# MPRA <br> Munich Personal RePEc Archive 

# Golden ratio of invisible hand: does the gravitation matter? 

Malakhov, Sergey
UPMF

14 November 2021

Online at https://mpra.ub.uni-muenchen.de/110674/
MPRA Paper No. 110674, posted 19 Nov 2021 06:34 UTC

Sergey Malakhov
Ph.D., Applied Economics
Pierre Mendes France University (Grenoble-Alpes)

## Golden Ratio of Invisible Hand: does the gravitation matter?


#### Abstract

The recent paper on the Invisible hand proves the fact that its equilibrium is mathematically perfect. If the producer allocates his time between production and delivery to the 'the door' of the buyer with zero search costs and unintentionally maximizes customer's consumption-leisure utility, both the marginal rate of transformation of production into delivery and the marginal rate of substitution of leisure for consumption become equal to the golden ratio conjugate whereas the sales-costs of production ratio becomes equal to the golden ratio itself, called once by Luca Pacioli, the founder of the modern accounting, as the divine proportion. Previous papers on Invisible hand formulated the hypothesis of the gravitation between sellers and buyers on commodity markets and between men and women in marriage markets. The golden ratio proves this hypothesis. At the equilibrium, gravitational fields of both seller and buyer as well as of both husband and wife are equal to the golden ratio conjugate. It means that any monopoly and monopsony really disappear. At the Invisible hand equilibrium, the transaction takes place between mutually attractive individuals. The equilibrium price stays competitive, but its economic nature is supported by the mutual respect of both parts in transaction.


[^0]
## JEL Classification: D11, D63, D83

Key words: golden ratio, invisible hand, gravitation, general competitive equilibrium

## Introduction

The general idea of this paper is to reproduce the reality of time when Adam Smith was working on the 'Wealth of Nations'. This reproduction can discover the grounds that resulted in his best guess on inner market mechanism, attributed to the Invisible hand. So, all the concept used on this paper - exchange, local equilibria, opportunity costs, general competitive equilibrium - represents the notions not in the sense of modern economics, but in the sense how they could be at times of the 'Wealth of Nations'.

Part I presents the mathematical proof of golden ratio proportions of the general competitive equilibrium. The proof uses only the datum of self-interested producer who allocates his time between production and delivery. The consumer enters this coordinate system with his pre-determined marginal rate of substitution of leisure for consumption. And both parts in transaction unintentionally optimize their allocation of time with respect to golden ratio proportions.

Part II dives much deeper in the epoque of the 'Wealth of Nations' and describes the work of Invisible hand by the tools of that time, i.e., by mathematics of the labor theory of value. The automatic successful meeting of uninformed buyers and sellers can be explained without help of the utility theory. That proof of Invisible hand could exist when Adam Smith formulated a hypothesis that selfinterested producers were led to the place where they could promote the public interest.

Part III comes back to the utility theory and shows at the general competitive equilibrium the intensity of consumption is also perfectly equilibrated with respect to golden ratio proportions.

Part IV revises Adam Smith's gravitation metaphor. He used that idea to explain the process of market prices' adjustment towards the natural price. This paper reconsiders the gravitation as the mutual interest of both parts in transaction - either on commodity market or in marriage market. We will see that the gravitation also respects the golden ratio.

## Part I. Production and delivery under wage dispersion

We take again the farmer who allocates his time between production on the farm and delivery to some point between the farm and the downtown. While his working time is constant, his total costs $T C$ are also constant. But he is indifferent where to sell because he gets advantage from the wage dispersion and his sales are always equal to his total costs:
(1) $T C=P Q=$ const

Choosing the place for sales, he determines the time for delivery $T_{d}$. It gives him as the residual the time for production $T_{f}$ and the quantity $Q$ to be produced and delivered under constant average $A C_{f}$ and marginal costs $M C_{f}$ on farming. Keeping in mind the constant $P Q=T C$ value, he gets the price $P$ for the quantity $Q$. This price gives him the total costs on delivery $T C_{d}$ with respect to the total costs on farming $T C_{f}$ :
(2) $T C_{d}=T C-T C_{f}$

And the price becomes equal to the total of both average and marginal costs on production and delivery:
(3) $P=\frac{T C}{Q}=\frac{T C_{f}}{Q}+\frac{T C_{d}}{Q}=A C_{f}+A C_{d}=M C_{f}+M C_{d}$

Really, there is a constant return on scale for any place of sales, i.e., for any quantity to be sold.

For any point of sale, the total costs on delivery are equal to the total costs given up on production. And we can get the following total differential for farming and delivery:
(4) $d T C_{F D}=d Q_{f} M C_{f}+d Q_{d} M C_{d}=0$
where the value $d Q_{f}$ is equal to the cut in production and the $d Q_{d}$ value means the final supply $Q$.

From here we get the transformation rate of farming for delivery:
(5) $R P T_{F D}=-\frac{d Q_{f}}{d Q_{d}}=\frac{M C_{d}}{M C_{f}}=\frac{A C_{d}}{A C_{f}}=\frac{T C_{d}}{T C_{f}}=\frac{T_{d}}{T_{f}}$

Equation 5 rearranges Equation 3 into the following form:

$$
\begin{equation*}
P=M C_{f}\left(1+\frac{M C_{d}}{M C_{f}}\right)=M C_{f}\left(1+\frac{T_{d}}{T_{f}}\right) \tag{6}
\end{equation*}
$$

The producer knows nothing about the consumer; neither his willingness to pay nor his allocation of time between labor $L$, search $S$, and leisure $H$. The producer wants only one thing - to sell the $P Q$ value. Both $P$ and $Q$ values are given by his $T_{d} / T_{f}$ allocation of time. In addition, his offer should be competitive. Here the value $M C_{f}$ becomes crucial. If we come back to Chapter VIII of the 'Wealth of Nations', we can follow Adam Smith's reasoning on the wage of independent producer who is both the master and the workman and who gets two distinct revenues - the profits of stocks and the wages of labor (Smith 2000, p.75). Here we get almost the same case. The profit is equal to the total costs on delivery, which rewards farmer's commercial skills, and the wages are equal to total costs on farming. But to be competitive, the marginal costs on farming $M C_{f}$ should be equal to wages of independent workmen, who are employed in neighbouring village. But these independent workmen should keep at the competitive market an option - either to work on the farm or to work on the factory. It means that at
the general competitive equilibrium farmer's marginal costs on production $M C_{f}$ are equal to the equilibrium wage rate $w_{e}$, or $M C_{f}=w_{e}{ }^{1}$

However, the competitive value of the marginal costs on farming $M C_{f}$ don't impede to sell fruits and vegetables to low-wage rate customers. For them, the farmer chooses the point of sale not far from the farm, and they spend some time $S$ on the search for the low price. As a result, the local equilibrium appears.

This equilibrium exists only in farmer's coordinate system. There, the values of the time of farming $T_{f}$, the quantity $Q$, and the price $P$ become dependent on the point of sales, i.e., on the time for delivery $T_{d}$. The consumer literally enters into this space with his pre-determined marginal rate of substitution of leisure for consumption MRS (H for Q).

It has been demonstrated with the help of l'Hopital rule that the value $M R S$ (H for $Q$ ) appears not at the moment of purchase but at the moment of intention to buy, when $Q, L, S \rightarrow 0$ (Malakhov 2020; 2021a). The application of l'Hopital rule for the moment of intention to buy result in the unit elasticity of consumption with respect to the total costs on purchase, or $e_{Q /(L+S)}=1$. This conclusion confirms consumer's stable preferences, which are exhibited here by the marginal rate of substitution of leisure for consumption in natural terms:
(7) $M R S(H$ for $Q)=\frac{Q}{L+S}$

To present it in monetary terms, we should come back to the inner mechanism of satisficing purchase (Malakhov 2021a).

Under the traditional problem of search for the fixed quantity demanded $Q$ (Stigler 1961), we get the intersection of $Q P(S)$ curve and labour income $w L(S)$ curve with regard to the time of search $S$ when the consumer chooses the first offer $Q P_{P}$ below his willingness to pay $W T P=w L_{0}$ :

[^1]

Fig.1. Behavioral satisficing choice
where $S$ - the search; $L$ - labor; $H$ - leisure; $T$ - time horizon until next purchase; $Q$ - quantity demanded; $w$ - wage rate; $w L_{0}$ - willingness to pay; $P_{P}$ - purchase price.

From here starts the process that goes beyond the consumer's mind. He makes the satisficing purchase $Q P=w L \leq w L_{o}$ and quits the market. He doesn't bother about recalculation of his usual working time into some piece rate because he knows nothing about producer's allocation of time. Moreover, he is not interested in it as well he is not interested whether his purchase is optimal from the point of view of economic theory. But his satisficing purchase implicitly launches the optimality process that we're going to follow.

The straight line with the slope $w$, passing the intersection point, i.e., the purchase, gives us the $Q P_{0}$ value on the $0 Y$ axis and $(S+L)$ value on the $0 X$ axis. The straight dotted line from the point $Q P_{0}$ with the slope $(-Q \partial P / \partial S)$, i.e., the tangent to the moment of purchase, gives us the value of the time horizon $T$ on the $0 X$ axis.

These considerations result in the following equation:
(8) $w(L+S)=-Q \frac{\partial P}{\partial S} T=Q P_{0}$

It is evident that Eq. 8 demonstrates consumer's willingness to accept $W T A$. But if he decides to re-sell the item, who can buy it? The answer also is evident. The consumer with positive search costs can re-sell the item to the consumer with zero search costs. Although the wage dispersion exists at the zero search level, there is no price dispersion. The $P_{0}$ value is the minimal price at the zero search level or the equilibrium price or $P_{0}=P_{e}$.

And we can take Eq. 8 as the budget constraint to some consumptionleisure utility function $U(Q, H)$, keeping in mind that for the given time horizon $T=L+S+H$ the value $\partial L / \partial H+\partial S / \partial H=-1$ :
(9.1) $\mathcal{L}=U(Q, H)+\lambda\left(w(L+S)-Q P_{e}\right)$
(9.2) $\frac{\partial \mathcal{L}}{\partial Q}=\frac{\partial U}{\partial Q}-\lambda P_{e}=0$
(9.3) $\frac{\partial \mathcal{L}}{\partial H}=\frac{\partial U}{\partial H}+\lambda w\left(\frac{\partial L}{\partial H}+\frac{\partial S}{\partial H}\right)=\frac{\partial U}{\partial H}-\lambda w=0$
(9.4) $\frac{\partial U / \partial H}{\partial U / \partial Q}=\operatorname{MRS}(H$ for $Q)=\frac{w}{P_{e}}$

It means that a satisficing choice with respect to the purchase price $P$ has its implicit optimal replication with respect to the equilibrium price $P_{e}$.

So, the consumer enters into farmer's space with the following preferences:
$M R S(H$ for $Q)=\frac{Q}{L+S}=\frac{w}{P_{e}}$
However, if he pays the $P Q$ value with satisfaction, we should confirm that his purchase is also optimal. But our means to prove this fact are very limited. The only thing we know that this $P Q$ value represents a point in farmer's coordinate system, where it is described by $\left(T_{d} ; T_{f}\right)$ allocation of his time.

The only mean, which can prove that the purchase is optimal, is the geometric normal to this point. But this normal tells us that the marginal rate of substitution of leisure for consumption is equal to the marginal rate of transformation of farming into delivery:


Fig.2.The optimal consumption-leisure choice for the given allocation of producer's time
(10) $M R S(H$ for $Q)=\frac{Q}{L+S}=\frac{w}{P_{e}}=\frac{T_{d}}{T_{f}}=\frac{M C_{d}}{M C_{f}}=R P T_{F D}$

By this way we can construct the set of multiple equilibria under wage and price dispersion along the way from the farm to the downtown.

Now we can rewrite Eq. 6 in the following form:

$$
\begin{equation*}
P=M C_{f}\left(1+\frac{T_{d}}{T_{f}}\right)=M C_{f}\left(1+\frac{Q}{L+S}\right)=M C_{f}\left(1+\frac{w}{P_{e}}\right) \tag{11}
\end{equation*}
$$

But the general competitive equilibrium also represents the local equilibrium, this time 'at the doors' of the consumer with zero search costs and with the equilibrium wage rate $w_{e}$ who pays the equilibrium price $P_{e}$ :

This consideration transforms Eq. 11:
(12) $P_{e}=M C_{f}\left(1+\frac{T_{d}}{T_{f}}\right)=M C_{f}\left(1+\frac{w_{e}}{P_{e}}\right)$

This equation also can be transformed, now with the help of the assumption that the competitive marginal costs of farming are equal to the equilibrium wage rate or $M C_{f}=w_{e}$ :

$$
\begin{equation*}
P_{e}=M C_{f}\left(1+\frac{w_{e}}{P_{e}}\right)=w_{e}\left(1+\frac{w_{e}}{P_{e}}\right) \tag{13}
\end{equation*}
$$

We're making two more steps and get the following result:

$$
\begin{equation*}
\frac{P_{e}}{1+\frac{w_{e}}{P_{e}}}=w_{e} ; \frac{P_{e}}{P_{e}+w_{e}}=\frac{w_{e}}{P_{e}} \tag{14.1}
\end{equation*}
$$

$$
\begin{equation*}
\frac{P_{e}}{P_{e}+w_{e}}=\frac{w_{e}}{P_{e}}=\frac{a}{a+b}=\frac{b}{a}=0,61803398 \ldots=\frac{1}{\varphi}=\Phi \tag{14.2}
\end{equation*}
$$

It means that at the general competitive equilibrium both the marginal rate of transformation of production into services and the marginal rate of substitution of leisure for consumption are equal to the golden ratio conjugate $\boldsymbol{\Phi}$ :

$$
\begin{equation*}
\frac{w_{e}}{P_{e}}=\frac{a}{a+b}=\frac{b}{a}=\frac{1}{\varphi}=\Phi=M R S_{e}(H \text { for } Q)=R P T_{e F D} \tag{14.3}
\end{equation*}
$$

We see that at the general competitive equilibrium the producer not only optimizes customer's consumption and leisure but also harmonizes them. But he doesn't stay aside from this process of harmonization. It is easy to show that his sales mark-up $m$ is equal to consumer's marginal rate of substitution of leisure for consumption. It means that sales-costs of production ratio is equal to the golden ratio itself:
(15.1) $m=\frac{P-A C_{f}}{A C_{f}}=\frac{P-M C_{f}}{M C_{f}}$

$$
\begin{align*}
& P=M C_{f}(1+m)=M C_{f}\left(1+\frac{w}{P_{e}}\right)  \tag{15.2}\\
& P_{e}=M C_{f}\left(1+\frac{w_{e}}{P_{e}}\right)=M C_{f}\left(1+m_{e}\right)=1,61803398 \ldots M C_{f}=\varphi M C_{f}
\end{align*}
$$

## Part II. The purchase of consumption and leisure

Making delivery, the farmer offers not only goods but also some leisure time. However, he doesn't produce leisure. It appears as a result of delivery. So, we cannot reproduce his goods-leisure production possibility frontier. But we can substitute it by some technical frontier, created by opportunity costs of leisure. While the concept of opportunity costs appeared much later, Adam Smith used it. We can find the illustration of opportunity costs approach just at the beginning of
the 'Wealth of Nations' in his considerations on the beaver-deer trade-off (Smith 2000, p.53).

The farmer knows nothing about customer's leisure time. Moreover, he is not interested in it. But when he raises the price with respect to the price 'on the farm', he is selling not only goods but also some leisure time. And we can evaluate the sale of leisure with the help of hic costs on delivery and production.

We can take the costs of leisure "production" to be equal to the costs on delivery:
(16) $T C_{d}=Q_{d} M C_{d}=H M C_{H}$

Eq. gives us the following value of marginal costs on delivery $M C_{d}$ :

$$
\begin{equation*}
M C_{d}=M C_{H} \frac{H}{Q_{d}} \tag{17}
\end{equation*}
$$

Keeping in mind that the quantity delivered is equal to the offer itself, or $\mathrm{Q}_{\mathrm{d}}=\mathrm{Q}$, we can rewrite Eq. in the following form:

$$
\begin{equation*}
\frac{T_{d}}{T_{f}}=\frac{M C_{d}}{M C_{f}}=\frac{M C_{H}}{M C_{f}} \frac{H}{Q} \tag{18}
\end{equation*}
$$

Here both $M C_{H}$ and $H$ values are unknown, but we can simulate them with the help of the opportunity costs on production lost with the delivery. This consideration results in the following equations: $M C_{H}=M C_{f} ; H=-d Q_{f}$. Of course, we can use and additional variable of productivity, but it can make the analysis more complicated. Here, we simply recalculate all values of time. If, for example, marginal costs on farming are equal to a half an hour, or $M C_{f}=0.5$, we divide wage rate $w$ and price reduction $\partial P / \partial S$ by two, whereas all absolute time values $-L, S$, and $H$ itself - are doubled. As a result, prices stay unchanged.

Here we get the shape of the technical production possibility frontier $P P F$. It is the straight line with the slope $(-1)$. But we will not search for the utility curve with the same slope because here the utility is unknown. The only thing we can cay that the meeting of farmer and his customer takes place at some $\left(T_{d} ; T_{f}\right)$
point on this technical frontier. Moreover, now this point can be presented as the total offer of both consumption and leisure:

$$
\begin{equation*}
\frac{T_{d}}{T_{f}}=\frac{M C_{d}}{M C_{f}}=\frac{M C_{H}}{M C_{f}} \frac{H}{Q}=\frac{H}{Q} \tag{19}
\end{equation*}
$$

This consideration results in the general meeting rule:
(20.1) $P_{\text {seller }}=M C_{f}\left(1+\frac{M C_{d}}{M C_{f}}\right)=M C_{f}\left(1+\frac{T_{d}}{T_{f}}\right)=M C_{f}\left(1+\frac{H}{Q}\right)=P_{\text {buyer }}$

Just here we discover the inner workings of Invisible hand. The producer offers both consumption $Q$ and leisure $H$. But paying high price with respect to the price 'on the farm', here we should not forget that the value $M C_{f}$ is competitive and it cannot affect the consumer's choice, the buyer has his own understanding how much leisure $H_{b}$ he can get with the given quantity $Q$ :
(20.2) $P_{\text {buyer }}<P_{\text {seller }} \rightarrow H_{b}<H$
(20.3) $P_{\text {buyer }}>P_{\text {seller }} \rightarrow H_{b}>H$

If the consumer evaluates the offer $H$ as an excessive, he doesn't accept the price and continues to search in order to diminish both price and leisure.

The consumer who needs more leisure, also can pay more. But he even doesn't come to the meeting point because it stays too far for his willingness to pay.

The consumer who evaluates $H$ as the 'just offer', also accepts the 'just price'. It means, that making the $P Q$ offer in some $T_{d}$ meeting point, the farmer unintentionally meets there the consumer who has just stopped the search for a low price in this meeting point. And the buyer takes the price for the given quantity as the 'just price'. Really, either something or someone has led the producer "to promote an end which was no part of his intentions" (Smith 2000, p.485).

Part III. The harmony of consumption-leisure choice

When the shopper, i.e., the consumer with zero search costs (Stahl 1989), pays the high price at his door, he intentionally buys some leisure time. His utility curve intersects the producer's technical frontier with the slope (-1). It happens because in reality the seller doesn't produce leisure. If he does it, his production possibility frontier could be closer to some usual Cobb-Douglas function, which could become tangent to the consumer's utility curve. But this curve is really imaginary. We can say nothing about it and should accept the technical frontier, derived from the opportunity costs on production itself. There the marginal costs of leisure are equal to the marginal costs of production and the quantity of leisure offered is equal to the quantity of goods lost for production.

When the consumer enters into the producer's coordinate system with the pre-determined marginal rate of substitution of leisure for consumption and accepts the $(H ; Q)$ offer, he increases his utility $U(Q ; H)$ but he cannot change his preferences. The new utility curve also will have the same optimal MRS (H for $Q)$. It means that the incremental value of the purchase also is optimal. And we derive from Eq. 19 the following consideration:

$$
\begin{equation*}
\frac{T_{d}}{T_{f}}=\frac{H}{Q}=\frac{Q}{L+S} \tag{21}
\end{equation*}
$$

But Eq. 21 tells us that the normal taken for the moment of sale $\left(T_{d} ; T_{f}\right)$ represents also the normal for the moment of purchase ( $H ; Q$ ) (Fig.3):


Fig.3.The optimal intensity of consumption
Here we understand that the given consumption-leisure utility function $U(Q ; H)$ doesn't represent the general utility itself. This is only the utility of the purchase. The general consumption-leisure utility function exists somewhere else, and here we get only its 'daughter' with the same stable preferences.

This consideration results in the following conclusion:
At the general competitive equilibrium, the intensity of consumption $Q / H$ is equal to the golden ratio $\varphi$.

Really, if the consumer enters into producer's space with the 'daughter' of his general utility function, he buys with respect to proportions provided by the general utility function. It means that the general utility function has been optimized by the golden ratio conjugate $\Phi$ before the purchase, and the consumption proportions have been equal to the golden ration $\varphi$ itself.

## Part IV. Gravitation

It is very difficult to rationally explain the ideal proportions of the perfect competitive equilibrium. However, the analysis of the Invisible hand provides an efficient tool for that.

Here we can use another Adam Smith's idea - to use the Newtonian concept of gravitation for the understanding of exchange. Smith applied that concept to the process of price adjustment:
"The natural price, therefore, is, as it were, the central price, to which the prices of all commodities are continually gravitating." (Smith 2000, p.53).

However, some authors pay attention to the fact that Smith's gravitation is not Newtonian because it says nothing about an opposite force. The Newtonian logic needs the gravitation of natural price toward market price (Cohen 1994).

If we take the simple idea that both parts in transaction are led to meet each other by their mutual interest, we can get more reliable idea of the gravitation in social processes. The only thing we need here is to justify the notion of gravitation
in the sense of mutual interest. And the model presented here gives us this justification. We can see that the mutual interest can be described by ratios of quantity and time, i.e., by real natural values, which looks like a reliable specific form of Newton's law of universal gravitation.

The model follows the assumption that leisure time $H$, 'sold' by the farmer is equal to his time on delivery $T_{d}$. But the time on production $T_{f}$ also plays some role in the sale of leisure. So, we can get a transformation rate of total farmer's time into consumer's leisure:

$$
\begin{equation*}
H=\delta\left(T_{f}+T_{d}\right) \tag{22}
\end{equation*}
$$

Eq. 21 and 22 give us the value of the transformation rate $\delta$ :

$$
\begin{align*}
& \delta\left(T_{f}+T_{d}\right)=\frac{Q^{2}}{L+S}  \tag{23.1}\\
& \delta=\frac{Q}{T_{f}+T_{d}} \frac{Q}{L+S}=\frac{Q}{T_{f}+T_{d}} \frac{q_{i}}{L_{i}+S_{i}}
\end{align*}
$$

where Q - total supply; $q_{i}$ - individual demand; $L_{i}$ - individual labor supply; $S_{i}-$ individual search.

Eq. 23.2 looks like a specific form of the equation on the gravitational force. Really, here we have 'masses' $Q$ and $q_{i}$ of both seller and buyer. We don't know the distance between them, but we know the time they have spent to meet each other. And we can formulate the hypothesis that there is the gravitation between buyer and seller, and its force is equal to the transformation rate of producer's time into consumer's leisure $\delta$ (Malakhov 2021a).

This hypothesis gives us the values of both gravitational fields of producer and consumer:
(24.1) $\quad \delta_{p}=\frac{Q}{T_{f}+T_{d}}$
(24.2) $\quad \delta_{c}=\frac{q_{i}}{L_{i}+S_{i}}=\frac{w_{i}}{P_{e}}$
where $w_{i}$ - individual wage rate.

From the economic point the gravitation force represents the mathematical product of sellers's productivity and buyer's purchasing power, or the product of their economic attractiveness. The productive seller is interesting for consumers like the wealthy consumer is interesting for producers.

Gravitational fields are not constant. When the producer spends almost all his time on production, his gravitational field is strong. But there the consumer spends much time on search that diminishes his gravitational field. While the local equilibrium takes place, the real equality doesn't. The producer's interest in a particular consumer is low, while the consumer's interest in the producer is so high that he is ready to cut his leisure time in favor of search. This situation exhibits an implicit monopoly power. In some sense, the consumer becomes dependent on the producer. Using the boxing analogy, we can say that the producer's 'weight' in the transaction if higher than the consumer's 'weight'.

The opposite local equilibrium changes these 'weights'. There, the gravitational field of consumer is much stronger. Now the producer becomes dependent on the high customer's purchasing power, and an implicit monopsony appears, where a customer gets much leisure time the enjoy the consumption.

Now we can turn to the general competitive equilibrium and to analyze the gravitational fields there.

Eq.24.2 reproduces Eq. 14.3 that gives us the value of consumer's gravitational field at the general competitive equilibrium. It is equal to the golden ratio conjugate $\Phi$. So, the equilibrium gravitational force $\delta_{e}$ is equal to

$$
\begin{equation*}
\delta_{e}=\frac{Q}{T_{f}+T_{d}} \frac{w_{e}}{P_{e}}=\Phi \frac{Q}{T_{f}+T_{d}} \tag{25}
\end{equation*}
$$

But we can also get the value of producer's gravitational field:

$$
\begin{equation*}
\frac{Q}{T_{f}+T_{d}}=\frac{Q}{T_{f}\left(1+\frac{T_{d}}{T_{f}}\right)}=\frac{M C_{f}}{M C_{f}} \frac{Q}{T_{f}\left(1+\frac{T_{d}}{T_{f}}\right)}=\frac{T C_{f}}{T_{f} P}=\frac{v}{P} \tag{26}
\end{equation*}
$$

where $v$ - is the cost of unit of time on production.

Here we understand the idea to recalculate piece rate into time rate that we used before. Here we really get the cost of farmer's time on production $v=T C_{f} / T_{f}$. And according to the recalculation rule we get $v=M C_{f}$, or the value of the unit of time is equal to the marginal costs on production. If one unit needs half an hour to be produced, the recalculation doubles the $T_{f}$ value, but the $v$ value becomes equal to a half of one-hour costs of production.

As a result, the $v$ value becomes equal to the unit of Ricardian 'corn wage', or $v=M C_{f}=w_{e}$. This conclusion gives as the value of producer's gravitational field:

$$
\begin{equation*}
v=M C_{f}=w_{e} ; P=P_{e} \rightarrow \frac{Q}{T_{f}+T_{d}}=\frac{w_{e}}{P_{e}}=\Phi \tag{27}
\end{equation*}
$$

It means that at equilibrium gravitational force is equal to the square of golden ratio conjugate:
(28) $\delta_{e}=\Phi^{2}$

But this harmonic result has very important economic sense. At the equilibrium, the gravitational fields of both producer and consumer, or their economic attractiveness, are equal. Both implicit monopoly and monopsony powers disappear. 'Weights' are equal, and nobody can get an advantage.

The common view on the general competitive equilibrium represents the meeting of infinite number of wordless price-takers. Here we get completely different picture. The mutual interest of both parts in transaction is equal. We observe the meeting of good men, who are pursuing their own interests, but they respect interests of others. The general competitive equilibrium really becomes socially perfect.

The same phenomenon emerges in marriage markets. Divorces are produced by corner solutions. However, good marriages don't mean that every good family is created by mutually attractive individuals. Local equilibria results either in an implicit monopoly, or implicit monopsony power, displaying the superiority either of a husband, or a wife. There is unique harmonic solution when a 'hunter' cuts his time in the 'forest' and reduces the 'quarry' in favor of
homemaking that can be redistributed between men and women, and his wife is making at home only woman's work. This allocation of time leaves them great leisure time to be shared. Mutual interests in this family are equal, and it really becomes happy.

These results prove the idea that the gravitation takes place in social relationships based on exchange. As an emotion, the gravitation represents the interest, or the feeling that causes attention to focus on counterpart in transaction. Now we can see that when interests of both parts in transaction are equal, the imputed value of an interest of every counterpart is equal to the golden ratio conjugate.

## Conclusion

The perfect equilibration of the original market and its following disequilibrating towards consumers' interests, where expanding services meet high wages rates and leisure time, give rise to the following question - either the golden ratio accidently enters social processes, or it appears as an abstract theoretical value? Unfortunately, any answer to this question doesn't cast doubt on the modern understanding of the perfect competitive equilibrium.

## References

Cohen, I. B. (1994) 'Newton and the social sciences, with special reference to economics, or, the case of the missing paradigm,' in Philip Mirowski, Natural Images in Economic Thought: "Markets Read in Tooth and Claw". Cambridge University Press, 55-90.

Malakhov, S. (2020) 'Proof of the Invisible hand: why the satisficing purchase becomes optimal?', Terra Economicus, 18 (2),70-84.

Malakhov, S. (2021a) 'Force of Invisible Hand, Labor, and Just Price: basic principles of sorting and matching under wage and price dispersion.' Journal of Institutional Studies, 13 (1),37-59.

Malakhov, S (2021b) 'Equilibrium of Invisible Hand in the Family: the gravitation between men and women in marriage markets'. Russian Journal of Economic Theory, 18(3), 357-373.

Ricardo, D. (1817) 'On the Principles of Political Economy and Taxation', J.Murray ed., 589 p.

Smith, A. (2000) 'The Wealth of Nations.' Random House, 1200 p.
Stahl, D. O. (1989) 'Oligopolistic Pricing with Sequential Consumer Search.' American Economic Review, 79(4),700-712.

Stigler, G. (1961) 'The Economics of Information.' Journal of Political Economy, 69(3), 213-225.


[^0]:    This paper represents the abridged version of the article "Divine Proportion of Invisible Hand: a new look at Adam Smith's natural theology", written in Russian for the Journal of Institutional Studies (forthcoming in JIS (2022) 14(1)).

[^1]:    ${ }^{1}$ The $M C_{f}=w_{e}$ assumption need the recalculation of the piece work into time work. David Ricardo used that recalculation for his 'corn wages' measured in quarters. It also takes place today in UK where piece workers must be paid at least the minimum wage for every hour worked. It means in our model that at the equilibrium piece costs of production are equal to some amount of time costs. If a piece needs half an hour of working time, we recalculate all time values ( $w ; \partial P / \partial S ; L ; S ; H ; T_{f} ; T d$ ) on the half-an-hour basis. So, the prices stay unchanged (S.M.).

