Democracy or Optimal Policy: Income Tax Decisions without Commitment

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Abstract

How do differences in the government's political and commitment structure affect the aggregate economy, inequality, and welfare? I analyze this question, using a calibrated Aiyagari's (1994) economy with wealth effects of labor supply wherein a flat tax rate and transfers are endogenously determined according to its political and commitment structure. I compare four economies: a baseline economy, an economy with the optimal tax with commitment in all steady states, an economy with the optimal tax without commitment, and a political economy with sequential voting. I obtain two main findings. First, the commitment structure shifts the government's weighting between redistribution and efficiency. A lack of commitment leads the government to pursue a more redistributive policy at the expense of efficiency. Second, given a lack of commitment, the political economy with voting yields greater welfare than the economy with the time-consistent optimal policy. In the latter case, a lack of commitment hinders the government from implementing a more frugal policy desirable in the long run; instead, it cares more for low-income and wealth households, resulting in a substantial efficient loss. However, in the political economy with voting, the government considers only the interests of the median voter, who is middle class and reluctant to bear larger distortions from a higher tax rate and larger transfers. These findings imply that in terms of welfare, policies targeting the middle class would possibly be better than those exquisitely designed for the general public.

JEL classification: E61, H11, P16.

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1 Introduction

Public and fiscal policies are essentially subject to a lack of government commitment because political procedures sequentially determine the policy executor. Previous studies have found that a lack of commitment can yield substantial differences in the implications of designing and implementing policies (Kydland and Prescott, 1977; Calvo, 1978; Barro and Gordon, 1983; Klein and Ríos-Rull, 2003; Klein, Krusell and Rios-Rull, 2008). However, relatively few studies have considered how the government's political structure affects the design of public and fiscal policies given the difficulty in devising a proper framework. Investigating this issue requires models that incorporate heterogeneous agents because political decisions—from selecting policymakers to implementing policies—widely interact at the individual level. In addition to heterogeneous agents, because a lack of commitment leads successive governments to make strategic choices, solving a dynamic game of consecutive governments is essential. A major obstacle in this direction of research is a substantial computational burden.

Political-economy models, originally developed by Krusell et al. (1996); Krusell and Rios-Rull (1999), have three types of equilibrium objects—individual decisions, the aggregate law of motion for the distribution of households, and the endogenous government policy function—that have to be consistent with one another in equilibrium. One might consider using Krusell and Smith's (1998) method to achieve their consistency; however, this approach is ineffective for this class of models. First, more than one aggregate law of motion increases the computational burden exponentially in this simulation-based method. The existence of the government policy function leads to adding another outer loop to the outer loop in their method. Second, the government policy function is severely nonlinear because political decisions, which shape the government policy function, are sensitive to the distribution of individuals. This nonlinearity is not well-captured by the parameterized law of motion in Krusell and Smith (1998).

In this paper, I develop a numerical solution method that allows solving a broad class of heterogeneous agent models with Markov perfect equilibria (MPE) in a dynamic game of consecutive governments. To handle the aforementioned computational issues, I take ideas from the backward induction method of Reiter (2010). Because the backward induction method is not designed for economies in the MPE but for those with aggregate uncertainty, I make variations to this method in accordance with the characteristics of the MPE while preserving its computational benefits. My method is a non-simulation-based approach as in Reiter (2010), which substantially improves computational efficiency. Furthermore, my solution method approximates the aggregate laws of motion, including the government policy function, through a non-parametric approach as in Reiter (2010), thereby enabling me to capture the nonlinearity.

Using this solution method, I explore how differences in the government's political and commitment structure play roles in the macroeconomy, inequality, and welfare. I apply this solution method to the canonical model of Aiyagari (1994) with wealth effects of labor supply, in which the government's tax/transfer system is endogenously determined according to its political and commitment structure. I assume a simple government financing rule to better understand the fundamental roles of the political and commitment structure: the government levies a flat tax from labor and capital income and redistributes its revenue to households through lump-sum transfers after covering a given size of government spending.

Specifically, I compare four economies: the baseline economy, an economy with the optimal policy with commitment in all steady states (the time-inconsistent case), an economy with the optimal policy without commitment (the time-consistent case), and a political economy with sequential voting (the voting case). In the economy with the optimal policy with commitment, because the government can commit to all future tax policy, it chooses a tax policy that maximizes the utilitarian welfare function in the long run. By contrast, in the time-consistent optimal case, the government can only decide a tax rate for the next period and cannot commit to it after that. Thus, the government sequentially chooses a tax policy maximizing the utilitarian welfare function under this commitment constraint, and this action continues perpetually. Finally, following a seminal study by Krusell and Rios-Rull (1999), the political economy with voting has two political parties whose unique goal is to win election in each period, meaning a lack of commitment, through the majority's support. The two parties propose tax rates on which households vote. Because the policy dimension is one in my policy exercise, the dominant strategy of the two parties is to offer the most preferred tax rate of the median voter. To prevent multiple equilibria, I assume that one party always wins when votes are tied.

Through these comparisons, I obtain two key findings. First, the commitment structure makes the government strike a different balance between redistribution and efficiency in determining its policy. A lack of commitment tends to lead the government to place greater weight on redistribution at the cost of efficiency. The government size, measured by the total tax revenue to output ratio, is the largest in the economy with the optimal policy without commitment, followed by the political economy with sequential voting, the economy with the optimal policy with commitment, and the baseline economy. The ranking on efficiency, measured by the aggregate output, is precisely the opposite. These are in line with findings in Krusell and Rios-Rull (1999); Klein and Ríos-Rull (2003); Klein et al. (2008); Corbae et al. (2009).

Second, and more notable, given a lack of commitment, the political economy with sequential voting shows a better welfare outcome than the economy with the optimal policy that is time-consistent. This result is driven by differences in the government's attitudes toward distortions according to the political structure. Under the optimal policy without commitment, the government is less sensitive to distortions from a higher income tax rate, resulting in a larger efficiency loss. Note that under the optimal policy without commitment, the government is allowed to choose a tax rate only in the next period because of a lack of commitment. It cannot select any policy after that, which is repeated throughout the game. Under this constraint, its optimal policy devotes more care to low-income individuals whose MPC is high, leading to a more redistributive policy: a higher tax rate resulting in larger lump-sum transfers. The change in the bottom 50 percent income share from before to after government increases by 31.7 percent in the optimal policy without commitment but by 25.4 percent in the optimal policy with commitment.

In the political economy with sequential voting, the government takes into account only the most preferred tax rate of the median voter, who acts like an individual in the middle class. This median voter tends to be more sensitive to distortions from a higher income tax. The endogenous income tax function shaped by the median voter's behavior indicates that in response to a rise in the market equilibrium wage, the median voter wants to reduce the income tax rate for the next period by more than does the government in the economy with the optimal policy without commitment. This finding means that the median voter prefers a lower tax resulting in lower transfers. The change in the bottom 50 percent income share from before to after government in this political economy increases by 28.1 percent but by 31.7 percent in the time-consistent optimal case. This mechanism alleviates efficiency loss, resulting in a better welfare outcome in the political economy. These findings imply that without commitment, following policies preferred by the middle class would possibly be better at certain times than exquisitely designed policies for the general.

This paper belongs to the stream of political macroeconomic literature that examines the implications of governments' political and commitment structure in designing public policies. Motivated by the seminal studies of Aiyagari and Peled (1995); Krusell, Quadrini and Rios-Rull (1996); Krusell and Rios-Rull (1999), several works have investigated the effects of the political procedure on policy decisions from a macroeconomic perspective. Corbae, D'Erasmo and Kuruscu (2009) studies how political governments make decisions on income taxation in response to the increased inequality in wages in the U.S. They find that the increased inequality in wages raises the equilibrium income tax rate without commitment. The study of Corbae et al. (2009) is similar to my work in the sense that both studies compare a series of economies with heterogeneous agents according to the political and commitment structure of the government. However, in contrast to my model, Corbae et al. (2009) employs the preference of Greenwood, Hercowitz and Huffman (1988), which lacks wealth effects of labor supply.[1] Note that the wealth effects of labor supply are crucial for the macroeconomy and welfare because changes in transfers affect efficiency through this channel. Although Corbae et al. (2009) did not analyze detailed equilibrium outcomes in the macroeconomy and welfare, this paper, by allowing for wealth effects of labor supply, examines the macroeconomic implications and welfare consequences of the political and commitment structure of governments. The inclusion of wealth effects of labor supply is enabled by the numerical

[1] The authors mention that this choice is made to mitigate the computational burden.

solution method that is another independent contribution of this paper.

Song, Storesletten and Zilibotti (2012) is another study using a political economy. Their goal is to understand intergenerational conflict through public policy instruments. The different objective leads to a different model selection. While they consider an overlapping generations model in partial equilibrium, this paper uses an infinite-horizon model in general equilibrium. Farhi, Sleet, Werning and Yeltekin (2012) is also related to my work because they address the choice of income tax without commitment. However, their approach is different from mine. Whereas Farhi et al. (2012) solves the planner's centralized problem in a dynamic Mirrleesian model, I solve households' decentralized problems in an incomplete markets model.

The solution method in this paper is a nonnegligible, independent contribution to the literature. Broadly, two types of methods are often used to solve macroeconomic models with Markov-perfect equilibria. The first is Klein, Krusell and Rios-Rull's (2008) approach that is a local solution method using the generalized Euler equation. This method is accurate and efficient but not general enough to handle the class of heterogeneous agent models. My method in this paper is a global solution method applicable to heterogeneous agent models. The other approach is Krusell and Smith's (1998) method, which applicable to heterogeneous agent models. For example, Corbae et al. (2009) used this approach in their heterogeneous agent economy. However, this simulation-based method is computationally costly in the political equilibrium because a political economy would have more than one aggregate law of motion (e.g., the law of motion for the distributions and the endogenous tax policy function). This political economy-specific structure increases the computational burden in an exponential manner. Additionally, the endogenous policy function is severe nonlinear that is not well-captured by the parameterized law of motion in Krusell and Smith's (1998) method. My method is an efficient non-simulation-based solution approach that captures the non-linearity through a non-parametric way as in Reiter (2010).

The remainder of this paper proceeds as follows. Section 2 presents the model and defines the equilibrium. Section 3 explains the core ideas of the numerical solution algorithm. Section 4 describes the calibration strategy. Section 5 presents the results of the policy analysis. Section 6 concludes this paper. Finally, Appendix A demonstrates the full details of the numerical solution algorithm.

2 Model

The quantitative model here builds upon the canonical model of Aiyagari (1994), incorporating wealth effects of labor supply. In this model, given a tax policy function, heterogeneous households make decisions on consumption, savings and labor supply at the intensive margin, as in standard incomplete markets models. A notable difference from the standard models is the setting of its tax policy function. The tax policy function is determined, according to the political and commitment structure of government. In equilibrium, the tax policy function, individual decisions, and the evolution of the distribution are consistent with one another under the political and commitment structure.

2.1 Environment

The model economy is populated by a continuum of infinitely lived households. Their preference follows

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t, 1-n_t)\right] \tag{1}$$

where c_t is consumption, $n_t \in [0,1]$ is labor supply in period t ($(1-n_t)$ refers to leisure), and β is the discount factor. Preferences are represented by

$$u(c_t, 1-n_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n_t)^{1-1/\chi}}{1-1/\chi} \tag{2}$$

where σ is the coefficient of relative risk aversion, B is the utility of leisure, and χ is the Frisch elasticity of labor supply.

It is worth spending more time on the above preference. Note that the preference here captures wealth effects of labor supply. By contrast, Corbae, D'Erasmo and Kuruscu (2009) employed the preference in Greenwood, Hercowitz and Huffman (1988) that lacks wealth effects of labor supply, to mitigate the computational burden. Such wealth effects are crucial for welfare analysis, closely related to efficiency loss. An increase in transfers, for example, decreases overall labor supply, shrinking the size of the aggregate economy and playing a role in reducing welfare.

The representative firm produces output with a constant return to scale. The firm's technology is represented by

$$Y_t = F(K_t, N_t) = K_t^{\theta} N_t^{1-\theta} \tag{3}$$

where K_t is the quantity of aggregate capital, N_t is the quantity of aggregate labor, and θ is the capital income share. Capital depreciates at the rate of δ each period.

In each period, households confront an uninsurable, idiosyncratic shock ϵ_t to their wage that follows an AR-1 process:

$$\log(\epsilon_{t+1}) = \rho_{\epsilon}\log(\epsilon_t) + \eta_{t+1}^{\epsilon} \tag{4}$$

where $\eta^{\epsilon}_{t+1} \sim N(0, \sigma^2_{\epsilon})$. Using the method in Rouwenhorst (1995), I approximate the AR-1 process as a finite-state Markov chain with transition probabilities π^{ϵ}_{ij} from state i to state j where N_{ϵ} is the total number of ϵ states. Households earn $w_t \epsilon_t n_t$ as their labor income where w_t is the market equilibrium wage. They can self insure through assets a_t. Such households have capital income of as much as $r_t a_t$ where r_t is the equilibrium risk-free interest rate.

The government obtains its tax revenue by levying taxes on household capital and labor income at the same proportional flat tax rate, τ_t. Given a tax revenue, the government covers government spending G_t, and the rest is used for lump-sum transfers T_t. The government runs a balanced budget each period:

$$G_t + T_t = \tau_t \left[r_t K_t + w_t N_t \right]. \tag{5}$$

2.2 Recursive Competitive Equilibrium, Exogenous Policy

It is convenient to present the household dynamic problems in a recursive manner. At the beginning of each period, households differ from one another in asset holdings a and labor productivity ϵ_i. In addition to the individual state variables a and ϵ_i, there are two aggregate state variables, including the distribution of households $\mu(a, \epsilon_i)$ over a and ϵ_i and income tax τ. A variable with a prime symbol denotes its value in the next period.

Let $v(a, \epsilon_i; \mu, \tau)$ denote the value of households associated with a state of $(a, \epsilon_i; \mu, \tau)$. They solve

$$v(a, \epsilon_i; \mu, \tau) = \max_{c>0,\ a' \geq \underline{a},\ 0 \leq n \leq 1} \left[\frac{c^{1-\sigma}}{1-\sigma} + B \, \frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta \sum_{j=1}^{N_{\epsilon}} \pi^{\epsilon}_{i,j} v(a', \epsilon'_j; \mu', \tau') \right] \tag{6}$$

such that

$$c + a' = (1-\tau)\, w(\mu)\, \epsilon_i \, n + (1 + r(\mu)(1-\tau))\, a + T$$

$$\tau' = \Psi(\mu, \tau)$$

$$\mu' = \Gamma(\mu, \tau, \tau') = \Gamma(\mu, \tau, \Psi(\mu, \tau))$$

where $\underline{a} \leq 0$ is a borrowing limit, $\tau' = \Psi(\mu, \tau)$ is the perceived law of motion of taxes, and $\mu' = \Gamma(\mu, \tau, \tau')$ is the law of motion for the distribution over households. Note that households here solve the above problem given an exogenous tax policy function $\tau' = \Psi(\mu, \tau)$.

Definition 2.2.1. ***Recursive Competitive Equilibrium (RCE).***
Given G and $\Psi(\mu, \tau)$, a recursive competitive equilibrium (RCE) is a set of prices $\{w(\mu), r(\mu)\}$, a set of decision rules for households $g_a(a, \epsilon_i; \mu, \tau)$ and $g_n(a, \epsilon_i; \mu, \tau)$, a value function $v(a, \epsilon_i; \mu, \tau)$, a distribution of households $\mu(a, \epsilon_i)$ over the state space, and the law of motion for the distribution

of households $\Gamma(\mu, \tau, \Psi(\mu, \tau))$ such that

(i) Given $\{w(\mu), r(\mu)\}$, the decision rules $a' = g_a(a, \epsilon_i; \mu, \tau)$ and $n = g_n(a, \epsilon_i; \mu, \tau)$ solve the household problem in (6), and $v(a, \epsilon_i; \mu, \tau)$ is the associated value function.

(ii) The representative agent firm engages in competitive pricing:

$$w(\mu) = (1 - \theta)\left(\frac{K}{N}\right)^{\theta} \tag{7}$$

$$r(\mu) = \theta\left(\frac{K}{N}\right)^{\theta-1} - \delta. \tag{8}$$

(iii) The factor markets clear:

$$K = \sum_{i=1}^{N_\epsilon} \int a\, \mu(da, \epsilon_i) \tag{9}$$

$$N = \sum_{i=1}^{N_\epsilon} \int \epsilon_i\, g_n(a, \epsilon_i; \mu, \tau)\, \mu(da, \epsilon_i) \tag{10}$$

(iv) The government budget constraint (5) is satisfied.

(v) The law of motion for the distribution of households $\mu' = \Gamma(\mu, \tau, \Psi(\mu, \tau))$ is consistent with individual decision rules and the stochastic process of ϵ_i.

2.3 Recursive Competitive Equilibrium, Endogenous Policy

In the spirit of Krusell et al. (1996); Krusell and Rios-Rull (1999); Corbae et al. (2009), I endogenize the tax choice in three ways: the optimal income tax with commitment in all steady states; the optimal income tax without commitment; and majority voting. Then, I compare the macroeconomic implications and welfare consequences of these economies.

Definition 2.3.1. ***A RCE with the Optimal Income Tax with Commitment in All Steady States.***

(i) A set of functions $\{w(\cdot), r(\cdot), g_a(\cdot), g_n(\cdot), v(\cdot), \Gamma(\cdot)\}$ satisfy the RCE definition 2.2.1.

(ii) For each (μ, τ), the government chooses $\tau^W(\mu, \tau)$ such that

$$\tau^W(\mu, \tau) = \underset{\tilde{\tau}}{\text{argmax}} \sum_{i=1}^{N_\epsilon} \int \hat{V}(a, \epsilon_i; \mu, \tilde{\tau}) \mu(da, \epsilon_i) \tag{11}$$

where

$$\hat{V}(a, \epsilon_i; \mu, \tilde{\tau}) = \max_{c>0,\, a' \geq \underline{a},\, 0 \leq n \leq 1} \left[\frac{c^{1-\sigma}}{1-\sigma} + B \frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta \sum_{j=1}^{N_\epsilon} \pi^\epsilon_{i,j} v(a', \epsilon'_j; \mu', \tilde{\tau}) \right]$$

such that

$$c + a' = (1-\tilde{\tau})\, w(\mu)\, \epsilon_i\, n + (1 + r(\mu)(1-\tilde{\tau}))\, a + T;$$

$$\tau' = \tilde{\tau}, \text{ and thereafter } \tau'' = \tau' = \tilde{\tau} = \Psi(\mu', \tau' = \tilde{\tau}); \tag{12}$$

$$\mu' = \Gamma(\mu, \tau = \tilde{\tau}, \tau' = \tilde{\tau}) \tag{13}$$

(iii) $a' = \hat{g}_a(a, \epsilon_i; \mu, \tilde{\tau})$ and $n = \hat{g}_n(a, \epsilon_i; \mu, \tilde{\tau})$ solve (11) at prices that clear markets and the government budget constraint, and Γ is consistent with individual decisions and the stochastic process of ϵ_i.

(iv) For each (μ, τ), the policy outcome function satisfies $\Psi(\mu, \tau) = \tau^W(\mu, \tau)$.

In the above economy with the optimal income tax with commitment, the government adopts the time-inconsistent optimal policy: a tax rate that is permanently committed for all the future periods while maximizing its utilitarian welfare function in the long run. Thus, once a tax rate $\tilde{\tau}$ is chosen, tax rates thereafter are persevered as $\tilde{\tau}$, as shown in (12), because the government can commit to the future tax policy all along. This approach has been broadly used in the public policy-related macroeconomics literature with heterogeneous agents (Conesa et al., 2009; Wu and Krueger, 2021; Holter et al., 2019; Jang, 2020; Heathcote et al., 2020).

Definition 2.3.2. ***A RCE with the Optimal Income Tax without Commitment.***

(i) A set of functions $\{w(\cdot), r(\cdot), g_a(\cdot), g_n(\cdot), v(\cdot), \Gamma(\cdot)\}$ satisfy the RCE definition 2.2.1.

(ii) For each (μ, τ), the government chooses $\tau^{WO}(\mu, \tau)$ such that

$$\tau^{WO}(\mu,\tau) = \underset{\tilde{\tau}'}{\operatorname{argmax}} \sum_{i=1}^{N_\epsilon} \int \hat{V}(a, \epsilon_i; \mu, \tau : \tilde{\tau}')\mu(da, \epsilon_i) \tag{14}$$

where

$$\hat{V}(a, \epsilon_i; \mu, \tau : \tilde{\tau}') = \max_{c>0,\ a' \geq \underline{a},\ 0 \leq n \leq 1} \left[\frac{c^{1-\sigma}}{1-\sigma} + B\frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta \sum_{j=1}^{N_\epsilon} \pi^{\epsilon}_{i,j} v(a', \epsilon'_j; \mu', \tilde{\tau}') \right]$$

such that

$$c + a' = (1-\tau)\, w(\mu)\, \epsilon_i\, n + (1 + r(\mu)(1-\tau))\, a + T$$

$$\tau' = \tilde{\tau}', \text{ and thereafter } \tau'' = \Psi(\mu', \tau' = \tilde{\tau}') \tag{15}$$

$$\mu' = \Gamma(\mu, \tau, \tilde{\tau}), \text{ and thereafter } \mu'' = \Gamma(\mu', \tilde{\tau}, \tau'' = \Psi(\mu', \tau' = \tilde{\tau}')) \tag{16}$$

(iii) $a' = \hat{g}_a(a, \epsilon_i; \mu, \tilde{\tau} : \tilde{\tau}')$ and $n = \hat{g}_n(a, \epsilon_i; \mu, \tilde{\tau} : \tilde{\tau}')$ solve (14) at prices that clear markets and the government budget constraint, and Γ is consistent with individual decisions and the stochastic process of ϵ_i.

(iv) For each (μ, τ), the policy outcome function satisfies $\Psi(\mu, \tau) = \tau^{WO}(\mu, \tau)$.

In the economy with the optimal income tax without commitment, the government implements the time-consistent optimal policy: a tax rate that is sequentially chosen only for the next period while maximizing its utilitarian welfare under this commitment constraint. Note that the government cannot commit to the future tax rate from the period after the next period. Thus, once a chosen tax rate $\tilde{\tau}'$ deviates from the equilibrium tax policy function $\Psi(\cdot)$, tax rates thereafter follow the equilibrium tax policy function $\Psi(\cdot)$ because the government cannot commit to the future tax policy after one period. (15) presents such dynamics. The law of motion for the distribution of households $\Gamma(\cdot)$ has to capture all the changes in the evolution of distributions caused by the deviation of the income tax from the equilibrium tax function, which is shown in (16). In equilibrium, for each aggregate state (μ, τ), the government's choice $\tau^{WO}(\mu, \tau)$ should be equal to the equilibrium tax function $\psi(\mu, \tau)$, which is presented in (iv).

Definition 2.3.3. ***A Political RCE with Voting.***

(i) A set of functions $\{w(\cdot), r(\cdot), g_a(\cdot), g_n(\cdot), v(\cdot), \Gamma(\cdot)\}$ satisfy the RCE definition 2.2.1.

(ii) For each $(a, \epsilon_i; \mu, \tau)$, households choose $\psi(a, \epsilon_i; \mu, \tau)$ such that

$$\psi(a, \epsilon_i; \mu, \tau) = \underset{\tilde{\tau}'}{\operatorname{argmax}} \sum_{i=1}^{N_\epsilon} \int \hat{V}(a, \epsilon_i; \mu, \tau : \tilde{\tau}')\mu(da, \epsilon_i) \tag{17}$$

where

$$\hat{V}(a, \epsilon_i; \mu, \tau : \tilde{\tau}') = \max_{c>0,\ a'\geq \underline{a},\ 0\leq n\leq 1} \left[\frac{c^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta \sum_{j=1}^{N_\epsilon} \pi^{\epsilon}_{i,j} v(a', \epsilon'_j; \mu', \tilde{\tau}')\right]$$

such that

$$c + a' = (1-\tau)\, w(\mu)\, \epsilon_i\, n + (1 + r(\mu)(1-\tau))\, a + T$$

$$\tau' = \tilde{\tau}',\ \text{and thereafter } \tau'' = \Psi(\mu', \tau' = \tilde{\tau}') \tag{18}$$

$$\mu' = \Gamma(\mu, \tau, \tilde{\tau}),\ \text{and thereafter } \mu'' = \Gamma(\mu', \tilde{\tau}, \tau'' = \Psi(\mu', \tau' = \tilde{\tau}')). \tag{19}$$

(iii) For each (μ, τ), the median voting outcome $\tau^M(\mu, \tau)$ satisfies

$$\sum_{i=1}^{N_\epsilon} \int_{\{\psi(a,\epsilon;\mu,\tau)\leq \tau^M(\mu,\tau)\}} \mu(da, \epsilon) \geq \frac{1}{2} \tag{20}$$

$$\sum_{i=1}^{N_\epsilon} \int_{\{\psi(a,\epsilon;\mu,\tau)\geq \tau^M(\mu,\tau)\}} \mu(da, \epsilon) \geq \frac{1}{2}. \tag{21}$$

(iv) For each (μ, τ), the policy outcome function satisfies $\Psi(\mu, \tau) = \tau^M(\mu, \tau)$.

The political economy with sequential voting follows a dynamic game between two political parties, as in Krusell and Rios-Rull (1999). These parties compete with one another to take power, and the winner is determined bymajority voting by households on income taxes that the two parties proposed for each period—a lack of commitment. One-dimensional voting with a single-peaked preference leads the most preferred policy of the median voters to be supported by the majority. As a result, the dominant strategy of these two parties is a policy preferred by the median voter. To avoid multiple equilibria, I assume that one party always wins when the votes are tied.

Condition (ii) implies that each household solves the one-time deviation problem in (17), resulting in $\psi(\cdot)$, the most preferred tax of households associated with a state of $(a, \epsilon_i; \mu, \tau)$. As in the case with the optimal policy without commitment, a lack of government commitment makes households believe that future tax rates after one period will follow a sequence of income taxes induced by the equilibrium tax policy function $\Psi(\cdot)$ as shown in (18). The law of motion for the distribution of households has to capture all changes in the evolution of these distributions caused by the one-time deviation problem of households, which is presented in (19).

Following Corbae et al. (2009), I use condition (iii) to define the median voter. I sort the agents by the most preferred tax rate of households $\psi(\cdot)$ and find $\tau^M(\cdot)$ for each (μ, τ). Condition (iv) implies that in the political equilibrium, the median voting outcome $\tau^M(\mu, \tau)$ should be equal to the equilibrium tax function $\Psi(\mu, \tau)$ for each (μ, τ).

3 Numerical Solution Algorithm

Here, I focus on conveying the key ideas of the numerical solution algorithm. Appendix A demonstrates each step of the algorithm with details.

Solving the model entails a substantial computational burden. The law of motion for the distribution of households $\Gamma(\cdot)$ has to be consistent with individual decisions. Additionally, because the labor supply is endogenous with wealth effects, the two factor markets—K and N—must clear. Furthermore, perhaps the most challenging part is finding the equilibrium policy function $\Psi(\cdot)$ that should be determined according to the political and commitment structure while consistent with individual decisions and the law of motion for the distribution of households. In other words, three equilibrium objects—individual decisions, the law of motion for the distribution $\Gamma(\cdot)$, and the policy function $\Psi(\cdot)$—interact and have to be consistent with one another in a Markov-perfect equilibrium.

I address the above computational issues by taking ideas from the backward induction method of Reiter (2010). The author introduced a non-simulation-based solution method to solve an incomplete markets economy with aggregate uncertainty. As in Krusell and Smith's (1998), Reiter's (2010) approach also reduces the dimension of distributions in the law of motion $\Gamma(\cdot)$ to some finite moments of the distribution, and it is defined across the aggregate finite grid points. However, the way of finding $\Gamma(\cdot)$ is differs substantially between the two methods. In Krusell and Smith (1998), their algorithm repeatedly simulates the model economy through the inner and outer loops. In the inner loops, the value is solved given a perceived law of motion for the distribution of households, and the law of motion is updated after a simulation in the outer loop. This procedure is repeated until the perceived law of motion is equal to the updated one.

By contrast, the backward induction method of Reiter (2010) does not simulate the economy to update the law of motion for the distribution of households $\Gamma(\cdot)$; rather, this is updated while solving for the value given a set of proxy distributions across the aggregate finite grid points. Given a proxy distribution, finding the law of motion for the distribution of households $\Gamma(\cdot)$ is feasible by using the moment-consistent conditions. For example, individual decision rules for assets allow me to obtain the information (e.g., mean or variance) on the aggregate capital in the next period. A simulation step is followed not to update the law of motion for the distribution of households $\Gamma(\cdot)$ but to update a set of proxy distributions across the finite nodes in the aggregate state. Simulations are much less required in Reiter (2010) than in Krusell and Smith (1998), which improves computational efficiency for the backward Induction method. Additionally, with these proxy distributions, the backward induction method allows me to approximate not only the aggregate law of motion for the distribution $\Gamma(\cdot)$ but also the tax policy function $\Psi(\cdot)$ consistent with the political and commitment structure. This is feasible because, with the value function, these endogenous tax functions can be directly obtained by solving (11), (14), and (17).

However, I wish to clarify that I cannot directly apply the Reiter's (2010) method to the model in this paper because of the existence of off-equilibrium paths. In the incomplete markets economy with aggregate uncertainty, for which Reiter's (2010) method is originally designed, the distribution of aggregate shocks (TFP) Z is stationary. Thus, all the aggregate states Z are not measure zero. With a positive probability, all the states in Z are realized on the equilibrium path. However, an economy in Markov-perfect equilibrium does not have this property. For example, in the political economy with sequential voting, the vote on policies are obtained by comparing one-time deviation policies. Some tax paths would not be reached on the equilibrium path.

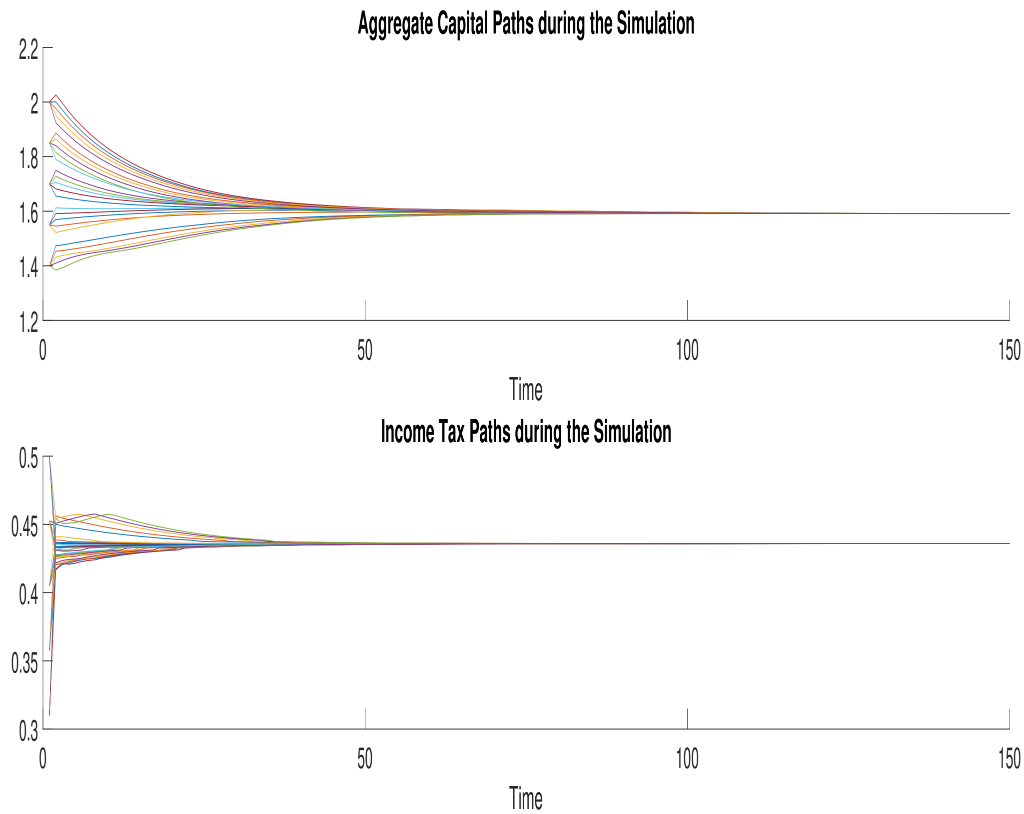

Figure 1: Transitions from off the Equilibrium to the Equilibrium

To cope with this issue, I make three variations to the original backward induction method of

Reiter (2010). First, as mentioned above, I approximate not only the aggregate law of motion for the distribution of households but also the endogenous tax policy function. I find these mappings in a nonparametric way as in Reiter (2010). Second, I arrange distributions for all types of off the equilibrium paths, taking the initial distribution of the simulations as the previous proxy distribution for each finite grid point of the aggregate state. Figure 1 shows various transitions from off the equilibrium to the steady-state equilibrium in the political economy with voting. Finally, I modify the way of constructing reference distributions, which is required to update the proxy distributions in Reiter (2010), by reflecting the features of the Markov-perfect equilibrium, in which how many times a tax rate off the equilibrium takes place is unknown before simulation. Appendix A demonstrates the full details of the solution method, with its performances in efficiency and accuracy.

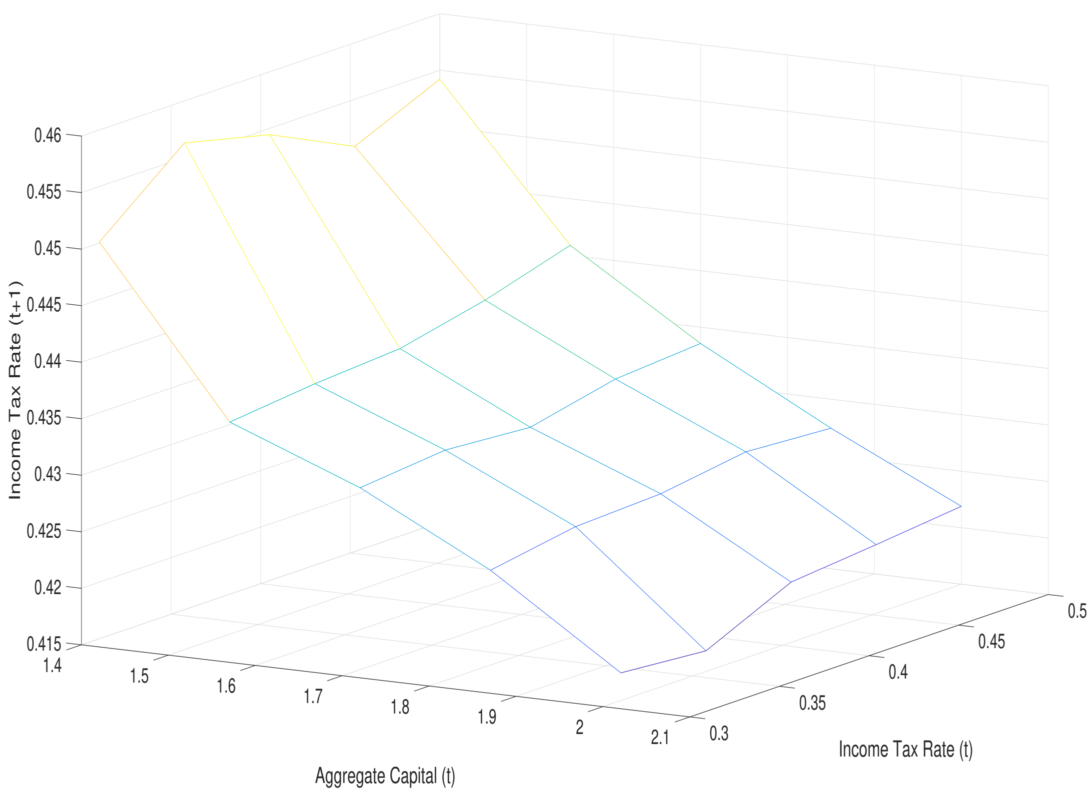

Figure 2: Income Tax Function $\Psi(m_i, \tau_k)$

Because of these somewhat complex variations in Reiter's (2010) method, one might consider simply using Krusell and Smith's (1998) method to solve this model. However, their approach would not be efficient in addressing this class of models in Markov-perfect equilibrium. First, finding the two aggregate laws of motion–Γ and Ψ—is computationally very costly when using this simulation-based solution method. When this method is employed to solve the economy in this paper, this process is the same as adding another outer loop to the outer loop in Krusell and Smith's (1998) original algorithm, thereby exponentially increasing the computational burden. Second, the parametric assumption of Krusell and Smith's (1998) approach would act as a barrier because the equilibrium tax function $\Psi(\cdot)$ could be severely nonlinear in the aggregate state. The parametric assumption works well when the law of motion for household distributions $\Gamma(\cdot)$ is close to linear. I find that although this linearity still appears in $\Gamma(\cdot)$, $\Psi(\cdot)$ shaped by the median voter's choice is severely nonlinear, as shown in Figure 2.[2]

4 Calibration

I calibrate the model to capture the features of the U.S. economy. I divide the parameters into two groups. The first set of the parameters requires solving the stationary distribution of the model to match moments generated by the model with their empirical counterparts. The other set of the parameters is determined outside the model. I take the values of these parameters from the macroeconomic literature and policies.

Table 1: Parameter Values of the Baseline Economy

	Description (Target)	Value
β	Discount factor ($K/Y = 3$)	0.951
B	Utility of leisure (AVG Wrk Hrs = 1/3)	3.803
σ	Relative risk aversion	2
χ	Frisch elasticity of labor supply	0.75
$\underline{a}$	Borrowing constraint	0
θ	Capital income share	0.36
δ	Depreciation rate	0.08
ρ_ϵ	Persistence of wage shocks	0.955
σ_ϵ	STD of wage shocks	0.20
G	Government spending	$G/Y = 0.19$
τ	AVG income tax	0.31

Table 1 displays the parameters. I internally calibrated two parameters: the discount factor β and the utility of leisure B. β is selected to match a capital to output ratio of 3, and B is chosen to reproduce an average hours worked of 8 hours a day. The other parameters are determined outside the model. The coefficient of relative risk aversion is set to 2. The Frisch elasticity of labor supply χ is taken to be 0.75. I set the borrowing constraint $\underline{a} = 0$. The capital income share θ is chosen to

[2] Corbae et al. (2009) employed Krusell and Smith's (1998) method to solve a similar economy to mine but without wealth effects of labor supply. Such difficulties might lead them to omit wealth effects of labor supply although adding more states to the forecasting rules.

reproduce the empirical finding that the share of capital income is 0.36. The annual depreciation rate δ is 8 percent. The persistence of wage shocks ρ_ϵ is set to be 0.955, and the standard deviation of wage shocks σ_ϵ is taken as 0.20. The values of ρ_ϵ and σ_ϵ lie in the range of those frequently used in the literature. Government spending G is set up so that the fraction of government spending out of GDP is equal to 19 percent. The flat income tax rate is chosen as 0.31 in the baseline economy.

5 Results

Table 2: Redistribution Outcomes According to Political and Commitment Structure

	Baseline	OPT w/ Commit	OPT w/o Commit	Voting
AVG income Tax	0.31	0.440	0.478	0.457
Trans/Y	0.046	0.156	0.190	0.171
Tax rev/Y	0.236	0.346	0.380	0.361
Change in BOT 50% Inc sh. from before to after govt.	+7%	+25.4%	+31.7%	+28.1%

Table 2 compares government size and redistribution-related outcomes across the four economies. I measure the size of governments as the total tax revenue to output ratio. The government size is the largest in the economy with the time-consistent optimal policy without commitment, followed by the political economy with voting (without commitment), the economy with the time-inconsistent optimal policy with commitment, and the baseline economy. The other government size-related measures (the average income tax rate and the total transfers to output ratio) also show consistent results. This result implies that a lack of commitment leads to a larger government. Given that the government in the baseline economy commits the future income tax to the baseline tax rate, economies with commitment (Baseline and OPT w/ Commit) present smaller governments than do those without commitment (OPT w/o Commit and Voting), in line with findings in Krusell et al. (1996); Krusell and Rios-Rull (1999); Klein and Ríos-Rull (2003); Klein et al. (2008); Corbae et al. (2009).

Table 2 also shows how much the government intends to redistribute income to households via its policy instrument, according to the political and commitment structure. To quantify this channel, I employ the change in the bottom 50 percent income share from before to after government to measure the extent of government-driven income redistribution. Income before government denotes the sum of labor and capital incomes before tax, excluding transfers; income after government refers to the sum of labor and capital incomes after tax, including transfers. A more significant gap between these two types of incomes, for instance, means a more substantial income redistribution intended by the government. Furthermore, the change in the bottom 50 percent of income share from before to after government indicates how much the government redistributes income, especially for low-income households.

Table 2 implies that the size of government is positively related to the extent of income redistribution driven by the policy executor. The magnitude of income redistribution, measured by the change in the bottom 50 percent income share from before to after government, is the largest in the economy with the time-consistent optimal policy without commitment, followed by the political economy with sequential voting, the economy with the time-inconsistent optimal policy with commitment, and the baseline economy. This order is the same as that in the size of government. This consistency suggests that the government uses its tax/transfer system to redistribute income to households the magnitude of which it targets.

Table 3: Efficiency Outcomes According to Political and Commitment Structure

	Baseline	OPT w/ Commit	OPT w/o Commit	Voting
Y	0.681	0.587	0.559	0.575
K/Y	3	2.67	2.562	2.623
H	0.333	0.295	0.282	0.289

Table 3 shows efficiency-related outcomes. For economic efficiency, I take output Y as the measure. The ranking for efficiency is precisely the opposite of that in the size of government. Efficiency is most significant in the baseline economy, followed by the economy with the time-inconsistent optimal policy with commitment, the political economy with sequential voting, and the case with the time-consistent optimal policy with commitment. Other efficiency-related measures——the capital to output ratio and the average hours worked——are also ranked in the same order. The opposite ranking between government size and efficiency implies that governments implement a more redistributive tax/transfer policy at the expense of efficiency.

The balance struck by the government in this trade-off differs according to its political and commitment structure. The policy executor with the optimal policy with commitment place greater weight on efficiency than that with the optimal policy without commitment. This tendency is closely related to the political and commitment constraint these policy executors confront. Thinking of a finite game is helpful to understand the government's behavior. With commitment, because the government must adopt the time-inconsistent policy (commitment), it does not follow the time-consistent policy at any time. This commitment is preserved in a backward manner. Therefore, the predecessors can also choose the time-inconsistent optimal tax policy maximizing the utilitarian welfare function in the long run.

In contrast, the policy executor with the optimal policy without commitment places greater weight on redistribution than that with the optimal policy with commitment. In this economy, the policy executor can choose a tax rate only for the next period and cannot commit to any policy

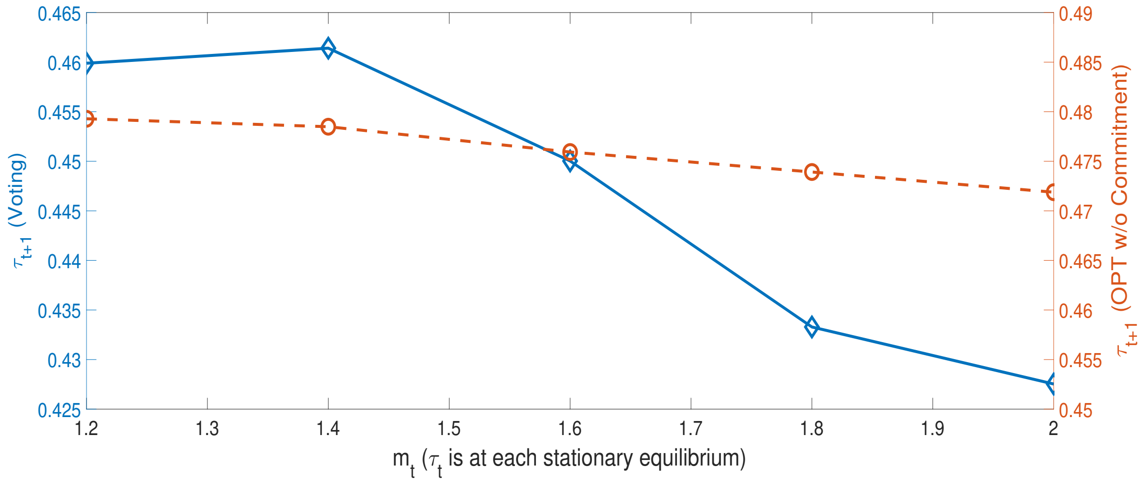

Figure 3: Response of the Income Tax Function to m_t

after that. Thinking of a finite game is helpful to understand the government's behavior. When the government is in the last period, a more generous policy is optimal because it does not have to be concerned about efficiency loss in the future. Because its predecessor knows this behavior of the successive government, the predecessor does not choose that highly frugal policy that is desirable in the long run with commitment. This strategic tendency is repeated in a backward manner throughout the game.

Note that the political economy with sequential voting places less weight on redistribution than the economy with the optimal policy without commitment. Although the political economy lacks commitment, as in the case with the optimal policy without commitment, the goal of political agents is very different. The objective of the two parties in this economy is to take power by winning majority support. When the voting-related policy instrument is one-dimensional, and preferences are single-peaked, offering a policy preferred by the median voter is the domestic strategy. As before, let us think of a finite repeated game. When the two parties are in the last period, their behavior is to provide the most preferred tax rate of the median voter, who is in the middle class and reluctant to accept a high tax rate with large transfers. This government behavior is repeated in a backward manner throughout the game.

Figure 3 shows how these strategic behaviors by the governments are reflected in their equilibrium income tax function $\Psi(m_i, \tau_k)$. I fix the current tax rate in each equilibrium and examine how the policy function changes in the current aggregate capital m_t. Because an increase in m_t leads to a rise in the market equilibrium wage in general equilibrium, Figure 3 shows how the governments' income tax rate endogenously responds to changes in the market equilibrium wage w. The dotted line is the income tax function in the economy with the time-consistent optimal policy. Their levels are overall higher than those in the political economy with voting, and its response to

$m_t\, w$ is smaller. These findings imply that this government embraces a large redistribution and is less sensitive to distortions. On the other hand, the solid line shows that the political economy with voting has lower levels of the income tax function and is more sensitive to distortions, reflecting the median voter's preference in the middle class. Therefore, given a lack of commitment, the policy in the political economy with voting tends to be less redistributive than the time-consistent optimal policy.

Table 4: Welfare Outcomes According to Political and Commitment Structure

	Baseline	OPT w/ Commit	OPT w/o Commit	Voting
Welfare (CEV)	-	+2.189%	+1.953%	+2.144%

This disparity results in a difference in welfare consequences, according to the government's political and commitment structure. Table 4 shows that welfare, measured by the consumption equivalent variation (CEV) of the utilitarian welfare function, is the highest in the economy with the time-inconsistent optimal policy (with commitment), followed by the political economy with voting and the economy with the time-consistent optimal policy. A notable result is that, given a lack of commitment, the political economy with sequential voting produces a better welfare outcome than the economy with the optimal, time-consistent policy. As highlighted in Figure 3, the time-consistent optimal policy tends to include that highly redistributive policy at the expense of efficiency, which plays a role in reducing welfare. However, in the political economy with voting, the government considers only the median voter's interest, who acts like an individual in the middle class and does not prefer enormous distortions from a more redistributive policy. These findings imply that, without commitment, democracy—pursuing policies supported by the middle class-—would be better than delicately-designed optimal policies for a broad group of people.

6 Conclusion

This paper examines how differences in the government's political and commitment structure affect the macroeconomy, inequality, and welfare. I develop a numerical solution method for models with a Markov-perfect equilibrium with heterogeneous agents and apply it to an Aiyagari's (1994) economy, in which its tax/transfer system is endogenously determined according to its government's political and commitment structure.

I find that, given a lack of commitment, the political economy with sequential voting shows a better welfare outcome than the economy with the optimal policy that is time-consistent. An absence of commitment leads the government to strategically adopt a more redistributive policy at the expense of efficiency. However, the government in the political economy with voting takes into account only the interest of the median voter, who is in the middle class and dislikes more distortions from a higher tax rate and larger transfers. These findings imply that, without commitment, adopting policies supported by the middle class would be better than implementing well-designed policies for a broad group of people.

Note that the solution method itself could provide many opportunities for studying unexplored research topics. Given the fundamental feature of Reiter (2010), this solution method can be compatible with aggregate uncertainty. This research direction would make it possible to revisit questions on fiscal policies according to the political and commitment structure. Another exciting application of the method is addressing the interactions between policies and life-cycle dimensions. Kim's (2021) method would make this direction reachable. She extends Reiter's (2010) backward induction method to solve an overlapping generations model with aggregate uncertainty. Such analyses are deferred to future work.

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Appendix A Numerical Solution Algorithm

Solving the Markov-Perfect Equilibria (MPE) of consecutive governments entails heavy computational burdens with heterogeneous agents. As in standard macroeconomic heterogeneous agent models, individual decisions should be consistent with the aggregate law of motion for the distribution of agents. On top of that, the aggregate tax policy function must be compatible with individual decisions and the aggregate law of motion for the distribution of agents. In other words, these three equilibrium objects—individual decisions, the law of motion for the distribution, and the tax policy function—have to be consistent with each other in the Markov-perfect equilibrium.

I address this computational issue by taking ideas from the Backward Induction method of Reiter (2010). This method discretizes the aggregate state into finite grid points. For each aggregate grid point, the Backward Induction algorithm allows updating the aggregate law of motion while solving the decision rules thanks to the existence of the proxy distribution. This means that for each aggregate grid point, the backward induction algorithm would make it possible to approximate not only the aggregate law of motion for the distribution; but also the tax policy function consistent with the voting outcome or optimal policy without government commitment. With the value function, this endogenous tax policy outcome can be directly obtained when the proxy distribution is explicitly available.

Unfortunately, the original Reiter's (2010) method cannot directly be applied to the MPE models because the existence of off the equilibrium paths makes it challenging to arrange the proxy distribution. In the model of Krusell and Smith (1998), for which Reiter's (2010) method is originally designed, the distribution of TFP shocks Z is stationary, thus all the aggregate states Z are not measure zero. With a positive probability, all the states Z are realized on the equilibrium path. However, the MPE economy does not have this property. Let us think about a political economy with sequential voting and its stationary distribution. In this political equilibrium, the voted policies are obtained by comparing among one-time deviation policies. Some tax paths would not be reached at all on the equilibrium path.

I have three variations from the original backward induction method. First, I have to approximate not only the aggregate law of motion for distributions but also the tax policy function that is endogenous. I find these mappings in a non-parametric way, as in Reiter (2010). Second, I arrange distributions for all types of off the equilibrium paths, taking the initial distribution of the simulations as the previous proxy distribution for each aggregate state. Finally, I modify the way of constructing the reference distributions in Reiter (2002, 2010), reflecting the features of economies in the MPE wherein how many times a policy off the equilibrium takes place is unknown before simulations.

Here, I show how to apply the algorithm to the political economy with sequential voting, which is the most complicated and informative in the three economies. Note that I solve all the value functions in the following steps with the Endogenous Grid Method of Carroll (2006).

A.1 Notation and Sketch of the Solution Method

The aggregate law of motion Γ and the tax policy function Ψ are evolved with the distribution μ that is an infinite dimensional equilibrium object, and thus it not not feasible in computations. To handle this issue, the Backward Induction method replaces μ with m, a set of moments from the distribution and discretize it. Here, I take the mean of the distribution and discretize it, $M = \{m_1, \cdots, m_{N_m}\}$. Furthermore, I discretze the tax policy, $T = \{\tau_1, \cdots, \tau_{N_\tau}\}$. This setting allows me to define the aggregate law motion and the tax policy function on each grid (m_{i_m}, τ_{i_τ}) such that $m' = G(m_{i_m}, \tau_{i_\tau}, \tau')$ where $\tau' = P(m_{i_m}, \tau_{i_\tau})$. Note that G and P do not rely on a parametric law.

Across a grid of aggregate states (m_{i_m}, τ_{i_τ}), each point selecting a proxy distribution, the Backward Induction method simultaneously solves for households' decision rules and an intratemporally consistent end-of-period distribution. This implies a future approximate aggregate state consistent with households' expectation ($m' = G(m_{i_m}, \tau_{i_\tau}, \tau')$). Likewise, the backward induction can find the tax policy function that is consistent with the voting outcome, by using household's value functions and the proxy distribution ($\tau^m = \tau' = P(m_{i_m}, \tau_{i_\tau})$). Theses mappings imply that G interacts with P. Given P, first, I find G during the iteration of value functions, and then update P with the value function and proxy distribution (voting). I repeat this until P is convergent.

Given a distribution over individual states at each aggregate grid point (m_{i_m}, τ_{i_τ}), my goal is to obtain the law of motion for households distribution G and the tax policy function P that are intratemporally consistent with the end-of-period distribution and the voting outcome. Explicitly,

$$m' = G(m_{i_m}, \tau_{i_\tau}, \tau') \tag{22}$$

$$\tau' = P(m_{i_m}, \tau_{i_\tau}) \tag{23}$$

$$\tau' = \tau^m(m_{i_m}, \tau_{i_\tau}) \tag{24}$$

$$w = W(m_{i_m}, \tau_{i_\tau}) \tag{25}$$

$$T = TR(m_{i_m}, \tau_{i_\tau}) \tag{26}$$

(22) is to approximate Γ, (23) is to do Ψ, (24) is for the voting outcome, (25) is the mapping for the market wage, and (26) is the mapping for trasnfers.

The backward induction method explicitly computes G, P, τ^m, W, and TR, given a set of proxy distributions before the simulation step. An issue is that computing $G(m_{i_m}, \tau_{i_\tau}, \tau')$ in solving the value is costly because it depends on τ' not only on the equilibrium path but also off the equilibrium path. To address this issue, I reduce $G(m_{i_m}, \tau_{i_\tau}, \tau')$ into $\tilde{G}(m_{i_m}, \tau_{i_\tau}) = G(m_{i_m}, \tau_{i_\tau}, P(m_{i_m}, \tau_{i_\tau}))$

while solving the value function; retrieve $G(m_{i_m}, \tau_{i_\tau}, \tau')$ with the converged value function and the proxy distribution. Note that $G(m_{i_m}, \tau_{i_\tau}, \tau')$ must also satisfy an intratemporal consistency.

A.2 Computing the Aggregate Mappings given a Set of Proxy Distributions

(1) Given $v^n(a, \epsilon; m, \tau)$ and $\tau' = P^q(m, \tau)$, where $n = 1, 2, \cdots$ and $q = 1, 2, \cdots$ denote the rounds of iteration, at grid (m_{i_m}, τ_{i_τ}), where $i_m = 1, \cdots, N_m$ and $i_\tau = 1, \cdots, N_\tau$ are grid indexes, solve for intratemporally consistent m'.

a) Guess m' . Using v^n and P^q, solve for $a' = g_a^{n+1}(a, \epsilon_i; m_{i_m}, \tau_{i_\tau})$ and $n = g_n^{n+1}(a, \epsilon_i; m_{i_m}, \tau_{i_\tau})$ using

$$v^{n+1}(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}) = \max_{c, a', n} u(c, 1-n) + \beta \sum_{j=1}^{N_\epsilon} v^n(a', \epsilon_j, m', \tau') \tag{27}$$

such that

$$c + a' = (1 - \tau_{i_\tau}) w(m_{i_m}, \tau_{i_\tau}) \epsilon_i n + (1 + (1 - \tau_{i_\tau}) r(m_{i_m}, \tau_{i_\tau})) a + T(m_{i_m}, \tau_{i_\tau})$$

$$\tau' = P^q(m_{i_m}, \tau_{i_\tau})$$

b) Using the proxy distribution, $\mu(a, \epsilon_i; m_{i_m}, \tau_{i_\tau})$, compute the distribution consistent with capital stock in the end of period $\tilde{m}'$, wage $\tilde{w}$, and transfers $\tilde{T}$.

$$\tilde{m}' = \sum_{i=1}^{N_\epsilon} \int g_a^{n+1}(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}) \mu(da, \epsilon_i; m_{i_m}, \tau_{i_\tau}) \tag{28}$$

$$\tilde{w} = (1 - \theta) \left(\frac{m_i}{N} \right)^{\theta} \tag{29}$$

$$\tilde{T} = \tau_{i_\tau} (r(m_{i_m}, \tau_{i_\tau}) m_i + w(m_{i_m}, \tau_{i_\tau}) N) \tag{30}$$

where

$$N = \sum_{i=1}^{N_\epsilon} \int g_n^{n+1}(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}) \epsilon_i \, \mu(da, \epsilon_i; m_{i_m}, \tau_{i_\tau})$$

c) If $\max \left\{ |\tilde{m}' - m'|, |\tilde{w} - w|, |\tilde{T} - T| \right\}$ >precision, update $m', w,$ and T; set $r = \theta \left(\frac{w}{1-\theta} \right)^{\frac{\theta - 1}{\theta}} - \delta$; and return to a).

(2) Having found $m' = \tilde{G}^q(m_{i_m}, \tau_{i_\tau})$, $w = W^q(m_{i_m}, \tau_{i_\tau})$, and $T = TR^q(m_{i_m}, \tau_{i_\tau})$, use (27)

to define $v^{n+1}(a, \epsilon; m, \tau)$ consistent with $v^n(a', \epsilon'; G^q(m_{i_m}, \tau_{i_\tau}), P^q(m_{i_m}, \tau_{i_\tau}))$. If $||v^{n+1} - v^n|| >$ precision, $n = n + 1$ and return to (1).

(3) For each aggregate grid $(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, retrieve $G^q(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ by solving for intratempollay consistent $\hat{m}'$.

a) For each $(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, guess $\hat{m}'$. With v^∞, solve for $a' = \hat{g}_a(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ and $n = \hat{g}_n(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ using

$$\hat{v}(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau}) = \max_{c, a', n} u(c, 1 - n) + \beta \sum_{j=1}^{N_\epsilon} v^\infty(a', \epsilon_j, m', \tau'_{i_\tau})$$

such that

$$c + a' = (1 - \tau_{i_\tau})\hat{w}(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})\epsilon_i n + (1 + (1 - \tau_{i_\tau})\hat{r}(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})a + \hat{T}$$

b) For each $(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, using the proxy distribution, $\mu(a, \epsilon_i; m_{i_m}, \tau_{i_\tau})$, compute the distribution consistent with the end of period aggregate capital stock.

$$\tilde{m}' = \sum_{i=1}^{N_\epsilon} \int \hat{g}_a(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})\mu(da, \epsilon_i; m_{i_m}, \tau_{i_\tau})$$

$$\tilde{w} = (1 - \theta)\left(\frac{m_i}{N}\right)^\theta$$

$$\tilde{T} = \tau_{i_\tau}(\hat{r} m_i + \hat{w} N)$$

where

$$N = \sum_{i=1}^{N_\epsilon} \int \hat{g}_n(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})\epsilon_i \, \mu(da, \epsilon_i; m_{i_m}, \tau_{i_\tau})$$

c) If $\max\left\{|\tilde{m}' - \hat{m}'|, |\tilde{w} - \hat{w}|, |\tilde{T} - \hat{T}|\right\} >$ precision, update $\hat{m}', \hat{w}$, and $\hat{T}$; set $\hat{r} = \theta\left(\frac{\hat{w}}{1-\theta}\right)^{\frac{\theta-1}{\theta}} - \delta$; and return to a).

(4) Having found $m' = G^q(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, keep $G^q(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$. Note that here is no update of the value.

(5) For each aggregate grid (m_{i_m}, τ_{i_τ}), find $\tau^{m,q}(m_{i_m}, \tau_{i_\tau})$.

a) Given $(a, \epsilon; m_{i_m}, \tau_{i_\tau})$, using $\hat{v}(a, \epsilon_i; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ in (3) - a), solve $\psi^q(a, \epsilon, m, \tau)$ as follows:

$$\psi^q(a,\epsilon;m_{i_m},\tau_{i_\tau}) = \underset{\tilde{\tau}'}{\operatorname{argmax}}\, \hat{v}(a,\epsilon_i;m_{i_m},\tau_{i_\tau},\tilde{\tau}') \tag{31}$$

The golden section search is used to find $\psi^q(a,\epsilon;m_{i_m},\tau_{i_\tau})$ with a cubic spline for $\hat{v}$ over τ'.

b) For each aggregate grid (m_{i_m},τ_{i_τ}), using the proxy distribution $\mu(a,\epsilon_i;m_{i_m},\tau_{i_\tau})$, compute the policy outcome $\tau^{m,q}(m_{i_m},\tau_{i_\tau})$ that satisfies

$$\sum_{i=1}^{N_\epsilon}\int_{\{\psi^q(a,\epsilon;\tau,\mu)\leq\tau^{m,q}(m_{i_m},\tau_{i_\tau})\}}\mu(da,\epsilon_i;m_{i_m},\tau_{i_\tau}) \geq \frac{1}{2} \tag{32}$$

$$\sum_{i=1}^{N_\epsilon}\int_{\{\psi^q(a,\epsilon;\tau,\mu)\geq\tau^{m,q}(m_{i_m},\tau_{i_\tau})\}}\mu(da,\epsilon_i;m_{i_m},\tau_{i_\tau}) \geq \frac{1}{2} \tag{33}$$

$$\tag{34}$$

c) For each aggregate grid (m_{i_m},τ_{i_τ}), if $P^q(m_{i_m},\tau_{i_\tau}) = \tau^{m,q}(m_{i_m},\tau_{i_\tau})$, G^q and P^q are the solutions, given the proxy distribution. Then, go to the next step. Otherwise, they are not the solutions. Take $P^{q+1} = \omega\cdot P^q + (1-\omega)\cdot\tau^{m,q}$, and go back to (1).

A.3 Constructing the Reference Distributions

Until now, I have solved G and P for a given set of proxy distributions. In the following step, I will simulate the economy and update the distribution selection function, as in Reiter (2002, 2010); but, the simulation step in this paper is substantially different from that in his method. He addresses Krusell and Smith (1998) model where aggregate uncertainty exists. Thus, what matters in his papers is to obtain the Ergodic set that is not affected by the initial distribution.

However, in economies without government commitment, it is important to obtain not only distributions on the equilibrium path but also those off the equilibrium path. For example, let us think of a political economy with sequential voting in the stationary equilibrium. Then, there will be a unique value of $\tau^* = P(m^*,\tau^*)$ and $m^* = G(m^*,\tau^*,\tau^*)$. In this case, I may not know the value of other alternatives because this economy has nothing but the unique equilibrium path. This difficulty might lead the previous studies to employ local solution methods in solving this type of the MPE. By constrast, my approach is a global solution method, which means I need proxy distributions over all types of off the equilibrium paths.

To reserve distributions off the equilibrium path, I use the proxy distributions in the previous step as the initial distribution for the simulation. For each (m_{i_m},τ_{i_τ}), I run a simulations for T

periods from the proxy distribution $\mu_0 = \mu(a, \epsilon; m_{i_m}, \tau_{i_\tau})$, implying the number of simulations is $N_m \times N_\tau$ and that of simulation outcomes is $T \times N_m \times N_\tau$. Note that any type of (m_{i_m}, τ_{i_τ}) will be observed at least once in the simulations. For each (m_{i_m}, τ_{i_τ}), using $\mu_0 = \mu(a, \epsilon; m_{i_m}, \tau_{i_\tau})$ and v^∞ from the previous step, I simulate the economy in a forward manner. I compute the market cleared w_t and r_t and transfers T_t satisfying the government budget condition for each simulation period $t = 1, \cdots, T$. In addition, I solve the median voting rule τ_t^m for each simulation period $t = 1, \cdots, T$ with the $m' = G(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ obtained in the previous step.

I gather all the simulated distributions and rearrange the index as $\tilde{t} = 1, \cdots, T \times N_m \times N_\tau$. In creating the reference distributions from the simulation, I need a measure of distance for the moments of a distribution. For (m, τ), define an inverse norm

$$d((m_0, \tau_0), (m_1, \tau_1)) = (m_0 - m_1)^{-4} + (\tau_0 - \tau_1)^{-4} \tag{35}$$

In contrast to an economy with uncertainty, the initial simulation results should be preserved, having to be used to construct the reference distributions off the equilibrium path (non-Ergodic set). For each t, when (m_t, τ_t) with $m_t \in [m_k, m_{k+1})$ and $\tau_t \in [\tau_s, \tau_{s+1})$,

$$\begin{aligned}
d_1(m_k, \tau_s) &= d_1(m_k, \tau_s) + (m_t - m_k)^{-4} + (\tau_t - \tau_s)^{-4} \\
d_1(m_{k+1}, \tau_s) &= d_1(m_{k+1}, \tau_s) + (m_t - m_{k+1})^{-4} + (\tau_t - \tau_s)^{-4} \\
d_1(m_k, \tau_{s+1}) &= d_1(m_k, \tau_{s+1}) + (m_t - m_k)^{-4} + (\tau_t - \tau_{s+1})^{-4} \\
d_1(m_{k+1}, \tau_{s+1}) &= d_1(m_{k+1}, \tau_{s+1}) + (m_t - m_{k+1})^{-4} + (\tau_t - \tau_{s+1})^{-4}
\end{aligned}$$

Above m_k (τ_s) is the k-th (s-th) grid point for m (τ). Note that distances between a given node and non-adjacent moments are not taken into account, which is different from the corresponding step in Reiter (2002, 2010).

I construct the reference distributions for each (m_{i_m}, τ_{i_τ}) using the above, when $(m_{\tilde{t}}, \tau_{\tilde{t}}) \in \Big([m_{i_m}, m_{i_m+1}), [\tau_{i_\tau}, \tau_{i_\tau+1})\Big)$,

$$\mu^r(a, \epsilon; m_{i_m}, \tau_{i_\tau}) = \sum_{\tilde{t}=1}^{T \times N_m \times N_\tau} \frac{d((m_{i_m}, \tau_{i_\tau}), (m_{\tilde{t}}, \tau_{\tilde{t}}))}{d_1(m_{i_m}, \tau_{i_\tau})} \mu_{\tilde{t}}(a, \epsilon). \tag{36}$$

Each reference distribution is a weighted sum of distributions over the simulation only when simulated moments are adjacent to a given pair of grid points (m_{i_m}, τ_{i_τ}). Since the simulation moments are not on an Ergodic set, this should be considered.

I arrange the finite grid, which is the distribution support, as explicit. The distribution over (a, ϵ) used below size $(N_a \times N_\epsilon)$ with $\epsilon \in E = \{\epsilon_1, \cdots, \epsilon_{N_\epsilon}\}$ and $a \in A = \{a_1, \cdots, a_{N_a}\}$.

I represent $\mu^r(a,\epsilon;m_{i_m},\tau_{i_\tau})$ using $\mu^r_{i_a,i_\epsilon}(i_m,i_\tau)$, indexing $(a_{i_a},\epsilon_{i_\epsilon})$ over $A\times E$ for (m_{i_m},τ_{i_τ}). The moment of a reference distribution, $\sum_{i_\epsilon}^{N_\epsilon}\mu^r_{i_a,i_\epsilon}(i_m,i_\tau)a_{i_a}$, will not be consistent with m_{i_m}. However, the proxy distribution at (i_m,i_τ) will have this property.

A.4 Updating the Proxy Distributions

Following Reiter (2002, 2010), for each aggregate grid (i_m,i_τ), I solve for μ_{i_a,i_ϵ}, the proxy distribution, as the solution to a problem that minimizes the distance to the reference distribution while imposing that each type of sums to its reference value and moment consistency.

$$\min_{\{\mu_{i_a,i_\epsilon}\}_{i_a=1,i_\epsilon=1}^{N_a,N_\epsilon}}\sum_{i_a=1}^{N_a}\sum_{i_\epsilon=1}^{N_\epsilon}\Big(\mu_{i_a,i_\epsilon}-\mu^r_{i_a,i_\epsilon}(i_m,i_\tau)\Big)^2 \tag{37}$$

$$\sum_{i_a=1}^{N_a}\mu_{i_a,1_\epsilon}=\sum_{i_a=1}^{N_a}\mu^r_{i_a,i_\epsilon}(i_m,i_\tau)\text{ for }i=1,\cdots,N_\epsilon \tag{38}$$

$$\sum_{i_\epsilon=1}^{N_\epsilon}\sum_{i_a=1}^{N_a}\mu_{i_a,i_\epsilon}\cdot a_{i_a}=m_{i_m} \tag{39}$$

$$\mu_{i_a,i_\epsilon}\geq 0 \tag{40}$$

The first-order condition for μ_{i_a,i_ϵ} with λ_i as the LaGrange multiplier for (38) and ω the multiplier (39) is

$$2(\mu_{i_a,i_\epsilon}-\mu^r_{i_a,i_\epsilon}(i_m,i_\tau))-\lambda_i-\omega\cdot a_{i_a}=0 \tag{41}$$

If I ignore the non-negative constraints for probabilities in (40), I have N_ϵ constraint in (38). 1 constraint in (39) and $N_a\times N_\epsilon$ first-order conditions in (40). These are a system of $N_a\times N_\epsilon+N_\epsilon+1$ linear equations in $\Big(\{\mu_{i_a,i_\epsilon}\}_{i_a=1,i_\epsilon=1}^{N_a,N_\epsilon},\{\lambda_{i_\epsilon}\}_{i_\epsilon}^{N_\epsilon},\omega\Big)$.

I construct a column vector **x**. The first block of **x** are the stack of the elements from the proxy distribution, such that $\mathbf{x}(j)=\mu_{i_a,i_\epsilon}$ where $j=(i_\epsilon-1)\times N_a+i_a$. Next are the N_ϵ multipliers λ_i, followed by one multiplier ω. I solve for **x** using a system of linear equations, $\mathbf{Ax}=\mathbf{b}$ in Figure 4. The non-zero element of **A** and **b** are described here. The coefficients for μ_{i_a,i_ϵ} are entered into **A** as

$$\mathbf{A}((i_\epsilon-1)\times N_a+i_a,(i_\epsilon-1)\times N_a+i_a)=2 \tag{42}$$

$$\mathbf{A}(N_\epsilon\times N_a+i_\epsilon,(i_\epsilon-1)\times N_a+i_a)=1\text{ for }i_\epsilon=1,\cdots,N_\epsilon \tag{43}$$

$$\mathbf{A}(N_\epsilon\times N_a+N_\epsilon+1,(i_\epsilon-1)\times N_a+i_a))=a_{i_a}. \tag{44}$$

The coefficient for λ_i are entered in $\mathbf{A}$, for $i_\epsilon = 1, \cdots N_\epsilon$ and $i_a = 1, \cdots, N_a$, as

$$\mathbf{A}((i_\epsilon - 1) \times N_a + i_a, N_\epsilon \times N_a + i_\epsilon) = -1 \tag{45}$$

The coefficients for ω sets the following elements of $\mathbf{A}$, for $i_\epsilon = 1, \cdots N_\epsilon$ and $i_a = 1, \cdots, N_a$,

$$\mathbf{A}((i_\epsilon - 1) \times N_a + i_a, N_\epsilon \times N_a + N_\epsilon + 1) = -a_{i_a}. \tag{46}$$

The elements of $\mathbf{b}$ are, for $i_\epsilon = 1, \cdots N_\epsilon$ and $i_a = 1, \cdots, N_a$,

$$\mathbf{b}((i_\epsilon - 1) \times N_a + i_a) = 2\mu^r_{i_a, i_\epsilon}(i_m, i_\tau) \tag{47}$$

$$\mathbf{b}(N_\epsilon \times N_a + i_\epsilon) = \sum_{i_a=1}^{N_a} \mu^r_{i_a, i_\epsilon}(i_m, i_\tau) \tag{48}$$

$$\mathbf{b}(N_\epsilon \times N_a + N_\epsilon + 1) = m_{i_m}. \tag{49}$$

I solve $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$ iteratively using an active set method corresponding to probabilities that are not set to 0.

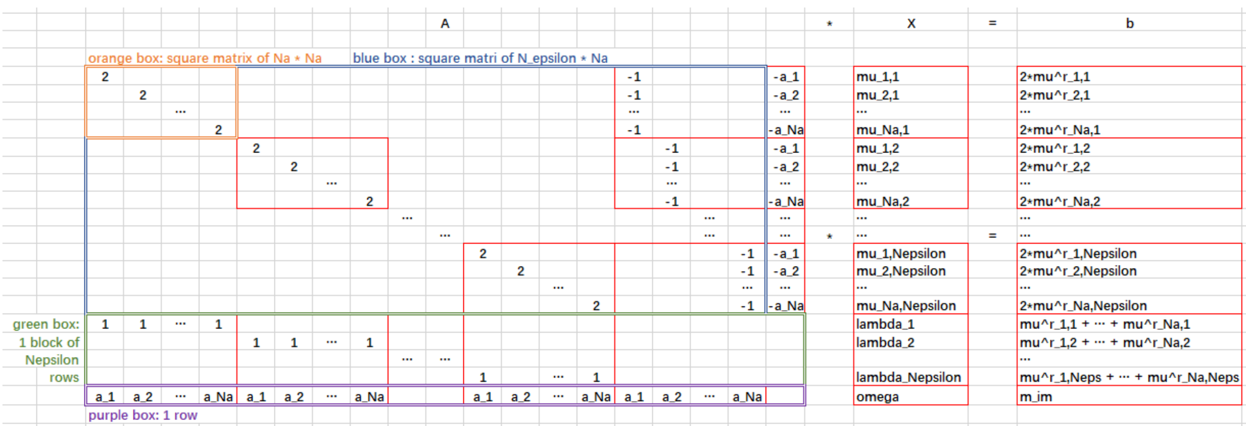

Figure 4: $\mathbf{A} \times \mathbf{x} = \mathbf{b}$

To solve the linear system, I use the active set approach to non-negative constraints in Reiter (2002, 2010). If any of the first $N_\epsilon \times N_a$ elements of $\mathbf{x}$ are negative, the constraint $\mu_{i_a, i_\epsilon} \geq 0$ has been violated for some $(i_\epsilon - 1)N_a + i_a = j \in J_0$ where

$$J_0 = \{j | 1 \leq j \leq N_a \times N_\epsilon \text{ and } \mathbf{x}(j) < 0\}. \tag{50}$$

For some $O > 0$, set the most negative O elements indexed in J_0 to 0, $\mu_{i_a, i_\epsilon} = 0$. Remove the $j - th$ row and column of A along with the $j - th$ element of $\mathbf{b}$. Solve the reduced system with O

less rows. If any of the $N_\epsilon \times (N_a - O)$ elements are negative, again discard the most negative O. I repeat this procedure until the most negative elements of $\mathbf{x}$ is larger than a precision level. This iteratively implements the non-negativity of probabilities (50).

Table 5: Setting for Computation

	num. of nodes	**Description**
N_a	400(400)	asset (distribution)
N_ϵ	10	persistence wage process
N_m	5	aggregate capital (aggregate)
N_τ	7	income tax (aggregate)

Table 5 shows the setting of the grids in this paper. With this setting, I continue to repeat the whole steps above until no improvement in accuracy statistic proposed by Den Haan (2010). I find that the mean errors on the equilibrium path are sufficiently small (considerably less than 0.6% for all cases) and the mean errors over transitions from off the equilibrium to the equilibrium are also reasonably small (not exceeding 0.6% for all cases). Furthermore, the method is substantially efficient in a usual personal computer.

Table 6: Accuracy and Efficiency of the Solution Method

	OPT w/o Commitment	Voting
Run time	11.1 min	15.8 min
DH of m at EQ	0.394%	0.539%
DH of w at EQ	0.048%	0.046%
DH of τ at EQ	0.153%	0.263%
AVG(DH) of m	0.668%	0.577%
AVG(DH) of w	0.251%	0.202%
AVG(DH) of τ	0.129%	0.244%
MAX(DH) of m	2.133%	2.44%
MAX(DH) of w	0.949%	0.935%
MAX(DH) of τ	0.415%	1.4%

$AVG(\cdot)$ and $MAX(\cdot)$ are computed with all of the results both on and off the equilibrium paths.
Processor: i7-10770 @ 2.9GHz, RAM: 16GB