Incentives and Strategic Behavior of Professional Boxers

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Abstract

This paper studies the strategic behaviour of professional boxers in choosing the opponent and sharing the revenues generated by the fight. In professional boxing, a higher-ranked boxer has an upper hand in choosing his opponent among many challengers varying in popularity and strength. We build a three-stage model of a professional boxing fight/bout between the chooser and the one of his challengers to examine the strategic incentives of a chooser in sharing the purse in Nash Bargaining framework and exerting proper level of effort within a contest theory model. More importantly, we endogenize the choice of the opponent and thus the purse to be generated by the bout. We characterize the factors affecting the choice of an "optimal" opponent and the effort level exerted by the chooser and the opponent. One interesting result of the paper is that an older chooser who is ready to cash in his reputation tends to choose a stronger opponent, but puts little effort into the fight. On the other hand, a young rising "star" in the boxing market prefers a match against weaker opponents in order to minimize his risk of losing and to maximize his record of the "winning" outcomes along with market values.

Keywords: boxing, incentives, contests, opponent choice, bargaining, game theory
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1 Introduction

Professional boxing industry is one of the most secretive ones in the world when it comes to the inaccurate records of money revenue inflows in it. There are several reasons for it. First of all,

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there is no uniform salary for all individuals involved in the organization of the bout. Especially boxers, acting as independent contractors (Chaplin et al. 2018), don't earn a regular salary and their earnings are not subject to minimum wage law. Second, the amount of the purse to be collected is mainly determined by the marketability of the fighters in the ring (Chaplin, 2012). Taxation, naturally, can be another reason. Last but not least, there is no specific calendar for the bout to organize. Usually, the top boxing "stars" don't fight in the ring more than twice a year and earn millions of dollars per fight, while those in the earlier careers may come to the ring 4 - 5 times a year and may not earn more than $1000 per fight.[1] Thus, the revenue from professional boxing varies greatly depending on experience, reputation or popularity, location etc. For example, Mayweather-Pacqiuao 2015 so-called "Fight of the Century" generated a total of $678 million revenue, from which Mayweather and Pacqiuao earned $223.5 million and $122 million, respectively, not counting the pay-per-view (PPV) and other cash sources.[2] In addition, HBO/Showtime PPVs broadcast between 2017 and 2018 generated approximately $700 million in the US market alone (Butler et al. 2020). Despite the popularity of pro-boxing sports along with the economy it generates, the literature in the sports economics using microeconomic tools to model it is surprisingly very limited.

In the sport of professional boxing one should be curious about the market value of the forthcoming bout between two fighters and its split between them. Put it differently, why do some bouts generate huge revenue and some less? Also, why does one fighter receive a larger share of the revenue than the other? Consider the purse history of Floyd Mayweather.[3] He faced De La Hoya, Canelo, and Pacquiao in 2007, 2013, and 2015, respectively. The respective total revenues from these fights are $187 mln, $214.7 mln and $678 mln. Excluding the cash from PPV and Gate entry, the purse splits (Mayweather vs opponent) were $20 mln vs $42 mln, $65 mln vs $9.7 mln, and $223.5 mln vs $122 mln, respectively. As noted earlier, there's no fixed revenue and fixed split though the rules for all fights are the same. The numbers clearly indicate how the revenue and purse split of Mayweather change for each opponent. Notice, the Mayweather-Pacquiao revenue in 2015 is more than three times that of Mayweather-De La Hoya in 2007 where the purse of Mayweather was twice less than that of his opponent. That is because De La Hoya seemed to be a bigger name than Mayweather at that time. On the one hand, upon announcing such big numbers in the cashier, one would expect that the boxers would put huge efforts in the fight in order to win (or earn or retain the title, say championship belt). On the other hand, once boxers' money shares from the fight is guaranteed, there are likely disincentives for extra effort

[1] https://themmaguru.com/how-much-money-do-professional-boxers-make/ and https://careertrend.com/salary-benefits-for-a-professional-boxer-13660492.html, accessed 25 October 2021.

[2] https://www.givemesport.com/1572094-floyd-mayweather-dominates-list-of-highest-grossing-boxing-fights-of-all-time, accessed 25 October 2021.

[3] https://www.dazn.com/en-GLOBAL/news/boxing/what-are-the-biggest-money-boxing-fights-in-history/1jevis9ex71na1xj24d5xgzzyz, accessed 25 October 2021.

in preparing for the fight, which create a moral hazard problem (Tenorio, 2000; Amegashie and Kutsati, 2005). Professional boxing matches such as these create interesting, possibly unexpected, scenarios that pose challenges to theoretical analysis.

Our paper is an attempt to understand the strategic incentives of boxers in sharing the money in Nash Bargaining framework and exerting proper level of effort within a contest theory model. More importantly, we take into consideration the decision to pick the optimal opponent among several alternatives challenging him. The existing literature is surprisingly silent about optimal opponent choice in spite of the fact that it is a very, if not the most, crucial decision of stakeholders in professional boxing.

Unlike other sports such as football, basketball, or tennis, there is neither a centralized governing body nor a structured and periodic tournament, league or match in professional boxing.[4] The existing problematic structure and ambiguous rules inevitably lead to strategic opponent choice not only at the top level but at all levels. Thus, in our paper we study the strategic opponent choice of a professional boxer and the relevant incentives shaping this choice. In professional boxing, usually the boxer who is higher in the rankings is challenged by the ones who are lower in the rankings. Then, the higher-ranked boxer chooses one of the challengers or refuses them all. Since the amount of money revenue is determined by reputation or popularity of both boxers, we argue that the higher-ranked boxer (we will call him[5] "chooser" or "champion" if he is at the top) decides whom to fight in the ring. Moreover, we intuitively assume that the fight with a more popular opponent will likely generate more money revenue than that with no-names. In supporting this argument, Butler et al. (2020) points out that fans prefer to watch more dominant boxers. In addition, Chaplin et al. (2017) finds that being a "superstar" is essential for live attendance in professional boxing. Thus, the main result of this paper highlights the importance of an opponent choice in money revenue, and shows that with the right names in the ring, the boxers (and promoters) are given the incentives to meaningfully negotiate on purse split and prepare for the upcoming bout accordingly.

We build a three-stage model of a professional boxing fight/bout between the chooser and one of his challengers from the opponent choice to the start of the bout. In the first stage, the chooser selects one of his challengers. Choosing a more popular challenger generates higher revenue, but it is also more costly in the sense that the likelihood of a loss is higher (Issabayev and Oskenbayev, 2019) since more popular opponents are usually stronger.[6] At the start of the second stage, the

[4] There are four major sanctioning bodies: the World Boxing Council (WBC), the World Boxing Association (WBA), the International Boxing Federation (IBF), and the World Boxing Organization (WBO). Since each of them organizes matches and ranks fighters in 17 different weight divisions within the body, there can be potentially four "world" boxing champions in each division at any given time, which reflects the unreasonableness of the current system of governance in professional boxing (Tenorio, 2006).

[5] We will call the boxers as "he" since professional boxing is dominantly a male combat sport. However, our analysis can be applied to women boxing in the same way.

[6] Although we assume this in the paper, being more popular does not necessarily imply being stronger. Choosers

names of the fighters are known. The shares of each fighter from the money revenue that will be generated from the fight is *endogenously* determined by the asymmetric Nash bargaining game (unlike Issabayev and Oskenbayev, 2019). We treat the popularity parameter as a *proxy* for bargaining power of each fighter, which are assumed to be asymmetric.

In the final stage, the fighters choose their efforts for the upcoming fight. We solve optimal efforts for each player. Given the names of the fighters in the first stage and the bargaining outcomes from the second stage, we consider the incentives of each fighter using a generalized Tullock contest (Tullock, 1980). We further assume the chooser has an edge over the opponent in transforming his efforts into a win probability as in Jia et al. (2013). It is also worth noting that the efforts can be considered as an investment with no return in the short term. In other words, the fighters are not paid for the efforts put in preparation for the forthcoming fight. However, the effort of a fighter is an investment with return in the long term. More precisely, the current efforts increase the probability of winning, which in turn would have a positive impact on future earnings.

Existing economics literature on boxing is very scarce and there are only two theoretical papers. Tenorio (2000) examines the boxer's incentives in choosing an effort level in a dynamic way. The reputation of the boxers determines the purse size and the payments to the boxers. However, due to the nature of the reward system in professional boxing, a boxer who is promised a fixed (especially high) payment has an incentive to shirk if his accumulated wealth is high enough. The boxers with many years ahead to fight are more likely to exert high effort, since the future payments will depend on past performances. Unlike ours, the focus of Tenorio (2000) is not to explain how the boxers are matched. Our paper, on the other hand, is modeling the strategic opponent choice process that in turn determines the purse size and characterizes the optimal sharing rule of this purse via Nash bargaining and optimal efforts.

The other paper studying the incentives in boxing is Amegashie and Kutsoati (2005) that points out the importance of the rematch clause. The authors show that if the winner is obliged to offer a rematch to the loser, i.e. a mandatory rematch clause, the aggregate effort level will be higher compared to the contracts that condition the rematch on the effort level. They consider neither opponent choice, nor purse size determination, nor optimal purse sharing. Although we are not considering the possibility of a rematch, as a future research, it would be interesting to see how the rematch possibilities (mandatory challenger or mandatory defence clauses that are common especially among high-ranked boxers) can change the strategic opponent choice and the optimal purse sharing rule.

Our paper has sought to add to the existing literature by focusing on the following research questions that are relevant in professional boxing: which factors affect the opponent choice in

such as Mayweather and Canelo, ceteris paribus, tend to avoid "no-name" formidable opponents like Paul Williams or Charlo.

professional boxing and how? How does the opponent choice impact the purse split decision between the fighters? For given fighters and purse split, how do fighters choose their optimal efforts to prepare for the match? Our model incorporates both strategic opponent choice and cooperative bargaining theory to explain the purse split, which lacks in the existing literature. Moreover, it is important to note that other than negotiating on purse split and deciding on how much effort to put, the opponent choice by a higher-ranked boxer is one of the most important decisions in professional boxing. The amount of purse, before it is split between the fighters, strongly depends on the popularity of the opponent. The steps of the model are dictated by the real-world anecdotes. To the best of our knowledge, it is the first paper that attempts to fill this missing gap in the existing literature.

While searching for answers to the above questions we came across with a number of interesting results, which obviously depend on the equilibrium concept we use. Under reasonable assumptions, we found that a higher minimum purse or outside option to the chooser implies a choice of relatively stronger opponent. But an additional victory premium (difference in the expected discounted future gains of a win and a loss) to both fighters or larger outside options to the opponent discourages the chooser to fight against a stronger opponent. We also documented that as long as the money purse of the chooser increases for picking a relatively stronger opponent, he is willing to sacrifice some share from the overall purse in favor of his opponent in order to make the fight happen. Furthermore, for a given opponent, the chooser is willing to put less effort for an additional victory premium of his opponent. However, the additional victory premium to the chooser tends to decrease the efforts of a given opponent. we also demonstrated that for a stronger opponent the chooser will be trying his best to win the bout. Finally, an older chooser in order to benefit from the purse tends to accept the challenge by a stronger opponent, but exerts lower efforts -abstracting from the age related physical deterioration. A young rising "star" in order to benefit more from a "winning record" in the long term is not ready to challenge a stronger opponent in the short term.

The rest of the paper is organized as follows. Section 2 introduces the model and characterization of the equilibrium. Section 3 describes the comparative statics and implications of the model. Finally, section 4 concludes with a discussion of the results.

2 The Model and its Analysis

We model the problem of a boxer as a three-stage game. Since the bout with the right opponent could generate big money revenue, the first and most important element of the professional boxing fight is the opponent.[7] Hence, in the first stage of the game, the higher-ranked boxer (the chooser) picks the right lower-ranked opponent among many challengers. Accordingly, the chooser has a

[7]https://bleacherreport.com/articles/2192887-making-money-how-floyd-mayweather-produces-boxings-biggest-fights, accessed 25 October 2021.

greater bargaining power in money share than the challenger.[8] Rankings are common knowledge. In the second stage, the boxers bargain over the total money amount to be shared. Finally, in the last stage, given the current purse, boxers choose their effort.

We now provide required definitions for the variables used in the model. Let us suppose that $0 \leq \alpha_i \leq 1$ is the popularity of a boxer for $i = c$ if the chooser and $i = o$ if the opponent. This α parameter can be considered as a proxy for popularity or inverse of ranking, for example, in International Boxing Federation (IBF) or World Boxing Association (WBA). To further simplify the analysis, we set the popularity of the chooser to one, $\alpha_c = 1$. Without loss of generality, we let $\alpha_o = \alpha$ so that the opponent's relative popularity or ability be $\frac{\alpha_o}{\alpha_c} = \alpha$. Thus, in the first stage, the chooser selects his opponent among his challengers, that is, he chooses an α. This choice essentially depends on the expected discounted values of future gains for the players based on the result of the bout. Namely,

$\overline{V}_c$ = Expected discounted value of future gains for the chooser if he wins,

$\underline{V}_c$ = Expected discounted value of future gains for the chooser if he loses,

$\overline{V}_o$ = Expected discounted value of future gains for the opponent if he wins,

$\underline{V}_o$ = Expected discounted value of future gains for the opponent if he loses.

We also define $\overline{V}_c - \underline{V}_c = V_c \geq 0$ and $\overline{V}_o - \underline{V}_o = V_o \geq 0$ for notational convenience. V_c (V_o), victory premium, is the extra premium that winning is expected to bring over losing for the chooser (opponent) that is always non-negative because winning the current fight cannot lower future prospects of a boxer. These can also be considered as proxies for the ambition or eagerness of a boxer to win. We also define v as the relative expected victory premium of the chooser, $v \equiv \frac{V_c}{V_o}$.

In the second stage, boxers play the cooperative Nash bargaining game. Total money amount to be shared is a function of α and takes the following form:

$$M(\alpha) = \underline{M}(1 + k\alpha)$$

where $M(0) = \underline{M}$, $M(1) = \overline{M}$, $M'(\alpha) > 0, M''(\alpha) = 0$ and $\frac{\overline{M}}{\underline{M}} = 1 + k$, $k \geq 1$.[9] $\underline{M}$ is the minimum purse that is obtained from the match if the weakest opponent is chosen. $\overline{M}$ is the maximum purse that can be expected from a match with the same ranked opponent ($\alpha = 1$).

We denote the share of the money revenue to the chooser as β, the outside option of the chooser as d_c, and the outside option of the opponent is assumed to be $d_o = d\alpha$ where $d > 0$. This

[8] Since the boxer who is lower in ranking usually challenges the one who is higher, the higher-ranked boxer has a greater bargaining power and the right to pick one of the lower-ranked challengers. However, there may be exceptions such that if the challenger is more popular than the chooser (E.g. challenger Canelo before 2017 fight with then chooser GGG), then the challenger has a greater bargaining power and a final say whether to fight the chooser.

[9] $M'(\alpha) > 0$ is intuitive since a stronger opponent brings a higher purse. Money function is assumed to be linear, $M''(\alpha) = 0$, mainly due to tractability purposes and to be able to obtain interior solution.

assumption is also plausible because a higher ranked boxer is more likely to have more alternatives to choose from that will increase his outside option relative to a lower ranked boxer. We assume that $d_c \geq d$, which is intuitive since the chooser has higher ranking than the opponent, and he can choose another equally or higher ranked opponent than the current one.[10]

The last stage is modelled as a contest game. We use one of the commonly used contest success functions in the literature. Probability of winning the contest depends on the efforts of both boxers and the relative popularity of the opponent α. Namely,

$$p_c(e_c, e_o, \alpha) = \frac{e_c}{e_c + \alpha e_o}; p_o(e_c, e_o, \alpha) = \frac{\alpha e_o}{e_c + \alpha e_o}$$

where e_c and e_o are the efforts of the chooser and the opponent, respectively (if $e_c + \alpha e_o = 0$, $p_c(e_c, e_o, \alpha) = p_o(e_c, e_o, \alpha) = 0.5$). Here, we assume that the chooser has an edge over the opponent in transforming his effort into a win probability (Jia et al., 2013).[11]

We now characterize how the chooser selects his opponent optimally, how the shares are determined, and how the boxers choose their optimal effort levels. We assume the utility of the boxers is linear in money purse (Amegashie and Kutsoati, 2005) and there is no incomplete information. We will write down each stage separately and then solve them by working backwards.

In the first stage, the chooser maximizes his expected utility by selecting an opponent, that is, by selecting an α such that

$$\max_{\{\alpha\}} (\beta M(\alpha) + \overline{V}_c) p_c(e_c, e_o, \alpha) + (\beta M(\alpha) + \underline{V}_c)(1 - p_c(e_c, e_o, \alpha))$$

The first and second parts represent the expected utility from winning and losing, respectively. This can be simplified as:

$$\max_{\{\alpha\}} \beta M(\alpha) + V_c p_c(e_c, e_o, \alpha) + \underline{V}_c \tag{1}$$

On top of the guaranteed expected gain, $\underline{V}_c$, the chooser gets his share from the generated purse, $\beta M(\alpha)$, and the expected victory premium if he wins the fight, $V_c p_c(e_c, e_o, \alpha)$. The purse is positively correlated with the popularity of the opponent as it increases with a more popular opponent. However, the expected victory premium is negatively correlated with the popularity of the opponent since a stronger opponent decreases the probability of winning. This essentially creates a trade-off in choosing the right opponent.

In the second stage, the boxers play the cooperative Nash bargaining game to decide how to share the total purse. We assume that the bargaining powers are not symmetric and are proportional to the relative popularity, α. We can easily extend this to a case where the opponent gets a higher share by relaxing the $\alpha \leq 1$ assumption. The main driver of the share of a boxer

[10] There might be exceptions for this condition and we will mention some of them.

[11] This assumed advantage can decline, even be reversed, as the boxer ages but in our model, we abstract from issues that may be driven by the boxer's age.

during the bargaining on the purse is his popularity and his share is positively correlated with his popularity. If the opponent is more popular than the chooser, he may well get a higher share. The Nash bargaining solution for the optimal share is found by solving the following problem:

$$\underset{\{\beta\}}{max}\ (\beta M(\alpha) - d_c)((1-\beta)M(\alpha) - d_o)^{\alpha} \tag{2}$$

In the third (last) stage, players make their effort choices non-cooperatively by taking the other party's effort as given. For tractability reasons, we find Nash equilibrium by assuming linear cost of effort (Amegashie and Kutsoati, 2005).[12] The equilibrium effort choices are found by simultaneously solving the following problems:

$$\underset{\{e_c\}}{max}\ V_c p_c(e_c, e_o, \alpha) + \underline{V}_c - e_c \tag{3}$$

$$\underset{\{e_o\}}{max}\ V_o p_o(e_c, e_o, \alpha) + \underline{V}_o - e_o \tag{4}$$

As a forward-looking rational agent, the chooser will work backwards and first start solving for the equilibrium effort levels as a function of α. Then, the equilibrium share is found in the second stage. Finally, he will choose the optimal opponent (α) given the optimal efforts and equilibrium shares that are found in the third and second stages, respectively. Trivial derivations are skipped and all proofs are relegated to the appendix.

STAGE 3: First order conditions for (3) and (4) are as follows:

$$e_c^*(e_o) = \sqrt{\alpha e_o V_c} - \alpha e_o \tag{5}$$

$$e_o^*(e_c) = \frac{\sqrt{\alpha e_c V_o} - e_c}{\alpha} \tag{6}$$

Solving (5) and (6) simultaneously implies that

$$e_c^* = \frac{\alpha V_o}{(1+\frac{\alpha V_o}{V_c})^2} = \frac{\alpha V_o V_c^2}{(V_c + \alpha V_o)^2} \tag{7}$$

$$e_o^* = \frac{\alpha V_o^2}{V_c(1+\frac{\alpha V_o}{V_c})^2} = \frac{\alpha V_o^2 V_c}{(V_c + \alpha V_o)^2} \tag{8}$$

Notice that the optimal efforts are proportional to the victory premiums, $\frac{e_c^*}{e_o^*} = \frac{V_c}{V_o}$.

STAGE 2: From the first order conditions for (2), we solve for the equilibrium of the Nash bargaining game over the total purse given the linear utility of money as follows:

$$\beta^* = \frac{M(\alpha) - d\alpha + d_c\alpha}{M(\alpha) + \alpha M(\alpha)} \tag{9}$$

[12] When convex cost is assumed as in Tenorio (2000), comparative statics qualitatively stay the same.

STAGE 1: The chooser selects α to maximize his expected payoff in (1). For any given α, we obtain the optimal share and efforts from stage 2 and 3, respectively. When we plug in these optimal values in (1), we get the following:

$$\underset{\{\alpha\}}{max} \beta M(\alpha) + V_c p_c(e_c, e_o, \alpha) + \underline{V}_c = \frac{\underline{M}(1 + k\alpha) - d\alpha + d_c\alpha}{1 + \alpha} + \frac{V_c^2}{V_c + \alpha V_o} + \underline{V}_c \tag{10}$$

First order condition for (10) is:

$$\frac{\underline{M}(k-1) - d + d_c}{(1 + \alpha^*)^2} - \frac{V_o V_c^2}{(V_c + \alpha^* V_o)^2} = 0 \tag{11}$$

If we plug in $v \equiv \frac{V_c}{V_o}$, FOC becomes:

$$\frac{\underline{M}(k-1) - d + d_c}{(1 + \alpha^*)^2} - \frac{vV_c}{(v + \alpha^*)^2} = 0$$

FOC represents the trade-off between the current monetary gain and expected future gains. The first part of this inequality refers to the *marginal benefit* of the chooser when he picks a stronger opponent because this increases his current money revenue. The second part refers to the *marginal cost* of choosing a stronger opponent due to the reduced winning probability. Solving for the α^* gives the following closed form solution for the optimal opponent choice:

$$\alpha^* = \frac{\sqrt{\frac{\underline{M}(k-1)-d+d_c}{V_o}} - 1}{1 - \sqrt{\frac{\underline{M}(k-1)-d+d_c}{vV_c}}}$$

Furthermore, second order condition is assumed to be negative for interior solution as follows:

$$-\frac{\underline{M}(k-1) - d + d_c}{(1 + \alpha^*)^3} + \frac{vV_c}{(v + \alpha^*)^3} < 0$$

A necessary condition for these two conditions to be satisfied together is $v \equiv \frac{V_c}{V_o} > 1$ or $V_c > V_o$. Of course, for some parameter values, there might be boundary solutions where the chooser picks the weakest or strongest opponent ($FOC < 0$ or $FOC > 0$ for all α, respectively). However, we focus on the interior solution and discuss the boundary cases in the last section.

We now continue our analysis with comparative statics assuming interior solution.

3 Comparative Statics and Implications

In this section, we derive some comparative statics from the explicitly found optimal level of efforts, bargaining share, and opponent choice in the previous section. These help us understand the motives of each boxer before coming to the ring. That is, whom to fight in the ring, how to come to negotiate regarding the money share and whether it is worth putting huge efforts in the upcoming fight. We will start with the optimal opponent choice.

3.1 Optimal Opponent Choice

The closed form solution for the optimal opponent choice is given as follows:

$$\alpha^* = \frac{\sqrt{\frac{\underline{M}(k-1)-d+d_c}{V_o}} - 1}{1 - \sqrt{\frac{\underline{M}(k-1)-d+d_c}{vV_c}}}$$

Note that the factors determining the optimal opponent are the multiplier parameter k, the minimum purse $\underline{M}$, the victory premiums for each fighter, V_c and V$_o$, and their outside options, d_c and d_o.

We can start the comparative statics analysis with respect to outside options of the fighters.

Proposition 1 *The greater outside option of the chooser (opponents) leads him to choose a relatively stronger (weaker) opponent* ($\frac{d\alpha^*}{dd_c} > 0, \frac{d\alpha^*}{d(d)} < 0$).

Proposition 1 implies that as the chooser's outside option gets higher, his marginal benefit today becomes larger. Hence, he may like to take advantage of this higher marginal gain by choosing a stronger opponent ($\frac{d\alpha^*}{dd_c} > 0$). Consider again the current champ Canelo. It seems that among his opponents the best choice for Canelo could be the rematch with GGG that almost every fan would like to see, especially the outcome of their 2018 bout in Las Vegas according to various experts is assessed to be very close to a draw or controversial. This shouldn't be surprising as Balmer et al. (2005) noticed that the scores for each round of the contest in professional boxing are subjectively judged by referees and judges. With this regard, they found that if there's no knockout, then the judge's decisions mostly end up with "home wins". In this case, Las Vegas was home for Canelo's fans. Thus, the trilogy between them could generate even a larger purse.

Moreover, as the opponents' outside options get higher, the chooser's marginal gain today gets lower and he chooses a weaker opponent to be able to increase his chance of winning the imminent fight to keep his future prospects high ($\frac{d\alpha^*}{d(d)} > 0$). For the larger outside option of the highest ranked opponent, say GGG, for Canelo it is optimal to pick a relatively weak opponent. This may be tied to the fact that the outside option of a fighter is part of his bargaining power. In other words, the larger the outside option of the challenger, the larger the bargaining power of him in negotiating the share from the purse. The negotiation about the purse split complicates the job of the two promoters representing their fighters (Tenorio, 2006). For the challenger GGG, the outside option could be a fight against the contender the second-highest ranked or lower-ranked ones. On the other hand, for Canelo, the fight with any of them could also increase his outside options and bring huge money to his team, especially if each of the contenders should prefer to challenge Canelo to GGG. Thus, the fight with a comparatively weak opponent will allow the champ to keep the championship title for a longer period in the future and generate a larger premium accordingly.

The opponent choice essentially depends on the expected victory premium of win over loss for the players. The next result specifies these relationships.

Proposition 2 *The additional expected premium of the chooser and the opponent encourages the chooser to pick a relatively weaker opponent* ($\frac{d\alpha^*}{dV_c} < 0; \frac{d\alpha^*}{dV_o} < 0$).

Remember that V_c is the extra premium of the chooser that winning is expected to bring over losing. Thus, an increase in V_c means that winning becomes more important because it brings more premium (marginal cost of choosing a stronger opponent increases). To be able to exploit the higher future premium, the chooser picks a weaker opponent to increase his chance of winning.

Since V_c is an expected discounted value of future gains for the chooser, it implicitly takes into account how the chooser discounts future payoffs. On the one hand, if a boxer heavily discounts future gains (if he has a high discount rate and/or is present biased), then this will reduce the value of future gains (a lower V_c) which in turn will make him choose a stronger opponent to increase current monetary gain. Majority of the boxers do not earn substantial amounts of money in their entire boxing career.[13] One reason that may force the chooser to pick a stronger opponent is the trouble with financial difficulties. An example of this is the former champ Mike Tyson who declared bankruptcy and attempted a desperate comeback in the early 2000s despite his damaged reputation and age. However, his professional career ended up with three consecutive losses. On the other hand, if a boxer is relatively patient and wants to slowly and consistently build a reputation of a professional record with no or very few lost fights (a high V_c), then he may prefer choosing weaker opponents. Earning a title is difficult, but retaining it is even more difficult. For the case of the current chooser, Canelo, it may be risky to lose the title and future gains against the strong opponent. Thus, instead of giving a rematch to the stronger opponent and risking of losing a title belt, it may be optimal for the current chooser to pick a comparatively lower ranked opponent, say the second or third-highest ranked contender, as his outside options keep increasing while it is still possible for the current chooser to milk his championship. In addition, a boxer's record strongly impacts his future prospects. Remaining undefeated becomes increasingly more important as he fights and remains undefeated. This implies an asymmetric decline in V_c (nonlinearity in incentives) for undefeated boxers relative to boxers with at least one defeat in their records.[14] Thus, undefeated boxers may have a higher tendency to hand pick relatively easy opponents to keep their records.[15]

[13]See https://themmaguru.com/how-much-money-do-professional-boxers-make, accessed 25 October 2021.

[14]These nonlinearities in incentives can also be observed in other contexts such as sumo wrestling (Duggan and Levitt, 2002; Dietl et al., 2010), chess (Howard, 2006), and football (Shepotylo, 2005).

[15]Throughout the paper, we assume there is a unique ranking of the boxers. However, in reality, the existence of multiple sanctioning bodies creates a perception of many "strong boxers." This in turn allows some top boxers, such as Canelo and Mayweather, to pick seemingly strong opponents who are actually not, and to conveniently ignore true strong opponents to be able to keep their records. This created perception in the eye of "naive" fans is yet another example of unreasonableness of the current governance system.

If the victory premium for the opponent, V_o, is expected to rise, then it's a sign of a rising "star" in the boxing market. If that is the case, then for the chooser it is better to avoid a match against a stronger opponent in order to minimize his risk of losing and to maximize his record of the "winning" outcomes along with market values. Because of this, such big fight events, in which only the two high-profile fighters may face each other, are rare (Tenorio, 2006). Even the promoters of big fighters are reluctant to bring them together in order not to risk their status in their early career. If that needs to happen, it is possible either when both fighters sign a contract with the same promoter company (e.g. GGG and Canelo now sign with DAZN) or when at least one fighter enters the later stages in his career. This is a case when GGG had been in his peak until age 35, Canelo's team had been just avoiding him. Finally, their first bout happened in 2017 when GGG's skills and speed had already deteriorated along with his age. Another good example is the case of Mike Tyson versus Lennox Lewis, who were supposed to meet in the late 1990's. For several reasons, each went their own way so that their promoters had to delay their bout till 2002. Thus, as the victory premium for the opponent increases, this increases the marginal cost of the chooser for picking a stronger opponent. A fight with a weaker opponent seems to increase the expected premium more than the foregone money revenue with a stronger opponent.

The minimum purse that can be obtained from a match, $\underline{M}$, and the multiplier, k, also have an impact on the choice of an opponent. **Both a higher minimum purse and a higher k implies a choice of a more popular opponent** ($\frac{d\alpha^*}{d\underline{M}} > 0$, $\frac{d\alpha^*}{dk} > 0$). An increase in both factors essentially increases the marginal benefit of choosing a stronger opponent and this creates an incentive for the chooser to pick a stronger opponent to exploit this increased revenue.

Our model sheds light on the moral hazard problem too, which arises when the chooser prepares for retirement and tries to milk his reputation before he quits, through the changes in the victory premium from winning over losing. In 1990, the world heavyweight boxing champion James Douglas met Evander Holyfield to defend his title, but since Douglas knew before the match that he guaranteed $20 million even in case of a loss, he was obviously unprepared and lost the match as a knockout in Round 3. Under "large" guaranteed purses (how large this is depends on each boxer's aspired wealth level), the victory premium for the boxer can be assumed to be very close to zero because he does not care much about the result and tries to milk this last opportunity to make as much money as possible. A very small V_c not only implies a very small effort (equation 7) but also implies an opponent choice with as high α as possible (equation 10). This is not implied by the incremental changes in V_c but is driven by the level of V_c (if V_c is close to zero, then FOC will likely be positive for any α, which implies choosing the strongest challenger). Thus, ***a boxer entering the later stages of his career may agree to fight with the stronger/strongest challenger in order to maximize his purse but put little effort.***[16] This does not have to be the last fight of the boxer. If the ratio of casual fans (defined

[16]We do not consider the effect of ability in our model. In reality, both effort and ability together drive the

as the ones who fail to account for the disincentive effect of large purses on the boxer's effort when deciding on their willingness to pay for a fight) is high enough (Tenorio, 2000), the boxer can make a comeback and continue his career (like James Douglas who did not fight for 6 years after Holyfield loss but then continued his career three more years). Here, we do not take into account other considerations that may affect the value of V_c such as pride, willingness to leave a legacy etc. When these are also incorporated, the moral hazard problem mentioned above may not be as likely to emerge.

3.2 Optimal Bargaining Shares

From the second stage of the game, we found that the optimal bargaining share from the money purse for each fighter depends on the choice of an opponent by the chooser and their disagreement payoffs.

$$\beta^*(\alpha^*; d, d_c, \underline{M}, k) = \frac{\underline{M}(1 + k\alpha^*) - d\alpha^* + d_c\alpha^*}{\underline{M}(1 + k\alpha^*)(1 + \alpha^*)}$$

The following proposition characterizes the relationship between the opponent choice and optimal share and money revenue.

Proposition 3 *As the chooser picks a more popular opponent, although his share decreases ($\frac{\partial \beta^*}{\partial \alpha^*} < 0$), his expected monetary gain increases ($\frac{\partial(\beta^* M(\alpha^*))}{\partial \alpha^*} > 0$).*

This proposition states that a match with a stronger opponent always generates a greater purse but his optimal share goes down. Since the total money revenue, not the share, is usually the priority for the chooser, and as long as the overall money to the chooser increases from the upcoming fight, the chooser is willing to sacrifice some share of the purse in favor of his challenger. A higher money revenue represents the benefit of choosing a stronger opponent when the boxer makes his optimal opponent choice.

Regarding the effect of outside options on the share, intuitively, a higher outside option of the chooser (opponent) should lead to a larger (lower) share of money revenue from bargaining, $\frac{\partial \beta}{\partial d_c} > 0$ ($\frac{\partial \beta^*}{\partial d} < 0$). However, this captures only the direct effect of outside options on the bargaining share. There is another indirect effect of outside options that works through the optimal opponent choice. As explained in the previous section, a higher outside option of the chooser (opponents) leads him to choose a stronger (weaker) opponent ($\frac{d\alpha^*}{dd_c} > 0$, $\frac{d\alpha^*}{d(d)} < 0$). This can be seen from the following expressions:

$$\frac{d\beta^*}{d(d_c)} = \frac{\partial \beta^*}{\partial d_c} + \frac{\partial \beta^*}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial d_c} \text{ and } \frac{d\beta^*}{d(d)} = \frac{\partial \beta^*}{\partial d} + \frac{\partial \beta^*}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial d}$$

performance of a boxer and his probability of winning. Although our argument about moral hazard is still valid, one can easily argue that as boxers age, they may not perform well even if they put high effort due to their deteriorated physical ability. This can be incorporated into our model as an extension.

The first and second parts refer to the direct and indirect effects, respectively. Given that $\frac{\partial \beta^*}{\partial \alpha} < 0$, since $\frac{\partial \beta^*}{\partial d_c} > 0$, $\frac{\partial \alpha^*}{\partial d_c} > 0$, and $\frac{\partial \beta^*}{\partial d} < 0$, $\frac{\partial \alpha^*}{\partial d} < 0$, direct and indirect effects will move in the opposite directions, which makes the relationships between the outside options and optimal share ambiguous.[17] However, ***the money revenue from the bout will unambiguously increase (decrease) if outside option of the chooser (opponent) gets higher since both direct and indirect effects move in the same direction***.[18]

Finally, it is easy to see that ***if the challenger is highly popular or close to the chooser in terms of his rank*** (α is close enough to 1 or if we relax the assumption we have, $d_c \geq d$)***, he can get a higher share*** (1-$\beta^* > 0.5$). Although the chooser is mostly the one who earns more than the challenger, this happens in rare cases. An example of this is the Canelo vs. Golovkin fight in 2017. Although GGG was the middleweight champion at that time, the purse split was in favor of Canelo because he had the bigger fan base (guaranteed base purse for Canelo and GGG was \$5 and \$3 million, respectively).[19] Another example is the heavyweight match in 2003 between the champion Ruiz and the challenger Jones Jr. who got a bigger share than Ruiz because he was more popular.

3.3 Optimal Efforts

Optimal efforts of the players in stage 3 depend on the choice of an opponent by the chooser and the expected victory premium of both players. The following propositions summarize the results we obtained from the comparative statics.

Proposition 4 *The chooser is willing to put higher effort into the fight against a stronger opponent* ($\frac{\partial e_c^*}{\partial \alpha} > 0$)*.*

Proposition 5 *The chooser (opponent) is willing to put more (less) effort into the fight as the expected victory premium of the opponent (chooser) increases* ($\frac{\partial e_c^*}{\partial V_o} > 0$ *and* $\frac{\partial e_o^*}{\partial V_c} < 0$)*.*

Proposition 4 states that the stronger the opponent, the higher effort the chooser puts. On the other hand, proposition 5 shows that for a given opponent[20], as the victory premium to the opponent increases, the chooser is willing to put more effort into the fight ($\frac{\partial e_c^*}{\partial V_o} > 0$). However, the

[17] Here, we implicitly assume the changes in outside options happen before the opponent choice. If the opponent is already set, there will be no indirect effect.

[18] On the one hand, as d_c goes up, it directly increases the money revenue, $\beta^* M(\alpha)$. But it also leads to choosing a stronger opponent that further increases the money revenue. On the other hand, as d goes up, it directly decreases the money revenue, $\beta^* M(\alpha)$. But it also leads to choosing a weaker opponent that further decreases the money revenue.

[19] https://www.cbssports.com/boxing/news/canelo-vs-ggg-2-fight-purses-how-much-each-fighter-on-main-card-will-take-home-on-saturday, accessed 25 October 2021.

[20] V_o can change after the opponent (α) has been chosen.

opposite is not true for the opponent. That is, the additional premium to the chooser demotivates the opponent ($\frac{\partial e_o^*}{\partial V_c} < 0$).

To give a concrete example of this, consider the current champ Saul Canelo Alvarez, who is in the peak of his professional boxing career and the opponent like Jaime Munguia. Just Canelo's "name" along with his past successful records until he became the champ (against big names such as Floyd Mayweather, Miguel Cotto, Gennady Gennadyevich Golovkin/GGG, Sergey Kovalev) already seem to generate huge money revenue in the boxing industry. On the other hand, as noted before, the higher victory premium to the opponent like Munguia may also imply another possible forthcoming star in the market. If this is so, then Canelo, who wants to remain a champion for a longer period, should be willing to put a huge effort into the fight. Hence, for an additional premium for his stronger opponent, the current chooser will do his best to remain as the "star", while Munguia, considering the dominance of the Canelo now and in the near future, may not be willing to work harder as he could for the additional expected premium to the Canelo.

One may intuitively expect that an increase in a player's own expected victory premium can motivate him to put more effort to earn this extra premium ($\frac{\partial e_c^*}{\partial V_c} > 0; \frac{\partial e_o^*}{\partial V_o} > 0$). Although this is the direct effect of changes in victory premium on the efforts, there is also an indirect effect if we assume that the changes in the victory premiums happen before the choice of the opponent. Changes in the victory premiums will make the chooser pick another opponent, which in turn will affect the resulting efforts of the boxers.

$$\frac{de_c^*}{dV_c} = \frac{\partial e_c^*}{\partial V_c} + \frac{\partial e_c^*}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial V_c} \text{ and } \frac{de_o^*}{dV_o} = \frac{\partial e_o^*}{\partial V_o} + \frac{\partial e_o^*}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial V_o}$$

It is easy to see that the direct effects are positive ($\frac{\partial e_c^*}{\partial V_c} > 0$, $\frac{\partial e_o^*}{\partial V_o} > 0$) and $\frac{\partial e_c^*}{\partial \alpha^*} > 0$ and $\frac{\partial e_o^*}{\partial \alpha^*} > 0$. Furthermore, $\frac{\partial \alpha^*}{\partial V_c} < 0$ and $\frac{\partial \alpha^*}{\partial V_o} < 0$ from propositions 2 and 3. Hence, since the direct and indirect effects move in the opposite directions, the effect of changes in the own victory premium on the boxers' own efforts are ambiguous. However, considering a fixed opponent as in proposition 5 eliminates the indirect effect. In this case, an increase in players' expected victory premium makes them put more effort into the fight.

Finally, the aggregate effort in a given match can be expected to positively influence the future prospects of fighters (Amegashie and Kutsati, 2005). The promoters will be able to generate a higher purse for the coming matches of a fighter who earned a reputation of being a tough nut to crack. Thus, it is always desirable for the promoters to extract as high effort as possible from the fighters to increase the purse. Our analysis showed that ***the total effort exerted by the boxers goes up as the chooser selects a stronger opponent*** ($\frac{\partial E}{\partial \alpha} > 0$, $E = e_c^* + e_o^*$).

4 Discussion and Conclusion

In this paper, we have developed a three-stage game-theoretic model to explain the incentives and strategic behavior of professional boxers. Our paper highlights the importance of the opponent choice by a higher-ranked boxer, which is missing in the existing literature. We model the fact that the amount of the purse should be determined endogenously by the popularity of the fighters. We then demonstrated how the optimal money share between the fighters and their respective efforts are strongly dependent on the popularity of the fighters. We also characterised the equilibrium in each stage to better understand the incentives of the boxers. It should be noted that since our paper is the first trying to understand the big picture in the professional boxing sport, we have substantially simplified the model. Therefore, we propose how the current work could be extended in a number of ways in the future.

Firstly, the expected discounted values of fighters' future gains can be a function of their spent efforts. As Tenorio (2000) and Amegashie and Kutsoati (2005) show, the expected victory premium to the fighters depends on their efforts, which we didn't consider for mainly tractability purposes. That is, if one is a tough nut to crack and puts a high effort then this most likely increases his future prospects. However, it is clear that this would complicate the calculations and lead to no closed form solution for the optimal efforts.

Secondly, one may argue that there is no formal connection between the second and the third stage in the model. Namely, the money guaranteed as a result of the bargaining game in the second stage does not affect the optimal effort levels of the players determined in the third stage. There is no direct link between guaranteed money and optimal effort in our model but the guaranteed share of the purse indirectly affects the expected discounted value of the future gains through the accumulated wealth. If this guaranteed money is large enough, it may lower this expected gain (and this, in turn, leads to less effort) because the boxer might have accumulated a high enough wealth with this large guaranteed purse so that winning or losing may not make too much of a difference for the boxer. This explains the naturally arising moral hazard problem that leads to low effort once the gains are fully insured. Besides, one may also consider the well-known prospect theory in this case that emphasizes how the losses can loom much larger than similar sized gains. This would be another interesting avenue for future research.

Third, we assume that the expected victory premium to the current higher-ranked fighter is independent of the chosen opponent. However, dependency of these two can easily be argued. For example, the WBO champ may be challenged by the IBF champ in order to unite the championship belts to qualify to announce his absolute championship. If the WBO champ accepts a challenge by the IBF champ, the future expected gain of the WBO champ will obviously be much higher than that with a lower-ranked contender in the WBO rank. Our model can be extended by relaxing this assumption.

In the paper, we focus on the interior solution. Namely, we consider the case where the boxer does not choose the weakest or the strongest opponent. These boundary choices may rarely happen in some real life cases that we have already touched upon. For example, if the outside options of opponents are high enough ($d_c << d$), this may make the chooser pick the weakest opponent. As we discussed, if the boxer has present biased preferences, then V_c tends to be very low and this may make him choose the strongest opponent possible to maximize the current money revenue and if he is future biased, he can go for the weakest one. We did not specifically focus on these boundary solutions but they are as interesting as characterizing equilibrium by assuming interior solutions.

Our paper can be considered as one of the first steps to better understand the complex incentive structure and opponent choice in this highly popular sport. Furthermore, we think that our framework can easily be extended to other combat sports such as UFC (Ultimate Fighting Championship) most of which have institutional structures similar to professional boxing.

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Appendix

Proof of Proposition 1. We need to show $\frac{d\alpha^*}{dd_c} > 0$ and $\frac{d\alpha^*}{d(d)} < 0$. Remember that the optimal choice α^* is defined by the following FOC:

$$F(\alpha^*; k, \underline{M}, V_c, V_o, d_c, d_o) = \frac{\underline{M}(k-1) - d + d_c}{(1+\alpha^*)^2} - \frac{V_o V_c^2}{(V_c + \alpha^* V_o)^2}$$

Although α^* is explicitly found, we will use Implicit Function Theorem (IFT) to prove the claims. IFT implies that

$$\frac{d\alpha^*}{dd_c} = -\frac{d\alpha^*}{d(d)}\left(= -\frac{\frac{\partial F(.)}{\partial d_c}}{\frac{\partial F(.)}{\partial \alpha^*}} = \frac{\frac{\partial F(.)}{\partial d}}{\frac{\partial F(.)}{\partial \alpha^*}}\right) = -\frac{\frac{1}{(1+\alpha^*)^2}}{-\frac{\underline{M}(k-1)-d+d_c}{(1+\alpha^*)^3} + \frac{vV_c}{(v+\alpha^*)^3}}$$

Since we assume interior solution, $\frac{\partial F(.)}{\partial \alpha^*} < 0$, which implies $\frac{d\alpha^*}{dd_c} > 0 > \frac{d\alpha^*}{d(d)}$. ■

Proof of Proposition 2. We need to show $\frac{d\alpha^*}{dV_c} < 0$ and $\frac{d\alpha^*}{dV_o} < 0$. Again, IFT implies that

$$\frac{d\alpha^*}{dV_c} = -\frac{\frac{\partial F(.)}{\partial V_c}}{\frac{\partial F(.)}{\partial \alpha^*}} = -\frac{-\frac{2\alpha V_o^2 V_c}{(V_c+\alpha^* V_o)^3}}{\frac{\partial F(.)}{\partial \alpha^*}}$$

Since $\frac{\partial F(.)}{\partial V_c} < 0$ and $\frac{\partial F(.)}{\partial \alpha^*} < 0$, this implies that $\frac{d\alpha^*}{dV_c} < 0$.

Again, by IFT, we write:

$$\frac{d\alpha^*}{dV_o} = -\frac{\frac{\partial F(.)}{\partial V_o}}{\frac{\partial F(.)}{\partial \alpha^*}} = -\frac{-\frac{V_c^2(V_c-\alpha V_o)}{(V_c+\alpha^* V_o)^3}}{\frac{\partial F(.)}{\partial \alpha^*}}$$

Remember that from the FOC and SOC of the optimal opponent choice problem, $V_c > V_o$ has to be satisfied to have an interior solution, which implies that $\frac{\partial F(.)}{\partial V_o} < 0$. Since $\frac{\partial F(.)}{\partial \alpha^*} < 0$, $\frac{d\alpha^*}{dV_o} < 0$. ■

Proof of Proposition 3. We need to show $\frac{\partial \beta^*}{\partial \alpha} < 0$ and $\frac{\partial(\beta^* M(\alpha))}{\partial \alpha} > 0$.

$$\frac{\partial \beta^*}{\partial \alpha} = \frac{\partial\left(\frac{\underline{M}(1+k\alpha)-d\alpha+\alpha d_c}{(\underline{M}(1+k\alpha))(1+\alpha)}\right)}{\partial \alpha} = \left(-\frac{\underline{M}}{(\underline{M}(1+k\alpha))(1+\alpha))^2}\right)(k\alpha(\underline{M}(1+k\alpha)-\alpha d)+\underline{M}k\alpha+\underline{M}+d+k\alpha d_c+\alpha d_c+2k\alpha^2 d_c)$$

Since $(\underline{M}(1+k\alpha)-\alpha d) > 0$, $\frac{\partial \beta^*}{\partial \alpha} < 0$.

$$\frac{\partial(\beta^* M(\alpha))}{\partial \alpha} = \frac{\partial(\frac{\underline{M}(1+k\alpha)-d\alpha+\alpha d_c}{1+\alpha})}{\partial \alpha} = \frac{\underline{M}(k-1)-d+d_c}{(1+\alpha)^2}$$

Since this is the first expression in the FOC, which has to be positive for the FOC to be satisfied, $\frac{\partial \beta^* M(\alpha)}{\partial \alpha} > 0$. ■

Proof of Proposition 4. We need to show $\frac{\partial e_c^*}{\partial \alpha} > 0$.

$$\frac{\partial e_c^*}{\partial \alpha} = \frac{\partial(\frac{\alpha V_o V_c^2}{(V_c+\alpha V_o)^2})}{\partial \alpha} = \frac{V_o V_c^2 (V_c+\alpha V_o)(V_c-\alpha V_o)}{(V_c+\alpha V_o)^4} = \frac{V_o V_c^2 (V_c-\alpha V_o)}{(V_c+\alpha V_o)^3}$$

Since $V_c > V_o$ is assumed for interior solution, $\frac{\partial e_c^*}{\partial \alpha} > 0$. ■

Proof of Proposition 5. We need to show $\frac{\partial e_c^*}{\partial V_o} > 0$ and $\frac{\partial e_o^*}{\partial V_c} < 0$. There is no indirect effect in this case because we consider a fixed opponent that eliminates the channel through which the victory premiums affect the opponent choice.

$$\frac{\partial e_c^*}{\partial V_o} = \frac{\partial(\frac{\alpha V_o V_c^2}{(V_c+\alpha V_o)^2})}{\partial V_o} = \frac{\alpha V_c^2 (V_c+\alpha V_o)(V_c-\alpha V_o)}{(V_c+\alpha V_o)^4} = \frac{\alpha V_c^2 (V_c-\alpha V_o)}{(V_c+\alpha V_o)^3}$$

$$\frac{\partial e_o^*}{\partial V_c} = \frac{\partial(\frac{\alpha V_o^2 V_c}{(V_c+\alpha V_o)^2})}{\partial V_o} = \frac{\alpha V_o^2 (V_c+\alpha V_o)(\alpha V_o-V_c)}{(V_c+\alpha V_o)^4} = \frac{\alpha V_o^2 (\alpha V_o-V_c)}{(V_c+\alpha V_o)^3}$$

Since $V_c > V_o$ is assumed for interior solution, $\frac{\partial e_c^*}{\partial V_o} > 0$ and $\frac{\partial e_o^*}{\partial V_c} < 0$. ■