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# Precautionary saving and un-anchored expectations*

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**Abstract**

This paper investigates monetary policy in a heterogeneous agent new Keynesian (HANK) model where agents face idiosyncratic income risk and use adaptive learning in order to form their expectations. Households experience different histories and observe different idiosyncratic variables. This gives rise to idiosyncratic learning processes, which naturally implies the existence of heterogeneous expectations. In HANK models, supply shocks generate precautionary saving. The learning setup amplifies this effect and can result in long-lasting disinflationary traps. Dovish Taylor rules focused on closing the output gap dampen the learning effects. Price level targeting improves the inflation and output stabilization trade-off by better anchoring expectations.

***Keywords*****:** Adaptive learning, precautionary saving, restricted perception equilibrium, heterogeneous expectations, heterogeneous agent.

***JEL codes*****:** E25, E31, E52 and E70.

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# 1 Introduction

The large supply disruptions generated by the Covid-19 pandemic have energized the debate around supply shocks' impact on the decisions and expectations of households. In the representative agent New Keynesian (RANK) benchmark, the optimal monetary policy (MP) response to adverse supply shocks should focus toward stabilizing inflation.[1] In RANK model, relaxing the rational expectations (RE) hypothesis allows expectations to lose their anchor and can trigger expectations driven slumps and deflationary/inflationary spirals. For this reason, under non-rational expectations RANK models, MP should even be more hawkish than under RE (Orphanides & Williams 2004, 2008).

However, it exists numerous evidence that aggregate supply shocks might have complex and heterogeneous impacts on households' consumption at the idiosyncratic level (Berger & Vavra 2015, Krueger, Mitman & Perri 2016a, Bayer, Lütticke, Pham-Dao & Tjaden 2019). In RE models which explicitly represent such processes,[2] the MP prescription is reversed. Macro policies should focus on stabilizing current and expected households' incomes in order to avoid unnecessary drops in aggregate demand due to precautionary saving (Guerrieri, Lorenzoni, Straub & Werning 2021).

Against this background, it is an open question if the MP results in RANK model under adaptive learning (AL) are model specific or are robust to HANK models. In this paper, I argue that the interplay between idiosyncratic income shocks, heterogeneous wealth and un-anchored expectations can be an important driver of precautionary saving. I show that in an imperfect unemployment insurance economy under adaptive learning, the HANK effects are amplified. Negative supply shocks, while being inflationary at first trigger long-lasting

[1] Supply shocks are inflationary thanks to positive prices adjustments due the increased in marginal cost and because of the low supply not being able to match the steady demand. In order to smooth its consumption, the representative household is reducing its saving. There is very little demand drop.

[2] Idiosyncratic shocks in New Keynesian models are a source of heterogeneity among agents. Those models are referred in the rest of the paper as heterogeneous agents new Keynesian models.

disinflationary periods. Those recessions are characterized by low consumption, excess saving and low interest rates. Under inflation targeting (IT), *i.e* Taylor rule, more aggressive response from monetary policy to output and a more *dovish* stance on inflation appear to neutralize the excess volatility generated by the non-rational expectations -*w.r.t* rational expectations. Finally, price level targeting (PLT) enables the model to converge back quickly to the steady-state and reduce the trade-off between output and inflation stabilization.

I develop a model based on the truncated histories of heterogeneous households in line with Challe & Ragot (2016), Challe, Matheron, Ragot & Rubio-Ramirez (2017), Ragot (2018) and Le Grand & Ragot (2021).[3] In this context, the novelty of this paper is to introduce adaptive learning under idiosyncratic restricted perceptions in a simple HANK model. Heterogeneity in risk realization, wealth holdings, information sets and idiosyncratic histories have important effects on households' expectations and macroeconomic dynamics. Thus, aggregate shocks have heterogeneous effects on households' decisions but also on their perception and learning processes.

I model expectations formation process using is a recursive least square (RLS) Euler equation learning process à la Evans & Honkapohja (2001).[4] Agents are assumed to forecast *as well as good econometricians* based on the information available to them. The presence of idiosyncratic dynamic naturally suggests the existence of idiosyncratic difference in perceptions and information sets. The perceptions of heterogeneous agents are restricted to their own idiosyncratic state variables. This leads to the introduction of heterogeneous perceived laws of motions (PLMs). Hence, agents hold heterogeneous beliefs about the economy that are not consistent in their form with the rational expectations solution - *called the Minimum State Variables (MSV) solution*

[3] With this method, it is possible to obtain a finite partition of households and an analytical expression of their policies functions by truncating their idiosyncratic histories and assuming wealth-sharing among agents sharing the same history.

[4] Euler equation learning is subject to the criticism that it could be inconsistent with the inter-temporal budget constraint. Yet, it has been shown in Honkapohja, Mitra & Evans (2013) the constraint holds *ex-post*. Moreover, Euler equation learning allows for a simple intuitive implementation.

*form*. Instead, they initially satisfy the least square orthogonality condition at the restricted perception equilibrium (RPE).[5] Agents constantly revise their beliefs about the economy by minimizing the square forecast error. I define temporary deviation of the PLMs from the RPE solution as expectations de-anchoring.

The first result of this paper is that learning properties are strengthened in the HANK setup in comparison with RANK models. This is due to the fact that the forecasts in the HANK model's Euler equations are more complex to learn than in the RANK model. This triggers constant revisions of the beliefs and thus deviations from the RPE. Assuming endogeneity between unemployment risk and productivity, the propagation of supply shocks is increased by the learning. Facing negative supply shock, precautionary saving is enlarged *w.r.t* the rational expectations benchmark which triggers deflationary pressure through the demand channel but also the marginal cost channel due to the excess capital supply.

Exploring the monetary policy options, it appears that a stronger stance on the output and a lower one on inflation in the Taylor rule neutralizes the excess volatility with respect to rational expectations HANK benchmark. Thus, monetary policy decreases the magnitude of the income risk generated by the uninsured unemployment risk and as consequence decreases the precautionary savings. By decreasing output volatility, the more aggressive monetary policy toward the output gap generates smaller idiosyncratic forecast errors and as consequence, smaller belief revisions too - *i.e.* it better anchors individual consumption PLMs thanks to smoother consumption patterns. This result in a HANK set-up contradicts previous ones in a RANK model under adaptive learning by the literature where inflation stabilization ought to be the main priority of monetary policy in order to achieve stable dynamics. Yet,

[5] It is also important to point out that this paper differs from previous heterogeneous expectations models such as Branch & McGough (2009), Massaro (2013) and Arifovic, Bullard & Kostyshyna (2013). Those models are based on an optimizing representative agent hypothesis where the heterogeneous expectations are combined in an aggregated representative expectation. This paper is based on heterogeneous New Keynesian agents with heterogeneous idiosyncratic expectations.

they are consistent with previous results established in HANK under rational expectations which emphasise on output stabilization.

A different policy scenario where the Central Bank (CB) targets price level and not inflation appears to solve this issue by better anchoring beliefs to the quasi rational expectation equilibrium (REE). Indeed, contrary to IT, PLT targets inflation rate symmetrically and enables agents to expect a smoother future income, and thus avoids a large amount of precautionary saving. Those phenomenons lessen the stabilization trade-off between inflation and output.

**Related literature.** There is increasing evidence of heterogeneity in households' responses to shock, uncertainty and economic conditions (see, e.g. Kaplan & Violante 2018, Crawley & Kuchler 2018). In response to this evidence, the literature has been developing heterogeneous agents models (see, e.g. Kaplan, Moll & Violante 2018, Den Haan, Rendahl & Riegler 2018, Bayer et al. 2019). In this literature and in my model, the permanent income hypothesis generates heterogeneous responses to change in aggregate economic conditions. The role of expected income in monetary policy transmission is enhanced at the expense of the inter-temporal substitution effect.

This concern for heterogeneity has produced new literature on optimal monetary policy. Using a degenerate distribution Ravn & Sterk (2020) and Challe (2020) investigate the optimal monetary policy in a rational expectations framework similar to this one - with a more elaborate labour market - where supply shocks lead to precautionary saving. By using constant absolute risk aversion (Bhandari, Evans, Golosov & Sargent 2018), a numerical algorithm (Acharya, Challe & Dogra 2020) or a reduced form (Bilbiie 2018) derive optimal reaction functions for the CB. All those papers, highlight the preeminent role of income and output stabilization at the expense of inflation as the welfare-maximizing option. I complete this research by incorporating an explicit belief dynamic in the problem.

Current HANK models rely exclusively on the rational expectations hypothesis. Yet, there exist extensive survey of forecasting data - at the idiosyncratic[6] and aggregate[7] levels - and laboratory[8] evidence of heterogeneous non rational expectations among economic agents (see the literature surveys by Coibion, Gorodnichenko & Kamdar 2018, Hommes 2021).

Against those facts, it exists important literature based on the hypothesis that economic agents do not know the rational expectation solution but learn to forecast in the most accurate manner based on past data (see Evans & Honkapohja 2001). This hypothesis has a non-trivial impact on optimal monetary policy design in RANK models. Due to the self-referential nature of the new Keynesian Philips Curve and expectations under adaptive learning, robust optimal MP in those models should stabilize inflation and inflation expectation at the expense of other variables (see e.g Orphanides & Williams 2008, Williams 2010).[9] To the best of my knowledge, New Keynesian models under adaptive learning have always been implemented using a representative agent consumption framework. My findings put in perspective those results by highlighting the potential impact of idiosyncratic shocks on beliefs' formation.

Finally, this paper relates to work on macro models with heterogeneous agents subject to bounded rationality such as Gobbi & Grazzini (2019) where the authors develop an agent-based HANK model with heterogeneous PLMs but without wealth heterogeneity. Honkapohja & Mitra (2006) and Radke & Wicknig (2020) use overlapping generation setups which can be envisioned as heterogeneous agents models under adaptive learning. Finally, Giusto (2014) proves that the Krusell & Smith (1998) equilibrium is E-stable under RLS.

The paper proceeds as follows: In Section 2, I develop the HANK model; the solution methods are presented in Section 3; the dynamic properties of the model are analysed in Section 4; Section 5 discusses the effects of different

[6]See for example Das & Van Soest (1999) using the Dutch Survey of consumers, Jappelli & Pistaferri (2000)using the Bank of Italy Survey of Household Income and Wealth and Souleles (2004) and Rozsypal & Schlafmann (2017) using the Michigan Surveys of Consumers.

[7]See for instance Carroll (2003), Branch (2004b), Del Negro & Eusepi (2011) and Malmendier & Nagel (2016).

[8]See the work by Hommes (2011)and Assenza, Heemeijer, Hommes & Massaro (2021).

[9]A notable exceptions of those results is the Eusepi & Preston (2018), where hawkish MP can be destabilizing for long-run expectations under the infinite horizon approach.

policy exercises; and Section 6 concludes.

# 2 The model

Time $t = 1, 2, ...$ is discrete. The economy is populated by a continuum of agents of measure $i$, distributed on an interval $\jmath$ according to measure $\ell(.)$. Following the literature, it is assumed that the law of large numbers holds. The economy is in a sequential competitive equilibrium and is described as a collection of individual allocations $(c_t^i, l_t^i, a_t^i)$, aggregate quantities $(K_t, L_t, Y_t)$ and price processes $(\pi_t, i_t, W_t, Z_t)$. Given an initial wealth distribution $(a_{-1}^i)_{i\in\jmath}$ and an initial value of aggregate capital stock $K_{-1} = \int_i a_{-1}^i \ell(di)$ it is possible to solve for equilibrium: solve the agents' optimization programs, clear the markets for goods, labour and capital and solve for prices.

## 2.1 The heterogeneous households problem

The household side is defined by the utility function $U(c, l)$ in the form of Greenwood, Hercowitz & Huffman (1988) (GHH) where households choose their consumption $c$ and labour supply $l$.[10] The utility reads as

$$U(c_t, l_t) = \begin{cases} \frac{1}{1-\sigma}(c_t - \frac{l_t^{1+1/\varphi}}{\chi(1+1/\varphi)})^{1-\sigma} \text{ if } \sigma \neq 1; \\ \log(c_t - \frac{l_t^{1+1/\varphi}}{\chi(1+1/\varphi)}) \text{ if } \sigma = 1; \end{cases} \tag{1}$$

where $\sigma > 0$ is the inter-temporal elasticity of substitution, $\varphi > 0$ is the Frish elasticity of labour supply, $\chi > 0$ scales labor disutility, and $U : \mathbb{R}^+ \to \mathbb{R}$ is twice continuously derivable , increasing, and concave.

Agents have additive inter-temporal preferences with a discount factor $\beta > 0$. They optimize their individual consumption $c_t$ and labour supply $l_t$ streams using inter-temporal utility criterion $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$. I consider a simplified set-up based on Krueger, Mitman & Perri (2016b) and Ragot (2018), where households face an idiosyncratic unemployment risk in order to create

[10] This functional form exhibits no wealth effect for the labour supply and therefore greatly simplifies our model by homogenising labour supply in the latter part of the paper.

heterogeneity in wealth and consumption decisions. At the beginning of each period, each agent faces an exogenous employment risk denoted $e_t^i$. $\mathcal{E} = \{e, u\}$ denotes the set of possible employment statuses. An agent with $e_t^i = e$ is considered as employed and free to choose her labour supply $l_t^i$. An agent with $e_t^i = u$ is considered as unemployed, cannot work and will suffer from a fixed disutility reflecting unemployment cost. The history of idiosyncratic states until $t$ is written $e^{i,t} = (e_0^i ... e_t^i)$. The employment status follows a discrete two state Markov process with transition matrix $M_t \in [0,1]^{2x2}$. The job separation rate in $t$ is written as $\Pi_{eu}$ and the job finding rate is symmetrically denoted as $\Pi_{ue,t}$. Hence, the transition matrix across employment status is

$$M_t = \begin{bmatrix} 1 - \Pi_{eu} & \Pi_{eu} \\ \Pi_{ue,t} & 1 - \Pi_{ue,t} \end{bmatrix}. \tag{2}$$

The probability to transition from unemployment to employment - the job-finding rate - $\Pi_{ue,t}$ comoves with the aggregate productivity shock $\varepsilon_t^p$ in this fashion

$$\Pi_{ue,t} = \Pi_{ue}^{SS} + \nu \varepsilon_t^p. \tag{3}$$

Productivity is following a basic a AR(1) process such that $e_t^{\varepsilon^p} = e^{\rho^p \varepsilon_{t-1}^p + \vartheta_t^p}$ with an exogenous shock $\vartheta_t^p$ i.i.d. This design is motivated by the finding of Shimer (2005), where the job separation rate is almost constant over time and unemployment dynamics is explained by variation in the job-finding rate. Here, an increase in productivity generates an increase in the job finding rate.[11] The total share of employed and unemployed agents are respectively defined as $S_{e,t}$ and $S_{u,t}$ with $S_{e,t} + S_{u,t} = 1$ and the steady-state satisfies $1 - \overline{\Pi}_{eu} = \overline{\Pi}_{uu} = \frac{(1-\Pi_{eu})S_u}{1-S_u}$.

The budget constraint of the agents is given by

$$c_t^i + a_t^i = (1 - \delta + Z_t) a_{t-1}^i + 1_{e_t^i = e} l_t^i W_t + \Delta_t^i, \tag{4}$$

where $1_{e_t^i = e}$ is a function equal to 1 when the agent is employed and 0 in the

[11] This setup could be envisioned as a reduce form of the search and matching model where the matching probability is 1 and wages are flexible.

opposite case. Thus, $1_{e_t^i=e} l_t^i W_t$ is a notation for the expected wage. $a_t^i$ is the net individual asset holding, $0 < \delta < 1$ the depreciation rate and $Z_t$ is the dividend paid by the firm in order to rent the capital from the households. $\Delta_t^i$ is a net transfer from the risk sharing agreement between agents with similar idiosyncratic histories after $N$ periods.

I should note that the no-arbitrage condition on the financial market between risk-free bonds and capital enables me to write

$$\mathbb{E}_t^* \frac{i_t}{\pi_{t+1}} = 1 + \mathbb{E}_t^* Z_{t+1} - \delta, \tag{5}$$

with $i_t$ the gross nominal interest rate, $\pi_t$ the gross inflation rate and $\frac{i_t}{\mathbb{E}_t^* \pi_{t+1}}$ the real interest rate. $\mathbb{E}_t^{*=\{RE,AL\}}$ is the subjective expectation operator that will be discussed in Section 3.

I considere an household $i \in \jmath$. She can save in an asset that pays a dividend $Z_t$. It is subject to the borrowing constraint such as her asset holding should be greater than a threshold $-\overline{a} = 0$. In $t = 0$, the household chooses consumption $c_t^i \geq 0$, labour supply $l_t^i \geq 0$ and saving $a_t^i \geq 0$ that maximize inter-temporal utility over an infinite horizon, subject to a budget constraint and the previous borrowing limit. If the household is credit constrained $a_t^i = 0$, it is said to belong the set $i \in \mathcal{C}$ of credit-constrained agents. For a given $a_{t-1}^i$, the problem of the household is given by

$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t=0}^{\infty}} \qquad \mathbb{E}_t^* \sum_{t=0}^{\infty} \beta^t U \left( c_t^i - \chi^{-1} \frac{(1_{e_t^i=e} l_t^i + 1_{e_t^i=u} \varsigma)^{1+1/\varphi}}{1+1/\varphi} \right) + v_t^i, \tag{6}$$

$$s.t.\ c_t^i + a_t^i = (1 - \delta + Z_t) a_{t-1}^i + 1_{e_t^i=e} l_t^i W_t,$$

$$s.t.\ a_t^i \geq -\overline{a}, \tag{7}$$

with $\varsigma$ the disutility implied by labour search or/and unemployment, and $v_t^i$ the Lagrange multiplier of the credit constraint of agent $i$.[12] The first order

[12] The Lagrange multiplier is null while $i$ is not credit constrained.

conditions for the employed and unemployed agents boil down to

$$U'(c_t^i, l_t^i) = \beta \mathbb{E}_t^* \left[ \frac{i_t}{\pi_{t+1}} U'(c_{t+1}^i, l_{t+1}^i) \right] + v_t^i, \tag{8}$$

$$l_t^{i,1/\varphi} = \chi W_t 1_{e_t^i = e}. \tag{9}$$

The aggregation for the model economy is straightforward: first, financial market clearing implies that the total sum of individual asset holdings equals the aggregate capital stock

$$\int_i a_t^i \ell(di) = K_t; \tag{10}$$

the labour is only supplied by employed agents, thus aggregate labour supply $L_t$ can be written as

$$\int_i l_t^i \ell(di) = L_t; \tag{11}$$

and aggregate consumption $C_t$ is the total sum of individual consumption

$$\int_i c_t^i \ell(di) = C_t. \tag{12}$$

Finally, using the transition matrix $M_t$ we can express the aggregate law of motion for the employed $S_{e,t}$ and unemployed $S_{u,t}$ agents as follow

$$S_{u,t} = 1 - S_{e,t} = \Pi_{eu} S_{e,t-1} + (1 - \Pi_{ue,t}) S_{u,t-1}. \tag{13}$$

## 2.2 A truncated history model

Following Ragot (2018) and Le Grand & Ragot (2021), I generate a discrete-time finite partition HANK model based on the truncated idiosyncratic histories of households. This method is appealing for adaptive learning implementation for four reasons: first, it allows for analytical expressions of household first order conditions; second, it enables to easily implement the adaptive learning algorithm and avoid the complication of working with continuous-time; it allows for an explicit expression of expectation of idiosyncratic state variables;

finally, it avoids the creation of complex abstract forward state variables.

At any date t, each agent $i \in \jmath$ is characterized by her personal history of idiosyncratic unemployment risk realizations $e^{i,t} = (e^i_t, e^i_{t-1}, e^i_{t-2}...)$. The main intuition is to sort agents in a finite number of families following their idiosyncratic unemployment history. Nonetheless, the agents being infinitely-lived, the number of idiosyncratic histories is infinite, which would lead to an infinite number of families. To overcome this issue, I impose $1 < N < +\infty$, a truncation of the idiosyncratic histories considered in the model.[13] In consequence, every family $\hbar$ is defined by a limited set of idiosyncratic realizations $\hbar \Leftrightarrow e^{\hbar,t} = (e^{\hbar}_t, e^{\hbar}_{t-1}...e^{\hbar}_{t-N-1})$.

I define a family $\hbar$ as a collection of households with the same finite sequence of idiosyncratic unemployment statues. A partition $\mathcal{H}$, is a finite collection of families such that at any date $t$, a sequence of employment $e^t$, truncated after $N$ periods, belongs to only one family $\hbar$ of the partition $\mathcal{H}$. Hence all families of the model are part of the partition such as $\hbar \in \mathcal{H}$. In this paper, a household belongs to $\hbar \in \mathcal{H}$ at date $t$ if her idiosyncratic employment history $e^{i,t}$ is the same that the one defining family $\hbar$.

When an agent $i$ is in family $\hbar$ corresponding to an history $e^{i,N}$ in $t-1$, the probability that it switches to another family $\tilde{\hbar}$ with history $\tilde{e}^{i,N}$ in $t$ is denoted by $\Pi^{\mathcal{E}}_{\tilde{e}^{i,N},e^{i,N},t}$ with

$$\Pi^{\mathcal{E}}_{e^{i,N},\tilde{e}^{i,N},t} = \Pi^{\mathcal{E}}_{e^{\hbar},e^{\tilde{\hbar}},t}. \tag{14}$$

$\Pi^{\mathcal{E}}_{\hbar,\tilde{\hbar},,t}$is the transition probability for an agent of moving from idiosyncratic history $e$ corresponding to family $\hbar$ to the history $\tilde{e}$ corresponding to family $\tilde{\hbar}$. Note that $\Pi^{\mathcal{E}}_{e^{\hbar},e^{\tilde{\hbar}},t} = \{1-\Pi_{eu}, \Pi_{eu}, \Pi_{ue,t}, 1-\Pi_{ue,t}\}$ depends on the idiosyncratic states of family $\hbar$ and $\tilde{\hbar}$. It is possible to define $\Pi^{\mathcal{E}}_{\hbar,\tilde{\hbar},,t}$ as the mass of agents in family $\hbar$ to transition to the family $\tilde{\hbar}$. Family size $\hbar$ in $t$ is denoted by $S_{\hbar,t}$.

[13] The $N = 0$ case is the RANK case where there is no idiosyncratic realisation and only one family exists. Thus, there is zero probability for an agent to transition from one family to another.

It is then possible to write

$$\Pi^{\mathcal{E}}_{\hbar,\tilde{\hbar},t} = \Pi^{\mathcal{E}}_{e^{\hbar},e^{\tilde{\hbar}},t} \frac{S_{\tilde{\hbar},t}}{S_{\hbar,t-1}}. \tag{15}$$

It is important to note that partitioning the households following their idiosyncratic histories generates a large number of families of very heterogeneous size. The number of families follows a geometric progression as a function of the number of the idiosyncratic states $\mathcal{E} = \{e, u\}$ to the power of the number of periods considered $N$.

I consider a partition $\mathcal{H}$ containing a finite number of family $\hbar \in \mathcal{H}$ of $N$ periods. The size of a family $\hbar \in \mathcal{H}$ in $t$ corresponds to the measure of agents $i$ with a idiosyncratic history $e^{i,t}$ belonging to family $\hbar$. Family size $S_{\hbar,t}$ boils down to

$$S_{\hbar,t} = \sum_{\tilde{\hbar}\in\mathcal{H}} \Pi^{\mathcal{E}}_{\tilde{\hbar}\hbar,t} S_{\tilde{\hbar},t-1}, \tag{16}$$

which simply denotes that the size of family $\hbar$ in $t$ is equal to the total number of households from other families $\tilde{\hbar}$ in $t-1$ transitioning to this family $\hbar$ in $t$.

In order to achieve similar preferences within each family $\hbar \in \mathcal{H}$, I assume a pooling mechanism of wealth as a risk-sharing arrangement between every member of the same family - see the term $\Delta^i_t$ in Equation 4.[14] This wealth pooling leads to homogeneous preferences and policy functions for the agents within the same family.

Figure 1 details the internal structure of the heterogeneous agent model in a truncated history setup with $N = 2$. It is possible to see from Figure 1 what families $\tilde{\hbar}$ are possible continuation of $\hbar$ and how the pooling mechanism assures homogeneity in preference. With $N = 2$, households in family $\hbar$ with history $\Leftrightarrow e^{\hbar,t} = (e, e)$ can transition in $t + 1$ to $\tilde{\hbar}$ with history $\Leftrightarrow e^{\tilde{\hbar},t} = (u, e)$ with a probability $\Pi_{eu}$ and stays in $\hbar \Leftrightarrow e^{\hbar,t} = (e, e)$ with a probability $1-\Pi_{eu}$.

[14] Ragot (2018), Challe et al. (2017) and the appendix in Le Grand & Ragot (2021) offer different justifications to achieve similar preferences within family. As in Lucas (1975), one justification is that the agents with the same idiosyncratic history for the last $N$ periods belong to a *family* and are located in the same *island*. They pool their resources and the optimal decision is taken by the *family head*. The other one is just a perfect risk-sharing arrangement between all members of the same family.

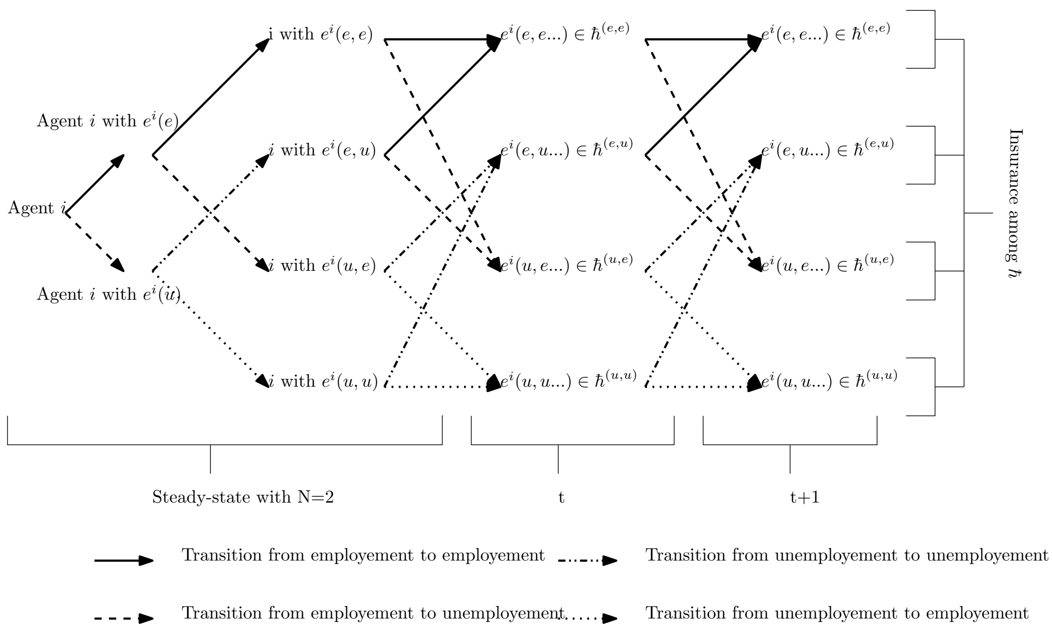



Figure 1: Idiosyncratic dynamics in the HANK model with $N = 2$

Symmetrically, households in bin $\hbar \Leftrightarrow e^{\hbar,t} = (u, e)$ can transition in $t + 1$ to $\tilde{\hbar} \Leftrightarrow e^{\tilde{\hbar},t} = (u, u)$ with a probability $1 - \Pi_{ue,t+1}$ and transfer in $\hat{\hbar} \Leftrightarrow e^{\hat{\hbar},t} = (e, u)$ with a probability $\Pi_{ue,t+1}$.

It is important to acknowledge the timing of the model. At the beginning of the period, exogenous aggregate shocks happen. Then the unemployment risk of all agents is realized. Agents transition to their new families and pool their wealth together. Afterwards, agents form their expectations about the future states of the economy. Finally, the decision problems are solved. If the model is under adaptive learning, agents observe their forecast errors and update their forecasting rules before the next period. Figure 2 summarizes the timing of events in the HANK model under adaptive learning.

## 2.3 The simulated model

**The steady-state.** The paper uses the Le Grand & Ragot (2021) routine to compute the parameters and the transition probabilities of the families'

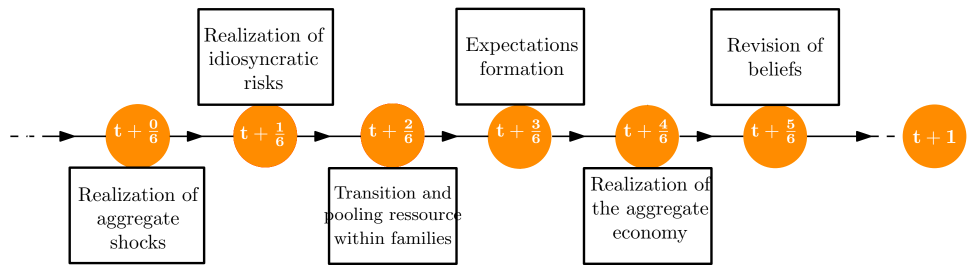



Figure 2: Intra-period timing of events in the HANK model under adaptive learning

equations in the model. In the absence of aggregate shocks of the model this routine uses the the steady-state equilibrium of the model.[15]

In order to achieve this distribution, the utility function is iterated through a *guess-and-verify* algorithm in order to directly obtain stable policy rules over every asset level of an exponential grid of 500 points. Given an initial endowment and an interest rate, it is possible to simulate the steady-state distribution under idiosyncratic risk and then aggregate saving and labour supply which leads to a new equilibrium interest rate. The process is iterated until the initial interest rate generates a distribution that leads to the same interest rate.

At any moment $t$, beginning of the period wealth holding of agent $i$, $a^i_{t-1}$, is a function of the realized idiosyncratic history $e^{i,t-1}$ up to date $t-1$. Thus $a^i_t = a(e^{i,t-1})$ defines a mapping between an initial wealth holding, idiosyncratic history realizations and the beginning of the period wealth to unique bin $\hbar$ such that $a(e^{t-1}) \in \hbar$. Setting an exogenous number of period $N$ in the idiosyncratic history, it is possible to express a model consistent with the above mentioned steady-state aggregate outcome as well as with the idiosyn-

[15] Aiyagari (1994) has demonstrated that given the initial wealth and idiosyncratic shocks distribution, it is possible to characterized the steady-state wealth distribution of this model in $[-\bar{a}, +\infty]$ in the absence of aggregate shocks.

cratic shock realisation during the $N$ periods.[16]

**The dynamic system.** The model considers a finite partition of idiosyncratic histories $\mathcal{H}$.[17]

First the Euler equations for all families $\hbar \in \mathcal{H}$ are given by

$$\forall \hbar \in \mathcal{H} \setminus \mathcal{C}, \quad \xi_{\hbar} U'\left(c_{\hbar,t}, l_{\hbar,t}\right) = \beta \mathbb{E}_t^* \left[ \frac{i_t}{\pi_{t+1}} \sum_{\tilde{\hbar} \in \mathcal{H}} \Pi^{\mathcal{E}}_{\hbar\tilde{\hbar},t+1} \xi_{\tilde{\hbar}} U'\left(c_{\tilde{\hbar},t+1}, l_{\tilde{\hbar},t+1}\right) \right], \quad (17)$$

where $U'$ is the marginal utility. $\xi_{\hbar}$ is a *preference shifter*, a coefficient correcting for consumption elasticity and level across the distribution and the non-linearity of the utility function. The shifters allow also for the steady state of the model to be consistent with aggregate outcomes of the model under idiosyncratic shocks.[18] In the end, this function is fairly simple, family $\hbar$ is trying to smooth its utility over an infinite horizon. The family is forecasting its expected utility according to the probability for each of its households to join the different families.[19]

The existence of unemployment risk and poorer families generates precautionary saving in order to smooth the marginal utility flow. An increase in income risk, *i.e.* probability to stay or become unemployed, would increase expected marginal utility by decreasing expected consumption and thus decrease current consumption. In the same way, a decrease in expected consumption in case of unemployment risk realization would also generate the same effect.

This equation is at the core of this paper. The discounted expected utility stream depends on expected inflation, expected transitions probabilities and sizes depending on the state of the employment. The employment risk itself

[16] With a large enough $N$, the cross-sectional distribution wealth of the model is similar to models simulated under Krusell & Smith (1998) or Reiter (2009) methods. See Le Grand & Ragot (2021) for numerical examples.

[17] see Le Grand & Ragot (2021) for a longer discussion about the difference between the *true* representation of the projected model and its approximation.

[18] Those coefficients are computed using the Le Grand & Ragot (2021) routine. See Le Grand & Ragot (2021) for a discussion about the computation of this model.

[19] In the other way, families are optimizing over their own expected mass of transitioning to other families.

is a function of the productivity, expected labour supply and expected consumption of every family that the households in family $\hbar$ can transition to. In consequence, consumption decisions follow the expected discounted stream of labour and consumption depending on the expected probability to transition to different idiosyncratic states.

The FOC for constrained families reads

$$\forall \hbar \in \mathcal{C}, \quad a_{\hbar,t} = -\overline{a}. \tag{18}$$

For the agents subject to the borrowing constraint, considering the fact that they have no savings, they fall into the "hand-to-mouth" category where they will consume all their endowments without any consideration for future consumption.[20] The FOC condition for labour supply means that it is only a function of the wage for the employed agents,

$$\forall \hbar \in \mathcal{H}, \quad l_{\hbar,t} = (\chi W_t 1_{e_\hbar = e})^{\varphi}. \tag{19}$$

The family-wide resources are equal to total wage (for employed agents) $W_t$, plus the total discounted assets $a_{t-1}$ and dividend $Z_t$ held in $t-1$ by agents staying and transferring to the family $\hbar$. Resources are at least equal to total current consumption and investment according to[21]

$$\forall \hbar \in \mathcal{H}, \quad c_{\hbar,t} + a_{\hbar,t} = (1 - \delta + Z_t) \sum_{\hbar \in \mathcal{H}} \Pi^{\mathcal{E}}_{\tilde{\hbar}\hbar,t-1} \frac{S_{\tilde{\hbar},t}}{S_{\hbar,t}} a_{\tilde{\hbar},t-1} + 1_{e_h = e} l_{\hbar,t} W_t. \tag{20}$$

Aggregation of all families means that the total asset holdings, labour inputs, consumption and employment statuses are respectively equal to the capital stocks, aggregate labour supply, aggregate consumption and unemployment

[20]The truncated histories method assumes that constrained families are always constrained and vice-versa. This is a reasonable assumption considering the fact that the model is linearized around a steady-state and is subject to only small shocks.

[21]In order not to complicate the family-wide resource constraint and deal with the distribution of price adjustment costs over the distribution of households, it is assumed that they are negligible. The results are robust to a lump-sum cost applied to all families. This is not an issue considering that the first-order approximation of the price adjustment costs in zero-inflation steady-state model is always equal to zero.

rate. Formally we can write

$$K_t = \sum_{\hbar \in \mathcal{H}} S_{\hbar,t} a_{\hbar,t}, \tag{21}$$

$$L_t = \sum_{\hbar \in \mathcal{H}} S_{\hbar,t} l_{\hbar,t}, \tag{22}$$

$$C_t = \sum_{\hbar \in \mathcal{H}} S_{\hbar,t} c_{\hbar,t}, \tag{23}$$

$$U_t = \sum_{\hbar \in \mathcal{H}} S_{\hbar,t} 1_{e_h = u}. \tag{24}$$

The rest of the model follows a standard two stages representative firm New Keynesian set-up. The production function is a simple Cobb-Douglas function with exogenous productivity, aggregate capital and aggregate labour as inputs,

$$Y_t = e^{\varepsilon_t^p} K_{t-1}^{\alpha} L_t^{1-\alpha}. \tag{25}$$

In a first step, a representative production firm minimizes costs through the following FOC

$$W_t = (1-\alpha) e^{\varepsilon_t^p} K_{t-1}^{\alpha} L_t^{-\alpha}, \tag{26}$$

and its marginal cost is

$$mc_t = \frac{1}{e^{\varepsilon_t^p}} (\frac{Z_t}{\alpha})^{\alpha} (\frac{W_t}{1-\alpha})^{1-\alpha}. \tag{27}$$

The production firm sells competitively its goods to a final firm operating under monopolistic competition. In a second step, this firm optimizes profit by setting its prices under the constraint of quadratic adjustment menu costs. Hence, the implied New Keynesian Phillips curve is

$$0 = 1 - (1 - \omega mc_t)\epsilon - \psi(\pi_t - 1)\pi_t + \psi\beta \mathbb{E}_t^* [(\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t}], \tag{28}$$

with $\epsilon > 1$ the elasticity of substitution among goods, $\psi > 0$ the price adjustment cost parameter and $\omega$ a technology scaling parameter.[22]

[22] $\omega$ tunes the marginal productivity of the retail sector in order to equalize marginal cost

The clearing of the financial market requires a no-arbitrage condition between bonds and assets is expressed in Equation 5. Hence dividends are managed by the CB's MP. The policy rate is set by the CB and is subject to a standard Taylor Rule as below,

$$i_t - \bar{i} = \phi^{\pi}(\pi_t - \overline{\pi}) + \phi^{y}(\frac{Y_t - \overline{Y}}{\overline{Y}}) + \varepsilon_t^r. \tag{29}$$

The MP shock $\varepsilon_t^r$ is an AR(1) exogenous process. Finally, there exist two exogenous stochastic AR(1) processes that shock the supply and demand sides of the economy

$$\varepsilon_t^r = \rho^r \varepsilon_{t-1}^r + \vartheta_t^r, \tag{30}$$

$$\varepsilon_t^p = \rho^p \varepsilon_{t-1}^p + \vartheta_t^p. \tag{31}$$

# 3 Expectations formation

In this section, I discuss how to solve the model based on $\mathbb{E}_t^{*=\{RE,AL\}}$, the subjective expectation operator. The expectations can be rational $* = \{RE\}$ or follow learning $* = \{AL\}$ and will be discussed in Sections 3.1 and 3.2. To solve the model, I perform through Dynare (Juillard et al. 1996) a first-order linearisation of the model around the steady-state. Therefore, in this paper, I denote with an $[\ \hat{\cdot}\ ]$ the log-linearised transformation of a variable.

I can collapse the state variables vector as $\hat{x}_t = [\hat{Y}_t, \hat{\pi}_t, \hat{i}_t, \hat{W}_t, ...]'$ and $\hat{x}_t^e \subset \hat{x}_t$ and $\hat{x}_t^s \subset \hat{x}_t$ the respective subsets of forward and backward looking variables of the model. I define $\hat{z}_t = [\varepsilon_t^r, \varepsilon_t^p]'$ and $\hat{u}_t = [\vartheta_t^r, \vartheta_t^p]'$ as respectively the shock processes and the exogenous variables. The state space representation of the linearised model is

$$A_0 + A_1 \hat{x}_{t-1}^s + A_2 \hat{x}_t + A_3 \mathbb{E}_t^* \hat{x}_{t+1}^e + A_4 \hat{z}_t = 0. \tag{32}$$

For the sake of clarity, I recapitulate the explicit difference between the

and the inverse of the retail sector mark-up at the steady-state.

RANK version (see Appendix A) and the HANK model with respect to expectations. From Appendix A, it is possible to define $\hat{x}^{e,RANK}$ the forward-looking variables in the RANK model as

$$\hat{x}_t^{e,RANK} = [\hat{Y}_t, \hat{\pi}_t, \hat{Z}_t, \hat{L}_t, \hat{C}_t]'. \tag{33}$$

On the other hand, the heterogeneity in the HANK model generated by the heterogeneous Euler Equations (17) creates a larger forward-looking variable vector $\hat{x}^{e,HANK}$ such as

$$\begin{aligned} \hat{x}_t^{e,HANK} =& [\hat{Y}_t, \hat{\pi}_t, \hat{Z}_t, \hat{l}_t, \hat{\Pi}_{ue,t} \quad \dots \\ & \hat{c}_{\hbar=1,t}, \hat{c}_{\hbar=2,t} \quad \dots \quad , \hat{c}_{\hbar=\mathcal{E}^N,t}, \quad \dots \\ & \hat{S}_{\hbar=1,t}, \hat{S}_{\hbar=2,t} \quad \dots \quad , \hat{S}_{\hbar=\mathcal{E}^N,t}]'. \end{aligned} \tag{34}$$

In the absence of a representative consumption state variable $\hat{C}_t$, the HANK model introduces through the heterogeneous households multiple forward-looking variables for expected disaggregated marginal utility flows. The new forward-looking variables are the set of all families' expected consumption $\hat{c}_{\hbar,t}$ and expected families' sizes $\hat{S}_{\hbar,t}$. The job-finding rate $\hat{\Pi}_{ue,t}$ is also introduced at the aggregate level. All of those variables are used in the set of Equations (17).

It should be clear that the heterogeneous expectations come initially from the heterogeneity in households. By symmetry, the number of state variables $\hat{x}_t$ in the HANK model is also much larger than in the RANK version which has an impact on the perceived law of motion (PLM) size and form.

Finally, for the sake of clarity, it is possible to define the whole set of backward state variables in the HANK model as

$$\begin{aligned} \hat{x}_t^{s,HANK} =& [\hat{K}_t, \quad \dots \\ & \hat{a}_{\hbar=1,t}, \hat{a}_{\hbar=2,t} \quad \dots \quad , \hat{a}_{\hbar=\mathcal{E}^N,t}, \quad \dots \\ & \hat{S}_{\hbar=1,t}, \hat{S}_{\hbar=2,t} \quad \dots \quad , \hat{S}_{\hbar=\mathcal{E}^N,t}]'. \end{aligned} \tag{35}$$

## 3.1 Rational Expectations

Assuming rational expectation $* = \{RE\}$, the MSV solution of the model follows

$$\hat{x}_t = A + P\hat{x}^s_{t-1} + Q\hat{z}_t. \tag{36}$$

Given Equation 36 and through perturbation methods, I can compute $A$, $P$, $Q$. Then I have,

$$\hat{x}_{t+1} = A + P\hat{x}^s_t + Xz_{t+1} \Leftrightarrow \hat{x}_{t+1} = A + P\hat{x}^s_t + X(\rho z_t + u_{t+1}),$$

with $u_t$ the exogenous i.i.d process. Thus I write $\mathbb{E}^{RE}_t(u_{t+1}) = 0$ and deduce

$$\mathbb{E}^{RE}_t \hat{x}^e_{t+1} = A + P\hat{x}^s_t + Q(\rho\hat{z}_t + 0).$$

Then, with the iteration $E^{RE}_t \hat{x}_{t+1}$ and $\hat{x}_t$,

$$\mathbb{E}^{RE}_t \hat{x}^e_{t+1} = (I + P)A + P^2 x^s_{t-1} + (PQ + \rho)\hat{z}_t).$$

Under RE, the forward-looking variables are thus

$$\mathbb{E}^{RE}_t \hat{x}^e_{t+1} = \alpha + \beta\hat{x}^s_{t-1} + \gamma z_t, \tag{37}$$

with

$$\begin{cases} \alpha = (I + P)A = 0 \\ \beta = P^2 \\ \gamma = PQ + \rho \end{cases}.$$

## 3.2 Adaptive Learning under restricted perception

In this section, I present how to solve the model under the restricted perception adaptive learning $* = \{AL\}$ based on the same notation as the rational expectations solution.

**Restricted perception learning.** In this paper, under AL and RE, the agents do not observe the same set of past state variables and exogenous shocks realizations. Under RE, agents have full information and observe the whole set of idiosyncratic and aggregate lagged state variables $x^s_{t-1}$ (35). It is worthwhile to point that this vector is very large and composed of a lot highly correlated aggregate and idiosyncratic variables. In a bounded rationality setup, especially under AL, the presence of idiosyncratic dynamic naturally suggest the existence of idiosyncratic perceptions and information sets. Hence, I restrict the perceptions of state variables for different forecasting rules.[23]

I assume that forecast regarding idiosyncratic variables $\forall \hbar \in \mathcal{H}$, $\hat{x}^e_{\hbar,t} = [\hat{c}_{\hbar,t}, \hat{S}_{\hbar,t}]$ use their own idiosyncratic variables $\hat{x}^s_{\hbar,t} = [\hat{a}_{\hbar,t}, \hat{S}_{\hbar,t}]$ and aggregate shocks $z_t$. In this model, it means that families observe their idiosyncratic asset holding and family size alongside aggregate shocks. I then assume that forecasts regarding aggregate states variables $\hat{x}^e_{AG,t} = [\hat{Y}_t, \hat{\pi}_t, \hat{Z}_t, \hat{l}_t, \hat{\Pi}_{ue,t}]$ use aggregate shocks $z_t$ and the aggregate state variables $\hat{x}^s_{AG,t} = [\hat{K}_t]$. In this model, it means that aggregate capital is the only observed aggregate state variable alongside shocks.

This information set restriction can be interpreted as a limited cognitive ability that forces agents to use only the most accessible state variables in order to form their forecasts. It seems reasonable that the aggregate forecaster $AG$ cannot observe idiosyncratic dynamics and only looks at macro-economic outcomes. In the same fashion family members $\hbar$, when forming their expectations, observe neither aggregate capital nor idiosyncratic variables in other families.

As consequence, the model acknowledges an set $\mathcal{I}$ composed of all of the model's information sets such as $\mathcal{I} = \{AG, \hbar = 1, \hbar = 2, \quad ... \quad, \hbar = \mathcal{E}^N\}$. Here, one aggregate forecaster produces forecasts using aggregate variables

[23] This is a common practice in learning models such as Sargent (1999), Branch (2004a), Hommes & Zhu (2014) or Hommes, Mavromatis & Ozden (2020) and heterogeneous agents models such as Krusell & Smith (1998). It is also used in Slobodyan & Wouters (2012b,a) as a way to avoid multi-collinearity among observed state variables that generates accuracy problems when inverting the second-moment matrix of the observed variables with near singular values. In this fashion, it limits the frequency of projection facilities and ridge regressions during MCMC sampling of large models during Bayesian estimation.

and heterogeneous households use private information to produce idiosyncratic forecasts. Hence, heterogeneity in expectations naturally arises from the heterogeneity in idiosyncratic realizations. All agents in the economy take those forecasts for granted for solving their optimization problems.[24]

**The perceived laws of motions.** Having restricted the information sets available, forecasts in the model are misspecified, but are consistent with the MSV form. Agents perceive the economy (36) through a linear forecasting model based on their information set $i \in \mathcal{I}$. Those PLMs can be written such as

$$\forall i \in \mathcal{I}, \quad \hat{x}^e_{i,t} = a_{i,t-1} + b_{i,t-1}\hat{x}^s_{i,t-1} + c_{i,t-1}z_t. \tag{38}$$

The subscript $t-1$ means that the forecasting coefficients and beliefs are subject are form in $t-1$. I now define $\phi_{i,t} = [a'_{i,t}, vec(b_{i,t}, c_{i,t})']'$ as the beliefs vector held by PLM $i$. $M_{i,t} = [1, \hat{x^s}'_{i,t-1}, z'_t]$ is the perceived moments matrix available for PLM $i$. I follow the econometric learning hypothesis, where the law of motion of those matrices follows a constant gain Recursive Least Square (RLS) process, which is standard in the literature,

$$\phi_{i,t} = \phi_{i,t-1} + gR^{-1}_{i,t}M_{i,t-1}(\hat{x}^e_{i,t} - M'_{i,t-1}\phi_{i,t-1}), \tag{39}$$

$$R_{i,t} = R_{i,t-1} + g(M_{i,t-1}M'_{i,t-1} - R_{i,t-1}). \tag{40}$$

Those equations describe the updating process of the beliefs by the model's agents. Here, agents behave like econometricians and minimize their forecasts' square error based on past data. The small $0 \leq g < 1$ means that the gain coefficient is constant and enables the learning to slowly discount the importance of past observations over time. I implement a ridge regression device (see Hoerl & Kennard 1970) that will trigger, when a near singular value in

[24] The assumption regarding aggregate forecast is rather strong but allow for a simple aggregation in expectations. In this model, each variable has a single forecast on the household side and the firm side is not affected by the heterogeneity on the household side. Implementing full heterogeneous expectations would involve tracking a family size and creating a weighted average of aggregate expectations on the firm side. Finally, it will introduce multiple aggregate variable forecasts for each heterogeneous Euler equation (17).

the second-moment matrix appears which would have lead to inaccuracy in the inversion as in Slobodyan & Wouters (2012b).

Finally, iterating (38), it is possible to write

$$\mathbb{E}^{AL}_{t|i}\hat{x}^e_{i,t+1} = (I + b_{i,t-1})a_{i,t-1} + b^2_{i,t-1}\hat{x}^s_{i,t-1} + (b_{i,t-1}c_{i,t-1} + \rho)z_t. \tag{41}$$

In order to obtain the whole vector of forecasts, outcomes of each PLM are aggregated in the final expectation vector such as

$$\hat{x}^e_{t+1} = vec(\hat{x}^e_{AG,t+1}, \hat{x}^e_{\hbar=1,t+1}, \hat{x}^e_{\hbar=2,t+1}, \quad \dots \quad , \hat{x}^e_{\hbar=\mathcal{E}^N,t+1}). \tag{42}$$

Then, plugging (42) into (32), I can simulate the model dynamic under constant gain recursive least square learning and heterogeneous PLMs. From the belief updating process, it is possible to see that the feedback loop between expectations formation and realizations of the model can drive the model out of the initial fixed point.

**Initial conditions and simulation protocol.** Following a procedure described in Slobodyan & Wouters (2012b), I initialize the moment matrices $R_{i,0}$ and the beliefs matrices $\phi_{i,0}$ using the analytical variance/covariance matrix generated by the REE solution of the model in Dynare Juillard et al. (1996). In a formal way, it is possible to write

$$R_{i,0} = \mathbb{E}^{RE}_0 M_{i,t-1}M'_{i,t-1}, \tag{43}$$

$$\phi_{i,0} = R^{-1}_{i,0}\mathbb{E}^{RE}_0 M_{i,t-1}\hat{x}^{e'}_{i,t}, \tag{44}$$

where $\mathbb{E}^{RE}_0 M_{i,t-1}M'_{i,t-1}$ is the second moment matrix under REE. This initialization method is equivalent to simulate under REE a long time series to sample from the ergodic distribution of beliefs and then using those as initial conditions. At initialization, those believe satisfy the least-squares orthogonality condition (Branch 2004a). Thus beliefs generate forecast errors which are orthogonal to PLMs - *i.e.* there is no correlation between forecast errors and PLM. The orthogonality condition guarantees that agents perceive their beliefs as consistent with the ALM. Thus, agents can have misspecified beliefs but within the context of their forecasting model are unable to detect their misspecification. At initialization, the PLMs are optimally misspecified and the equilibrium concept is defined as a RPE.

This initialization method is the closest in the literature to the REE solution.[25] As consequence, the deviations from the RPE dynamic under AL are only due to temporary deviation from the RPE because of the constant learning process and the above-mentioned restrictions on perceptions imposed on beliefs.

# 4 Model dynamics

In this section, I present the dynamic response from the model to a productivity shock. This section first discusses the calibration of the model and then presents a productivity shock impulse response function (IRF).[26] The main takeaway of this section is that learning dramatically amplifies the impact of productivity shock and precautionary saving by temporally pushing beliefs outside the RPE solution.

## 4.1 Calibration

**Standard parameters.** I choose a discount factor of $\beta = 0.99$ (Woodford 2003) and an inter-temporal elasticity of substitution of $\sigma = 1.5$ consistent with Smets & Wouters (2007). The Taylor rule coefficient on inflation $\phi^{\pi} = 1.50$ and output gap $\phi^{y} = 0.125$ are standard. The price elasticity of demand is $\epsilon = 10$ (Smets & Wouters 2007) and the menu cost $\psi = 50$ is consistent with fairly flexible prices but allows for a large determinacy zone. $\alpha = 0.2$ is consistent with Smets & Wouters (2007) estimate. I assume a zero inflation target $\overline{\pi} = 1$ as in Woodford (2003). The same calibration $\delta = 0.025$ as in Smets & Wouters (2007) is used for the depreciation rate of capital. I use

[25] Using the second-moment matrix under REE for the initialization also rules out the existence of potential multiple equilibrium.

[26] IRFs for a nominal rate shock is displayed in the Appendix C.

$\varphi = 0.5$ for the Frish elasticity which is consistent with Ragot (2018), Le Grand & Ragot (2021) and Smets & Wouters (2007) and literature on HANK model with GHH utility function. Finally I scale labour supply with $\chi = 0.04$ as in Le Grand & Ragot (2021).

**Shocks.** With regards to the shocks, I define $\nu = 10$ in the realm of the literature, which implies than an increase in productivity of 1% increases the job finding rate by 10%. In order for the model to yield realistic dynamics, I calibrate the shock processes with $\{\sigma(\vartheta_t^R), \rho^r, \sigma(\vartheta_t^p), \rho^p\} = \{0.01, 0.8, 0.01, 0.8\}$, which are values within the boundaries of the literature. Results are robust to different calibrations, especially shock calibrations.

**Labour market.** In order to generate large heterogeneity between agents without introducing more idiosyncratic states, I set the unemployment rate at the steady-state $S_u = 10\%$. I then use the estimated job finding rate $\Pi_{ue}^{SS} = 0.8$ by Krueger et al. (2016b). This means that unemployed households have a 80% probability to exit unemployment every quarter at the steady-state. There is no standard calibration for the disutility of unemployment in the literature. Therefore, I set $\varsigma = \frac{1}{2}\bar{l}$ to be equal to half the steady-state labour supply of employed agent. This calibration enables the model to avoid negative consumption for poor unemployed agents and a reasonable level of aggregate capital and investment. I set the borrowing constraint to $-a = 0$ to avoid agents with negative wealth and to have a one to one mapping between aggregate savings and capital stock.

**Truncation and implication for heterogeneity.** I truncate the idiosyncratic history after 2 quarters $N = 2$. This leads to the creation of a partition $\mathcal{H}$ of 4 different families $\hbar$. The number $N$ before truncation is motivated by the trade-off between the need to have accurate distributions and tractability.[27]

[27] A small $N$ helps with computation speed, implementation and the accuracy problems encountered while inverting the very large second-moment matrix during the learning process. The results are robust to larger $N$. Nonetheless, $N = 2$ has a clear advantage, it is easier to interpret. Moreover, in the simulations, the model experience less ridge regressions

At the steady-state, each family at their steady states can be characterized by a set of values including their size $\overline{S}_{\hbar}$, asset holdings $\overline{a}_{\hbar}$ and consumption levels $\overline{c}_{\hbar}$. In consequence, we have

$$\begin{Bmatrix} \overline{S}_{\hbar(uu)} & \overline{a}_{\hbar(uu)} & \overline{c}_{\hbar(uu)} \\ \overline{S}_{\hbar(ue)} & \overline{a}_{\hbar(ue)} & \overline{c}_{\hbar(ue)} \\ \overline{S}_{\hbar(eu)} & \overline{a}_{\hbar(eu)} & \overline{c}_{\hbar(eu)} \\ \overline{S}_{\hbar(ee)} & \overline{a}_{\hbar(ee)} & \overline{c}_{\hbar(ee)} \end{Bmatrix} = \begin{Bmatrix} 0.02 & 1.40 & 0.178 \\ 0.08 & 1.60 & 0.184 \\ 0.08 & 1.59 & 0.265 \\ 0.82 & 1.79 & 0.275 \end{Bmatrix}.$$

Because of the small value of $N = 2$, heterogeneity among families is rather limited and differences can be mostly observed between employed and unemployed families.[28] Nonetheless, there is a ratio of 1.54 in consumption and 1.28 in assets between $\hbar^{(e,e)} = (e_t, e_{t-1})$ and $\hbar^{(u,u)} = (u_t, u_{t-1})e$ respectively the richest and poorest families of the model. At the steady-state, $\hbar^{(e,e)}$ and $\hbar^{(u,e)}$ respectively include 82% and 8% of the households. Those families can be considered as representative of the employed and unemployed agents populations. Obviously, this distribution is not realistic. However, the main objective of this paper is to investigate the interplay between adaptive learning and a model which acknowledge the explicit impact of idiosyncratic levels and risk on households' expectations.

**Learning parameters.** Finally, I set $g = 0.01$ which is within the bound of the literature [.01 .03](Milani 2007) . This value allows us to display relatively large variation between the RE and AL economy without a high risk of instability or triggering ridge regression. Results are robust within the bounds of the literature. Sensitivity to the gain parameter can be observed in Appendix B. The ridge regression is triggered when any eigenvalues of the second moments matrix is below $10^{-7}$ and results are robust within the interval $[10^{-5}, 10^{-8}]$.

during the updating of the PLMs. It does not require the implementation of a projection facility.

[28] In order to converge to a Krusell & Smith (1998) economy, the literature often suggests $N > 8$ which implies at the least 256 families which is beyond the scoop of this paper in term of managing heterogeneous PLMs.

## 4.2 Impulse response functions from a supply shock

In this model, household heterogeneity and the endogenous dynamic of the unemployment risk make the supply shock propagation more complex. Indeed when productivity decreases, the job-finding probability decreases and the unemployment rate and duration increase. Nonetheless, finding a benchmark to compare the HANK model under AL is not easy. I present here the dynamic of the HANK model with respect to the RANK counterpart under adaptive learning - *i.e* in the absence of idiosyncratic risk (see Appendix A for the detailed equations of the RANK model).[29] The HANK model is simulated under rational expectations and adaptive learning in order to disentangle the effect generated by the learning, the heterogeneity and the interaction between both features. It worth mentioning, that in the RANK model, with current calibration and initialization procedure, and under a fairly small gain, there is only a small difference between the rational expectations and adaptive learning simulations. Therefore, the RANK-RE responses are not displayed.

Figure 3 presents the IRFs of the HANK and RANK models under rational expectations and adaptive learning to a negative supply side shock represented by $-1\%$ productivity $AR(1)$ process.[30]

The RANK AL model presents the standard response to a negative supply shock. The decrease in productivity creates a drop in supply and increases the marginal cost while demand is relatively steady. Thus, the representative firm increases its price in order to balance the supply and demand sides (Figure 3-a). The change in expected inflation decreases the ex-ante real interest rate. Reacting to the surge in inflation, the CB increases nominal rate (Figure 3-c). This rate hike depresses demand and tempers down the prices hike by the supply side (Figure 3-a). In consequence, the representative household cuts down on saving (Figure 3-e), consumption (Figure 3-f) and labour (Figure 3-g) until the productivity level is back to its steady-state. To sum up, a negative

[29] The AL in the RANK model is a simple MSV learning *i.e.* no restrictions on the information set have been implemented.

[30] Due to the absence of variation in idiosyncratic risk, demand sided shocks have less interesting propagation. IRFs of a nominal rate shock are displayed in Appendix C.

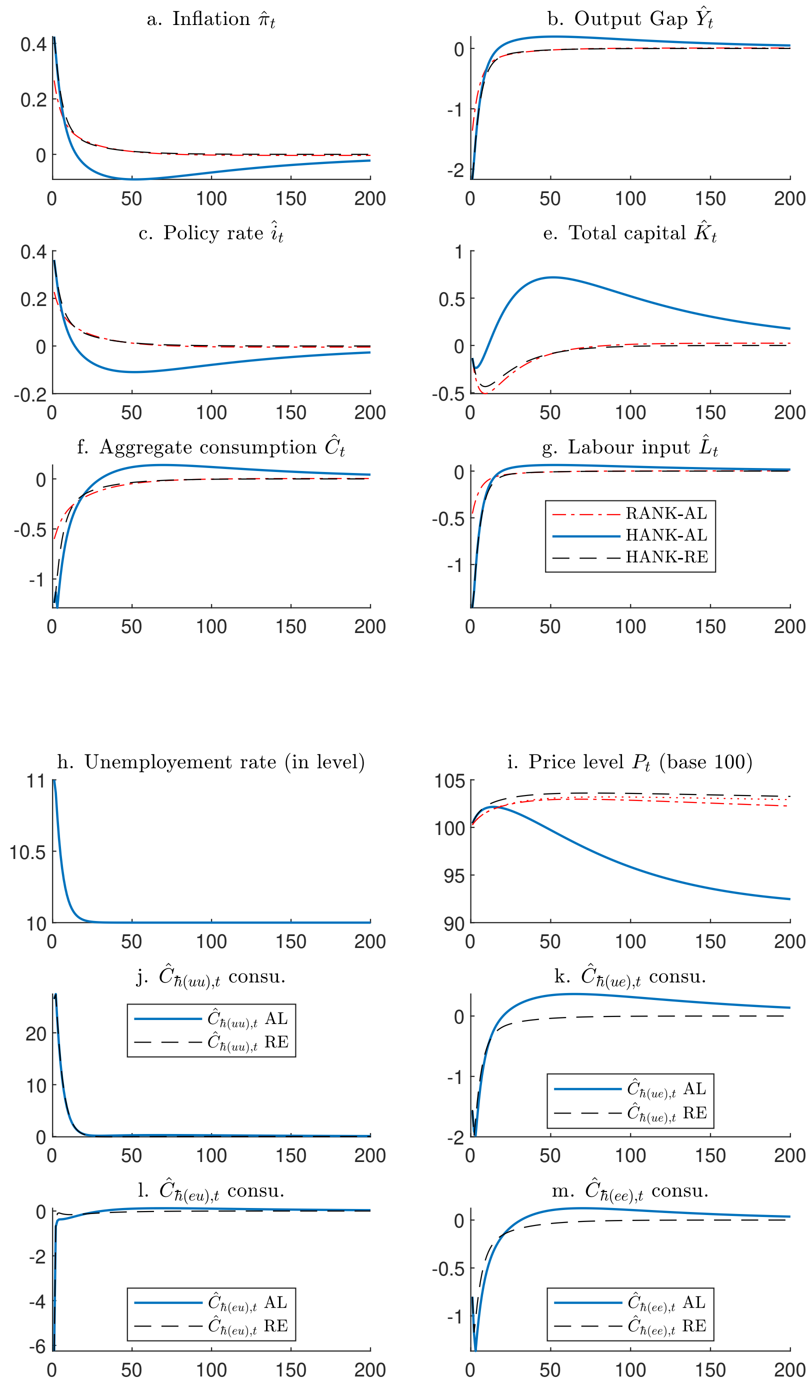
supply shock in a RANK model is inflationary even though it creates a drop in output.

Comparing the RANK AL, the HANK model under rational expectations presents already a different dynamic. First of all, the decrease in productivity implies a lower job finding rate and thus a higher unemployment (Figure 3-h) and lower aggregate labour supply by households (Figure 3-g) relative to the steady-state. Even though initially the supply-side effects are the same, the aggregate demand behaves very differently (Figure 3-f) between the two models. First, the consumption drops twice as much and the aggregate saving by households does not drop as much (Figures 3 e and f). This is due to the precautionary saving implied in the households' FOCs (see Equation 17). Indeed, the increased probability and duration of unemployment triggers precautionary saving in order to smooth future utility flows.

In order to be exhaustive, the panels j, k, l and m in Figure 3 present the consumption behaviours of all families. $\hbar^{(e,e)}$ and $\hbar^{(e,u)}$ are the employed households families and respectively include at the steady-state, 82% and 8% of the households. $\hbar^{(u,e)}$ and $\hbar^{(u,u)}$ are the unemployed households families and respectively include at the steady-state, 8% and 2% of the households.

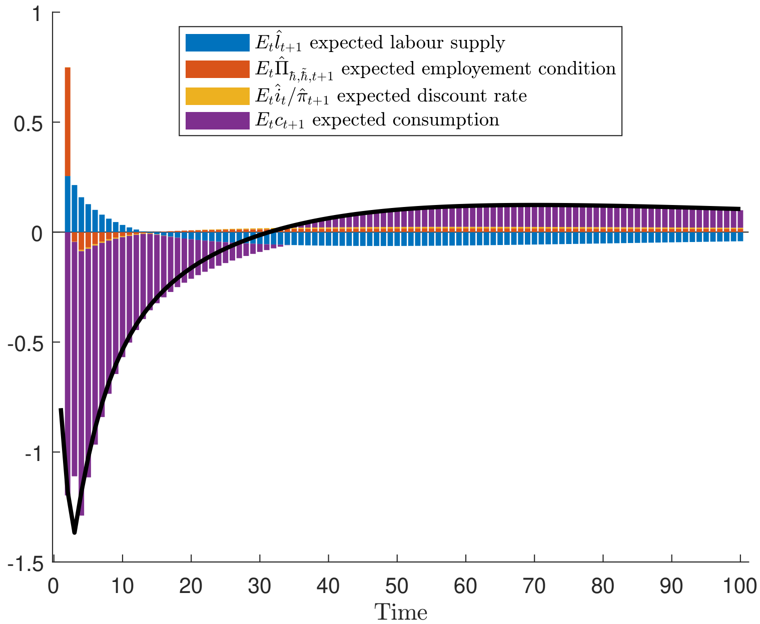
The precautionary saving in the HANK RE can be disentangled into two effects. For unemployed households, the drop in job-finding rate implies an increased weight in the probability to stay unemployed in $t+1$. Hence, a higher probability to stay in an unemployed family where the expected income and consumption are lower. This drop in consumption has also implications for employed households. It lowers their own expected consumption level due to the lower consumption in case of the realization of the unemployment risk. Precautionary saving and demand contraction appear to be larger for richer households than the poorer ones (Figure 3 k and m). This is logical because richer households' income is less a function of dividends and saving and more a function of wages and labour which are adversely affected by the shock. Moreover being richer, it is easier for them to allocate part of their income into future utility/saving.

Finally, it is important to acknowledge the strange positive wealth effects

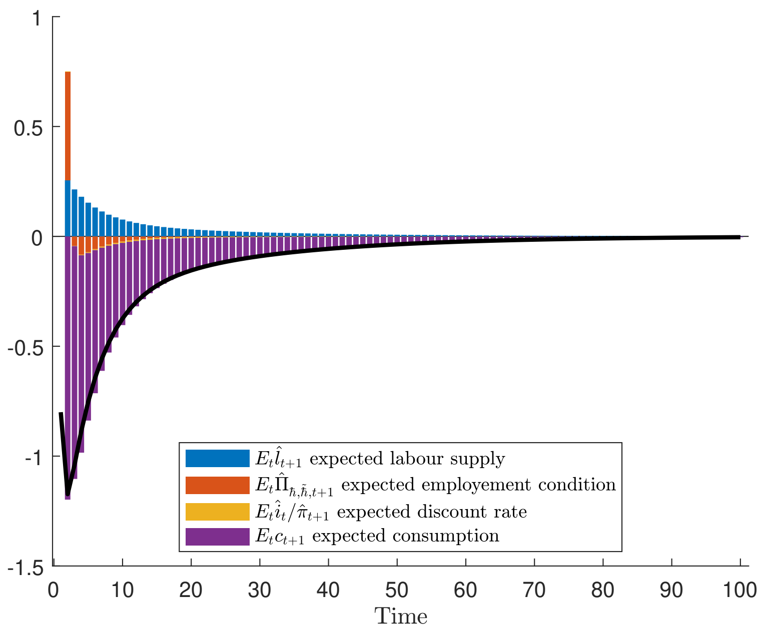



Notes: Results are in percentage point of the log deviation from the steady states. Price level and unemployment rate are in level. No projection facilities and no ridge regressions were implemented during those simulations.

Figure 3: Deflationary episode in the HANK model under learning (in blue) after a $-1\%$ productivity shock




(a) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under AL in the HANK




(b) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under RE in the HANK shock

Notes: $\hat{C}_{\hbar(e,e),t}$ is the consumption of $\hbar(e,e)$, the family with agents employed in the last two periods. At the steady-states 82% of the agents are in this family. In order to build this figure, I take advantage of the linearization of Euler Equation (17) of $\hbar(e,e)$. Thanks to that, it is possible to decompose the aggregate output equation by computing the movement generated by each term displayed in the legend. Notice that for the sake of clarity, the contribution of present labour supply is not displayed.



Figure 4: Expectation contribution of the employed agents' consumption after a $-1\%$ productivity shock

(especially in Figure 3 j) generated by the variation in the size of the family. This effect is an artefact of the HANK solution methods. Indeed, an increase in unemployment means a larger intake of wealthier households in poor unemployed families, through the family-wide insurance mechanism redistributes wealth to all family members.

In figures 4, I present under RE and AL the expectations contribution to consumption of family $\hbar^{(e,e)}$ which include 80% of the total households population and 88% of the employed population.[31]

Looking at Figure 4b , it is possible to observe that the main driver of consumption drop for employed agents is an expected drop in their individual consumption (in purple) through an expected drop of consumption in case of unemployment and employment. Another phenomenon is the effect of expected labour condition (in red) which initially generates an increase in consumption (less poor agents find employment, join the family and reduce the family's wealth) and then a decrease due to the longer than expected unemployment duration. The expected labour supply (in blue), drives consumption up due to the construction of the GHH utility function.

The HANK model under adaptive learning broadly exhibits in the short run the same responses as its rational expectations counterpart. Nonetheless, the aggregate consumption drops more (see Figure 3-f) while labour supply is comparable (see Figure 3 g), which leads to a large increase in individual saving and aggregate capital (see Figure 3-e). The large difference between the HANK-RE and HANK-AL is mostly due to the large forecast error on shock's impact that trigger important revisions in beliefs, and thus larger deviation from the HANK RE model.[32]

It is striking to see in Appendix B, that in the absence of updating in the model $g = 0$, the heterogeneous RPE and REE models exhibit very similar dynamics (black and green lines). This suggests that idiosyncratic/aggregate

[31] In AppendixD , I display expectations contribution to the representative household consumption.

[32] This is different from the REE solution of the RANK model that generates smaller forecast errors on shock impact, thus less PLM's revision and almost no deviation between the RANK-AL and RANK-RE models.

observations are redundant for aggregate/idiosyncratic forecasts. Moreover, it also means that the following results in the paper are mostly driven by the learning processes and not by the assumptions on the information sets. Finally, it is important to note that the learning model under full information assumption and $g = 0$ would be equivalent to the REE model. In a sense, the REE model is a special case of the RPE model under adaptive learning.

Relative to the HANK RE counterpart, in the HANK AL model the excess drop in consumption (Figure 3-f) is the biggest within the employed agent families (see Figure 3 m). Figure 4a displays the consumption of $\hbar_{(e,e)}$, the family, which represents 80% of the households and 88.88% of the employed households at the steady-state. It can be considered as a good proxy for employed agents behaviour. Figure 4a shows that employed agents under AL, after the shock impact struggle to learn their level of consumption. The over-optimism of the forecast error on impact creates a downward revision of the beliefs - especially for the intercept of the PLMs. This introduces a pessimistic bias in the PLM and thus in the families' beliefs. In consequence, it amplifies the precautionary saving. In the medium run, after 10 quaters, it is possible to observe a negative contribution from expected labour supply, which is higher than the RE counterpart (see Figure 4b). This higher expected labour supply is more than compensated in the longer run by higher than the steady-state expected consumption.

It is possible to observe the same effect for $\hbar_{(u,e)}$ (Figure 3-k), the family which represents 8% of the households and 88.88% of the unemployed households at the steady-state. The unexpected decrease in job-finding rate generates a pessimistic revision of beliefs and a larger than rational drop in consumption.

On the medium run and at the aggregate level, expectations lose their anchorage to the rational expectations path and drift toward a disinflationary transient state that can last a very long time (see Figure 3-a). The model eventually converges back to its steady-state after over 200 periods. What happens in the model is that the lower consumption expectations generate a lower aggregate consumption that leads to excess aggregate capital accumulation (see Figure 3-e). This reduce down marginal cost, the wage and pushes up labour supply (see Figure 3-g). All those aggregate effects are disinflationary.

The large differences in the aggregate supply and demand dynamics between the HANK AL and the other two models naturally raise the question about the design of stabilization policies. In the context of this model, what could and what should the CB do in order to jointly stabilize output and inflation?

# 5 Monetary policy implications

After describing the implications of supply-side shocks in a HANK model under adaptive learning, the purpose of this section is to investigate the monetary policy consequences of those modelling choices and the change with respect to a HANK model under rational expectations and a RANK model under adaptive learning.

## 5.1 The ambiguous effects of monetary policy

Primarily, it is important to observe the models statistical moments under a standard IT regime. The first column of Table 1 presents the business cycle statistics of the HANK-AL model with respect to the RANK-AL and HANK-RE models with a standard calibration of the CB's reaction function. Relative to the RANK-AL model, both HANK models exhibit higher volatility in output, aggregate consumption and aggregate capital. Those effects come from the fact that households from the HANK models use capital as a way to smooth their consumption and insure themselves again expected idiosyncratic future drop in income due to unemployment risk. Hence, contrary to the RANK household which uses its capital as the adjustment variable of the resource constraint, HANK households keep a buffer of capital as an insurance mechanism against idiosyncratic risk which is evolving over time.

For the sake of clarity, I include in the table the variances of the consump-

| *Monetary policy regime*: *Calibration* | *Standard IT* $\phi^{\pi} = 1.5$ $\phi^{y} = 0.125$ | *Hawkish IT* $\phi^{\pi} = 2.50$ $\phi^{y} = 0.125$ | *Dovish IT* $\phi^{\pi} = 1$ $\phi^{y} = 1$ | *PLT* $\phi^{p} = 0.25$ $\phi^{y} = 1$ |
|---|---|---|---|---|
| ***Inflation Variance*** $var(\hat{\pi}_t)$: | | | | |
| RANK-AL | 9.7981 (0.0102) | 1.2012 (0.0013) | 360.1683 (0.2629) | 15.6534 (0.0076) |
| HANK-RE | 8.8602 (0.0092) | 1.1892 (0.0013) | 31.4332 (0.0283) | 9.1578 (0.0049) |
| **HANK-AL** | **9.4717** (0.0104) | **1.2705** (0.0028) | **35.3716** (0.0353) | **9.3219** (0.0050) |
| ***Output Gap Variance*** $var\left(\hat{Y}_t\right)$: | | | | |
| RANK-AL | 6.6628 (0.0092) | 6.6354 (0.0092) | 4.0162 (0.0035) | 7.0446 (0.0105) |
| HANK-RE | 18.5623 (0.0263) | 16.7225 (0.0219) | 7.1004 (0.0048) | 8.4133 (0.0075) |
| **HANK-AL** | **21.5123** (0.0356) | **20.5573** (0.1229) | **7.1108** (0.0047) | **11.5376** (0.0228) |
| ***Aggregate Consumption Variance*** $var\left(\hat{C}_t\right)$: | | | | |
| RANK-AL | 3.5478 (0.0062) | 2.5443 (0.0056) | 34.7914 (0.0248) | 2.2172 (0.0058) |
| HANK-RE | 12.9919 (0.0269) | 8.3221 (0.0148) | 5.9788 (0.0078) | 2.9514 (0.0076) |
| **HANK-AL** | **17.3224** (0.0362) | **13.8411** (0.0486) | **5.2057** (0.0077) | **2.7051** (0.0161) |
| ***Employed Consumption Variance*** $var\left(\hat{c}_{\hbar(eu),t}\right)$: | | | | |
| HANK-RE | 12.6535 (0.0268) | 8.0756 (0.0148) | 5.3385 (0.0074) | 2.9291 (0.0074) |
| **HANK-AL** | **16.9594** (0.0352) | **13.5146** (0.0433) | **5.1895** (0.0088) | **2.8206** (0.0143) |
| ***Unemployed Consumption Variance*** $var\left(\hat{c}_{\hbar(ue),t}\right)$: | | | | |
| HANK-RE | 23.6877 (0.0414) | 18.6654 (0.0289) | 4.2653 (0.0056) | 4.5186 (0.0073) |
| **HANK-AL** | **36.7748** (0.0864) | **34.1853** (0.1978) | **4.7815** (0.0093) | **14.1958** (0.0679) |
| ***Capital Variance*** $var\left(\hat{K}_t\right)$: | | | | |
| RANK-AL | 7.8065 (0.0223) | 6.9323 (0.0208) | 6.2092 (0.0107) | 10.1193 (0.0304) |
| HANK-RE | 45.2120 (0.1254) | 13.1407 (0.0371) | 68.3600 (0.0898) | 45.9711 (0.0878) |
| **HANK-AL** | **84.4807** (0.3680) | **62.1031** (1.6309) | **70.9396** (0.0958) | **86.4607** (0.3956) |

Notes: Every moment of the table is the result of 10,000 Monte-Carlo simulations over 800 periods. Except for one time in the HANK-AL PLT runs, no crashes happens in any simulations.

Table 1: Moments under different CB's reaction function calibrations

tion of families $\hbar_{(e,e)}$[33] and $\hbar_{(u,e)}$.[34] Unemployed agents' consumption volatility is higher than employed due to their lower saving buffer and income (only dividends) which doesn't allow them to smooth their consumption as much as employed agents.

With respect to the HANK under rational expectations, the HANK under adaptive learning exhibits higher volatility in all its state variables. The most striking ones are the ones driven by the households' sides where the variance in capital stock is almost twice as big as in the rational expectations counterpart. This excess variance in aggregate capital is due to the excess volatility in de-aggregated consumption. This excess volatility is driven by the learning dynamic and the restriction on perceptions. Both unemployed and employed

[33] This family represents 80% of the households and 88.88% of the employed households at the steady-state. It is a good proxy for employed agents behaviour.

[34] This family represents 8% of the households and 88.88% of the unemployed households at the steady-state. It is a good proxy for unemployed agents behaviour.

agents are subject to an increase in consumption variances due to the constant revision of their misspecified beliefs.

In the second and third columns of Table 1, I present two cases of non-standard calibration for monetary policy. The first one is name *hawkish* with $\{\phi^{\pi}, \phi^{y}\} = \{2.5, 0.125\}$, a harsh response to inflation deviation and the other one is *dovish* with $\{\phi^{\pi}, \phi^{y}\} = \{1, 1\}$ a strong reaction to output gap.

In the *hawkish* case, the change in all three models is fairly similar. The inflation is less volatile in the same magnitude. Output is relatively unchanged even though the difference between the HANK-RE and HANK-AL models increases. The difference in output volatility (+22%) and the aggregate capital variances (+372%) are very different. It can be explained by the fact that the increase in the volatility of the nominal rate makes the complex expected utility streams of Equations 17 more volatile. Hence, the expected incomes are harder to forecast.

On the other hand, the *dovish* policy creates a large difference between the HANKs and RANK-AL models. Inflation in the RANK-AL model explodes *w.r.t* to the standard Taylor rule calibration case. The only large difference between the HANK-AL and HANK-RE is the inflation variance. Nonetheless, those ones are similar in magnitude to the standard calibration scenario which make up for the case that a more *dovish* monetary policy is not harder to learn for heterogeneous households. Those results make up for the case that stability in HANK models is driven through the stability of the expected income channel and not through the fast adjustment of the discounting process. Hence, it explains also why the discrepancy between the HANK-RE and HANK-AL is so small. Indeed stabler expected consumption and discounting process ease the complex learning dynamic of heterogeneous families. Finally, it is noteworthy to point that the overall and desegregated volatilities of consumption in the HANK-AL are smaller than under RE. This puzzling data point might be generated by drifts from the RE perceived law of motion to a less volatile one under learning and restricted perception.

## 5.2 Monetary policy trade-off

In Figure 5 a and b, I observe the evolution of the output and inflation variances over the parameter space of the Taylor rule under supply shocks.[35] In Appendix E, it is possible to observe the excess volatility of the HANK-AL model *w.r.t* the other models.

In order to formalize monetary policy trade-off, it is possible to give a standard loss function to the CB. Thus, the CB tries to stabilize the economy by minimizing its loss function $\mathcal{L}$ as follow

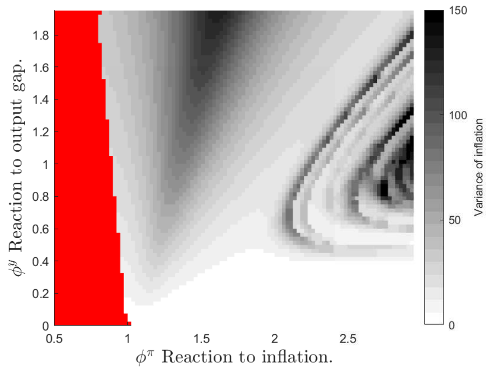
$$min_{\{\phi^\pi,\phi^y\}} \qquad \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \lambda \hat{\pi}_t^2 + (1-\lambda)\hat{y}_t^2 \right), \tag{45}$$

where $0 \leq \lambda \leq 1$ defines the relative preference of the CB between output and inflation stabilization. Due to the complexity of the model, it is beyond the scope of this paper to solve analytically this problem. Nonetheless, given a defined preference $\lambda$, it is possible to find by a simple grid search the optimal point on a given simulated grid based on different parameter sets such as in Figures 5. It is then possible to extract the combinations of output and inflation variances for 1000 values of $\lambda \in [0 \quad 1]$. In Figure 6, the results of this exercise for the different models are displayed. The curves in Figure 6 are the combinations of output and inflation volatility satisfying the CB's loss minimization problem as a function of $\lambda \in [0 \quad 1]$ and illustrate the monetary policy stabilization trade-off.

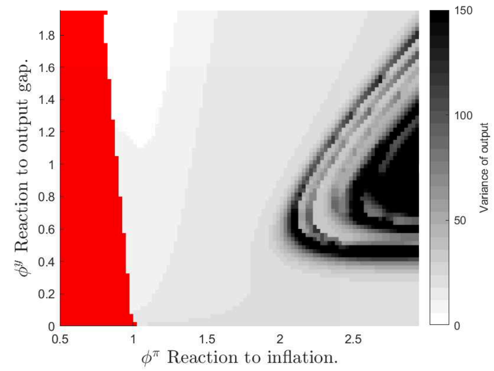
The RANK-AL model yields, in Figures 5 e-f and 6, the standard result that only an aggressive monetary policy relative to the inflation generates low inflation variance in a NK model under learning. This result is in line with Orphanides & Williams (2008) and Eusepi & Preston (2018) analysis where a conservative CB should be preferred under Euler equation learning in a RANK model.[36] As illustrated by its very steep curve in Figure 6, the trade-

[35] Not incorporating monetary policy shocks in the simulations allows to only extract situations where the output/inflation stabilization trade-off is more acute and allows to draw more explicit results.

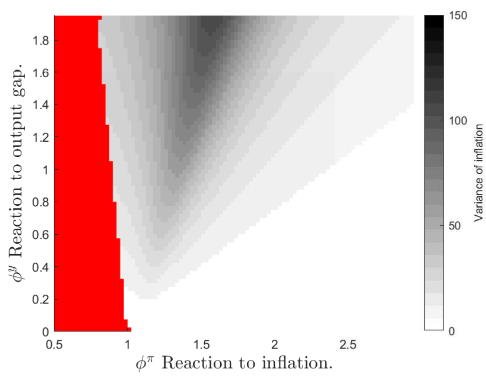
[36] It is important to point out that those results are not valid when considering longer horizon adaptive learning/anticipated utility such as in Preston (2005) or Eusepi, Giannoni



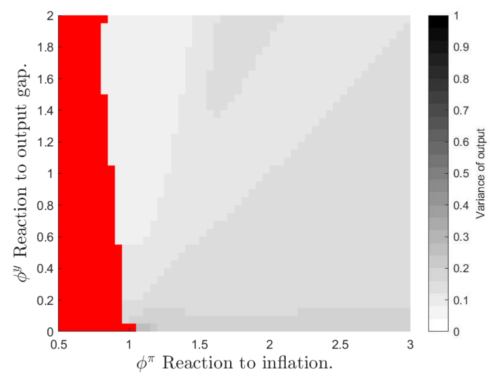

(a) Variance in inflation HANK AL



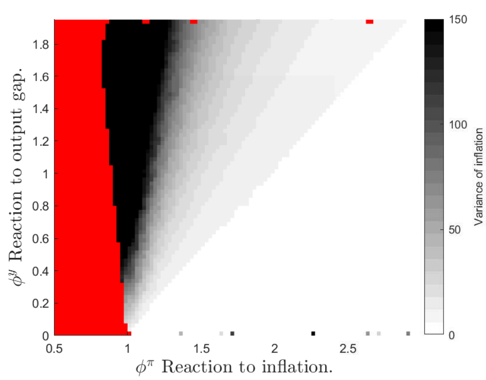

(b) Variance in output HANK AL



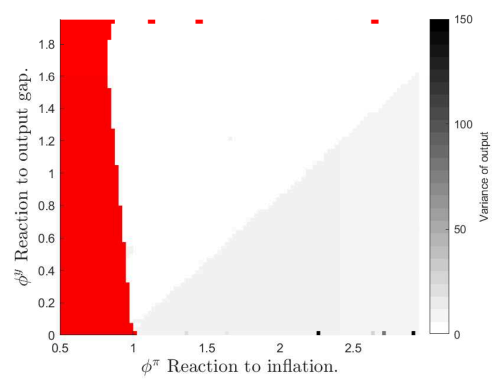

(c) Variance in inflation HANK RE




(d) Variance in output HANK RE



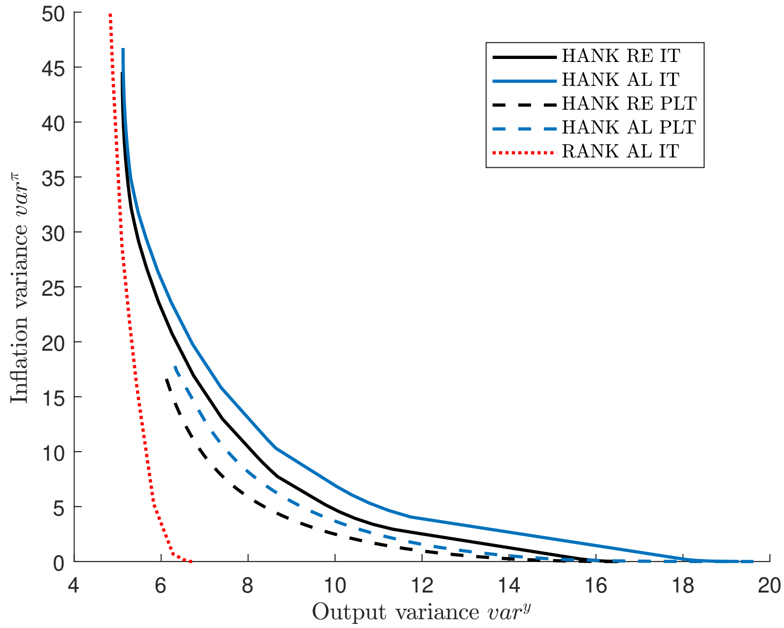

(e) Variance in inflation RANK AL




(f) Variance in output RANK AL

<u>Notes</u>: Every data points of the map is the mean result of 100 Monte-Carlo simulations (800,000 total) over 800 periods. The red zone is where respectively the probability of the time series to crash is over 20% in the adaptive learning models and indeterminate zone in the rational expectations model. In the RANK model, the red zone is also where inflation o' output variances are over 150 (in order to keep the same legend). A crash is defined when the standard deviation of output or inflation is more than 4 times its rational expectations counterpart. For the adaptive learning models, the noise from the map is filtered out by taking for a point $x[i,j]$ the median value between $filter(x[i,j]) = median\{x[i,j], x[i-1,j], x[i-1,j-1], x[i,j-1], x[i+1,j], x[i+1,j+1], x[i,j+1], x[i-1,j+1], x[i+1,j-1]\}$. The variance of MP shocks is null in order to clarify the results.

Figure 5: Policy trade-off under supply shocks over the monetary policy space




Notes: Values are computed using a grid search over the parameter spaces and results displayed in Figure 5 and 8. Each point on each curve correspond to a different value of $\lambda$.

Figure 6: Policy frontiers under supply shocks for the monetary policy as a function of CB preferences

off between output and inflation stabilization is very strong in the RANK adaptive learning model. Reducing the volatility from the output gap implies increasing a lot the one from inflation.[37] The intuition behind this problem is relatively straightforward, due to the self-referential nature of the new Keynesian Phillips curve, the de-anchoring from the inflation expectations will trigger more volatile inflation which will self-reinforce the de-anchoring of inflation expectations. On the other side, a de-anchoring of output expectation will generate a co-movement in the same direction in inflation which would be self-correction through the intervention of the CB.

In all three models there exist trade-offs between inflation and output gap

& Preston (2018).

[37]The inflation/output gap stabilization sacrifice ratio is said to be larger in the RANK-AL than in the RANK-RE model.

stabilization *i.e* the higher/lower the reaction to inflation $\phi^{\pi}$/output $\phi^{y}$ the more/less stable inflation is and the more/less volatile output gap is. Nonetheless, in the HANK-RE and HANK-AL, it is crucial to remark that the top left-hand zones of both maps *i.e* where the monetary policy reacts aggressively to the output gap and lightly to inflation; is not the high volatility inflation unstable zone like the RANK-AL model (Figures 5 a,b,c and d). It is striking to note that a *dovish* monetary policy stance appears to be a viable low volatility scenario. Moreover, it is the zone where the HANK-AL variances are the closest to the HANK-RE scenario (see Appendix E). In Appendix E, it is even possible to observe that in the Dovish zone, the output is even less volatile under AL with restricted perceptions.

Despite the resemblance between the HANK-AL and HANK-RE maps, there is an obvious difference. The top right-hand side of the HANK-AL maps appears very volatile (Figures 5 a,b,c and d). Strong reaction to both output and inflation by the CB appears to destabilize the learning model. In the HANK model, strong policy reaction to unexpected shocks under learning is destabilizing for expectations by increasing the size of the forecast errors in the Euler equations and through that increases the idiosyncratic beliefs adjustments speed. All of that can drive the model out of near rational expectations dynamics. This effect has been observed in Preston (2005) or Eusepi et al. (2018) in a different context where strong policy reaction can be destabilizing for long-run expectations of the real rate.

In Figure 6, it is possible to observe the monetary policy efficiency frontiers for the HANK model under RE and AL. First of all, both curves are shallower and are to the right of the RANK AL curve. The position of the curves explains itself by the additional friction generated by the existence of the reduced form labour market and its impact on labour supply and income which increase the impact of supply shocks on output. The shallower slope of the curves represents the lesser trade-off between inflation and output stabilization. The shallower shape can be explained by the fact that in the absence of output stabilization, precautionary saving has a very large effect. Moreover, the implementation of a very volatile nominal rate in order to stabilize inflation might be destabilizing for the expected income/marginal utility and thus for inflation-induced through output and marginal cost volatility.

Finally, it is important to notice in Figure 6 that when the CB focuses on output stabilization, it the where the outcomes of optimal policy exercises under RE and AL are the most similar. This straightens the argument that the destabilizing factor in the HANK model under learning is the de-anchoring of the idiosyncratic income/marginal utility expectations. By better anchoring idiosyncratic income/marginal utility expectation, the CB neutralizes the effect of the learning. It reduces the volatility of the idiosyncratic forward variables and thus limits the updating of the PLMs.

## 5.3 A price level targeting experiment

Despite some promising results, the alternative policy does not appear to solve the slow convergence issue. Hawkish policies while avoiding long disinflation increase the magnitude of large precautionary saving and low consumption periods after a supply shock under learning. In the meanwhile, dovish policy despite the smaller differences between the rational case creates large precautionary savings and high variance in inflation.

A burgeoning strand of the literature has been developed in order to analyse possible ways to escape the deflationary trap at the Effective Lower Bound (ELB) through PLT when agents are learning.[38] PLT appears to be an efficient way to drive expectations out of those sunspots driven liquidity traps. Despite this paper's model not including the ELB, it can still generate disinflationary episodes and PLT appears to be a promising policy treatment to avoid those.

I introduce a PLT reaction function instead of the canonical IT rule. Based on the definition of inflation $\pi_t = \frac{P_t}{P_{t-1}}$, it is possible to write

$$P_t = \pi_t P_{t-1}. \tag{46}$$

[38] On one side Williams (2010) and Honkapohja & Mitra (2019) argue in favour of PLT in order escape expectations driven recessions when Mele, Molnár & Santoro (2020) point toward its destabilizing effects on the long run.

The PLT rule is then implemented. The reaction function reads as

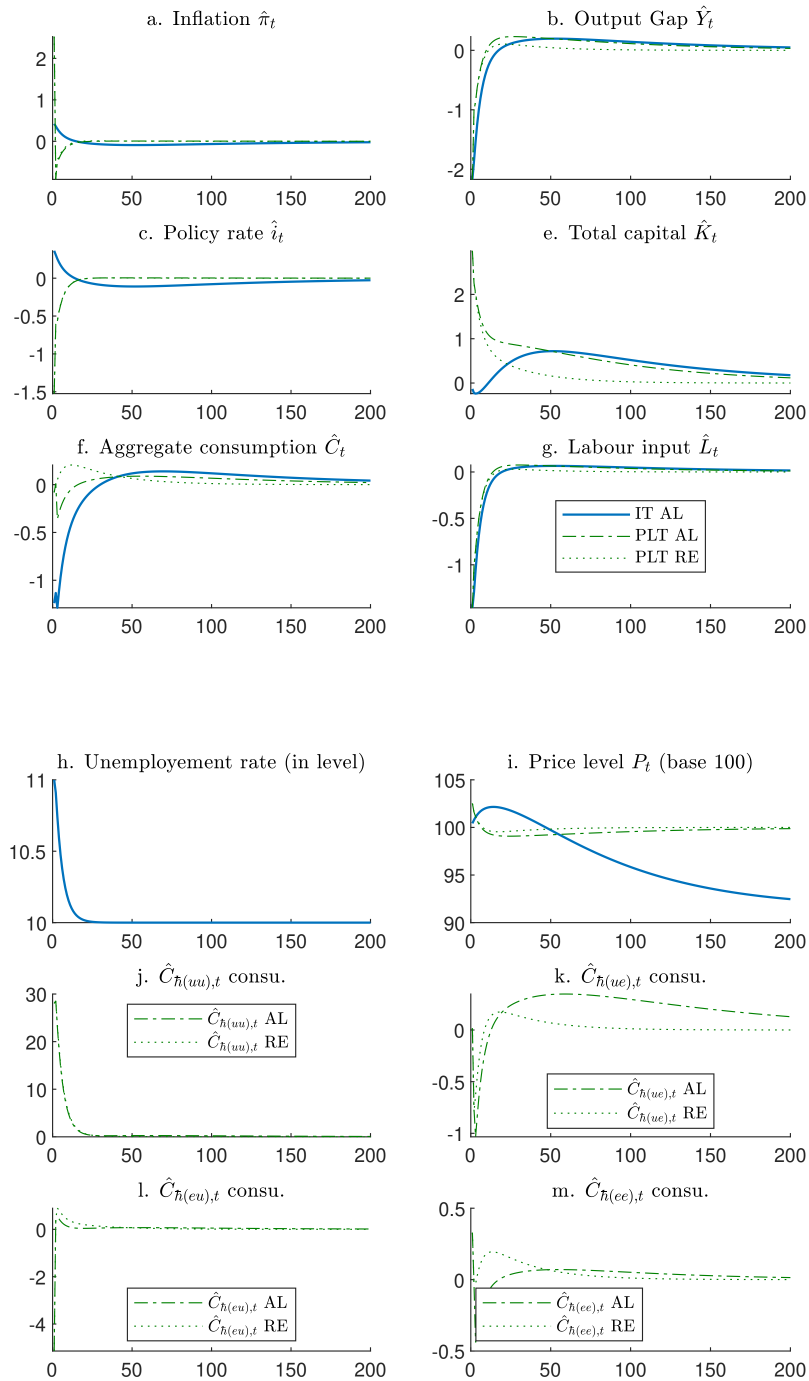
$$i_t - \bar{i} = \phi^p \left( \frac{P_t - \overline{P}}{\overline{P}} \right) + \phi^y \left( \frac{Y_t - \overline{Y}}{\overline{Y}} \right) + \varepsilon_t^r, \tag{47}$$

with $\{\phi^p, \phi^y\} = \{0.25, 1\}$ in line with previous works under learning by Williams (2010) or Honkapohja & Mitra (2019). The intuition behind PLT is that the CB commits to *make-up* for the past under/overshoot of inflation by generating a symmetric over/undershoot. In a zero inflation steady state model like this one, this implies keeping the price level at an arbitrary level (here $\overline{P} = 100$). With respect to the restricted perception, I assume that all PLMs observe the price level.

Observing Figure 7, the most striking fact is the change in the policy rate reaction. While under IT the reaction function leads to a tightening, PLT eases which is inflationary and then quickly deflationary for a dozen of quarters. Nonetheless, the decrease in nominal and real rate boost consumption for employed agent (Figure 7 j and l and Appendix F). It seems that unemployed consumption decisions tend to drift less from the rational expectations solution under PLT (Figure 7 j and l). This is due to the smoother and stabler dynamic implied by the more aggressive policy generated in case of deflationary trap. It seems that those stabler dynamics are also much easier to learn for unemployed agent and their excess forecast errors implied by the adaptive learning are reduced which reduce beliefs' revision and thus deviation from the RPE solution.

In Figure 7, we can see that after the initial productivity shock and following policy rate cut, aggregate consumption and saving converge much faster to their steady-state. Consumption in the HANK-AL model under PLT converges back to the steady-state faster. Poorer households appear to be better anchored in their saving and consumption decisions thanks to smoother expected utility streams. Richer employed households are better guided by the expected change in the real rate which allows for less drift in the PLM and ALM (see Appendix E).

The last column of Table 1 presents the main statistical moments of this




Notes: Results are in percentage point of the log deviation from the steady states. Price level and unemployment rate are in level. No projection facilities and no ridge regressions were implemented during those simulations.

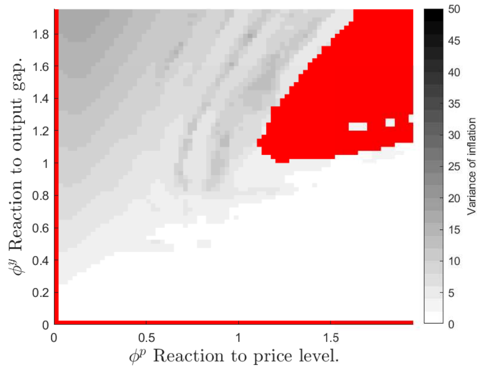
Figure 7: Convergence after a $-1\%$ productivity shock under PLT (in green)

policy experiment and Figures 8 displays the results for a large range of the parameters space. First, we can observe that PLT reduces inflation's variance in the HANK cases but not in the RANK (*w.r.t* to the benchmark calibration). This effect is due to the fact that prices adjustment by the representative firm become pointless if output is stable enough. In fact, they will trigger symmetric price adjustment in the opposite direction. Yet in all cases and models, regarding inflation, it is less efficient than the *hawkish* calibration.

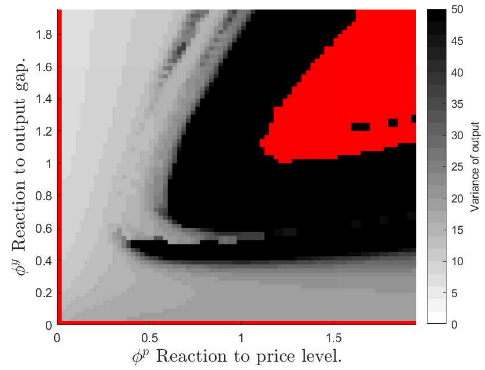
The PLT treatment increases the volatility of output in the RANK-AL -and thus inflation- but not in the HANK models.[39] In the RANK-AL model, the trade-off between inflation and output stabilization is stronger, in consequence, the current PLT calibration trades output stability for price/inflation volatility. In the HANK models, inflation is more driven by the output volatility and the trade-off is less clear. This is why a decline in output volatility is possible. This is due to the same effect discussed previously: in the same fashion as the *dovish* policy, PLT generates a stabler future discounted expected marginal utility flow/income and thus decreases precautionary saving and consumption variances. PLT by averaging inflation over time smooths expected marginal utility streams much more than the IT framework. By smoothing the income streams the CB is able to reduce precautionary saving by generating a symmetric boom after a bust.

As in Figures 5a and 5b under IT, excessive reaction from the CB yields an unstable zone under PLT (Figures 8a and 8b ). Finally, PLT's efficiency curves of the HANK models in Figure 6 are mostly to the left of the IT counterparts and slightly shallower. Apart from when CB's preference is very skewed toward output stabilization, the leftward position of the efficiency curves suggests that PLT is welfare improving in this setup.

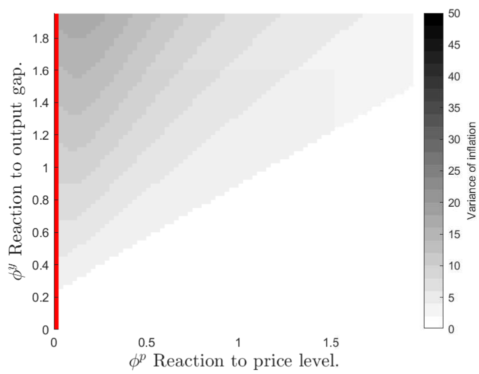
[39] In the absence of projection facility a large share of the parameter space of the RANK-AL model is unstable. Because the RANK model is not the point of paper but a benchmark. I do not include the RANK-AL model in Figure 8.



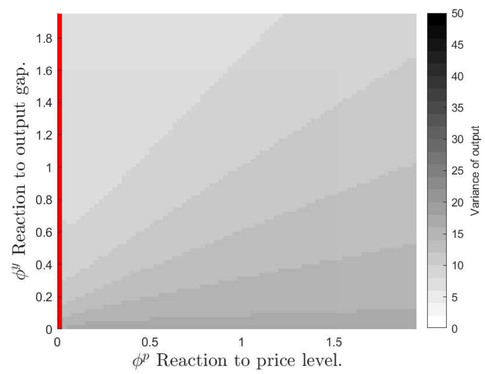

(a) Variance in inflation HANK AL




(b) Variance in output HANK AL




(c) Variance in inflation HANK RE




(d) Variance in output HANK RE

<u>Notes</u>: Every data points of the map is the mean result of 100 Monte-Carlo simulations (800,000 total) over 800 periods. The red zone is where respectively the probability of the time series to crash is over 20% in the adaptive learning models and indeterminate zone in the rational expectations model. In the RANK model, the red zone is also where inflation o' output variances are over 150 (in order to keep the same legend). A crash is defined when the standard deviation of output or inflation is more than 4 times its rational expectations counterpart. For the adaptive learning models, the noise from the map is filtered out by taking for a point $x[i,j]$ the median value between $filter(x[i,j]) = median\{x[i,j], x[i-1,j], x[i-1,j-1], x[i,j-1], x[i+1,j], x[i+1,j+1], x[i,j+1], x[i-1,j+1], x[i+1,j-1]\}$. The variance of MP shocks is null in order to clarify the results.

Figure 8: Policy trade-off under supply shocks over the monetary policy space in the HANK-AL and HANK-RE models under PLT

# 6 Conclusion

This paper investigates the implication of supply shocks when heterogeneous households are subject to an imperfect unemployment insurance market and form their expectations under bounded rationality. This paper is built on a HANK model based on the truncated idiosyncratic histories of heterogeneous households. Heterogeneous expectations are explicitly modelled through an adaptive learning system based on restricted perceptions and the RLS algorithm.

The model shows that negative supply shocks can trigger very long disinflationary episodes characterized by excess precautionary saving and depressed consumption. On one hand, monetary policy focused on inflation tends to increase the difference between the adaptive learning model and its counterpart. On the other hand, Taylor rules with more emphasis on output gap deviation decrease the discrepancy between the rational and the learning model. Those results are not in line with the literature with representative agent models under learning which favours more inflation oriented policies. PLT appears to be a policy that increases the speed of convergence, limits the beliefs' updates generated by the AL and enhances expected consumption anchorage to the RE solution at the disaggregated level. The results suggest that commitment to some kind of PLT or average inflation targeting might help to lessen the trade-off between output and inflation stabilization.

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# A The RANK model

In this appendix, the equations describing the RANK model are displayed.

$$U'(C_t, L_t) = \beta \mathbb{E}_t^*[\frac{i_t}{\pi_{t+1}} U'(C_{t+1}, L_{t+1})], \tag{48}$$

$$L_t = (\chi W_t)^{\varphi}, \tag{49}$$

$$C_t + K_t = Y_t + (1-\delta)K_{t-1} - \frac{\psi}{2}(\pi_t - 1)^2, \tag{50}$$

$$0 = 1 - (1 - mc_t)\epsilon - \psi(\pi_t - 1)\pi_t + \psi\beta\mathbb{E}_t^*(\{\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\}, \tag{51}$$

$$W_t = (1-\alpha)e^{\varepsilon_t^p} K_{t-1}^{\alpha} L_t^{-\alpha}, \tag{52}$$

$$Y_t = e^{\varepsilon_t^s} K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{53}$$

$$mc_t = \frac{1}{e^{\varepsilon_t^s}}(\frac{Z_t}{\alpha})^{\alpha}(\frac{W_t}{1-\alpha})^{1-\alpha}, \tag{54}$$

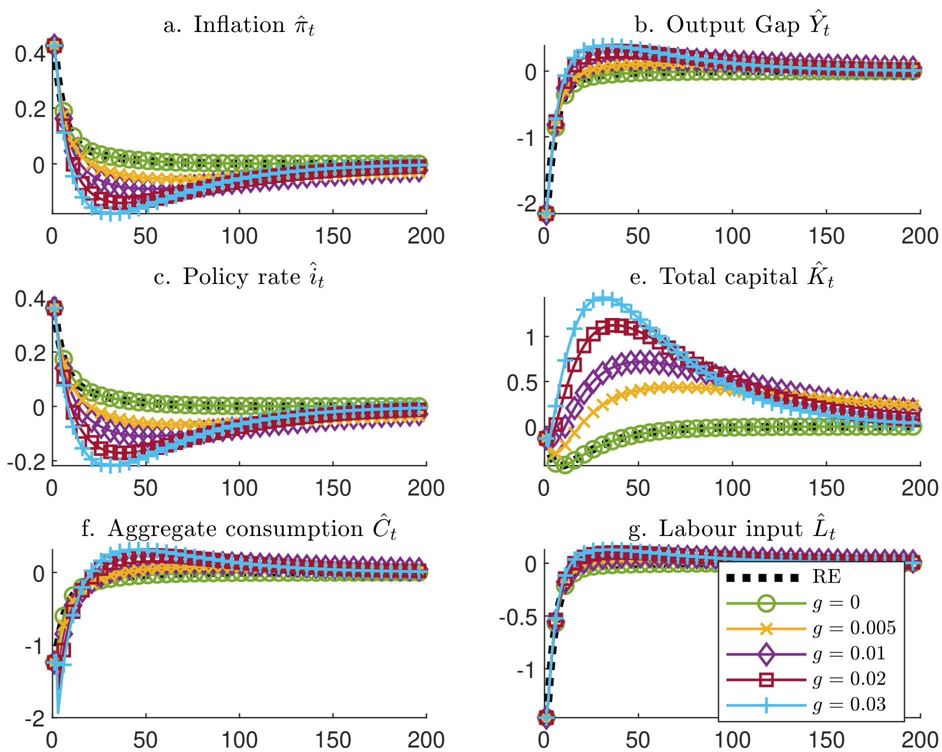
$$\mathbb{E}_t^* \frac{i_t}{\pi_{t+1}} = \mathbb{E}_t^* Z_{t+1} - \delta - 1, \tag{55}$$

$$i_t - \bar{i} = \phi^{\pi}(\pi_t - \overline{\pi}) + \phi^{y}(\frac{Y_t - \overline{Y}}{\overline{Y}}) + \varepsilon_t^r, \tag{56}$$

$$\varepsilon_t^r = \rho^r \varepsilon_{t-1}^r + \vartheta_t^r, \tag{57}$$

$$\varepsilon_t^p = \rho^p \varepsilon_{t-1}^p + \vartheta_t^p. \tag{58}$$

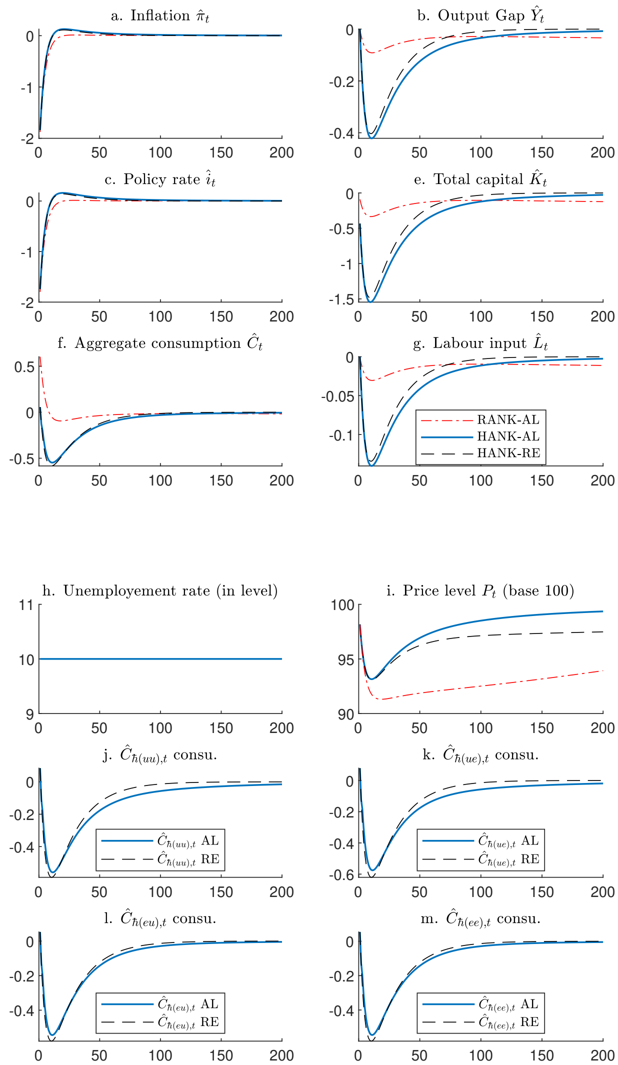
# B Sensitivity to the gain parameter




Notes: Results are in percentage point of the log deviation from the steady states. Price level and unemployment rate are in level. No projection facility was detected in any simulations. Ridge regressions were triggered 5 times at $g = 0.02$, and 11 times at $g = 0.03$ over the 5 PLMs.

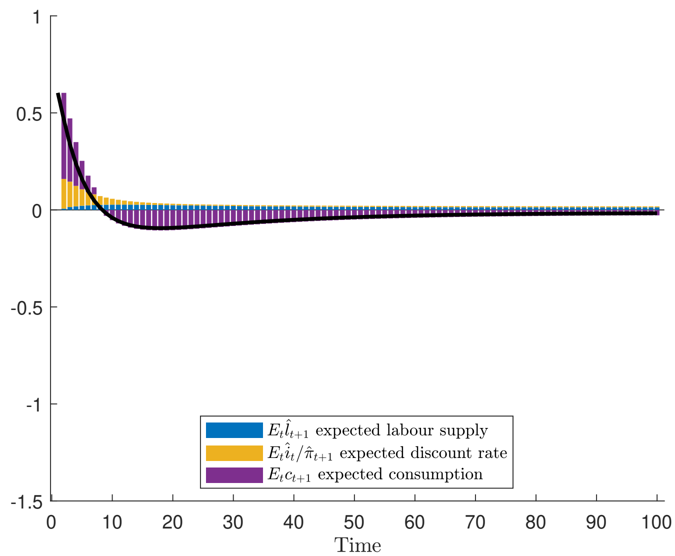
Figure 9: Sensitivity of the model to gain parameter after a $-1\%$ productivity shock

# C Nominal rate shock IRFs

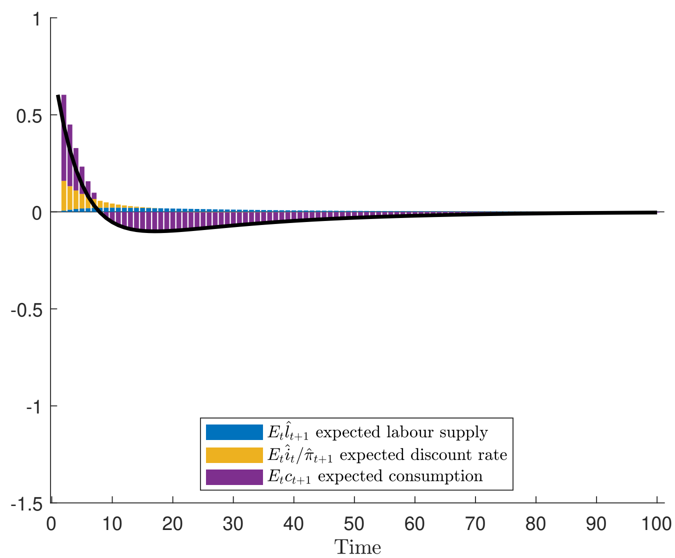



Notes: Results are in percentage point of the log deviation from the steady states. Price level and unemployment rate are in level.

Figure 10: Aggregate response to a +1% nominal rate shock



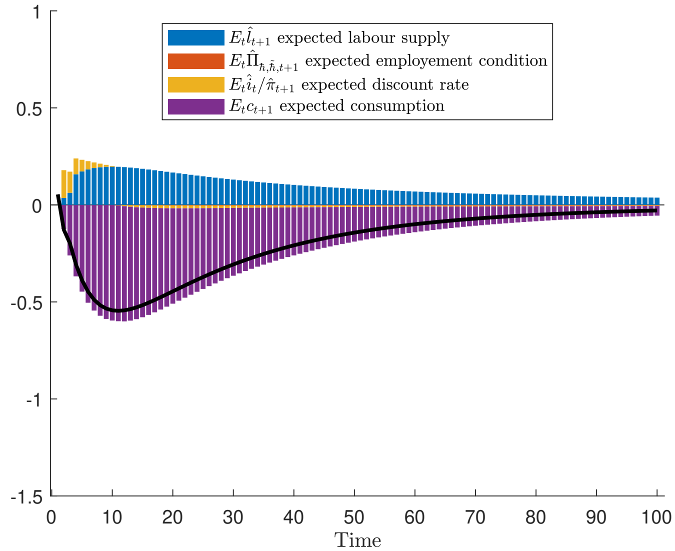

(a) Expectations contribution to $\hat{C}_t$ under AL in the RANK



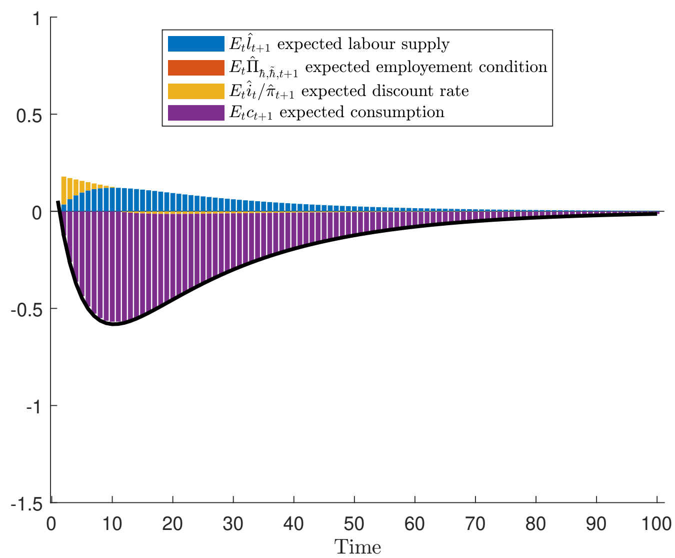

(b) Expectations contribution to $\hat{C}_t$ under RE in the

Notes: In order to build this figure, I take advantage of the linearization of Euler Equation. Thanks to that, it is possible to decompose the aggregate output equation by computing the movement generated by each term displayed in the legend. Notice that for the sake of clarity, the contribution of present labour supply is not displayed.

Figure 11: Expectation contribution of the representative agent' consumption after a +1% nominal rate shock




(a) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under AL in the HANK

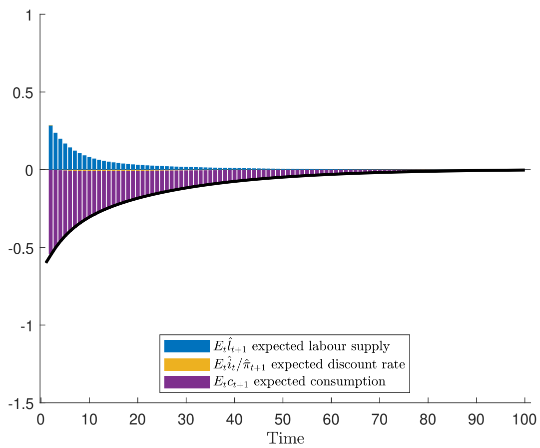



(b) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under RE in the HANK shock

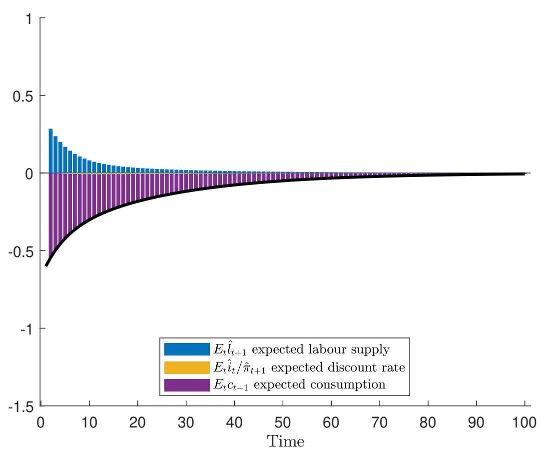
Notes: $\hat{C}_{\hbar(e,e),t}$ is the consumption of $\hbar(e,e)$, the family with agents employed in the last two periods. At the steady-states 82% of the agents are in this family. In order to build this figure, I take advantage of the linearization of Euler Equation 17 of $\hbar(e,e)$. Thanks to that, it is possible to decompose the aggregate output equation by computing the movement generated by each term displayed in the legend. Notice that for the sake of clarity, the contribution of present labour supply is not displayed.

Figure 12: Expectation contribution of the employed agents' consumption after a +1% nominal rate shock

# D Expectations and consumption in the RANK




(a) Expectations contribution to $\hat{C}_t$ under AL in the RANK

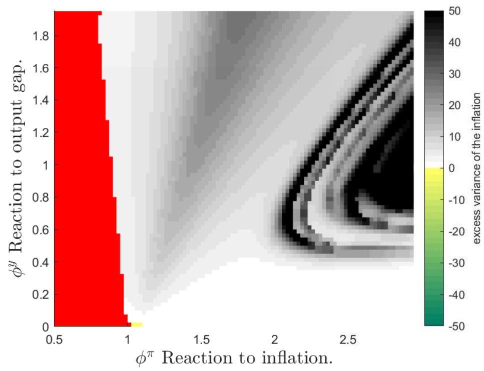



(b) Expectations contribution to $\hat{C}_t$ under RE in the RANK

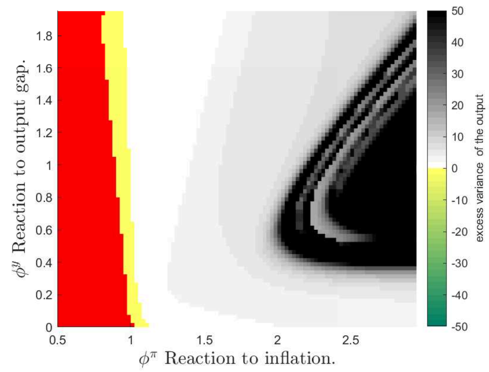
Notes: In order to build this figure, I take advantage of the linearization of Euler Equation. Thanks to that, it is possible to decompose the aggregate output equation by computing the movement generated by each term displayed in the legend. Notice that for the sake of clarity, the contribution of present labour supply is not displayed.

Figure 13: Expectations contribution of the representative agent' consumption

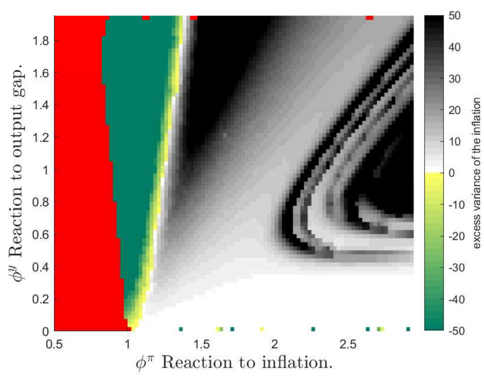
# E Excess variances of the HANK AL



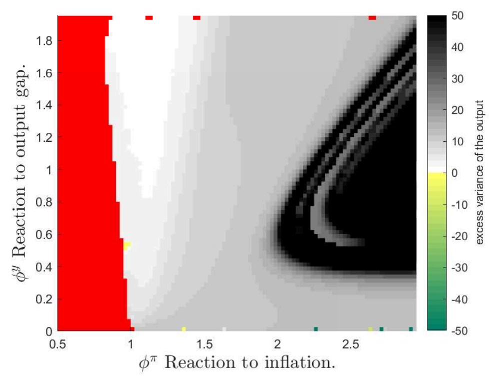

(a) Excess variance in inflation w.r.t HANK RE




(b) Excess variance in output w.r.t HANK RE



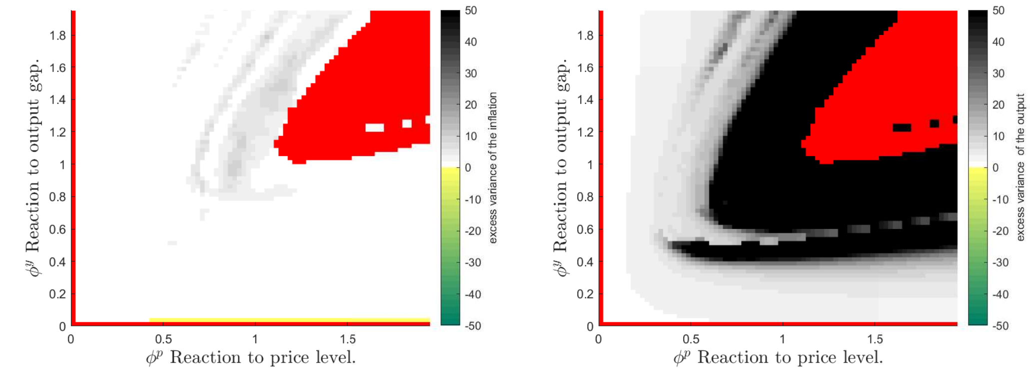

(c) Excess variance in inflation w.r.t RANK AL




(d) Excess variance in output w.r.t RANK AL

Notes: The data is the absolute difference between the models and the HANK adaptive learning model. See Figure 5 for more detail.

Figure 14: Excess output and inflation variances of the HANK-AL model w.r.t the other models under IT



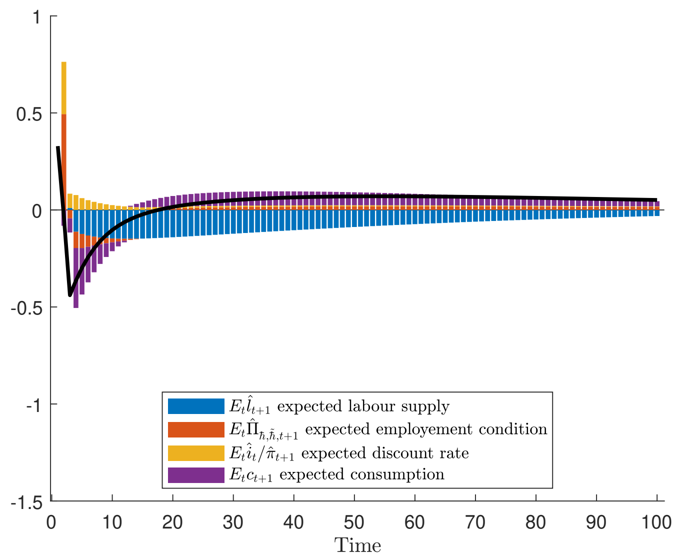

(a) Excess variance in inflation w.r.t HANK RE

(b) Excess variance in output w.r.t HANK RE

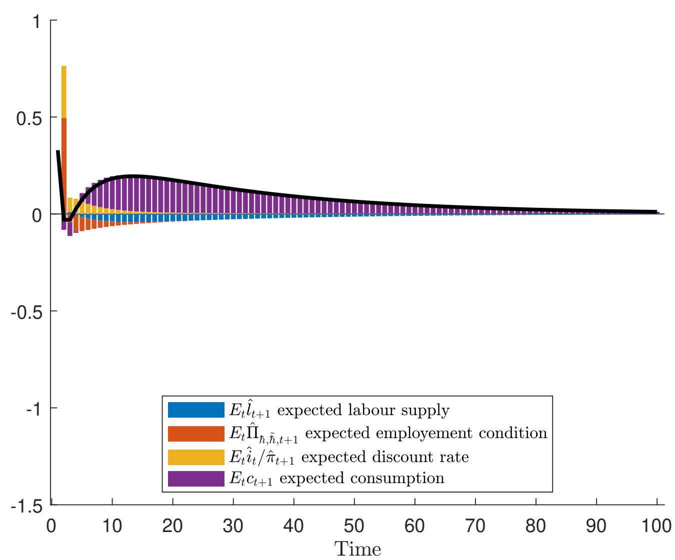
Notes: The data is the absolute difference between the models and the HANK adaptive learning model. See Figure 5 for more detail.

Figure 15: Excess output and inflation variances of the HANK-AL model w.r.t the HANK RE under PLT

# F Employed agents' expectations and consumption under PLT




(a) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under AL in the HANK




(b) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under RE in the HANK

Notes: $\hat{C}_{\hbar(e,e),t}$ is the consumption of $\hbar(e,e)$, the family with agents employed in the last two periods. At the steady states 82% of the agents are in this family. In order to build this figure, I take advantage of the linearization of Euler Equation 17 of $\hbar(e,e)$. Thanks to that, it is possible to decompose the aggregate output equation by computing the movement generated by each term displayed in the legend. Notice that for the sake of clarity, the contribution of present labour supply is not displayed.

Figure 16: Expectations contribution of the employed agents' consumption after a $-1\%$ productivity shock under PLT