	5.6916	3.2993
M4	Divisia	3.7002	5.7348	3.3514
	CA-Divisia	3.7017	5.7326	3.3550

Note. Full sample: 2007.12-2020.2. Great Recession: 2007.12-2009.6. After Great Recession: 2009.7-2020.2.

When we consider M1 as a monetary indicator, RMSFE of the original Divisia is 3.8607, of the credit-card-augmented Divisia is 3.8169, and of the credit-card-augmented Divisia inside money aggregate is 3.6567. The RMSFE for M2 of the original Divisia is 3.8330, of the credit-card-augmented Divisia is 3.8070, and of the credit-card-augmented Divisia inside money aggregate is 3.7834. The RMSFE for MZM of original Divisia is 3.7828, of the credit-card-augmented Divisia is 3.7598, and of the credit-card-augmented Divisia inside money aggregate is 3.7304. The RMSFE for the ALL level of aggregation of the original Divisia is 3.8243, of the credit-card-augmented Divisia is 3.8065, and of the credit-card-augmented Divisia inside money aggregate is 3.7853. For each of the narrow money levels of aggregation, M1, M2, MZM, and ALL, the credit-card-augmented Divisia aggregate produces smaller forecast errors than the original Divisia, while the credit-card-augmented Divisia inside money aggregate performs best in forecasting inflation.

The following are the result when the model uses M3 as a monetary indicator for forecasting inflation. The RMSFE of the original Divisia is 3.6903, of the credit-card-augmented Divisia is 3.6902, and of the credit-card-augmented Divisia inside money aggregate is 3.6920. For M4-, the RMSFE of the original Divisia is 3.6491, of the credit-card-augmented Divisia is 3.6482, and of the credit-card-augmented Divisia inside money aggregate is 3.6492. The RMSFE of M4 for original Divisia is 3.7002 and of the credit-card-augmented Divisia is 3.7017. Credit-card-augmented Divisia inside money does not exist for M4 separately from M4-, since M4 is defined to be M4- plus T-Bills. But credit-card-augmented Divisia inside money must exclude T-Bills, which are not produced outputs of bank services. Hence, credit-card-augmented Divisia inside money for M4 is identical to credit-card-augmented Divisia for M4-.

For broad money, we find little difference in RMSFEs among the three Divisia indices. Unlike narrow money, RMSFEs of credit-card-augmented Divisia inside money for M3, and M4- are not the smallest among the three Divisia indices. The demand side credit-card-augmented Divisia aggregates tend to perform best with the broad aggregates. Also, we find that RMSFEs of the broad aggregates are smaller than those of the narrow aggregates. Overall, the M4-level of aggregation performs best for forecasting inflation, especially when the monetary aggregate is credit-card-augmented. Similarly, Barnett, Offenbacher, and Spindt (1984), with much earlier data, found that the most stable demand-for-money functions were acquired with the broadest Divisia monetary aggregates, although credit-card-augmented aggregates were not known at that time.

Our other forecasting method is rolling regression with a fixed forecasting window. Table 3 reports the rolling regression results for forecasting annual inflation. We find that RMSFEs with all types of money are smaller than with recursive regression. When the forecasting window is fixed, credit-card-augmented Divisia inside money works better than the other two Divisia indices for narrow money, but not for the broad money. The larger the forecasting window, the smaller the forecasting error. To be specific, when the forecasting window is 120 months (N=120), the M4- component grouping is the best for forecasting inflation with both rolling regression and recursive regression.

Table 3: Forecasting Inflation (rolling regression): Root Mean Square Forecast Error

Forecasting window (months)		60	84	120
M1	Divisia	2.4742	2.3209	2.0679
	CA-Divisia	2.4282	2.3120	2.0404
	CA-Divisia Inside	2.4066	2.3051	2.0348
M2	Divisia	2.4815	2.3333	2.0565
	CA-Divisia	2.4548	2.3258	2.0438
	CA-Divisia Inside	2.4488	2.3229	2.0414
MZM	Divisia	2.5006	2.3195	2.0688
	CA-Divisia	2.4795	2.3165	2.0558
	CA-Divisia Inside	2.4751	2.3127	2.0541
ALL	Divisia	2.5245	2.3180	2.0917
	CA-Divisia	2.5066	2.3169	2.0774
	CA-Divisia Inside	2.5033	2.3116	2.0755
M3	Divisia	2.3943	2.3820	2.0188
	CA-Divisia	2.4089	2.4003	2.0152
	CA-Divisia Inside	2.4069	2.4115	2.0160
M4-	Divisia	2.3824	2.3629	2.0127
	CA-Divisia	2.3909	2.3747	2.0057
	CA-Divisia Inside	2.3877	2.3831	2.0049
M4	Divisia	2.3672	2.3374	2.0299
	CA-Divisia	2.3743	2.3452	2.0270

In particular, we find that M4- and M4 work best for forecasting inflation by the recursive regression and rolling regression, respectively. Although the results in this paper show the importance of broad money in the prediction of inflation, the Federal Reserve no longer publishes M3, M4-, and M4 data, but the Center for Financial Stability does.

5.3.2. Output Growth Forecasting

Table 4 contains the RMSFE results with recursive regression for forecasting output growth. The RMSFE of M1 is 2.5595 for the original Divisia, 2.5039 for the credit-card-augmented Divisia, and 2.5519 for credit-card-augmented Divisia inside money aggregate. For M2, the RMSFE of the original Divisia is 2.5393, of the credit-card-augmented Divisia is 2.5028, and of the credit-card-augmented Divisia inside money aggregate is 2.5299. The RMSFE of the original Divisia for MZM is 2.6219, of the credit-card-augmented Divisia is 2.5907, and of the credit-card-augmented Divisia inside money aggregate is 2.6194. The RMSFE of the ALL component grouping for the original Divisia is 2.6771, for the credit-card-augmented Divisia is 2.6461, and for the credit-card-augmented Divisia inside money aggregate is 2.6730. When we consider the broad M3 as a monetary indicator for forecasting output growth, the RMSFE of the original Divisia is 2.7523, for the credit-card-augmented Divisia is 2.7357, and for the credit-card-augmented Divisia inside money aggregate is 2.7510. For M4-, the RMSFE of the original Divisia is 2.7576, for the credit-card-augmented Divisia is 2.7448, and for the credit-card-augmented inside money aggregate is 2.7563. The RMSFE of M4 for the original Divisia is 2.7761 and for the credit-card-augmented Divisia is 2.7682.

Table 4: Forecasting Output Growth (recursive regression): Root Mean Square Forecast Error

		Full sample	Great Recession	After Great Recession
M1	Divisia	2.5595	2.4544	2.0181
	CA-Divisia	2.5039	2.5270	1.8894
	CA-Divisia Inside	2.5519	2.4096	2.0050
M2	Divisia	2.5393	2.5888	1.8876
	CA-Divisia	2.5028	2.6029	1.8048
	CA-Divisia Inside	2.5299	2.5994	1.8571
MZM	Divisia	2.6219	2.5190	2.0604
	CA-Divisia	2.5907	2.5353	1.9931
	CA-Divisia Inside	2.6194	2.5283	2.0474
ALL	Divisia	2.6771	2.5691	2.1254
	CA-Divisia	2.6461	2.5817	2.0583
	CA-Divisia Inside	2.6730	2.5770	2.1093
M3	Divisia	2.7523	2.6267	2.2512
	CA-Divisia	2.7357	2.6306	2.2106
	CA-Divisia Inside	2.7510	2.6305	2.2447
M4-	Divisia	2.7576	2.6356	2.2662
	CA-Divisia	2.7448	2.6390	2.2319
	CA-Divisia Inside	2.7563	2.6392	2.2597
M4	Divisia	2.7761	2.6177	2.3092
	CA-Divisia	2.7682	2.6221	2.2861

Note. Full sample: 2007.12-2020.2. Great Recession: 2007.12-2009.6. After Great Recession: 2009.7-2020.2.

The RMSFEs of each of the Divisia indices for M2 are the lowest among the seven types of money, and the narrow money performs better than the broad money. Also, the results show that for each of the component groupings, credit-card-augmented Divisia works best for forecasting output growth among all three Divisia indices.

When we compare the results of forecasting inflation and output growth during the Great Recession, the RMSFEs of inflation forecasting during the Great Recession are greater than during other periods. The Great Recession between 2007-2009 was associated with the financial crisis, which was deeply related to money.

Table 5 focuses on forecasting output growth using rolling regression. Contrary to the results when forecasting inflation, the smaller the forecasting window the smaller the forecasting errors. Consistent with the results from recursive regression, we see that the M2 component grouping works best for forecasting output growth with rolling regression.

Table 5: Forecasting Output Growth (rolling regression): Root Mean Square Forecast Error

Forecasting window (months)		60	84	120
M1	Divisia	2.6484	2.9967	3.7988
	CA-Divisia	2.5683	2.9278	3.7748
	CA-Divisia Inside	2.6377	2.9904	3.8031
M2	Divisia	2.5703	2.9767	3.8593
	CA-Divisia	2.5237	2.9336	3.8408
	CA-Divisia Inside	2.5581	2.9708	3.8617
MZM	Divisia	2.6212	3.0134	3.8916
	CA-Divisia	2.5796	2.9728	3.8743
	CA-Divisia Inside	2.6161	3.0117	3.8980
ALL	Divisia	2.7072	3.0894	3.9364
	CA-Divisia	2.6599	3.0420	3.9240
	CA-Divisia Inside	2.7017	3.0859	3.9439
M3	Divisia	2.9046	3.2254	3.9306
	CA-Divisia	2.8653	3.1955	3.9363
	CA-Divisia Inside	2.8990	3.2287	3.9487
M4-	Divisia	2.9499	3.2641	3.9165
	CA-Divisia	2.9160	3.2384	3.9205
	CA-Divisia Inside	2.9445	3.2684	3.9297
M4	Divisia	2.9418	3.2541	3.9261
	CA-Divisia	2.9160	3.2358	3.9332

6. Bayesian Vector Autoregression (BVAR) Approach

In this section, we examine the forecasting performance of credit-card-augmented Divisia money in another way. We estimate a Bayesian Vector Autoregression (BVAR) model for forecasting U.S. annual inflation and output growth.

6.1. Model Specification

Let $\boldsymbol{Y}_t$ be the vector containing the M variables at time *t* with the model having p lags of the variables as follows:

$$\boldsymbol{Y}_t = \boldsymbol{B}_0 + \boldsymbol{B}_1\boldsymbol{Y}_{t-1} + \boldsymbol{B}_2\boldsymbol{Y}_{t-2} + \cdots + \boldsymbol{B}_p\boldsymbol{Y}_{t-p} + \boldsymbol{\varepsilon}_t\,, \tag{15}$$

where $\boldsymbol{\varepsilon}_t \sim N\left(\mathbf{0},\boldsymbol{\Psi}\right)$, $\boldsymbol{B}_0$ is an $M \times 1$ vector of intercepts, $\boldsymbol{B}_1,\cdots,\boldsymbol{B}_p$ are $M \times M$ autoregressive matrices of coefficients, and $\boldsymbol{\Psi}$ is an $M \times M$ covariance matrix. In this application, we set $M = 3$. The three variables for forecasting inflation are the h-period inflation, the change in monthly inflation, and the change in the growth rate of a Divisia monetary aggregate, or alternatively for forecasting output growth the three variables are the h-period output growth, the monthly output growth, and the change in the growth rate of a Divisia monetary aggregate. We still set h = 12 in estimating the annual inflation and the annual output growth.

Equation (15) can equivalently be written as

$$\boldsymbol{y}_t = \boldsymbol{x}_t\boldsymbol{\beta} + \boldsymbol{\varepsilon}_t\,, \tag{16}$$

where $\boldsymbol{y}_t = \boldsymbol{Y}_t$, $\boldsymbol{x}_t = \boldsymbol{I}_M \otimes \left[\mathbf{1},\boldsymbol{Y}'_{t-1},\cdots,\boldsymbol{Y}'_{t-p}\right]$, $\boldsymbol{\beta} = vec\left(\left[\boldsymbol{B}_0,\boldsymbol{B}_1,\cdots,\boldsymbol{B}_p\right]\right)$, and $\boldsymbol{\varepsilon}_t \sim N\left(\mathbf{0},\boldsymbol{\Psi}\right)$.

In BVAR estimation, the choice of prior is central to the procedure. We adopt the Minnesota prior. Litterman (1980) introduced the Minnesota prior, which assumes that each variable follows a random walk process. The Minnesota prior then assumes that the covariance matrix is known and diagonal, $\hat{\boldsymbol{\Psi}} = \mathrm{diag}\left(\sigma_1{}^2,\cdots,\sigma_M{}^2\right)$, and its hyperparameters are fixed. Kadiyala and Karlsson (1997) investigate BVAR forecasting. They observe that the Minnesota prior is frequently used in forecasting applications under this prior distribution specification:

$$\boldsymbol{\beta} \mid \boldsymbol{y}, \boldsymbol{\Psi} \sim N\left(\overline{\boldsymbol{\beta}}, \overline{\boldsymbol{\Omega}}\right), \tag{17}$$

so that the prior distribution of $\boldsymbol{\beta}$ is normal with the prior mean $\overline{\boldsymbol{\beta}}$ and covariance matrix $\overline{\boldsymbol{\Omega}}$.

Given the prior distribution and the observations, the posterior distribution was produced numerically using the Gibbs sampling algorithm with 1000 iterations of the Gibbs sampler. As in the previous forecasting methods, we use the same period of the data, from 2006:07 to 2020:02, for all variables, since the credit-card-augmented Divisia indices are provided from 2006:07. In our BVAR estimation, we set 120 months as the in-sample period and forecast annual inflation and output growth for 35 months. Unlike in the previous forecasting methods, the optimal number of lags result in our estimation of the VAR model with 3 lags for forecasting inflation and 2 lags for forecasting output growth.

6.2. Results with BVAR

Table 6 displays Root Mean Squared Forecast Errors (RMSFEs) from the results of BVAR estimation in forecasting annual inflation and output growth. Conditionally on the prior specification over parameters, the credit-card-augmented Divisia performs better than the original Divisia in forecasting inflation with narrow money.

In the BVAR estimation, credit-card-augmented Divisia forecast output growth better than original Divisia with broad money, M3, M4-, and M4. However, all Divisia indices show very small differences in RMSFEs as indicators for output growth.

Table 6: BVAR Forecasting Inflation and Output Growth: Root Mean Square Forecast Error

		Inflation	Output Growth
M1	Divisia	2.1351	3.7814
	CA-Divisia	2.1163	3.7887
	CA-Divisia Inside	2.1363	3.7833
M2	Divisia	2.1690	3.7654
	CA-Divisia	2.1616	3.7674
	CA-Divisia Inside	2.1710	3.7688
MZM	Divisia	2.1166	3.7671
	CA-Divisia	2.1157	3.7708
	CA-Divisia Inside	2.1269	3.7713
ALL	Divisia	2.1333	3.7642
	CA-Divisia	2.1303	3.7671
	CA-Divisia Inside	2.1390	3.7689
M3	Divisia	2.2358	3.7776
	CA-Divisia	2.2361	3.7761
	CA-Divisia Inside	2.2358	3.7767
M4-	Divisia	2.2286	3.7788
	CA-Divisia	2.2290	3.7772
	CA-Divisia Inside	2.2264	3.7778
M4	Divisia	2.2528	3.7746
	CA-Divisia	2.2571	3.7743

7. Conclusion

Credit-card-augmented Divisia monetary aggregates provide valuable measures of monetary service flows in the economy. We assess the credit-card-augmented Divisia monetary aggregates, provided by the CFS, as indicators in forecasting annual US inflation and output growth. We estimate an Autoregressive Distributed Lag (ADL) model using in-sample data with an expanding forecast window and recursively predict the change in annual inflation and output growth in the out-of-sample period. The model is also estimated with a fixed forecast window as a robustness check. In each case, we evaluate the forecasts by calculating Root Mean Square Forecast Error (RMSFE).

We find that credit-card-augmented Divisia is a valuable indicator in predicting annual inflation. Moreover, the credit-card-augmented Divisia inside money aggregates are even better than the credit-card-augmented Divisia monetary aggregates for forecasting inflation, when the monetary aggregates use narrow money component clusters. The broad money aggregates are very effective indicators in predicting annual inflation with all seven component groupings, with the M4- component cluster being especially successful.

Also, we investigate the ability of credit-card-augmented Divisia as an indicator in forecasting annual output growth. For all types of money this paper examines, we see that credit-card-augmented Divisia is consistently the best indicator in forecasting output growth among the original Divisia, the demand-side credit-card-augmented Divisia, and the supply-side credit-card-augmented Divisia inside money aggregate. In addition, narrow money component groupings are more effective than the broad money component clusterings as indicators in forecasting output growth, with the M2 being the best.

We also examine the forecasting performance of credit-card-augmented Divisia in an alternative approach. We use Bayesian vector autoregression and find that the narrow credit-card-augmented Divisia measures forecast U.S. annual inflation well, while the broad credit-card-augmented Divisia are best in forecasting output growth. But in all cases, the forecasting errors are very small.

Barnett, Chauvet, Leiva-Leon, and Su (2016) are working on an indicator-optimized variant of the credit-card-augmented monetary aggregates, but that variant is not currently available to the public. As a result, we are not yet able to determine its performance in forecasting.

REFERENCES

BARNETT, WILLIAM A. 1980. "Economic Monetary Aggregates: An Application of Index Number and Aggregation Theory", Journal of Econometrics, 14(1), 11-48. Reprinted in Barnett, W. A. and Serletis, A. (eds.) (2000) The Theory of Monetary Aggregation, North Holland, Amsterdam. 11–48.

BARNETT, WILLIAM A. 1987. "The Microeconomic Theory of Monetary Aggregation", New Approaches to Monetary Economics, Barnett, W. A. and Singleton, K. (eds.) 115-168. Reprinted in Barnett, W. A. and Serletis, A. (eds.) (2000) The theory of Monetary Aggregation, North Holland, Amsterdam, 49-99.

BARNETT, WILLIAM A., AND LITING SU. 2016. "Joint Aggregation over Money and Credit Card Services under Risk", Economics Bulletin, 36(4), 2301-2310.

BARNETT, WILLIAM A., AND LITING SU. 2018. "Financial Firm Production of Inside Monetary and Credit Card Services: An Aggregation Theoretic Approach", Macroeconomic Dynamics, 24(1), 1-31.

BARNETT, WILLIAM A., MARCELLE CHAUVET, AND DANILO LEIVA-LEON. 2016. "Real-time Nowcasting of Nominal GDP with Structural Breaks", Journal of Econometrics, 191(2), 312-324.

BARNETT, WILLIAM A., JIA LIU, AND RYAN S. MATTSON. 2013. "The New CFS Divisia Monetary Aggregates: Design, Construction, and Data Sources", Open Economies Review, 24(1), 101-124.

BARNETT, WILLIAM A., MARCELLE CHAUVET, DANILO LEIVA-LEON AND LITING SU. 2016. "The Credit-Card-Services Augmented Divisia Monetary Aggregates," MPRA Paper 73245, University Library of Munich, Germany.

BARNETT, WILLIAM A., DOUGLAS FISHER, AND APOSTOLOS SERLETIS. 1992. "Consumer Theory and the Demand for Money", Journal of Economic Literature, 30(4), 2086-2119.

BARNETT, WILLIAM A., EDWARD K. OFFENBACHER, AND PAUL A. SPINDT. 1984. "The New Divisia Monetary Aggregates", Journal of Political Economy, 92(6), 1049-1085.

BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM. 2019. "The 2019 Federal Reserve Payment Study", A Federal Reserve System Publication.

CHIN, KUO. H., AND XUE LI. 2019. "Bayesian Forecast Combination in VAR-DSGE Models", Journal of Macroeconomics, 59, 278-298.

DOAN, THOMAS, ROBERT B. LITTERMAN, AND CHRISTOPHER A. SIMS. 1984. "Forecasting and Conditional Projection Using Realistic Prior Distribution", Econometric Reviews, 3, 1-100.

ELLINGTON, MICHAEL. 2018. "The Case of Divisia Monetary Statistics: A Bayesian Time-Varying Approach", Journal of Economic Dynamics and Control, 96, 26-41.

KADIYALA, K. RAO, AND SUNE KARLSSON. 1997. "Numerical Methods for Estimation and Inference in Bayesian VAR-Models", Journal of Applied Econometrics, 12(2), 99-132.

KIM, LAURA, RAYNIL KUMAR, AND SHAUN O'BRIEN. 2020. "2020 Findings from the Diary of Consumer Payment Choice", FedNotes, Federal Reserve Bank of San Francisco.

LITTERMAN, ROBERT B. 1980. "A Bayesian Procedure for Forecasting with Vector Autoregression", Working Paper, Massachusetts Institute of Technology, Department of Economics, Cambridge, MA.

LIU, JINAN, COSMAS DERY, AND APOSTOLOS SERLETIS. 2020. "Recent Monetary Policy and the Credit Card-Augmented Divisia Monetary Aggregates", Journal of Macroeconomics, 64, 103203.

ROSSI, BARBARA, AND TATEVIK SEKHPOSYAN. 2010. "Have Economic Models' Forecasting Performance for US Output Growth and Inflation Changed Over Time, and When?", International Journal of Forecasting, 26(4), 808-835.

ROSSI, BARBARA, AND TATEVIK SEKHPOSYAN. 2014. "Evaluating Predictive Densities of US Output Growth and Inflation in a Large Macroeconomic Data Set", International Journal of Forecasting, 30(3), 662-682.

SCHUNK, DONALD. L. 2001. "The Relative Forecasting Performance of the Divisia and Simple Sum Monetary Aggregates", Journal of Money, Credit, and Banking, 33(2), 272-283.

STOCK, JAMES H, AND MARK W. WATSON. 1999. "Forecasting Inflation", Journal of Monetary Economics, 44(2), 293-335.

STOCK, JAMES H, AND MARK W. WATSON. 2003. "Forecasting Output and Inflation: The Role of Asset Prices", Journal of Economic Literature, 41(3), 788-829.

Appendix A. Tables

Table A.1: Description of Data Set

	Description	Source
CPI	Consumer Price Index (1982-1984=100, Seasonally Adjusted)	BLS
GDP	Gross Domestic Product (Real)	HIS Markit
DivisiaM1	Divisia monetary aggregates: M1	CFS
DivisiaM1A	Credit card-augmented Divisia: M1	CFS
DivisiaM1AI	Credit card-augmented Divisia inside money: M1	CFS
DivisiaM2	Divisia monetary aggregates: M2	CFS
DivisiaM2A	Credit card-augmented Divisia: M2	CFS
DivisiaM2AI	Credit card-augmented Divisia inside money: M2	CFS
DivisiaMZM	Divisia monetary aggregates: MZM	CFS
DivisiaMZMA	Credit card-augmented Divisia: MZM	CFS
DivisiaMZMAI	Credit card-augmented Divisia inside money: MZM	CFS
DivisiaALL	Divisia monetary aggregates: ALL	CFS
DivisiaALLA	Credit card-augmented Divisia: ALL	CFS
DivisiaALLAI	Credit card-augmented Divisia inside money: ALL	CFS
DivisiaM3	Divisia monetary aggregates: M3	CFS
DivisiaM3A	Credit card-augmented Divisia: M3	CFS
DivisiaM3AI	Credit card-augmented Divisia inside money: M3	CFS
DivisiaM4-	Divisia monetary aggregates: M4-	CFS
DivisiaM4A-	Credit card-augmented Divisia: M4-	CFS
DivisiaM4AI-	Credit card-augmented Divisia inside money: M4-	CFS
DivisiaM4	Divisia monetary aggregates: M4	CFS
DivisiaM4A	Credit card-augmented Divisia: M4	CFS

Note. BLS is Bureau of Labor Statistics. CFS is Center for Financial Stability.

Table A.2: Components of Aggregates

Asset	Divisia M1	Divisia M2M	Divisia MZM	Divisia M2	Divisia M2 ALL	Divisia M3	Divisia M4-	Divisia M4
Currency	V	V	V	V	V	V	V	V
Travelers Checks	V	V	V	V	V	V	V	V
Demand Deposits	V	V	V	V	V	V	V	V
OCD Commercial	V	V	V	V	V	V	V	V
OCD Thrift	V	V	V	V	V	V	V	V
Savings Deposits Commercial		V	V	V	V	V	V	V
Savings Deposits Thrift		V	V	V	V	V	V	V
Retail Money Market Funds		V	V	V	V	V	V	V
Small Time Deposits Commercial				V	V	V	V	V
Small Time Deposits Thrift				V	V	V	V	V
Institutional Money Market Funds			V		V	V	V	V
Large Time Deposits						V	V	V
Overnight and Term Repos						V	V	V
Commercial Paper							V	V
T-Bills								V

Source: Barnett, Liu, and Mattson (2013).

Table A.3: Augmented Dickey-Fuller (ADF) Test Results

Variable	ADF test statistic (ϕ)	p-value	Decision
CPI (Log difference)	-7.3912	0.0000	Stationary
GDP (Log difference)	-11.2480	0.0000	Stationary
DivisiaM1 (Log difference)	-14.1586	0.0000	Stationary
DivisiaM1A (Log difference)	-14.5593	0.0000	Stationary
DivisiaM1A (Log difference)	-15.3340	0.0000	Stationary
DivisiaM2 (Log difference)	-9.1965	0.0000	Stationary
DivisiaM2A (Log difference)	-10.0913	0.0000	Stationary
DivisiaM2AI (Log difference)	-10.5895	0.0000	Stationary
DivisiaMZM (Log difference)	-6.6509	0.0000	Stationary
DivisiaMZMA (Log difference)	-7.2618	0.0000	Stationary
DivisiaMZMAI (Log difference)	-7.5137	0.0000	Stationary
DivisiaALL (Log difference)	-6.0977	0.0000	Stationary
DivisiaALLA (Log difference)	-6.6952	0.0000	Stationary
DivisiaALLAI (Log difference)	-6.8814	0.0000	Stationary
DivisiaM3 (Log difference)	-10.7024	0.0000	Stationary
DivisiaM3A (Log difference)	-10.6399	0.0000	Stationary
DivisiaM3AI (Log difference)	-10.4619	0.0000	Stationary
DivisiaM4- (Log difference)	-10.0059	0.0000	Stationary
DivisiaM4A- (Log difference)	-9.9153	0.0000	Stationary
DivisiaM4AI- (Log difference)	-9.7785	0.0000	Stationary
DivisiaM4 (Log difference)	-9.8832	0.0000	Stationary
DivisiaM4A (Log difference)	-10.0719	0.0000	Stationary