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21 October 2021

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ABSTRACT

Presented here is a simplified mathematical model exploring dynamic factors of the economic growth. In particular, it examines two factors affecting the economic growth – first factor is an increase of the rates of product investment and another factor is an increase of the rates of product demand. Economic growth is presented through an increase of the rates of monetary demand. If the rates of monetary demand evolve there is an economic growth. If the rates of monetary demand decrease there is an economic decline. Both an increase of the product investment and increase of the product demand have similar effects with regard to equilibrium between the product supply and product demand. For investment and demand increases the model explores scenarios of constant-rate and constant-accelerated product amendments.

JEL Classification Numbers: C61, E32, O42

Keywords: economic growth, investment increase, demand increase

1 Introduction

The work is devoted to the field of theoretical economics and involves building appropriate mathematical models and consecutive analysis and exploration of economic processes with the help of said models.

In (Krouglov, 2006) there was described a framework of dynamic mathematical models where economic values of supply, demand and price were balanced through the system of ordinary differential equations. In particular, the framework stipulated that in equilibrium state the values of supply and demand were equal. When the equality of supply and demand was broken then third value, the price, was adjusting in order to bring the market economic system back into equilibrium by matching the values of supply and demand.

In (Krouglov, 2017) the framework was applied to the concept of an economic growth. Particularly, there were explored the phenomena of economic crises and produced a simplified mathematical model to outline some aspects of the financial crises. The economic growth was viewed via a situation where some amount of supply (intended for the investment purposes) was removed from the market. Removal from the market of some amount of supply caused the supply and demand equilibrium to be broken, the price was increasing and demand was declining. Economic growth was analyzed as the rates of monetary demand (associated with the product earnings). Increase of the rates of monetary demand was viewed as an economic growth, and decrease of the rates of monetary demand was viewed as an economic decline.

In this article I supplement phenomena of economic growth as result of the product investment increase with circumstances of economic growth as consequence of the product demand increase. I build mathematical models and explore the similarities and differences in said two distinct situations that produce an economic growth. Additionally, in the model I examine two simplified scenarios for product investment increase and product demand increase of constant-rate and constant-accelerated product amendments.

2 Outline of a Simplified Mathematical Model

I deploy a simplified mathematical model of the single-product market economy, which allows me explicitly comparing different economic variables and explicitly researching interactions among different economic variables. Economic forces on the market acting behind the economic variables reflect in particular inherent forces of economic demand and economic supply. The forces of demand and supply are complemented with other economic forces, which in particular produce the phenomenon of economic growth. The actions of the economic variables on market are expressed through the system of ordinary differential equations.

When there are no disturbing economic forces, the market stays in an equilibrium position, i.e., the supply of and demand for product are equal, the values of product supply and product demand are developing with a constant rate and a price of product is also constant.

As it was mentioned, I use a simplified model of the single-product economy. I investigate the impact on economic growth a phenomenon of product investment increase and phenomenon of product demand increase. I restrict the presented research with scenarios where both a product investment increase and a product demand increase are developing with either a constant rate or a constant acceleration.

2.1. A Model of Economy in the Undisturbed State

To turn to mathematical descriptions, when there are no disturbing economic forces, the market is in an equilibrium position, i.e., the product's supply and demand are equal, and they are developing with a constant rate and the product's price is also constant.

I assume the market had been in an equilibrium until time $t = t_0$, volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was also constant,

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{1}$$

$$V_S(t) = V_D(t) \tag{2}$$

$$P(t) = P^0 \tag{3}$$

where $V_D(t_0) = V_D^0$.

When the balance between the product's supply and demand is broken, the market is experiencing economic forces, which act to bring the market to a new equilibrium position.

2.2. A Model of the Product Investment Increase

These economic forces are described by the following ordinary differential equations regarding to the product's supply $V_S(t)$, demand $V_D(t)$, price $P(t)$, and investment $S_{II}(t)$ increase (Krouglov, 2006; 2017),

$$\frac{dP(t)}{dt} = -\lambda_P \left(\left(V_S(t) - S_{II}(t) \right) - V_D(t) \right) \tag{4}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{5}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P(t)}{dt^2} \tag{6}$$

In Eq. (4 – 6) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants.

I present two scenarios describing the situation with product's investment increase.

Model of the Constant-Rate Investment Increase

According to the first scenario, I assume an amount of investment increase $S_{II}(t)$ on the market develops since time $t = t_0$ according to the following formula,

$$S_{II}(t) = \begin{cases} 0, & t < t_0 \\ \delta_{II}(t - t_0), & t \geq t_0 \end{cases} \tag{7}$$

where $S_{II}(t) = 0$ for $t < t_0$.

Let me use a new variable $D(t) \equiv \left(\left(V_S(t) - S_{II}(t)\right) - V_D(t)\right)$ representing the volume of product surplus (or shortage) on the market due to an investment amount. The behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \, \lambda_D \frac{dD(t)}{dt} + \lambda_P \, \lambda_S \, D(t) = 0 \tag{8}$$

with the initial conditions, $D(t_0) = 0$, $\dfrac{dD(t_0)}{dt} = -\delta_{II}$.

We can observe values for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, for the investment amount $S_{II}(t)$ when $t \to +\infty$ (see Piskunov, 1965; Petrovski, 1966),

$$D(t) \to 0 \tag{9}$$

$$P(t) \to P^0 + \frac{1}{\lambda_S} \delta_{II} \tag{10}$$

$$V_D(t) \to r_D^0 (t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_{II} \tag{11}$$

$$V_S(t) \to \left(r_D^0 + \delta_{II}\right)\left(t - t_0\right) + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{II} \tag{12}$$

$$V_S(t) - S_{II}(t) \to r_D^0\left(t - t_0\right) + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{II} \tag{13}$$

$$S_{II}(t) = \delta_{II}\left(t - t_0\right) \tag{14}$$

I use variables $r_D(t) \equiv \frac{dV_D(t)}{dt}$ and $r_S(t) \equiv \frac{dV_S(t)}{dt}$ representing the rates of product demand and product supply on the market at time t and the rate of investment increase $r_{II}(t) \equiv \frac{dS_{II}(t)}{dt}$ at time t.

Then it takes place for the rate of product demand and the rate of product supply, and the rate of investment increase when $t \to +\infty$,

$$r_D(t) \to r_D^0 \tag{15}$$

$$r_S(t) \to r_D^0 + \delta_{II} \tag{16}$$

$$r_S(t) - r_{II}(t) \to r_D^0 \tag{17}$$

I use variables $E_D(t) \equiv P(t)\, r_D(t)$ and $E_S(t) \equiv P(t)\, r_S(t)$ that represent the rates of monetary demand and monetary supply on the market at time t.

Then it takes place for the rates of monetary demand and monetary supply when $t \to +\infty$,

$$E_D(t) \to \left(P^0 + \frac{1}{\lambda_S}\delta_{II}\right) r_D^0 \tag{18}$$

$$E_S(t) \to \left(P^0 + \frac{1}{\lambda_S}\delta_{II}\right)\left(r_D^0 + \delta_{II}\right) \tag{19}$$

Model of the Constant-Accelerated Investment Increase

According to the second scenario, I assume an amount of investment increase $S_{II}(t)$ on the market develops since time $t = t_0$ according to the following formula,

$$S_{II}(t) = \begin{cases} 0, & t < t_0 \\ \delta_{II}(t - t_0) + \dfrac{\varepsilon_{II}}{2}(t - t_0)^2, & t \geq t_0 \end{cases} \tag{20}$$

where $S_{II}(t) = 0$ for $t < t_0$.

I use a variable $D(t)$ representing the volume of product surplus (or shortage) on the market due to an investment amount. The behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \lambda_D \frac{dD(t)}{dt} + \lambda_P \lambda_S D(t) + \varepsilon_{II} = 0 \tag{21}$$

with the initial conditions, $D(t_0) = 0$, $\dfrac{dD(t_0)}{dt} = -\delta_{II}$.

We can observe values for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, for the investment amount $S_{II}(t)$ when $t \to +\infty$ (see Piskunov, 1965; Petrovski, 1966),

$$D(t) \to -\frac{\varepsilon_{II}}{\lambda_P \lambda_S} \tag{22}$$

$$P(t) \to \frac{\varepsilon_{II}}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_{II}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_{II} \tag{23}$$

$$V_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{II} \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{II} + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_{II} \tag{24}$$

$$V_S(t) \to \left(r_D^0 + \delta_{II} - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right)(t - t_0) + \frac{\varepsilon_{II}}{2}(t - t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_{II} - \frac{\varepsilon_{II}}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_{II} \tag{25}$$

$$V_S(t) - S_{II}(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_{II} - \frac{\varepsilon_{II}}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_{II} \tag{26}$$

$$S_{II}(t) = \delta_{II}(t - t_0) + \frac{\varepsilon_{II}}{2}(t - t_0)^2 \tag{27}$$

I use variables $r_D(t)$ and $r_S(t)$ representing the rates of product demand and supply on the market at time t and the rate of investment increase $r_{II}(t)$ at time t. Then it takes place for the rate of product demand and the rate of product supply, and the rate of investment increase when $t \to +\infty$,

$$r_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right) \tag{28}$$

$$r_S(t) \to \left(r_D^0 + \delta_{II} - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right) + \varepsilon_{II}(t - t_0) \tag{29}$$

$$r_S(t) - r_{II}(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right) \tag{30}$$

I use variables $E_D(t)$ and $E_S(t)$ that represent the rates of monetary demand and monetary supply on the market at time t. Then it takes place for the rates of monetary demand and monetary supply when $t \to +\infty$,

$$E_D(t) \to \left(\frac{\varepsilon_{II}}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_{II}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_{II} \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right) \tag{31}$$

$$E_S(t) \to \left(\frac{\varepsilon_{II}}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_{II}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_{II} \right)\left(\left(r_D^0 + \delta_{II} - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right) + \varepsilon_{II}(t - t_0) \right) \tag{32}$$

I use a variable $e_D(t) \equiv \frac{dE_D(t)}{dt}$ representing the change of monetary demand rate on the market at time t. It takes place for the variable $e_D(t)$,

$$e_D(t) \to \frac{\varepsilon_{II}}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{II} \right) \tag{33}$$

We can notice that $E_D(t)$ has a maximum trend e_D when $\varepsilon_{II} = \frac{\lambda_S}{2\lambda_D} r_D^0$ and the value e_D of maximum trend is equal to $e_D = \frac{1}{4\lambda_D}\left(r_D^0\right)^2$.

2.3. A Model of the Product Demand Increase

These economic forces are described by the following ordinary differential equations regarding to the product's supply $V_S(t)$, demand $V_D(t)$, price $P(t)$, and demand $S_{DI}(t)$ increase (Krouglov, 2006; 2017),

$$\frac{dP(t)}{dt} = -\lambda_P\left(V_S(t) - \left(V_D(t) + S_{DI}(t)\right)\right) \tag{34}$$

$$\frac{d^2V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{35}$$

$$\frac{d^2V_D(t)}{dt^2} = -\lambda_D \frac{d^2P(t)}{dt^2} \tag{36}$$

In Eq. (34 – 36) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants.

I present two scenarios describing the situation with product's demand increase.

Model of the Constant-Rate Demand Increase

According to the first scenario, I assume an amount of demand increase $S_{DI}(t)$ on the market develops since time $t = t_0$ according to the following formula,

$$S_{DI}(t) = \begin{cases} 0, & t < t_0 \\ \delta_{DI}(t - t_0), & t \geq t_0 \end{cases} \tag{37}$$

where $S_{DI}(t) = 0$ for $t < t_0$.

Let me use a new variable $D(t) \equiv \left(V_S(t) - \left(V_D(t) + S_{DI}(t)\right)\right)$ representing the volume of product surplus (or shortage) on the market due to a demand increase. The behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \lambda_D \frac{dD(t)}{dt} + \lambda_P \lambda_S D(t) = 0 \tag{38}$$

with the initial conditions, $D(t_0) = 0$, $\frac{dD(t_0)}{dt} = -\delta_{DI}$.

We can observe values for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, for the demand increase amount $S_{DI}(t)$ when $t \to +\infty$ (see Piskunov, 1965; Petrovski, 1966),

$$D(t) \to 0 \tag{39}$$

$$P(t) \to P^0 + \frac{1}{\lambda_S} \delta_{DI} \tag{40}$$

$$V_S(t) \to \left(r_D^0 + \delta_{DI}\right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_{DI} \tag{41}$$

$$V_D(t) \to r_D^0(t-t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{DI} \tag{42}$$

$$V_D(t) + S_{DI}(t) \to \left(r_D^0 + \delta_{DI}\right)(t-t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{DI} \tag{43}$$

$$S_{DI}(t) = \delta_{DI}(t-t_0) \tag{44}$$

I use variables $r_D(t)$ and $r_S(t)$ representing the rate of product demand and the rate of product supply on the market at time t, and the rate of demand increase $r_{DI}(t) \equiv \frac{dS_{DI}(t)}{dt}$ at time t.

Then it takes place for the rate of product demand and the rate of product supply, and the rate of demand increase when $t \to +\infty$,

$$r_S(t) \to r_D^0 + \delta_{DI} \tag{45}$$

$$r_D(t) \to r_D^0 \tag{46}$$

$$r_D(t) + r_{DI}(t) \to r_D^0 + \delta_{DI} \tag{47}$$

I use variables $E_D(t)$ and $E_S(t)$ that represent the rates of monetary demand and monetary supply on the market at time t.

Then it takes place for the rates of monetary demand and monetary supply when $t \to +\infty$,

$$E_D(t) \to \left(P^0 + \frac{1}{\lambda_S}\delta_{DI}\right) r_D^0 \tag{48}$$

$$E_S(t) \to \left(P^0 + \frac{1}{\lambda_S}\delta_{DI}\right)\left(r_D^0 + \delta_{DI}\right) \tag{49}$$

Model of the Constant-Accelerated Demand Increase

According to the second scenario, I assume an amount of demand increase $S_{DI}(t)$ on the market develops since time $t = t_0$ according to the following formula,

$$S_{DI}(t) = \begin{cases} 0, & t < t_0 \\ \delta_{DI}(t-t_0) + \frac{\varepsilon_{DI}}{2}(t-t_0)^2, & t \geq t_0 \end{cases} \tag{50}$$

where $S_{DI}(t) = 0$ for $t < t_0$.

I use a variable $D(t)$ representing the volume of product surplus (or shortage) on the market due to a demand increase. The behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \lambda_D \frac{dD(t)}{dt} + \lambda_P \lambda_S D(t) + \varepsilon_{DI} = 0 \tag{51}$$

with the initial conditions, $D(t_0) = 0$, $\frac{dD(t_0)}{dt} = -\delta_{DI}$.

We can observe values for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, for the demand increase amount $S_{DI}(t)$ when $t \to +\infty$ (see Piskunov, 1965; Petrovski, 1966),

$$D(t) \to -\frac{\varepsilon_{DI}}{\lambda_P \lambda_S} \tag{52}$$

$$P(t) \to \frac{\varepsilon_{DI}}{\lambda_S}(t-t_0) + P^0 + \frac{\delta_{DI}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_{DI} \tag{53}$$

$$V_S(t) \to \left(r_D^0 + \delta_{DI} - \frac{\lambda_D}{\lambda_S}\varepsilon_{DI} \right)(t-t_0) + \frac{\varepsilon_{DI}}{2}(t-t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{DI} - \frac{\varepsilon_{DI}}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_{DI} \tag{54}$$

$$V_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_{DI} \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_{DI} + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_{DI} \tag{55}$$

$$V_D(t) + S_{DI}(t) \to \left(r_D^0 + \delta_{DI} - \frac{\lambda_D}{\lambda_S} \varepsilon_{DI} \right)(t - t_0) + \frac{\varepsilon_{DI}}{2}(t - t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_{DI} - \frac{\varepsilon_{DI}}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_{DI} \tag{56}$$

$$S_{DI}(t) = \delta_{DI}(t - t_0) + \frac{\varepsilon_{DI}}{2}(t - t_0)^2 \tag{57}$$

I use variables $r_D(t)$ and $r_S(t)$ representing the rate of product demand and the rate of product supply on the market at time t, and the rate of demand increase $r_{DI}(t)$ at time t. Then it takes place for the rate of product demand, the rate of product supply and the rate of demand increase when $t \to +\infty$,

$$r_S(t) \to \left(r_D^0 + \delta_{DI} - \frac{\lambda_D}{\lambda_S} \varepsilon_{DI} \right) + \varepsilon_{DI}(t - t_0) \tag{58}$$

$$r_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_{II} \right) \tag{59}$$

$$r_D(t) + r_{DI}(t) \to \left(r_D^0 + \delta_{DI} - \frac{\lambda_D}{\lambda_S} \varepsilon_{DI} \right) + \varepsilon_{DI}(t - t_0) \tag{60}$$

I use variables $E_D(t)$ and $E_S(t)$ that represent the rates of monetary demand and monetary supply on the market at time t. Then it takes place for the rates of monetary demand and monetary supply when $t \to +\infty$,

$$E_D(t) \to \left(\frac{\varepsilon_{DI}}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_{DI}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_{DI} \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_{DI} \right) \tag{61}$$

$$E_S(t) \to \left(\frac{\varepsilon_{DI}}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_{DI}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_{DI} \right)\left(\left(r_D^0 + \delta_{DI} - \frac{\lambda_D}{\lambda_S} \varepsilon_{DI} \right) + \varepsilon_{DI}(t - t_0) \right) \tag{62}$$

I use a variable $e_D(t)$ representing the change of monetary demand rate on the market at time t. It takes place for the variable $e_D(t)$,

$$e_D(t) \to \frac{\varepsilon_{DI}}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{DI} \right) \tag{63}$$

We can notice that $E_D(t)$ has a maximum trend e_D when $\varepsilon_{DI} = \frac{\lambda_S}{2\lambda_D} r_D^0$ and the value e_D of maximum trend is equal to $e_D = \frac{1}{4\lambda_D}\left(r_D^0\right)^2$.

3 Economic Discussion

To build and investigate a mathematical model of the economic growth I employ a simplified model of the market with a single product. Economic forces acting on the market reflect in particular the forces of demand and supply. Besides the forces of demand and supply, I describe the economic forces that create an economic growth.

When there are no disturbing economic forces, the market is in an equilibrium position, i.e., the supply of and demand for product is equal, the supply and demand are developing with constant rates, and the price of product is constant. When the balance between the quantities of product supply and product demand is broken, market is experiencing an impact of the forces, which act to bring the market to a new equilibrium position.

An economic growth is caused when the disturbing economic forces break an equilibrium position on the market. Here I explore two distinct situations – an economic growth caused by the product investment and an economic growth caused by the market expansion (i.e., caused by the product demand increase).

Discussion of a Situation of the Product Investment Increase

In order to bring the market economic system into equilibrium state the market needs to adjust the product demand and product supply. Relationships between the product demand and product supply on the market are described with a system of second order differential equations. The product's prices act as an economic mechanism to reconcile the rates of product demand and product supply on the market.

Now I describe an economic construction depicting how the differential equations induce a mechanism of the economic growth on the market. I assume that a part of the product supply is not consumed in economic system but saved and used in the form of the product investment. Therefore, instead of the product supply and product demand I have already three economic values in a model – the product supply, demand, and investment. Thus, I have to consider the equilibrium between product supply minus savings/investment and product demand. When the equilibrium is breached the economic forces try to bring the market back into another equilibrium state.

To summarize, when the product investment increases, the product demand prevails over the product supply minus investment and the product’s price increases. Increases in the product's prices indicate to entrepreneurs that product demand prevails over product supply minus investment at this moment. That unsatisfied product demand tells an entrepreneur to increase the product supply. It would cause the economic system to match the product supply and product demand and move to another equilibrium point.

Discussion of a Situation of the Product Demand Increase

As before, in order to bring the market economic system into equilibrium state the market needs to adjust the product demand and product supply. Relationships between the product demand and product supply on market are described with a system of second order differential equations. The product's prices act as an economic mechanism to reconcile the rates of product demand and product supply on market.

I describe an economic construction depicting how differential equations induce a mechanism of economic growth caused by the product demand increase. Before I assumed that a part of the product supply was not consumed in economic system but was saved and used in the form of the product investment. Thus, instead of the product supply and product demand I had three economic values in a model – the product supply, product demand, and product amendment. I consider two situations – the first one is equilibrium between product supply minus product amendment (i.e., product investment) and product demand; the second one is equilibrium between product supply and product demand plus product amendment (i.e., product demand increase). I notice that decrease of the product supply (supply minus amendment) and increase of the product demand (demand plus amendment) has similar effects with regard to the equilibrium state between the product supply and product demand. When the equilibrium is breached the economic forces try to bring the market back into another equilibrium state.

Therefore, when an amendment of the product demand increases, the product demand prevails over product supply and the product's price increases. Increment of the product's price indicates to an entrepreneur that the product demand plus demand amendment prevails over the product supply at this moment. That unsatisfied product demand tells an entrepreneur to increase the product supply. It would cause the economic system to match the product supply and product demand and move to another equilibrium point.

Explanation of Economic Growth through a Monetary Demand Increase

To explore an economic growth experienced by market economic system (defined as the rates of product demand multiplied by the product's prices) I make use of the rates of monetary demand amendments.

As it was discussed, instead of the product supply and product demand I have three economic values in a model – the product supply, product demand, and product amendment. For an exploration of the economic growth I employ the fourth economic value – the product's price. As I discussed before, a decrease of the product supply (supply minus amendment) and increase of the product demand (demand plus amendment) has similar effects with regard to equilibrium between the product supply and product demand.

Since the product's prices and the rates of product demand for both the product supply (product supply minus product amendment) and the product demand (product demand plus product amendment) has similar effects with regard to equilibrium between the product supply and product demand, they are to provide the same rates of monetary demand and to deliver an equal economic growth for the market economic system.

In a model of the market economic system I have examined the rates of monetary demand (i.e., the product earnings) defined as the product's prices multiplied by the rates of product demand. If the rates of monetary demand are growing I detect an economic growth on the market. Likewise, if the rates of monetary demand are declining I detect an economic decline on the market.

For simplicity, I have explored in the model two scenarios of constant-rate and constant-accelerated amendments for both the product investment increase and the product demand increase.

The cases of both constant-rate investment increase and constant-rate demand increase imply that rates of the product investment increase and rates of the product demand increase both have permanent values in the model. There is no long-term economic growth induced by a constant-rate investment increase or by a constant-rate demand increase on the market.

The cases of constant-accelerated investment increase and constant-accelerated demand increase imply that changes in the rates of product investment increase and rates of product demand increase are permanent. There are possibilities of either a long-term economic growth or a long-term economic decline induced by the constant-accelerated product investment increase or constant-accelerated product demand increase. If the product investment increase or product demand increase is made with a modest constant acceleration then the rates of monetary demand (i.e., the product earnings) are growing and I detect an economic growth on the market. If the product investment increase or product demand increase is made with a large constant acceleration then the rates of monetary demand (i.e., the product earnings) are declining and I detect an economic decline on the market.

4 Conclusion

The article is devoted to the mathematical modeling of phenomenon of the economic growth.

An economic growth is viewed as increase of the rates of monetary demand. The monetary demand in market economic system is defined as the rates of product demand multiplied by the product’s prices.

It is shown that an economic growth could be induced by two distinct economic factors. The first one is an increase in the product investments. The second one is an increase of the product demand. It is also shown that both these factors have similar effects with regard to equilibrium between the product supply and product demand. Since the product’s prices and the rates of product demand for the product supply and product demand have similar effects with regard to equilibrium between the product supply and product demand, they provide equal rates of monetary demand and deliver the equal economic growth for market economic system.

In a model of the market economic system there are examined the rates of monetary demand defined as the product's prices multiplied by the rates of product demand. If the rates of monetary demand are growing the effect is an economic growth on the market.

It is explored in the model two scenarios of constant-rate and constant-accelerated amendments for both the product investment increase and the product demand increase. There has no long-term economic growth induced by a constant-rate investment increase or by a constant-rate demand increase.

There are possibilities of a long-term growth or a long-term decline induced by the constant-accelerated product investment increase or product demand increase. If product investment increase or product demand increase is made with a modest acceleration the rates of monetary demand are growing and there is an economic growth on market. If product investment increase or product demand increase is made with a large acceleration the rates of monetary demand are decreasing and there is an economic decline on market.

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