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Generational Distribution of Fiscal Burdens: A Positive Analysis*

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Abstract

This study presents a political economy model with overlapping generations to analyze the effects of population aging on fiscal policy formation and the resulting distribution of the fiscal burden across generations. The analysis shows that both an increased life expectancy and a decreased population growth rate increase the ratios of government debt and labor income tax revenue to GDP. However, they decrease the ratio of capital income tax revenue to GDP. Furthermore, it also shows that the increased political weight of the elderly creates an increase in the ratios of public debt and labor income tax revenue to GDP, as well as an initial decrease followed by an increase in the ratio of capital income tax revenue to GDP.

- Keywords: Generational burden, Overlapping generations, Political economy, Population aging, Public debt
- JEL Classification: D70, E24, E62, H60

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1 Introduction

How is the burden of fiscal policy distributed across generations? How do demographic changes affect the pattern of generational burdens? To answer these questions, several studies explored the political determinants of fiscal policy in overlapping generations models. Examples are works by [Renström \(1996\)](#), [Beauchemin \(1998\)](#), [Boldrin and Rustichini \(2000\)](#), [Razin et al. \(2004\)](#), and [Razin and Sadka \(2007\)](#), which are based on tractable models of the economy and voting process. Recently, [Forni \(2005\)](#), [Bassetto \(2008\)](#), [Gonzalez-Eiras and Niepelt \(2008, 2012\)](#), [Mateos-Planas \(2008, 2010\)](#), [Ono and Uchida \(2016\)](#), and [Bishnu and Wang \(2017\)](#) studied taxation and public expenditure in a framework in which voting yields time consistent policies. All these works assume a balanced government budget in each period, and thus ignore the possibility of a shift of fiscal burdens onto future generations via public debt issuance.

Several researchers address the political economy of public debt, such as [Cukierman and Meltzer \(1989\)](#), [Song et al. \(2012\)](#), [Müller et al. \(2016\)](#), [Röhrs \(2016\)](#), [Arawatari and Ono \(2017\)](#), [Arai et al. \(2018\)](#), [Ono and Uchida \(2018\)](#), and [Andersen \(2019\)](#). In these studies, labor income tax on the working generation is the only tax instrument; capital income tax on the retired elderly, which is a possible additional instrument, is abstracted away from the analyses. An exception is [Arcalean \(2018\)](#), who considers dynamic fiscal competition over public spending financed by labor and capital taxes and public debt. He focuses on the effects of fiscal cross-border externalities on welfare and growth.¹ In other words, these studies do not fully address the generational conflict over age-specific taxes and the resulting distribution of the fiscal burden across generations. However, as [Mateos-Planas \(2010\)](#) indicates, demographic changes, such as increasing longevity and declining birth rates, affect voters' interests in taxing different factors, and thus drive the change in the mix of capital and labor income taxes over periods.

To address the generational conflict over taxes and public debt, we present an overlapping generations model in which labor supply is elastic and public goods expenditure, which benefits both middle-aged and old people, is financed by labor and capital taxes as well as public debt issue. Following [Song et al. \(2012\)](#) and the subsequent literature, we assume probabilistic voting (e.g., [Lindbeck and Weibull \(1987\)](#), [Persson and Tabellini \(2002\)](#)) in which fiscal policy in each period is determined to maximize the weighted sum of utility of the middle-aged and elderly. Within this setting, we analyze the effects of population aging on the fiscal policy formation and the resulting fiscal burdens on current and future generations.

For the analysis, we discuss two representative factors of aging: life expectancy and population growth rate. Following [Song et al. \(2012\)](#), [Lancia and Russo \(2016\)](#), and [Katagiri et al. \(2020\)](#), we consider the political weight of the elderly as an additional representative factor of aging, because the voter turnout of older people is significantly higher than that of younger ones

¹[Uchida and Ono \(2021\)](#) also touch on this association, but limit their analysis to the case of inelastic labor supply and productive public expenditure that benefits only the young, so they do not fully address the issue of the fiscal burden across generations. Instead, their analysis focuses on the effects of debt ceilings on policy formation and economic growth.

in developed countries. For example, [OECD \(2007\)](#) reports that the voter turnout of individuals aged 15-24 (25-50) is on average 20 (9) points lower than that of individuals aged 65 and more. We focus on the ratios of public debt, capital income tax revenue, and labor income tax revenue to GDP as measures of the fiscal burdens and investigate how the three aging factors affect these fiscal burdens across generations.

In the present framework, the current policy choice affects debt and physical capital in the next period through the government budget and the capital market, in turn affecting the return from savings and the next period's policy choice. In the presence of such an intertemporal effect, most previous studies relied on numerical methods to derive the solutions of the policy functions (e.g., [Song et al. \(2012\)](#), [Lancia and Russo \(2016\)](#), [Katagiri et al. \(2020\)](#)). Unlike these studies, we obtain analytical solutions of the policy functions, and show that the government takes account of the intertemporal effect in presenting the policy to voters because the utility of the middle-aged depends on that effect. Thus, the intertemporal effect plays an important role in shaping the fiscal policy.

Using the policy functions of the model, we take a numerical approach to investigate the effect of population aging on the fiscal burdens across generations. In particular, we calibrate the parameter that governs the degree of preferences for public goods to match the average ratio of government expenditure to GDP during 1995–2016 for OECD countries. Based on this calibration, we investigate the aging effects and find that increased life expectancy and decreased rate of population growth both lead to an increase in the ratios of government debt and labor income tax revenue to GDP. However, they lower the ratio of capital income tax revenue to GDP.

The abovementioned result, which is consistent with the evidence observed in OECD countries in [Figure 1](#), is brought about by the following. The government, representing the middle-aged and elderly, tends to shift the fiscal burdens from the elderly to the middle-aged as the population ages. Having given increased fiscal burdens, the middle-aged choose to reduce their labor supply and savings with a utility maximization viewpoint. This change in the private choice of the middle-aged leads to an increase in the interest rate and thus the debt repayment costs, thereby inducing the government to issue further public debt to finance increased repayment costs.

The numerical investigation also shows that the increased political weight of the elderly has a qualitatively similar effect on the ratios of labor income tax revenue and government debt to GDP as the effect of increased life expectancy and decreased population growth rate. However, it has a different effect on the ratio of capital income tax revenue to GDP. That is, the increased political weight of the elderly produces an initial decrease followed by an increase in the ratio of capital income tax revenue to GDP. This U-shaped pattern can explain the existence of countries, such as Korea, Ireland, Sweden, and the United States, that deviate from the negative association, as observed in [Figure 1](#). The result, therefore, suggests that the increased political

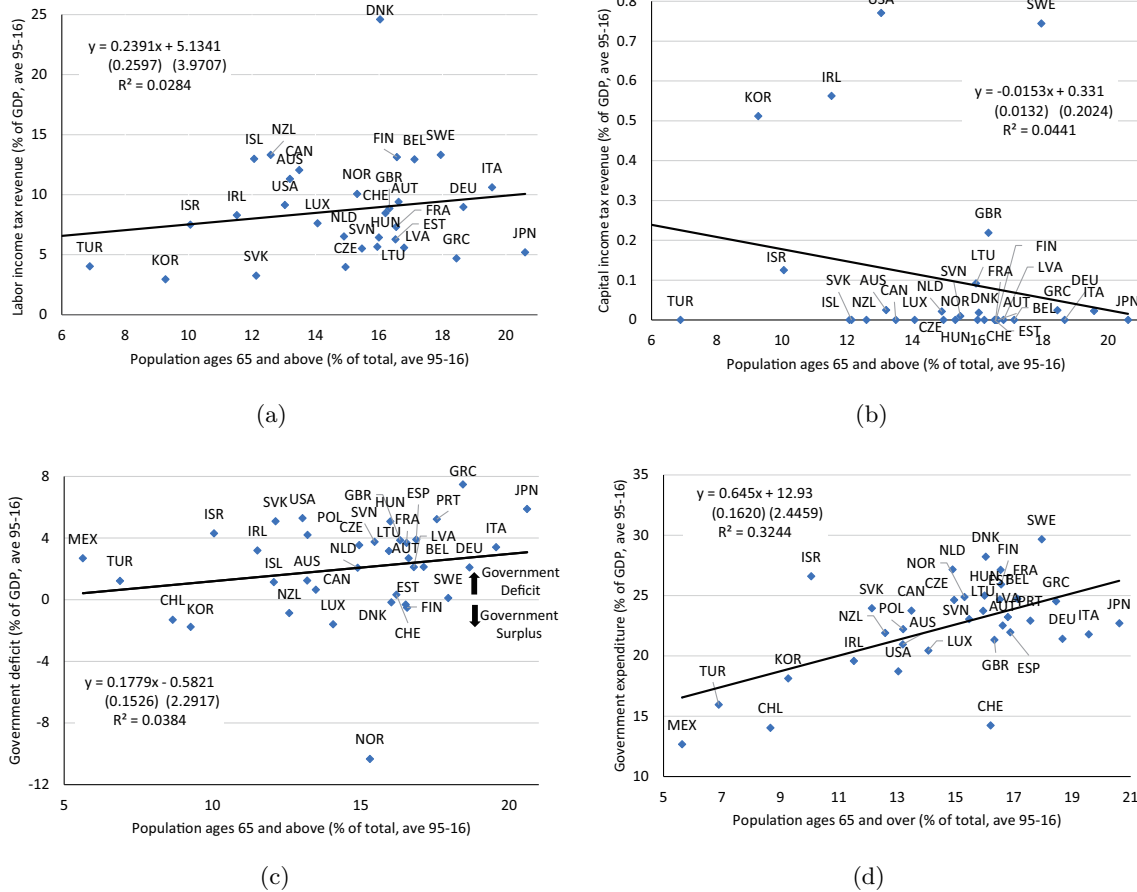


Figure 1: Each panel plots the data for OECD countries during 1995–2016. The horizontal axis represents the average share of the population aged 65 years and over. The vertical axis represents the average ratio of labor income tax revenue to GDP (Panel (a)), the average ratio of capital income tax revenue to GDP (Panel (b)), the average ratio of deficit to GDP (Panel (c)), and the average ratio of government expenditure to GDP (Panel (d)). In Panel (c), the budget deficit is an approximate variable for the public debt. Each panel presents the OLS equation estimated results. The numbers in parentheses represent the standard errors. Source: OECD.Stat (<https://stats.oecd.org/>) (accessed on September 25, 2019).

weight of the elderly is the key to account for the different patterns of the ratio observed in the OECD countries.²

The mechanism behind the U-shaped pattern is as follows. The ratio of capital income tax revenue to GDP is the product of two factors: the ratio of capital income to GDP and the capital income tax rate. These factors are influenced by the increased political weight of the elderly in the following ways. First, as the weight increases, the government raises the labor income tax rate and increases public debt issues to pass the fiscal burden onto the middle-aged. A higher labor income tax rate reduces labor supply and saving, and a higher level of public debt strengthens the crowding-out effect in the capital market. These two effects, in turn, slow down capital accumulation and raise the interest rate, and thereby increase the ratio of capital income

²We also investigate its effect on the ratio of government expenditure to GDP and show that the result fits well with the evidence observed in Panel (d) of Figure 1.

to GDP. This is the positive effect on the ratio. Second, as the weight increases, the government lowers the capital income tax rate to reduce the fiscal burden on the elderly. However, as the weight increases further, the government chooses to raise the capital income tax rate in response to public debt accumulation. Thus, aging produces an initial decrease followed by an increase in the capital income tax rate, yielding the U-shaped effect on the ratio.

The present study is related to recent theoretical contributions on fiscal politics. [Razin et al. \(2004\)](#), [Razin and Sadka \(2007\)](#), [Bassetto \(2008\)](#), and [Mateos-Planas \(2008, 2010\)](#) focus on the association between population aging and income taxation. In particular, [Mateos-Planas \(2010\)](#) examines the United States based on a median voter framework and shows that the tax rate initially decreases and then increases as the population ages. The present study instead uses a probabilistic voting framework that reflects the preferences of all voters, and shows that the keys to this U-shaped pattern are elastic labor supply and the increased political weight of the elderly. [Song et al. \(2012\)](#) and [Lancia and Russo \(2016\)](#) also consider the role of the political weight of the elderly, but focus on public debt finance ([Song et al. \(2012\)](#)) or public investment ([Lancia and Russo \(2016\)](#)); they abstract capital income taxation away from their analysis. Thus, our study bridges the gap in the literature by comprehensively evaluating the effects of population aging on fiscal policy formation and the resulting fiscal burden on current and future generations.

The present study also contributes to the literature on time-consistent fiscal policy ([Klein and Ríos-Rull \(2003\)](#), [Klein et al. \(2008\)](#), [Martin \(2009, 2010\)](#), and [Ortigueira et al. \(2012\)](#)). In their frameworks with long-lived agents, the government chooses the Markov strategy in each period. That is, current policies depend on payoff-relevant state variables. The present study follows the policy strategy of these works, but departs from theirs by assuming a short-lived government representing only existing generations. Under this alternative assumption, we consider the conflict of interest among generations and its generational consequences.

The organization of the rest of this paper is as follows. Section 2 presents the model. Section 3 gives the characterization of the political equilibrium and then investigates the policy response to population aging. Section 4 provides concluding remarks. The Appendix provides proofs of propositions and supplementary explanations of the analytical and numerical methods.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live at most for two periods: middle and old age.³ Individuals face uncertain lifetimes in the second period of life. Let $\pi \in [0, 1]$ denote life expectancy (i.e., the probability of living in old age). This is idiosyncratic for all individuals and is

³In conventional terminology, the first period of life is called youth. We refer to it as middle-aged instead of young, because in Section 3.2.2, we introduce pre-employment young into the model and extend it to a three-period version to apply the model for the numerical analysis. This extension does not intrinsically affect the structure of the model because the young do not make any decisions; they depend on their parents.

constant across periods. Each middle-aged individual gives birth to $1 + n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n (> -1)$: $N_{t+1} = (1 + n)N_t$.

Individuals

Individuals have the following economic behavior over their life cycles. During middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. In old age, they retire, and receive and consume returns from savings.

Consider a middle-aged individual in period t . The individual is endowed with one unit of time. He/she supplies it elastically in the labor market and obtains labor income $w_t l_t$, where w_t is wage rate per unit of labor and $l_t \in (0, 1)$ is the amount of labor supply. After paying tax $\tau_t w_t l_t$, where τ_t is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings held as an annuity and invested in physical capital, s_t . Therefore, the period- t budget constraint for the middle-aged becomes

$$c_t + s_t \leq (1 - \tau_t)w_t l_t. \quad (1)$$

The period $t + 1$ budget constraint in old age is

$$d_{t+1} \leq (1 - \tau_{t+1}^K) \frac{R_{t+1}}{\pi} s_t, \quad (2)$$

where d_{t+1} is consumption, τ_{t+1}^K is the period- $t + 1$ capital income tax rate, and R_{t+1} is the gross return from savings. If an individual dies at the end of the middle-aged period, then his or her annuitized wealth is transferred to the individuals who live throughout old age via annuity markets. Therefore, the return from saving becomes R_{t+1}/π under the assumption of perfect annuity markets.

The preferences of a middle-aged individual in period t are specified by the following expected utility function:

$$\ln \left(c_t - \frac{(l_t)^{1+1/v}}{1 + 1/v} \right) + \theta \ln g_t + \beta \pi (\ln d_{t+1} + \theta \ln g_{t+1}),$$

where g_t is per-capita public goods in period t , $\beta \in (0, 1)$ is a discount factor, and $\theta (> 0)$ is the degree of preferences for public goods. Following [Greenwood et al. \(1988\)](#) and [Müller et al. \(2016\)](#), we assume that the disutility from labor effort is $(l_t)^{1+1/v} / (1 + 1/v)$, where $v (> 0)$ parameterizes the Frisch elasticity of labor supply.

We substitute the budget constraints (1) and (2) into the utility function to form the unconstrained maximization problem:

$$\max_{\{s_t, l_t\}} \ln \left((1 - \tau_t)w_t l_t - s - \frac{(l_t)^{1+1/v}}{1 + 1/v} \right) + \theta \ln g_t + \beta \pi (\ln (1 - \tau_{t+1}^K) R_{t+1} s_t + \theta \ln g_{t+1}).$$

By solving this problem, we obtain the following labor supply and savings functions:

$$l_t = [(1 - \tau_t)w_t]^v, \quad (3)$$

$$s_t = \frac{\beta \pi}{1 + \beta \pi} \cdot \frac{1/v}{1 + 1/v} [(1 - \tau_t)w_t]^{1+v}. \quad (4)$$

The labor supply and savings decrease as the labor income tax rate, τ_t , increases, but they increase as the wage rate, w_t , increases.

Firms

There is a continuum of identical firms that are perfectly competitive profit maximizers and that produce the final output Y_t with a constant-returns-to-scale Cobb–Douglas production function, $Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}$. Here, $A(> 0)$ is total factor productivity, which is constant across periods, K_t is aggregate capital, L_t is aggregate labor, and $\alpha \in (0, 1)$ is a constant parameter representing capital share in production.

In each period, a firm chooses capital and labor to maximize its profit, $A(K_t)^\alpha (L_t)^{1-\alpha} - R_t K_t - w_t L_t$, where R_t is the gross return on physical capital and w_t is the wage rate. The firm's profit maximization leads to

$$K_t : R_t = \alpha A(k_t)^{\alpha-1} (l_t)^{1-\alpha}, \quad (5)$$

$$L_t : w_t = (1 - \alpha) A(k_t)^\alpha (l_t)^{-\alpha}, \quad (6)$$

where $k_t \equiv K_t/N_t$ is per-capita capital and $l_t \equiv L_t/N_t$ is per-capita labor. Capital fully depreciates in a single period.

Government budget constraint

Government expenditure is financed by both taxes on capital and labor income and public debt issues. Let B_t denote aggregate inherited debt. The government budget constraint in period t is $\tau_t w_t l_t N_t + \tau_t^K (R_t/\pi) s_{t-1} \pi N_{t-1} + B_{t+1} = R_t B_t + G_t$, where $\tau_t w_t l_t N_t$ is aggregate labor income tax revenue, $\tau_t^K R_t s_{t-1} N_{t-1}$ is aggregate capital income tax revenue, B_{t+1} is newly issued public debt, $R_t B_t$ is debt repayment, and G_t is aggregate public expenditure. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt.

By dividing both sides of the constraint by N_t , we can obtain a per-capita expression of the government budget constraint:

$$\tau_t w_t l_t + \frac{\tau_t^K R_t s_{t-1}}{1+n} + (1+n)b_{t+1} = R_t b_t + \frac{(1+n) + \pi}{1+n} g_t, \quad (7)$$

where $b_t \equiv B_t/N_t$ is per-capita debt and $g_t \equiv G_t/(N_t + \pi N_{t-1})$ is the per-capita public expenditure.

Capital market-clearing condition

Public debt is traded in the domestic capital market. The market-clearing condition for capital is $K_{t+1} + B_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged agents in period t , $N_t s_t$, to the sum of the stocks of aggregate physical capital and aggregate public debt at the beginning of period $t + 1$. We can rewrite this condition as

$$(1+n)(k_{t+1} + b_{t+1}) = s_t. \quad (8)$$

Economic Equilibrium

Hereafter, we drop the usage of time subscripts and use the notation z' (e.g., k' , b' , g' , τ' , and $\tau^{K'}$) to denote the next period of z . To define an economic equilibrium in the present framework, we reduce the conditions (1) - (8) to a system of two difference equations, one representing the government budget constraint and the other representing the capital market-clearing condition, for two state variables, physical capital k and public debt b . To show this, consider the labor supply in (3), the savings in (4), and factor prices in (5) and (6). We write these as functions of physical capital, k , and the labor income tax rate, τ as follows:⁴

$$l = l(\tau, k) \equiv [(1 - \tau)(1 - \alpha)A(k)^\alpha]^{v/(1+\alpha v)}, \quad (9)$$

$$s = s(\tau, k, l(\tau, k)) \equiv \frac{\beta\pi}{1 + \beta\pi} \cdot \frac{1/v}{1 + 1/v} [(1 - \tau)w(k, l(\tau, k))]^{1+v}, \quad (10)$$

$$w = w(k, l(\tau, k)) \equiv (1 - \alpha)A(k)^\alpha [l(\tau, k)]^{-\alpha}, \quad (11)$$

$$R = R(k, l(\tau, k)) \equiv \alpha A(k)^{\alpha-1} [l(\tau, k)]^{1-\alpha}. \quad (12)$$

Using the labor supply function in (9) and the factor prices in (11) and (12), we can reformulate the government budget constraint in Eq. (7) in terms of the state variables, k and b , and the government policy variables, τ , τ^K , and g as follows:

$$TR(\tau, k) + TR^K(\tau^K, \tau, k, b) + (1 + n)b' = R(k, l(\tau, k))b + \frac{(1 + n) + \pi}{1 + n}g,$$

or,

$$b' = b'(\tau^K, \tau, g, k, b) \equiv \frac{1}{1 + n} \left[\frac{(1 + n) + \pi}{1 + n}g + R(k, l(\tau, k))b - TR(\tau, k) - TR^K(\tau^K, \tau, k, b) \right], \quad (13)$$

where we define $TR(\tau, k)$ and $TR^K(\tau^K, \tau, k, b)$, representing the tax revenues from labor and capital income, respectively, as follows:

$$\begin{aligned} TR(\tau, k) &\equiv \tau w(k, l(\tau, k)) l(\tau, k), \\ TR^K(\tau^K, \tau, k, b) &\equiv \tau^K R(k, l(\tau, k)) (k + b). \end{aligned}$$

We can also reformulate the capital market-clearing condition in (8) as follows:

$$(1 + n)(k' + b') = s(\tau, k, l(\tau, k)) = \frac{\beta\pi}{1 + \beta\pi} \cdot \frac{1/v}{1 + 1/v} [(1 - \tau)(1 - \alpha)A(k)^\alpha]^{1+v},$$

or,

$$k' = k'(\tau^K, \tau, g, k, b) \equiv \frac{1}{1 + n} \cdot \frac{\beta\pi}{1 + \beta\pi} \cdot \frac{1/v}{1 + 1/v} [(1 - \tau)(1 - \alpha)A(k)^\alpha]^{1+v} - b'(\tau^K, \tau, g, k, b). \quad (14)$$

Thus, the economic equilibrium in the present framework is defined as follows:

⁴The derivation of (9) - (12) is as follows. First, we substitute (6) into (3) to write the optimal labor supply as a function of τ_t and k_t , as in (9). Second, we reformulate the saving function in (4) using (6) and (9), as in (10). Third, we use firms' profit maximization with respect to L_t in (6) and the labor supply function in (9) to obtain the labor market-clearing wage rate, as in (11). Finally, firms' profit maximization with respect to K_t in (5) and the labor supply function in (9) lead to (12).

Definition 1 Given the current state variables (k, b) and the government policy variables (τ, τ^K, g) , an economic equilibrium is a mapping Ψ^{ECON} from the current state (k, b) to the next period's state (k', b') , and a conditional on the mapping that governs policy Ψ^{POL} , defined in Definition 2 below. That is,

$$(k', b') = \Psi^{ECON}(k, b; \tau, \tau^K, g)$$

with $(\tau, \tau^K, g) = \Psi^{POL}(k, b)$, where Ψ^{ECON} is defined by (13) and (14).

In the economic equilibrium, we can express the indirect utility of the current middle-aged, V^M , and that of the current elderly, V^O , as functions of policy variables, physical capital, and public debt. V^M becomes:

$$V^M = \ln \left[c(\tau, k) - \frac{(l(\tau, k))^{1+1/v}}{1 + 1/v} \right] + \theta \ln g + \beta \pi [\ln d'(\tau^{K'}, \tau, \tau', k, k') + \theta \ln g'], \quad (15)$$

where we define $c(\tau, k)$ and $d'(\tau^{K'}, \tau, \tau', k, k')$, representing consumption in middle and old ages, respectively, as follows:

$$\begin{aligned} c(\tau, k) &\equiv (1 - \tau)w(k, l(\tau, k))l(\tau, k) - s(\tau, k, l(\tau, k)), \\ d'(\tau^{K'}, \tau, \tau', k, k') &\equiv (1 - \tau^{K'}) \frac{R(k', l(\tau', k'))}{\pi} s(\tau, k, l(\tau, k)). \end{aligned}$$

The indirect utility function of the elderly in period t , V^O , is

$$V^O = \ln d(\tau^K, \tau, k, b) + \theta \ln g, \quad (16)$$

where $d(\tau^K, \tau, k, b)$ is defined as⁵

$$d(\tau^K, \tau, k, b) \equiv (1 - \tau^K) \frac{R(k, l(\tau, k))}{\pi} (1 + n)(k + b).$$

3 Political Equilibrium

In this section, we consider voting on fiscal policy. We employ probabilistic voting à la [Lindbeck and Weibull \(1987\)](#). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As [Persson and Tabellini \(2002\)](#) demonstrate, the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

In the present framework, both the middle-aged and elderly have an incentive to vote. Thus, the political objective is the weighted sum of the utility of the middle-aged and elderly, given by $\pi \omega V^O + (1 + n)(1 - \omega)V^M$, where $\omega \in (0, 1)$ and $1 - \omega$ are the political weights placed on the elderly and middle-aged, respectively. A larger value of ω implies greater political power

⁵The arguments of $d'(\cdot)$ differs from those of $d(\cdot)$. The reason for the difference is that in the expression of d , s is replaced by $(1 + n)(k + b)$ by using the capital market clearing condition.

of the elderly. We use the gross population growth rate, $(1 + n)$ to adjust the weight of the middle-aged and life expectancy (i.e., the probability of living in old age), π to adjust the weight of the elderly, to reflect their share of the population. To obtain the intuition behind this result, we divide the objective function by $(1 + n)(1 - \omega)$ and redefine it, denoted by Ω , as follows:

$$\Omega = \frac{\pi\omega}{(1+n)(1-\omega)} V^O + V^M,$$

where the coefficient $\pi\omega/(1+n)(1-\omega)$ of V^O represents the relative political weight of the elderly.

We substitute V^M in (15) and V^O in (16) into Ω and obtain

$$\begin{aligned} \Omega &= \frac{\pi\omega}{(1+n)(1-\omega)} V^O(\tau, \tau^K, g, k, b) + V^M(\tau, g, \tau', \tau^{K'}, g', k', k) \\ &= \frac{\pi\omega}{(1+n)(1-\omega)} [\ln d(\tau^K, \tau, k, b) + \theta \ln g] + \ln \left[c(\tau, k) - \frac{(l(\tau, k))^{1+1/v}}{1+1/v} \right] + \theta \ln g \\ &\quad + \beta\pi [\ln d'(\tau^{K'}, \tau, \tau', k, k') + \theta \ln g']. \end{aligned} \quad (17)$$

The political objective function in (17) suggests that the current policy choice affects decisions on future policy via physical capital accumulation. In particular, the current choice of τ^K , τ , and g affect the formation of physical capital in the next period. This in turn influences decision making on the next period's fiscal policy. Thus, the expected provision of the public goods and the rates of labor and capital income taxes for the next period, g' , τ' , and $\tau^{K'}$, respectively, could be given by the functions of the next period's state variables, k' and b' : $g' = G(k', b')$, $\tau' = T(k', b')$, and $\tau^{K'} = T^K(k', b')$. Let Ω denote the political objective function, where k' and b' are replaced with k and b by substituting (13), (14), $g' = G(k', b')$, $\tau' = T(k', b')$, and $\tau^{K'} = T^K(k', b')$. We can define a political equilibrium in the present framework as follows.

Definition 2 A political equilibrium is a mapping Ψ^{POL} that solves the fixed point problem

$$\Psi^{POL}(k, b) = \arg \max_{\tau, \tau^K, g} \Omega(\tau, \tau^K, g, k, b; \Psi^{POL}) \text{ for all } k, b.$$

Each period, the government chooses τ , τ^K , and g , given state variables k and b . The public debt issue, b' , is determined as a residual from the government budget constraint.

3.1 Characterization of Political Equilibrium

To obtain the set of policy functions in Definition 2, we conjecture the following policy functions in the next period:

$$1 - \tau^{K'} = \frac{\bar{T}^K}{\alpha} \cdot \frac{1}{1 + b'/k'}, \quad (18)$$

$$\tau' = 1 - \bar{T}, \quad (19)$$

$$g' = \bar{G} \cdot [A(k')^\alpha]^{\frac{1+v}{1+\alpha v}}, \quad (20)$$

where \bar{T}^K , \bar{T} , and \bar{G} are positive constant parameters. The conjecture in (18) implies that the capital income tax rate is set such that the difference between the ratio of capital income to GDP and that of capital income tax revenue to GDP is constant. This is confirmed by reformulating (18) as $\alpha(1 + b'/k') - \tau^{K'}\alpha(1 + b'/k') = \bar{T}^K$. The same argument applies to the conjecture in (19): the labor income tax rate is set such that the difference between the ratio of labor income to GDP and that of labor income tax revenue to GDP is constant. Finally, the conjecture in (20) states that public goods are provided at a fixed percentage rate of GDP.

Given the conjectures in (18)–(20), we consider the optimization problem described in Definition 2. We solve the problem and obtain the following first-order conditions:

$$\begin{aligned} \tau : & \frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_\tau}{d} + \frac{c_\tau - (l)^{1/v} l_\tau}{c - \frac{(l)^{1+1/v}}{1+1/v}} + \beta\pi \frac{d'_\tau}{d'} + \beta\pi \left(\frac{d'_{\tau K'} \tau_{k'}^{K'} + d'_{k'}}{d'} + \theta \frac{g'_{k'}}{g'} \right) k'_\tau \\ & + \beta\pi \left(\frac{d'_{\tau K'} \tau_{b'}^{K'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} \right) b'_\tau = 0, \end{aligned} \quad (21)$$

$$\tau^{K'} : \frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_{\tau K'}}{d} + \beta\pi \left(\frac{d'_{\tau K'} \tau_{b'}^{K'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} \right) b'_{\tau K'} = 0, \quad (22)$$

$$g : \left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} + \beta\pi \left(\frac{d'_{\tau K'} \tau_{b'}^{K'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} \right) b'_g = 0, \quad (23)$$

where a variable with a subscript x represents a derivative with respect to x (e.g., $d_\tau = \partial d / \partial \tau$).

Given the definition of b' in (13), we have

$$\begin{aligned} b'_\tau &= \frac{1}{1+n} (-TR_{R_\tau} - TR_{\tau K'} + R_\tau b), \\ b'_{\tau K'} &= -\frac{1}{1+n} TR_{\tau K'}, \\ b'_g &= \frac{1}{1+n} \cdot \frac{(1+n) + \pi}{1+n}. \end{aligned}$$

Thus, we can summarize the first-order conditions in (21)–(23) by focusing on the policy trade-offs as follows:

$$\frac{\frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_\tau}{d} + \frac{c_\tau - (l)^{1/v} l_\tau}{c - \frac{(l)^{1+1/v}}{1+1/v}} + \beta\pi \frac{d'_\tau}{d'} + \beta\pi \left(\frac{d'_{\tau K'} \tau_{k'}^{K'} + d'_{k'}}{d'} + \theta \frac{g'_{k'}}{g'} \right) k'_\tau}{TR_{R_\tau} + TR_{\tau K'} - R_\tau b} = \frac{\beta\pi}{1+n} \left(\frac{d'_{\tau K'} \tau_{b'}^{K'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} \right), \quad (24)$$

$$\frac{\frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_{\tau K'}}{d}}{TR_{\tau K'}} = \frac{\beta\pi}{1+n} \left(\frac{d'_{\tau K'} \tau_{b'}^{K'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} \right), \quad (25)$$

$$\left(\frac{\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} = (-1) \frac{\beta\pi}{1+n} \left(\frac{d'_{\tau K'} \tau_{b'}^{K'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} \right). \quad (26)$$

The expressions in (24)-(26) suggest that the following three effects shape policy: the effect through the next period's state variables k' and b' ; the effect through the capital income tax rate, $\tau^{K'}$; and the effect through public goods provision, g' . To understand how these effects work, first consider (24). The numerator on the left-hand side, representing the net marginal benefits of the labor income tax cut, includes the following four effects. First, the term $[\pi\omega/(1+n)(1-\omega)] \cdot (d_\tau/d)$ shows the marginal benefit of the tax cut for the elderly; lowering the tax rate induces the middle-aged to increase labor supply. This in turn raises the return from savings and thus increases the consumption of the elderly.

Second, the term $(c_\tau - (l)^{1/v} l_\tau) / (c - (l)^{1+1/v} / (1 + 1/v))$ includes the marginal costs and benefits for the middle-aged. A tax cut causes disposable income to rise, and thus increases the consumption of the middle-aged, as represented by the term c_τ . At the same time, the cut promotes the labor supply of the middle-aged and thus increases their disutility of labor, as the term $(l)^{1/v} l_\tau$ represents. Third, the term $\beta\pi d'_\tau/d'$ shows the marginal benefit of the labor income tax cut for the middle-aged. The cut in the labor income tax rate increases the disposable income of the middle-aged and thus their savings, which in turn increases consumption in their old age.

Finally, the term $\beta\pi \left((d'_{\tau K'} \tau^{K'}_{k'} + d'_{k'}) / d' + \theta g'_{k'} / g' \right) k'_\tau$ includes the marginal costs and benefits of the labor income tax cut for the middle-aged through the next period physical capital, k' . As mentioned above, the cut in the labor income tax rate increases savings, which stimulates physical capital accumulation. This, in turn, produces the following three effects on the middle-aged: a reduction in the capital income tax rate in the next period, thus increasing their consumption in their old age, as represented by the term $d'_{\tau K'} \tau^{K'}_{k'} / d'$. Secondly, a decrease in the return from savings, i.e., the consumption in their old age, as represented by the term $d'_{k'} / d'$. Lastly, an increase in public goods provision in the next period, as represented by the term $\theta g'_{k'} / g'$. The left-hand side of (24) evaluates the above four effects based on the change in the tax revenue through labor income taxation, as represented by the term $TR_\tau + TR_\tau^K - R_\tau b$ in the denominator.

The right-hand side of (24) shows the marginal net costs of public debt issuance associated with the decision on the labor income tax. As we can see in (24), the issue of public debt increases the capital income tax rate in the next period, $\tau^{K'}$. Simultaneously, the issue of public debt crowds out physical capital accumulation, thus has the opposite effect to that observed on the fourth term in the left-hand side of (24). The expression in (24) suggests that the government chooses the labor income tax rate to balance the abovementioned costs and benefits.

Next, consider the expression in (25). The left-hand side shows the marginal benefits of a capital income tax cut. The cut increases the consumption of the elderly and thus makes them better off. We evaluate this effect based on the change in the tax revenue from capital income taxation represented by the term TR_τ^K in the denominator. The right-hand side represents the net marginal costs of public debt issuance associated with the decision on the capital income

tax. The expression in (25) therefore suggests that the government chooses the capital income tax rate to balance these costs and benefits.

Finally, consider the expression in (26). The left-hand side shows the marginal benefit of public goods provision, normalized by the dependency (i.e., beneficiary-contributor) ratio. The right-hand side is equal to that of (24) multiplied by minus one, and so represents the net marginal costs of public debt reduction associated with the decision on public goods provision. The expression in (26) suggests that the government chooses public goods provision to balance the costs and benefits arising from the choice of public goods provision.

We can obtain the policy functions that are the solutions to the government optimization problem by solving (24)-(26) and the government budget constraint in (13) for τ , τ^K , g , and b' . To simplify the presentation of the policy functions, we introduce the following notations:

$$\begin{aligned}\bar{T}^K &\equiv \frac{1 - \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) \frac{D_3}{D_1}}{\left[1 + \theta + \frac{(1+n)(1-\omega)}{\pi\omega} \left(\theta + \frac{\beta(1+\theta)\alpha(1+v)}{1+\alpha v} \right) \right] - \left(\frac{\beta}{1+\beta} \frac{1/v}{1+1/v} - 1 \right) \frac{D_2}{D_1}}, \\ \bar{T} &\equiv \frac{1}{1-\alpha} \cdot \frac{D_2 \bar{T}^K - D_3}{D_1}, \\ \bar{G} &\equiv \frac{1+n}{(1+n)+\pi} \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta [(1-\alpha)\bar{T}]^{(1-\alpha)v/(1+\alpha v)} \bar{T}^K, \\ \bar{B} &\equiv [(1-\alpha)\bar{T}]^{(1-\alpha)v/(1+\alpha v)} [\bar{T}^K + (1-\alpha)\bar{T} - 1] + \frac{(1+n)+\pi}{1+n} \bar{G},\end{aligned}$$

where D_1, D_2 , and D_3 are defined by

$$\begin{aligned}D_1 &\equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) - \beta\pi(1+\theta)\alpha(1+v) \left[(-1) \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} \frac{1+v}{1+\alpha v} + 1 + \frac{v(1-\alpha)}{1+\alpha v} \right], \\ D_2 &\equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \left[1 + \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta \right] + \frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}, \\ D_3 &\equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] + \frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.\end{aligned}$$

The following proposition describes the optimal policy functions in the present framework.

Proposition 1 *There is a political equilibrium characterized by the following policy functions:*

$$\begin{aligned}\tau^K &= 1 - \frac{\bar{T}^K}{\alpha} \cdot \frac{1}{1+b/k}, \\ \tau &= 1 - \bar{T}, \\ g &= \bar{G} \cdot [A(k)^\alpha]^{(1+v)/(1+\alpha v)}, \\ (1+n)b' &= \bar{B} \cdot [A(k)^\alpha]^{(1+v)/(1+\alpha v)},\end{aligned}$$

Proof. See Appendix A.1.

Proposition 1 implies that the policy functions have the following features. First, the capital income tax rate is increasing in public debt, but decreasing in physical capital. A higher level of public debt increases the burden of debt repayment. The government responds to the increased burden by raising the capital income tax rate. By contrast, a higher level of physical capital

lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate. Second, the levels of public goods provision and public debt issues are linear functions of the output. This implies that the government finds it optimal to provide public goods and to issue public debt in proportion to the output. Third, the government borrows in the capital market as long as $\bar{B} > 0$; if this is the case, then the government finds it optimal to shift a part of the burden onto the future generations.

Having established the policy functions, we are now ready to demonstrate the accumulation of physical capital. We substitute the policy functions of b' and τ in Proposition 1 into the capital market-clearing condition in (14) and obtain

$$k' = \frac{1}{1+n} \left\{ \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} [(1-\alpha)\bar{T}]^{(1+v)/(1+\alpha v)} - \bar{B} \right\} [A(k)^\alpha]^{(1+v)/(1+\alpha v)}, \quad (27)$$

where k' denotes the next-period capital stock. Given the initial condition k_0 , Eq. (27) determines the equilibrium sequence $\{k_t\}$. A steady state is defined as an equilibrium sequence with $k = k'$. In other words, per-capita capital is constant in a steady state. Eq. (27) indicates that there is a unique, stable steady-state equilibrium of k .

3.2 Policy Response to Population Aging

The result established in Proposition 1 indicates that an increase in π (i.e., an increased life expectancy), a decrease in n (i.e., a decreased population growth rate), and an increase in ω (i.e., an increased political weight of the elderly) affect the policy functions. As mentioned in the introduction the voter turnout of older people is higher than that of younger ones in OECD countries. This implies that the aging of society works in the direction of increasing the political weight of the elderly. Therefore, we focus on ω , as well as π and n , to analyze the effects of population aging on the ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP. We analyze the cases of inelastic and elastic labor supply in turn.

3.2.1 Inelastic Labor Supply

We first consider the case of inelastic labor supply, $v = 0$, and obtain the following result.

Proposition 2 *Suppose that labor supply is inelastic: $v = 0$.*

(i) *If $\alpha(1+\theta) < 1$ such that the government borrows in the capital market (i.e., $b' > 0$), then the ratio of government debt to GDP is increasing in the life expectancy and the population growth rate, and decreasing in the political weight of the elderly: $\partial(B_{t+1}/Y_t)/\partial\pi > 0$, $\partial(B_{t+1}/Y_t)/\partial n > 0$, and $\partial(B_{t+1}/Y_t)/\partial\omega < 0$.*

(ii) *The ratio of capital income tax revenue to GDP is decreasing in the life expectancy and the political weight of the elderly, and increasing in the population growth rate: $\partial(\tau_t^K R_{r s_{t-1}} N_{t-1}/Y_t)/\partial\pi < 0$, $\partial(\tau_t^K R_{r s_{t-1}} N_{t-1}/Y_t)/\partial n > 0$, and $\partial(\tau_t^K R_{r s_{t-1}} N_{t-1}/Y_t)/\partial\omega < 0$.*

(iii) *The ratio of labor income tax revenue to GDP is increasing (decreasing) in the life expectancy if $\omega/(1+n)(1-\omega) > (<)\beta(1-\alpha)$, decreasing in the population growth rate, and increasing in the political weight of the elderly: $\partial(\tau_t w_t N_t/Y_t)/\partial\pi \geq 0$ if and only if $\omega/(1+n)(1-\omega) \geq \beta(1-\alpha)$, $\partial(\tau_t w_t N_t/Y_t)/\partial n < 0$, and $\partial(\tau_t w_t N_t/Y_t)/\partial\omega > 0$.*

(iv) *The ratio of government expenditure to GDP is decreasing in the population growth rate and the life expectancy, and increasing in the political weight of the elderly: $\partial(G_t/Y_t)/\partial\pi < 0$, $\partial(G_t/Y_t)/\partial n < 0$, and $\partial(G_t/Y_t)/\partial\omega > 0$.*

Proof. See Appendix A.2.

Proposition 2 shows that when labor supply is inelastic, the ratio of labor income tax revenue to GDP increases as the population growth rate declines and the political weight of the elderly increases. Moreover, when the political weight of the elderly is large, the ratio increases with the increase in life expectancy. These results are generally consistent with the evidence from Figure 1. However, there is a discrepancy between theory and evidence for the ratios of government expenditure and debt to GDP. Proposition 2 reports that the ratio of government expenditure to GDP decreases as the life expectancy increases. The ratio of government debt to GDP decreases as the population growth rate decreases and the political power of the elderly increases. These results appear to be inconsistent with the evidence from Figure 1.

Proposition 2 also suggests a discrepancy between theory and evidence for the ratio of capital income tax revenue to GDP. The result in Proposition 2 shows that the ratio decreases as the life expectancy and the political weight of the elderly increase and the population growth rate decreases. This result, implying a negative association between the ratio and aging, seems to be intuitive at first glance, because such changes in demographic factors lead to an increase in the political weight of the elderly, which in turn provides incentives for the government to choose policies favoring the elderly who bear the capital income tax burden. However, the evidence from Figure 1 shows that the negative association does not hold for some countries. In particular, Ireland, Korea, and the United States show population aging rates below the OECD average, while they show higher ratios of capital income tax revenue to GDP than other countries, except for Sweden. In the following analysis, we show that assuming elastic labor supply could solve the discrepancy between theory and the empirical findings.

3.2.2 Elastic Labor Supply

For the analysis, we take a numerical approach owing to the limitations of the analytical approach in the presence of elastic labor supply. Our strategy is to calibrate the model economy in such a manner that the steady-state equilibrium matches some key statistics of the average OECD country over the time period 1995–2016. We then use the calibrated economy to run some quantitative experiments.

We fix the share of capital at $\alpha = 1/3$ following Song et al. (2012) and Lancia and Russo (2016). We introduce young age into the model: during youth, individuals make no economic

decision and depend on their parents for their livelihood. Each period lasts 30 years; this assumption is standard in quantitative analyses of two- or three-period overlapping-generations models (e.g., [Gonzalez-Eiras and Niepelt \(2008\)](#), [Song et al. \(2012\)](#), and [Lancia and Russo \(2016\)](#)). Our selection of β is 0.99 per quarter, which is also standard in the literature (e.g., [Kydland and Prescott \(1982\)](#), [de la Croix and Doepke \(2003\)](#)). Since agents in the present model plan over a generation that spans 30 years, we discount the future by $(0.99)^{120}$. Following [Lancia and Russo \(2016\)](#), we set the relative political weight of the elderly before adjustment for the population ratio, $\omega/(1 - \omega)$, to 0.8. In line with [Trabandt and Uhlig \(2011\)](#), we set $v = 3/2$ such that the top of the labor income tax Laffer curve is 60% (see [Appendix A.3](#) for the derivation).

The probability of living in old age, π , is taken from the average life expectancy at birth. The average life expectancy in OECD countries is 78.052 years, so individuals will, on average, live 18.05(=78.05–60) years into old age. In other words, individuals are expected to live 18.052/30 of their 30 years of old age, so $\pi = 0.602$. The net population growth rate, n , is taken from the average annual (gross) population growth rate, 1.00548. The net population growth rate for one period is $(1.00548)^{30} - 1 \simeq 0.178$. The preference weight of public goods provision, θ , is chosen such that the simulated version of the model matches the average ratio of government expenditure to GDP.⁶ [Table 1](#) summarizes the parameters.

α : Capital share of output	1/3
β : Discount factor	$(0.99)^{120}$
$\omega/(1 - \omega)$: Relative political weight of the elderly	0.8
v : Frisch elasticity of labor supply	3/2
π : Probability of living in old age	0.602
n : Population growth rate	0.178
θ : Preferences for public goods	0.667

Table 1: Calibration

We numerically investigate the effects of aging factors, π and n as well as the political weight of the elderly, ω , on the ratios of government debt, capital income tax revenue, labor income tax revenue, and government expenditure to GDP. The numerical results in [Figure 2](#) show that the ratio of labor income tax revenue to GDP increases as the life expectancy π increases, the population growth rate n declines, and the political weight of the elderly ω increases. These results are almost qualitatively similar to those in the case of inelastic labor supply, presented in [Proposition 2](#), and are consistent with the evidence observed in [Figure 1](#).⁷

As for the ratio of government expenditure to GDP, the result in [Figure 2](#) shows that the ratio

⁶We define government expenditure as the sum of general government consumption expenditure and general government gross fixed capital formation. Data on the average life expectancy, the average annual population growth rate, and government expenditures, is sourced from OECD.stat. Source: OECD.Stat (<https://stats.oecd.org/>) (accessed on April 6, 2021).

⁷We also investigate the effects of π , n , and ω on the labor and capital income tax rates as depicted in [Figure 5](#) in [Appendix A.4](#). The figure shows that the responses of the labor (capital) income tax rate to changes in π , n , and ω are qualitatively similar to those of the ratio of labor (capital) income tax revenue to GDP.

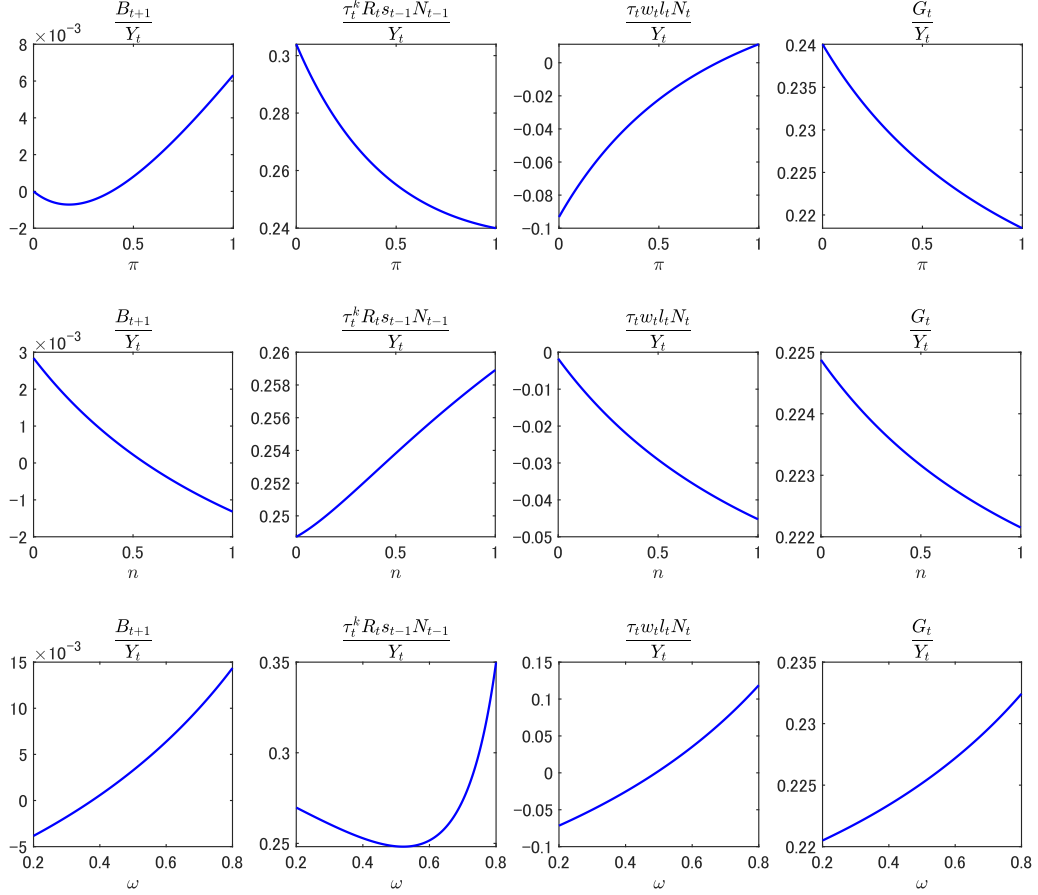


Figure 2: Predicted ratios of government debt, capital income tax revenue, labor income tax revenue, and government expenditure to GDP against changes in π , n , and ω .

increases as the population growth rate declines and the political weight of the elderly increases, but it decreases as life expectancy increases. These results are also similar to those in the case of the inelastic labor supply presented in Proposition 2, so the discrepancy between theory and evidence still remains, in terms of life expectancy. This implies that the discrepancy does not stem from the assumption of an inelastic labor supply. Rather, it is fitting to interpret the discrepancy as an outcome of the effects of the declining population growth rate and increased political weight of the elderly outweighing the effect of increased life expectancy.

The comparative statics for the ratio of public debt to GDP differs significantly from those obtained under the assumption of inelastic labor supply. The ratio of debt to GDP increases as the population growth rate declines and the political weight of the elderly increases; the ratio increases with the rise in the life expectancy when its initial level exceeds around 0.2. The effects of declining population growth rates and the increasing political weight of the elderly are very different from those obtained under inelastic labor supply, but appear to be consistent with the evidence observed in Figure 1.

The comparative statics for the ratio of capital income tax revenue to GDP also differ from those obtained under inelastic labor supply. The ratio of capital income tax revenue to

GDP shows a monotone decline against the declining population growth rate and increasing life expectancy. However, it shows a U-shaped pattern against the increasing political weight of the elderly. The former result is qualitatively similar to that demonstrated in the case of inelastic labor supply, but the latter result differs from that under inelastic labor supply. However, the latter result seems to be consistent with the non-monotonic relationship between aging and the ratio of capital income tax revenue to GDP observed in Figure 1. The numerical results presented in Figure 2 suggest that the elastic labor supply assumption and the political weight of the elderly are the keys to obtaining comparative statics consistent with the evidence. In the following, we examine the implications and mechanisms of the elastic labor supply assumption in detail, focusing on the effects of the political weight of the elderly, ω .

Ratio of Debt to GDP

To understand the role of the endogenous labor supply assumption, we first consider the ratio of public debt to GDP, B'/Y . From the government budget constraint in (13), the ratio when $v \geq 0$ (including both elastic and inelastic labor supply cases) is

$$\frac{B'}{Y} = (-1)\frac{TR(\tau, k)}{Y} + (-1)\frac{TR^K(\tau, \tau^K, k, b)}{Y} + \frac{G}{Y} + \frac{R(k, l(\tau, k))B}{Y}. \quad (28)$$

Equation (28) indicates that the ratio depends on the four terms on the right-hand side, TR/Y , TR^K/Y , G/Y , and RB/Y . Figure 3 plots the changes in these four terms, as well as B'/Y against the changes in ω to help clarify the mechanism behind the difference between the elastic ($v > 0$) and inelastic ($v = 0$) cases.

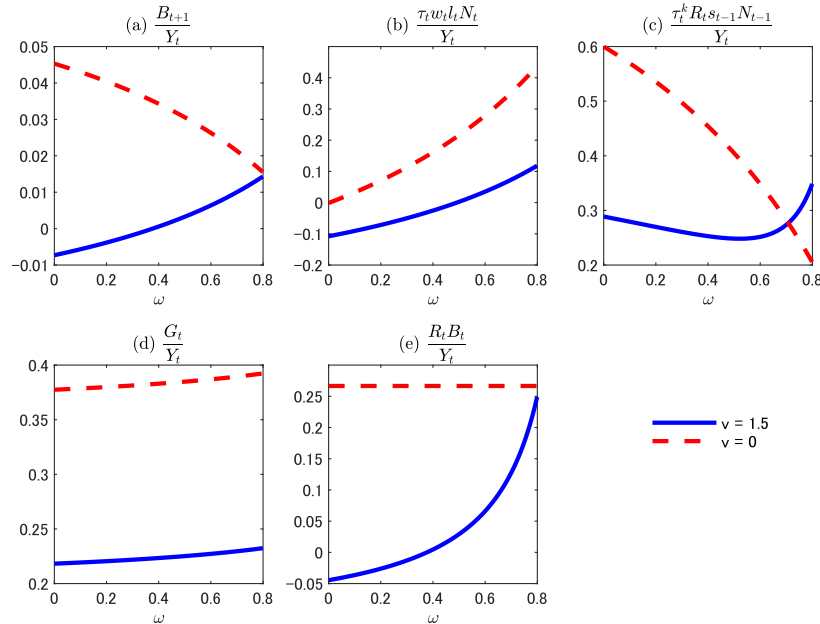


Figure 3: Numerical illustration of the effects of ω on B_{t+1}/Y_t , $\tau_t w_t l_t N_t / Y_t$, $\tau_t^K R_t s_{t-1} N_{t-1} / Y_t$, G_t / Y_t , and $R_t B_t / Y_t$. The dotted and solid curves plot the results when $v = 0$ and 1.5, respectively.

As Panel (a) shows, the ratio B'/Y decreases as ω increases when labor supply is inelastic, whereas it increases when labor supply is elastic. The elasticity of labor supply leads to these contrasting results. When the labor supply is elastic, an increase in ω gives the government an incentive to raise the labor income tax rate, which leads to an increase in the interest rate, R , through the household's choice of labor supply. This leads to an increase in the ratio B'/Y through an increase in the debt repayment, RB (Panel (e)). However, when the labor supply is inelastic, this positive effect through the interest rate is absent, so an increase in ω leads to a decrease in the ratio B'/Y .

Ratio of Capital Income Tax Revenue to GDP

Next, we consider the ratio of capital income tax revenue to GDP, $\tau_t^K R_t s_{t-1} N_{t-1} / Y_t$. The ratio depends on two factors: the capital income tax rate, τ_t^K , and the ratio of capital income to GDP, $R_t s_{t-1} N_{t-1} / Y_t$. Figure 4 plots changes in $R_t s_{t-1} N_{t-1} / Y_t$, τ_t^K , and $\tau_t^K R_t s_{t-1} N_{t-1} / Y_t$ against changes in ω . In each panel, the solid (dashed) curve represents the changes in a concerned variable when labor supply is elastic (inelastic). We first assess the effect through the ratio of capital income to GDP, $R_t s_{t-1} N_{t-1} / Y_t$, and then assess the effect through the capital income tax rate, τ_t^K .

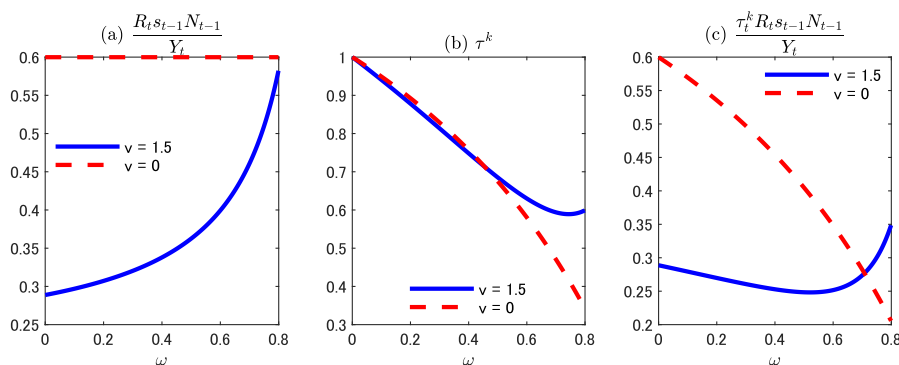


Figure 4: Numerical illustration of the effects of ω on $R_t s_{t-1} N_{t-1} / Y_t$, τ_t^K , and $\tau_t^K R_t s_{t-1} N_{t-1} / Y_t$. The dotted and solid curves plot the results when $v = 0$ and 1.5, respectively.

When the labor supply is inelastic, such that $v = 0$ holds, the ratio of capital income to GDP reduces to $R_t s_{t-1} N_{t-1} / Y_t = (1 - \alpha\theta) / (1 + \theta)$, which is independent of the political weight of the elderly, ω . However, when the labor supply is elastic, such that $v > 0$ holds, a change in ω affects the ratio $R_t s_{t-1} N_{t-1} / Y_t$ through the labor supply decision as follows. The government chooses a higher labor income tax rate as the political weight of the elderly increases. A higher labor income tax rate reduces the supply of labor, resulting in lower labor income and thus lower savings. This leads to a decrease in the ratio of capital income to GDP through the term s_{t-1} . On the other hand, a decrease in savings leads to an increase in the interest rate, R_t , through a decrease in the capital level, which leads to an increase in the ratio of capital income to GDP. In total, the latter effect dominates the former one, implying that the ratio of capital income to GDP increases as ω increases.

Next, consider the effect of ω through the capital income tax rate, $\tau_t^K = 1 - \bar{T}^K / [\alpha(k_t + b_t)/k_t]$. The expression shows that a change in ω affects the tax rate through \bar{T}^K and $\alpha(k_t + b_t)/k_t$. The term \bar{T}^K increases and thus, τ_t^K decreases as ω increases, irrespective of the status of labor supply, v . The negative effect on the capital income tax rate reflects the preferences of the elderly who want to reduce their fiscal burden of capital income taxation. The term $\alpha(k_t + b_t)/k_t$, which is equal to $R_t s_{t-1} N_{t-1} / Y_t$, is independent of ω when $v = 0$, while it is increasing in ω when $v > 0$, as we argued in the last paragraph. The effects through the two terms suggest that when $v = 0$, the capital income tax rate decreases as ω increases. However, when $v > 0$, the negative effect through the term \bar{T}^K outweighs the positive effect through the capital income to GDP ratio, $\alpha(k_t + b_t)/k_t$, for low values of ω ; the opposite result holds for high values of ω . Thus, an increase in the political power of the elderly produces an initial decrease followed by an increase in the capital income tax rate.

Up to now, the results have the following implications for the ratio of capital income tax revenue to GDP. When the labor supply is inelastic, $v = 0$, the effect through the ratio of capital income to GDP does not appear; the effect through the capital income tax rate remains and works to lower the ratio of capital income tax revenue to GDP as ω increases. However, when the labor supply is elastic, $v > 0$, the positive effect through the ratio of capital income to GDP may outweigh the negative effect through the capital income tax rate for high values of ω . Which effect dominates depends on the initial value of ω . Therefore, the political weight of elderly and the elastic labor supply play important roles in determining the ratio of capital income tax revenue to GDP.

4 Conclusion

This study analyzed the distribution of the fiscal burden across generations in a political economy model of fiscal policy. The model includes (i) two tax instruments: capital and labor income taxes, accompanied by debt finance; and (ii) household decisions on labor supply. The first element enables us to investigate the impact of population aging on the distribution of the fiscal burden across generations; the second element allows us to present the effects of aging on policy variables via households' labor decisions.

Given these features, we showed that aging, which implies increased political weight of the elderly, leads to (i) an increase in the ratios of debt to GDP and labor income tax revenue to GDP; and (ii) an initial decrease followed by an increase in the ratio of capital income tax revenue to GDP. These model predictions fit well with the evidence observed in OECD countries. In particular, the latter result suggests that the political weight of the elderly is a key factor in the different patterns of the ratio observed among the OECD countries sharing similar demographic characteristics.

The results of this study provide important and useful information to predict the generational burden of fiscal policy in aging societies. Population aging first produces a shift of the tax

burden from older to younger generations. As the population ages further, the tax burden on both younger and older generations increases. This suggests a U-shaped pattern of the fiscal burden on the elderly. [Mateos-Planas \(2010\)](#) predicts this pattern, but limits his analysis to the balanced government budget case. The present study instead allowed for government deficits, showing that the ratio of public debt to GDP increases as population ages. The result suggests that when debt finance is allowed, a shift of the fiscal burden from older to younger generations could prove to be stronger in the early stage of population aging. However, further aging leads to an increased fiscal burden on both younger and older generations. The increased fiscal burden is an inevitable consequence of population aging in the long run.

A Mathematical Appendix

A.1 Proof of Proposition 1

Based on the specification of the utility and production functions, we can reformulate the first-order conditions in (21)-(23) as follows:

$$\frac{(-1)\pi\omega}{(1+n)(1-\omega)} \frac{1}{1-\tau^K} + \frac{\beta\pi(1+\theta)\alpha(1+v)}{1+\alpha v} \times \frac{\alpha[(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \left(1 + \frac{b}{k}\right)}{(1+n)k'} = 0, \quad (\text{A.1})$$

$$\begin{aligned} & (-1) \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \frac{1}{1+\alpha v} \frac{1}{1-\tau} \\ & + \frac{\beta\pi(1+\theta)\alpha(1+v)}{1+\alpha v} \cdot \frac{(1-\tau)^{(1-\alpha)v/(1+\alpha v)} [(1-\alpha)A(k)^\alpha]^{(1+v)/(1+\alpha v)}}{(1+n)k'} \\ & \times \left\{ (-1) \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} \frac{1+v}{1+\alpha v} + \frac{1}{1-\tau} \frac{v}{1+\alpha v} \left[\alpha(1-\tau^K) \left(1 + \frac{b}{k}\right) - \alpha - (1-\alpha)\tau \right] + 1 \right\} = 0, \end{aligned} \quad (\text{A.2})$$

$$\left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} - \beta\pi(1+\theta) \frac{\alpha(1+v)}{1+\alpha v} \frac{\frac{(1+n)+\pi}{1+n}}{(1+n)k'} = 0. \quad (\text{A.3})$$

We present the derivation of (A.1)–(A.3) in Appendix B.

The procedure to find the optimal policy functions is as follows. First, substitute the first-order condition with respect to τ^K in (A.1) into the first-order condition with respect to g in (A.3) to write g as a function of τ^K and τ : $g = g(\tau^K, \tau)$. Second, substitute $g = g(\tau^K, \tau)$ into the capital market-clearing condition in (14) to write k' as a function of τ^K and τ : $k' = k'(\tau^K, \tau)$. Third, substitute $k' = k'(\tau^K, \tau)$ into the first-order condition with respect to τ^K in (A.1) and τ in (A.2) to obtain the two optimal relations between τ^K and τ , and solve them for τ^K and τ . Fourth, substitute the solutions for τ^K and τ into $g = g(\tau^K, \tau)$ to obtain the optimal policy function of g . Finally, substitute the optimal policy functions of τ^K , τ , and g into the government budget constraint in (13) to obtain the optimal policy function of b' .

Recall the first-order condition with respect to g in (A.3), which we rewrite as

$$\frac{(1+n)+\pi}{1+n} g = \left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \theta \frac{1+\alpha v}{\beta\pi(1+\theta)\alpha(1+v)} (1+n)k'.$$

We substitute the first-order condition with respect to τ^K in (A.1) into the above expression to obtain $g = g(\tau^K, \tau)$, or

$$\frac{(1+n)+\pi}{1+n} g = \left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \theta \frac{\alpha[(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \left(1 + \frac{b}{k}\right)}{\frac{\pi\omega}{(1+n)(1-\omega)} \frac{1}{1-\tau^K}}. \quad (\text{A.4})$$

Next, we substitute (A.4) into the capital market-clearing condition in (14) to obtain

$$\begin{aligned}
(1+n)k' &= \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} [(1-\tau)(1-\alpha)A(k)^\alpha]^{(1+v)/(1+\alpha v)} \\
&\quad - \alpha [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \frac{b}{k} \\
&\quad - \left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \theta \frac{\alpha [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \left(1 + \frac{b}{k}\right)}{\frac{\pi\omega}{(1+n)(1-\omega)} \frac{1}{1-\tau^K}} \\
&\quad + \frac{\tau}{1-\tau} [(1-\tau)(1-\alpha)A(k)^\alpha]^{1/(1+\alpha v)} \cdot [(1-\tau)(1-\alpha)A(k)^\alpha]^{v/(1+\alpha v)} \\
&\quad + \tau^K \alpha [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \left(1 + \frac{b}{k}\right).
\end{aligned}$$

Rearranging the terms, we have

$$\begin{aligned}
(1+n)k' &= [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \\
&\quad \times \left\{ \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) (1-\tau)(1-\alpha) \right. \\
&\quad \left. - \left[1 + \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta \right] \alpha (1-\tau^K) \left(1 + \frac{b}{k} \right) + 1 \right\}. \tag{A.5}
\end{aligned}$$

Eq. (A.5) shows that we can express $(1+n)k'$ as a function of τ^K and τ .

Third, we substitute (A.5) into the first-order condition with respect to τ^K in (A.1) and obtain

$$\begin{aligned}
&\frac{\pi\omega}{(1+n)(1-\omega)} \frac{1}{1-\tau^K} \\
&= \frac{\beta\pi(1+\theta)\alpha(1+v)}{1+\alpha v} \times \frac{\alpha [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \left(1 + \frac{b}{k}\right)}{X} \\
&= \frac{\beta\pi(1+\theta)\alpha(1+v)}{1+\alpha v} \\
&\quad \times \frac{\alpha \left(1 + \frac{b}{k}\right)}{\left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) (1-\tau)(1-\alpha) - \left[1 + \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta \right] \alpha (1-\tau^K) \left(1 + \frac{b}{k} \right) + 1},
\end{aligned}$$

where X is defined by

$$\begin{aligned}
X &\equiv [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \\
&\quad \times \left\{ \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) (1-\tau)(1-\alpha) - \left[1 + \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta \right] \alpha (1-\tau^K) \left(1 + \frac{b}{k} \right) + 1 \right\}.
\end{aligned}$$

Rearranging the terms, we have

$$\begin{aligned}
&\left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) (1-\tau)(1-\alpha) + 1 \\
&= \left[1 + \theta + \frac{(1+n)(1-\omega)}{\pi\omega} \left(\theta + \frac{\beta\pi(1+\theta)\alpha(1+v)}{1+\alpha v} \right) \right] \alpha (1-\tau^K) \left(1 + \frac{b}{k} \right). \tag{A.6}
\end{aligned}$$

This equation describes the optimal relationship between τ^K and τ .

Third, we substitute (A.5) in the first-order condition with respect to τ into (A.2) to obtain

$$\begin{aligned}
& \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \left\{ \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) (1-\tau)(1-\alpha) \right. \\
& \left. - \left[1 + \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta \right] \alpha (1-\tau^K) \left(1 + \frac{b}{k} \right) + 1 \right\} \\
& = \beta\pi (1+\theta) \alpha(1+v) (1-\tau) (1-\alpha) \\
& \times \left\{ (-1) \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} \frac{1+v}{1+\alpha v} + \frac{v}{1+\alpha v} \frac{1}{1-\tau} \left[\alpha (1-\tau^K) \left(1 + \frac{b}{k} \right) - \alpha - (1-\alpha)\tau \right] + 1 \right\} \\
& = \beta\pi (1+\theta) \alpha(1+v) (1-\alpha) \\
& \times \left\{ \left[(-1) \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} \frac{1+v}{1+\alpha v} + 1 + \frac{v(1-\alpha)}{1+\alpha v} \right] (1-\tau) + \frac{v}{1+\alpha v} \alpha (1-\tau^K) \left(1 + \frac{b}{k} \right) - \frac{v}{1+\alpha v} \right\}.
\end{aligned}$$

Rearranging the terms, we have

$$\begin{aligned}
& \underbrace{\left\{ \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) - \beta\pi (1+\theta) \alpha(1+v) \left[(-1) \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} \frac{1+v}{1+\alpha v} + 1 + \frac{v(1-\alpha)}{1+\alpha v} \right] \right\}}_{D_1} \\
& \times (1-\tau)(1-\alpha) \\
& = \underbrace{\left\{ \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \left[1 + \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta \right] + \frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v} \right\}}_{D_2} \alpha (1-\tau^K) \left(1 + \frac{b}{k} \right) \\
& - \underbrace{\left\{ \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] + \frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v} \right\}}_{D_3}, \tag{A.7}
\end{aligned}$$

where D_1 , D_2 , and D_3 are defined by

$$\begin{aligned}
D_1 & \equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) - \beta\pi (1+\theta) \alpha(1+v) \left[(-1) \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} \frac{1+v}{1+\alpha v} + 1 + \frac{v(1-\alpha)}{1+\alpha v} \right], \\
D_2 & \equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] \left[1 + \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta \right] + \frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}, \\
D_3 & \equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v) \right] + \frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.
\end{aligned}$$

Eqs. (A.6) and (A.7) characterize the optimal τ and τ^K . Substituting (A.7) into (A.6) yields

$$1 - \tau^K = \frac{1 - \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) \frac{D_3}{D_1}}{\underbrace{\left[1 + \theta + \frac{(1+n)(1-\omega)}{\pi\omega} \left(\theta + \frac{\beta\pi(1+\theta)\alpha(1+v)}{1+\alpha v} \right) \right]}_{=\bar{T}^K} - \left(\frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} - 1 \right) \frac{D_2}{D_1}} \cdot \frac{1}{\alpha} \cdot \frac{1}{1+b/k}, \tag{A.8}$$

thereby verifying the conjecture of τ^K in (18). In addition, we substitute (A.8) into (A.7) to obtain

$$1 - \tau = \frac{1}{1-\alpha} \cdot \frac{D_2 \bar{T}^K - D_3}{D_1} \equiv \bar{T}, \tag{A.9}$$

thereby verifying the conjecture of τ in (19).

Fourth, we substitute (A.8) and (A.9) into (A.4) to derive the policy function of g :

$$g = \underbrace{\frac{1+n}{(1+n)+\pi} \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} \right) \theta}_{=\bar{G}} \left[(1-\alpha) \bar{T} \right]^{(1-\alpha)v/(1+\alpha v)} \bar{T}^K [A(k)^\alpha]^{(1+v)/(1+\alpha v)}, \tag{A.10}$$

thereby verifying the conjecture in (20).

Finally, substituting τ^K , τ , and g into the government budget constraint in (13) leads to the following policy function of b' :

$$(1+n)b' = \bar{B} [A(k)^\alpha]^{(1+v)/(1+\alpha v)}, \quad (\text{A.11})$$

where \bar{B} is defined by

$$\bar{B} \equiv [(1-\alpha)\bar{T}]^{(1-\alpha)v/(1+\alpha v)} [\bar{T}^K + (1-\alpha)\bar{T} - 1] + \frac{(1+n) + \pi}{1+n} \bar{G}.$$

■

A.2 Proof of Proposition 2

Suppose that $v = 0$ holds. The policy functions of b_{t+1} , τ_t^K , and τ_t presented in Proposition 1 then reduce to

$$\begin{aligned} b_{t+1} &= \frac{1}{1+n} \cdot \frac{\beta\pi(1-\alpha(1+\theta)) \frac{(1+n)(1-\omega)}{\pi\omega}}{(1+\theta) \left(1 + \frac{(1+n)(1-\omega)}{\omega} (1+\alpha\beta\pi)\right)} A(k)^\alpha, \\ \tau_t^K &= 1 - \frac{1}{(1+\theta) \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} (1+\alpha\beta\pi)\right)} \cdot \frac{1}{\alpha(1+b_t/k_t)}, \\ \tau_t &= 1 - \frac{\frac{1+\beta\pi}{1-\alpha} \cdot \frac{(1+n)(1-\omega)}{\pi\omega}}{(1+\theta) \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} (1+\alpha\beta\pi)\right)}. \end{aligned}$$

The ratio of B_{t+1} to Y_t becomes

$$\frac{B_{t+1}}{Y_t} = \frac{(1+n)b_{t+1}N_t}{A(k)^\alpha N_t} = \frac{\beta\pi(1-\alpha(1+\theta))}{(1+\theta) \left(\frac{\pi\omega}{(1+n)(1-\omega)} + (1+\alpha\beta\pi)\right)}.$$

The expression above indicates that the ratio of B_{t+1}/Y_t is positive if $\alpha(1+\theta) < 1$, and that the ratio is increasing in π and decreasing in ω and n if $\alpha(1+\theta) < 1$.

The ratio of $\tau_t^K R_r s_{t-1} N_{t-1}$ to Y_t becomes

$$\begin{aligned} \frac{\tau_t^K R_r s_{t-1} N_{t-1}}{Y_t} &= \frac{\tau_t^K \alpha A(k)^{\alpha-1} (k_t + b_t)(1+n)N_{t-1}}{A(k)^\alpha N_t} \\ &= \alpha \left(1 + \frac{b_t}{k_t}\right) - \frac{1}{(1+\theta) \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} (1+\alpha\beta\pi)\right)}. \end{aligned}$$

In period 0, the ratio becomes

$$\frac{\tau_0^K R_0 s_{-1} N_{-1}}{Y_0} = \alpha \left(1 + \frac{b_0}{k_{t0}}\right) - \frac{1}{(1+\theta) \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} (1+\alpha\beta\pi)\right)}.$$

Given the initial conditions of k_0 and b_0 , the equation indicates that the ratio $\tau_0^K R_0 s_{-1} N_{-1}/Y_0$ is decreasing in π and ω and increasing in n . In period $t \geq 1$, we have

$$1 + \frac{b_t}{k_t} = \frac{1}{\alpha(1+\theta)}.$$

Thus, the ratio becomes

$$\frac{\tau_t^K R_r s_{t-1} N_{t-1}}{Y_t} = \frac{1}{(1+\theta)} - \frac{1}{(1+\theta) \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} (1 + \alpha\beta\pi)\right)},$$

showing that $\tau_t^K R_r s_{t-1} N_{t-1}/Y_t$ is decreasing in π and ω and increasing in n .

The ratio of $\tau_t w_t N_t$ to Y_t becomes

$$\frac{\tau_t w_t N_t}{Y_t} = \frac{\tau_t (1-\alpha) A(k)^\alpha N_t}{A(k)^\alpha N_t} = (1-\alpha) - \frac{1+\beta\pi}{(1+\theta) \left(\frac{\pi\omega}{(1+n)(1-\omega)} + (1+\alpha\beta\pi)\right)}.$$

The equation indicates that the ratio of $\tau_t w_t N_t/Y_t$ is decreasing in n and increasing in ω . To see the effect of a higher π on the ratio, we take the first derivative of the ratio with respect to π and obtain

$$\partial \left(\frac{\tau_t w_t N_t}{Y_t} \right) / \partial \pi = \frac{(-1)}{(1+\theta) \left(\frac{\pi\omega}{(1+n)(1-\omega)} + (1+\alpha\beta\pi)\right)^2} \left[-\frac{\pi\omega}{(1+n)(1-\omega)} + \beta(1-\alpha) \right].$$

Thus, $\partial (\tau_t w_t N_t/Y_t) / \partial \pi \geq 0$ if $\frac{\pi\omega}{(1+n)(1-\omega)} \geq \beta(1-\alpha)$ holds.

When $v = 0$, the ratio of G_t to Y_t becomes

$$\frac{G_t}{Y_t} = \frac{g_t}{A(k_t)^\alpha} \left(1 + \frac{\pi}{1+n}\right) = \frac{\left(1 + \frac{(1+n)(1-\omega)}{\pi\omega}\right) \theta}{(1+\theta) \left(1 + \frac{(1+n)(1-\omega)}{\pi\omega} (1 + \alpha\beta\pi)\right)}.$$

Direct calculation leads to $\partial (G_t/Y_t) / \partial n < 0$, $\partial (G_t/Y_t) / \partial \pi < 0$, and $\partial (G_t/Y_t) / \partial \omega > 0$. ■

A.3 Calibration of v

The labor income tax revenue is

$$\begin{aligned} \tau_t w_t l_t N_t &= \tau_t (1-\alpha) A(k_t)^\alpha (l_t)^{-\alpha} l_t N_t \\ &= [(1-\alpha) A(k_t)^\alpha]^{(1-\alpha)v/(1+\alpha v)} N_t \tau_t (1-\tau_t)^{(1-\alpha)v/(1+\alpha v)}, \end{aligned}$$

where we obtain the equality in the second line by substituting the labor supply function in Eq. (9) into the expression in the first line. The revenue-maximizing tax rate, denoted by τ_{\max} , satisfies the following first-order condition:

$$(1-\tau_{\max})^{(1-\alpha)v/(1+\alpha v)} - \tau_{\max} \frac{(1-\alpha)v}{1+\alpha v} (1-\tau_{\max})^{(1-\alpha)v/(1+\alpha v)-1} = 0,$$

which leads to

$$\tau_{\max} = \frac{1+\alpha v}{1+v}.$$

Following Trabandt and Uhlig (2011), we set v such that the top of the labor income tax Laffer curve is at 60%. Setting $\tau_{\max} = 0.6$ and $\alpha = 1/3$, we obtain $v = 3/2$. ■

A.4 Effects of π , n , and ω on τ and τ^K

We numerically investigate the effects of π , n , and ω on the labor and capital income tax rates, τ and τ^K , depicted in Figure 5, which supplements the analysis in Section 3.2.2. We use the parameter values calibrated in Section 3.2.2. The figure shows that the responses of the labor (capital) income tax rate to changes in π , n , and ω are qualitatively similar to those of the ratio of labor (capital) income tax revenue to GDP.

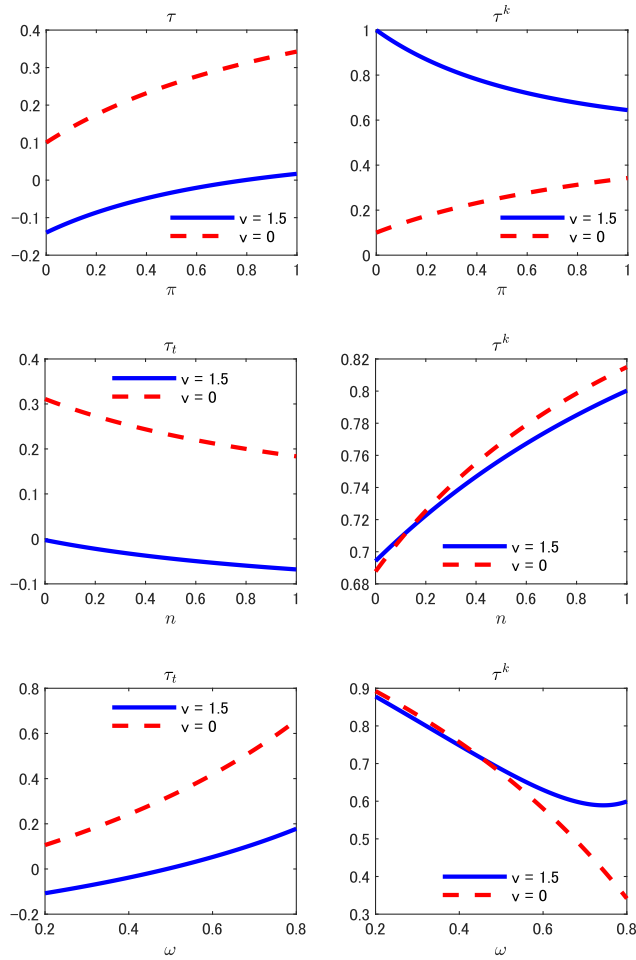


Figure 5: Numerical illustration of the effects of π , n , and ω on τ and τ^K .

■

B Online Appendix

B.1 Reformulation of V^M in (15) and V^O in (16)

The utility function for the middle-aged in period t , V^M , is

$$V^M = \ln \left(c - \frac{(l)^{1+1/v}}{1+1/v} \right) + \theta \ln g + \beta\pi (\ln d' + \theta \ln g').$$

We rewrite the term $c - (l)^{1+1/v} / (1 + 1/v)$ as follows:

$$\begin{aligned} c - \frac{(l)^{1+1/v}}{1+1/v} &= (1-\tau)wl - s - \frac{(l)^{1+1/v}}{1+1/v} \\ &= (1-\tau)w(k, l(\tau, k))l(\tau, k) - s(\tau, k, l(\tau, k)) - \frac{(l(\tau, k))^{1+1/v}}{1+1/v}, \end{aligned} \quad (\text{B.1})$$

where the first line comes from the budget constraint in middle age in (1), and the second line comes from the labor market-clearing wage rate in (11), the labor supply function in (9), and the saving function in (10). Rearranging the terms, we can reduce the expression in (B.1) to

$$c - \frac{(l)^{1+1/v}}{1+1/v} = \frac{1}{1+\beta\pi} \cdot \frac{1/v}{1+1/v} [(1-\tau)(1-\alpha)A(k)^\alpha]^{(1+v)/(1+\alpha v)}. \quad (\text{B.2})$$

We rewrite the term d' as follows:

$$\begin{aligned} d' &= (1-\tau^{K'})R's \\ &= (1-\tau^{K'})\frac{R(k', l(\tau', k'))}{\pi}s(\tau, k, l(\tau, k)), \end{aligned} \quad (\text{B.3})$$

where the equality in the second line comes from (10) and (12).

With (9), (10), (11), and (12), we can reformulate the equation in (B.3) further as follows:

$$\begin{aligned} d' &= (1-\tau^{K'}) \cdot \frac{\alpha}{\pi} [(1-\tau')(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k')^\alpha]^{(1+v)/(1+\alpha v)} \frac{1}{k'} \\ &\times \frac{\beta\pi}{1+\beta\pi} \cdot \frac{1/v}{1+1/v} [(1-\tau)(1-\alpha)A(k)^\alpha]^{1/(1+\alpha v)}. \end{aligned} \quad (\text{B.4})$$

Thus, with (B.2) and (B.4), we can reformulate the expression in (15) as

$$\begin{aligned} V^M &= V^M(\tau, g, \tau', \tau^{K'}, g', k'; k) \\ &\simeq \underbrace{(1+\beta\pi)\frac{1+v}{1+\alpha v} \ln(1-\tau) + \theta \ln g + \beta\pi \ln(1-\tau^{K'})}_{(\#1)} \\ &\quad + \underbrace{\beta\pi \frac{(1-\alpha)v}{1+\alpha v} \ln(1-\tau')}_{(\#2)} + \underbrace{(-1)\beta\pi \frac{1-\alpha}{1+\alpha v} \ln k' + \beta\pi \theta \ln g'}_{(\#3)}, \end{aligned} \quad (\text{B.5})$$

where we omit the irrelevant terms from the expression in (B.5). Term (#1) includes the effects of the period- t labor income tax rate on $c - (l)^{1+1/v} / (1 + 1/v)$ and s_t ; term (#2) includes the effect of the period- $t + 1$ labor income tax rate on the interest rate R' through the labor supply l_{t+1} ; and term (#3) includes the effect of physical capital on the interest rate R' .

Using (9) and (12), we reformulate the expression in (16) as follows:

$$V^O = V^O(\tau, \tau^K, g, k, b) \simeq \ln(1 - \tau^K) + \frac{(1 - \alpha)v}{1 + \alpha v} \ln(1 - \tau) + \theta \ln g, \quad (\text{B.6})$$

where we omit the irrelevant terms from the expression. ■

B.2 Derivation of (A.1)

We reformulate the terms d_{τ^K}/d , $TR_{\tau^K}^K$, $(d'_{\tau^K}, \tau_b^{K'} + d'_{b'})/d'$, and $g'_{b'}/g'$ in (25) as follows. First, consider the terms d_{τ^K}/d and $TR_{\tau^K}^K$. Given $d = (1 - \tau^K)R(k, l(\tau, k))(1 + n)(k + b)$ and $TR^K = \tau^K R(k, l(\tau, k))(k + b)$, we have

$$d_{\tau^K} = -R(k, l(\tau, k))(1 + n)(k + b) \Rightarrow \frac{d_{\tau^K}}{d} = \frac{-1}{1 - \tau^K}, \quad (\text{B.7})$$

$$TR_{\tau^K}^K = R(k, l(\tau, k))(k + b). \quad (\text{B.8})$$

Next, consider the term $(d'_{\tau^K}, \tau_b^{K'} + d'_{b'})/d'$. Note that we can rewrite d' as

$$\begin{aligned} d' &= (1 - \tau^{K'}) \frac{R(k', l(\tau', k'))}{\pi} s(\tau, k, l(\tau, k)) \\ &= \frac{\bar{T}^K}{\alpha} \frac{(1 + n)k'}{s(\tau, k, l(\tau, k))} \frac{1}{\pi} \alpha A(k')^{\alpha-1} [(1 - \tau')(1 - \alpha)A(k')^\alpha]^{(1-\alpha)v/(1+\alpha v)} s(\tau, k, l(\tau, k)) \\ &= \frac{\bar{T}^K}{\alpha} (1 + n) \frac{\alpha A}{\pi} [(1 - \tau')(1 - \alpha)A]^{(1-\alpha)v/(1+\alpha v)} (k')^{\alpha(1+v)/(1+\alpha v)}, \end{aligned} \quad (\text{B.9})$$

where the equality in the second line comes from (10), (12), and (18). The capital market clearing condition, $(1 + n)(k' + b') = s$, implies $\partial k'/\partial b' = -1$. Thus, we have

$$\frac{d'_{\tau^K}, \tau_b^{K'} + d'_{b'}}{d'} = \frac{\partial d'}{\partial k'} \frac{\partial k'}{\partial b'} \frac{1}{d'} = (-1) \frac{(1 + v)\alpha}{1 + \alpha v} \frac{1}{k'}. \quad (\text{B.10})$$

Finally, consider the term $g'_{b'}/g'$. Based on the conjecture of the policy function in (20), we have

$$\frac{g'_{b'}}{g'} = \frac{\partial g'}{\partial k'} \frac{\partial k'}{\partial b'} \frac{1}{g'} = (-1) \frac{(1 + v)\alpha}{1 + \alpha v} \frac{1}{k'}. \quad (\text{B.11})$$

With (B.10) and (B.11), we obtain

$$\frac{d'_{\tau^K}, \tau_b^{K'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} = (-1) \frac{(1 + v)\alpha}{1 + \alpha v} (1 + \theta) \frac{1}{k'}. \quad (\text{B.12})$$

By using (B.7), (B.8), (B.10), and (B.12), we can reformulate (25) as

$$\frac{\frac{\pi\omega}{(1+n)(1-\omega)} \frac{-1}{1-\tau^K}}{R(k, l(\tau, k))(1+n)(k+b)} + \frac{\beta\pi}{1+n} \frac{(1+v)\alpha}{1+\alpha v} (1+\theta) \frac{1}{k'} = 0,$$

or as in (A.1). ■

B.3 Derivation of (A.2) and (A.3)

Equation (A.3) is immediate from substituting (B.12) in (26). The derivation of (A.2), which is equivalent to (24), is as follows.

We reformulate the terms in (24) as follows. First, consider the term $\frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d\tau}{d}$, which expresses the first derivative of $\frac{\pi\omega}{(1+n)(1-\omega)} \ln(1-\tau^K) R(k, l(\tau, k))s$ with respect to τ . From (9) and (12), R is given by

$$R = \alpha [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \frac{1}{k}.$$

We substitute this into the term $\frac{\pi\omega}{(1+n)(1-\omega)} \ln(1-\tau^K) R(k, l(\tau, k))s$ and obtain

$$\begin{aligned} & \frac{\pi\omega}{(1+n)(1-\omega)} \ln(1-\tau^K) R(k, l(\tau, k))s \\ &= \frac{\pi\omega}{(1+n)(1-\omega)} \ln(1-\tau^K) \alpha [(1-\tau)(1-\alpha)]^{(1-\alpha)v/(1+\alpha v)} [A(k)^\alpha]^{(1+v)/(1+\alpha v)} \frac{1}{k} (1+n)(k+b). \end{aligned}$$

Differentiation with respect to τ leads to

$$\frac{\pi\omega}{(1+n)(1-\omega)} \frac{d\tau}{d} = (-1) \frac{\pi\omega}{(1+n)(1-\omega)} \frac{(1-\alpha)v}{1+\alpha v} \frac{1}{1-\tau}. \quad (\text{B.13})$$

Next, consider the term $[c_\tau - (l)^{1/v} l_\tau] / [c - (l)^{1+1/v} / (1+1/v)]$, which expresses the first derivative of $\ln \left\{ c(\tau, k, l(\tau, k)) - \frac{[l(\tau, k)]^{1+1/v}}{1+1/v} \right\}$ with respect to τ . Using (B.2), we have

$$\ln \left\{ c(\tau, k, l(\tau, k)) - \frac{[l(\tau, k)]^{1+1/v}}{1+1/v} \right\} = \ln \frac{1}{1+\beta\pi} \frac{1/v}{1+1/v} [(1-\tau)(1-\alpha)A(k)^\alpha]^{(1+v)/(1+\alpha v)}.$$

Differentiating $\ln \left\{ c(\tau, k, l(\tau, k)) - \frac{[l(\tau, k)]^{1+1/v}}{1+1/v} \right\}$ with respect to τ leads to

$$\frac{c_\tau - (l)^{1/v} l_\tau}{c - (l)^{1+1/v} / (1+1/v)} = \frac{1+v}{1+\alpha v} \frac{-1}{1-\tau}. \quad (\text{B.14})$$

Third, consider the term $\beta\pi d'_\tau / d' + \beta\pi ((d'_{\tau K}, \tau_{k'}^{K'} + d'_{k'}) / d' + \theta g'_{k'} / g') k'_\tau$, which expresses the first derivative of $\beta\pi \ln d' + \beta\pi \theta \ln g'$ with respect to τ . Using (9) and (12) and the conjecture of the policy function in (20), we can write

$$\begin{aligned} \beta\pi \ln d' + \beta\pi \theta \ln g' &= \beta\pi \ln \frac{\bar{T}^K (1+n)k' R'}{\alpha s \pi} + \beta\pi \theta \ln (k')^{\alpha(1+v)/(1+\alpha v)} \\ &\simeq \beta\pi \frac{\alpha(1+v)}{1+\alpha v} (1+\theta) \ln k'. \end{aligned}$$

Thus, we have

$$\beta\pi \frac{d'_\tau}{d'} + \beta\pi \left(\frac{d'_{\tau K}, \tau_{k'}^{K'} + d'_{k'}}{d'} \right) k'_\tau = \beta\pi (1+\theta) \frac{\alpha(1+v)}{1+\alpha v} \frac{k'_\tau}{k'}. \quad (\text{B.15})$$

Fourth, consider the term $TR_\tau + TR_\tau^K - R_\tau b$, which expresses the first derivative of $TR + TR^K - Rb$ with respect to τ . Using (9), (11), and (12), we can reformulate the term $TR +$

$TR^K - Rb$ as follows:

$$\begin{aligned} TR + TR^K - Rb &= \tau w(k, l(\tau, k)) l(\tau, k) + \tau^K R(k, l(\tau, k)) (k + b) - R(k, l(\tau, k)) b \\ &= [(1 - \alpha) A(k)^\alpha]^{(1+v)/(1+\alpha v)} (1 - \tau)^{(1-\alpha)v/(1+\alpha v)} \left[\tau + \tau^K \frac{\alpha}{1 - \alpha} \left(1 + \frac{b}{k} \right) - \frac{\alpha}{1 - \alpha} \frac{b}{k} \right]. \end{aligned} \quad (\text{B.16})$$

Differentiating $TR + TR^K - Rb$ in (B.16) with respect to τ leads to

$$\begin{aligned} TR_\tau + TR_\tau^K - R_\tau b &= \left\{ (-1) \frac{(1 - \alpha)v}{1 + \alpha v} \frac{1}{1 - \tau} \left[\tau + \tau^K \frac{\alpha}{1 - \alpha} \left(1 + \frac{b}{k} \right) - \frac{\alpha}{1 - \alpha} \frac{b}{k} \right] + 1 \right\} \\ &\quad \times (1 - \tau)^{(1-\alpha)v/(1+\alpha v)} [(1 - \alpha)A(k)^\alpha]^{(1+v)/(1+\alpha v)}. \end{aligned} \quad (\text{B.17})$$

Using (B.12) and (B.13)–(B.17) derived so far, we can rewrite (24) as

$$\begin{aligned} &(-1) \frac{\pi\omega}{(1+n)(1-\omega)} \frac{(1-\alpha)v}{1+\alpha v} \frac{1}{1-\tau} + \frac{1+v}{1+\alpha v} (-1) \frac{1}{1-\tau} + \beta\pi(1+\theta) \frac{\alpha(1+v)}{1+\alpha v} \frac{k'_\tau}{k'} \\ &= \left\{ (-1) \frac{(1-\alpha)v}{1+\alpha v} \frac{1}{1-\tau} \left[\tau + \tau^K \frac{\alpha}{1-\alpha} \left(1 + \frac{b}{k} \right) - \frac{\alpha}{1-\alpha} \frac{b}{k} \right] + 1 \right\} \\ &\quad \times (1-\tau)^{(1-\alpha)v/(1+\alpha v)} [(1-\alpha)A(k)^\alpha]^{(1+v)/(1+\alpha v)} (-1) \frac{\beta\pi}{1+n} \frac{\alpha(1+v)}{1+\alpha v} (1+\theta) \frac{1}{k'}. \end{aligned} \quad (\text{B.18})$$

The remaining task is to compute k'_τ . Recall the capital market clearing condition in (14). Differentiating k' with respect to τ yields

$$k'_\tau = (-1) \frac{1+v}{1+\alpha v} \frac{1}{1+n} \frac{\beta\pi}{1+\beta\pi} \frac{1/v}{1+1/v} (1-\tau)^{(1-\alpha)v/(1+\alpha v)} [(1-\alpha)A(k)^\alpha]^{(1+v)/(1+\alpha v)}. \quad (\text{B.19})$$

Substituting (B.19) into (B.18) and rearranging the terms, we obtain (A.2). ■

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