Semi-Endogenous or Fully Endogenous Growth? A Simple Unified Theory.

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Abstract

Is growth ultimately fully endogenous or semi-endogenous? A quarter-century theoretical and empirical growth economics has still kept both possibilities open. Consequently, I assume that R&D-driven growth is a general combination of both semi-endogenous and fully endogenous mechanisms.

I here prove that if the semi-endogenous growth component is essential to the actual growth mechanism, the long-run growth rate will follow the semi-endogenous growth predictions. On the other hand, if the semi-endogenous is not essential, the fully endogenous growth mechanism may dictate the long run if the world population does not grow too fast.

This result holds regardless of whether fully endogenous growth is essential.

I also prove that if no other (third) growth mechanism exists, it suffices to prove that less research always leads to fewer innovations to ascertain semi-endogenous growth essentiality.

JEL classification: O30, O40.
Keywords: Strong scale effect; Semi-endogenous growth; Fully endogenous growth.

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1 Introduction

Will growth be eventually endogenous or semi-endogenous? This question haunts the modern growth theory since Jones (1995b) path-breaking article. After a quarter-century of debate and one Nobel price, both growth approaches pervade macroeconomics.

Consequently, it is safe to say that the true aggregate growth process envisaged by theory so far is either semi-endogenous or fully endogenous, or a combination of the two. No other growth mechanism exist, according to R&D-driven growth theory.

The semi-endogenous growth theory keeps the door open to the very non-rivalry of ideas in the innovation process: any individual researcher's stock of cumulated ideas can facilitate new ideas. Yet, in Romer (1990), as in Grossman and Helpman (1991) and Aghion and Howitt (1992) "creative destruction" version, this implied a strong scale effect[1] (Jones, 2005) that conflicted with data, as Jones (1995a and b) has highlighted. This finding motivated Jones (1995b), Kortum (1997), and Segerstrom (1998) semi-endogenous growth variant: the knowledge of the existing stock of existing ideas facilitates innovation, but with decreasing returns. An important implication of such a solution is that per-capita GDP could not grow without a growing population. Key to this solution is the idea that R&D's (TFP impact-adjusted) productivity declines the more knowledge accumulates, as recently empirically confirmed by Bloom, Jones, Van Reenen, and Webb (2020).[2]

A different group of scholars, such as Smulders and Van de Klundert (1995), Dinopoulous and Thompson (1998), Peretto (1998), Young (1998), and Howitt (1999), have proposed another class of endogenous growth models immune to the "strong scale effect", despite dynamic returns to ideas to be constant. In these models, it is not the single researcher that matters, but research as a fraction of the population (Jones, 1999 and 2005). Several microfoundations exist to this solution of the scale effect problem. Still, the basic structure is common to all: a higher population, by diluting the individual research contribution on a larger population reduces the effect of their research effort in proportion. This idea stresses the industrial dynamics and cross-sector dilution of innovation, with population partitioning and specializing innovative efforts.[3]

Both theories are well motivated, and both illuminate an essential aspect of the innovative process. The importance of cross-sectoral non-rivalry - which characterized semi-endogenous growth - is undeniable, as is the concept of an increase in specialization accompanying population growth - which marks the endogenous growth dilution approach. In this paper, I study the consequence of assuming that both aspects are essential to the innovation process: TFP growth rate would be zero if one of them were absent. I will show that endogenous growth essentiality plays no role in dictating the long-run growth endogeneity property. Instead, the semi-endogenous growth essentiality is the key to a long term prediction.

I will also propose a simple rule to discriminate which growth rate will eventually win in the steady-state. If a reduction in the number of researchers always reduces the growth rate, semi-endogenous growth has to be essential and dictate the long run.

[1] Which counterfactually predicted that the steady-state per-capita GDP growth rate would increase with the economy's population size.

[2] Also see Venturini (2012).

[3] For recent evidence favouring scale-free endogenous growth theory, see Minniti and Venturini (2017). Also, see Madsen (2008).

Both theories are highly credited in growth macroeconomics. However, I claim that the time has come to settle this divide and get used to the idea that both approaches coexist in a unique framework. Hence, I have run two parametric exercises in Cozzi (2017a and b) in which I assumed that the real-world technology growth rate aggregates the semi-endogenous and the fully endogenous solution linearly (in Cozzi, 2017a) and with a CES (in Cozzi, 2017b). Surprisingly, I obtain that the steady-state growth rate of these two growth mechanisms' aggregation is not the aggregate of the two growth steady-state growth rates of each of them. Instead, only one prevails.[4]

Given the importance of this issue, this paper claims that the long term predictions should be robust to much more general aggregations. I will here show that significant results do not rely on any functional form. Therefore, only natural aggregation properties will be assumed to get striking long-term results analytically. Section 2 sets up the general model. Section 3 will show that if the semi-endogenous engine of growth is essential, the steady-state will only follow the semi-endogenous growth part. This result holds regardless of whether the fully endogenous growth is essential or not. This case is helpful because, for example, no CES aggregator allows the essentiality of only one input. Section 3.2 nests this general steady-state result into a standard Romer (1990) model.

Section 4 proves that if semi-endogenous growth is not essential, growth will eventually be fully endogenous if and only if the population growth rate is low enough. Section 4.1 applies this new result to Romer's (1990) model we used in section 3.2. Section 5 concludes.

2 Growth Mechanics

Let us assume the following aggregate production function:

$$Y_t = A_t L_{Yt}, \tag{1}$$

where Y_t is output at time t, A_t is total factor productivity, and L_{Yt} is labor employed in manufacturing. By definition, L_{Yt} is a fraction $0 < s_{Yt} < 1$ of the total labour force L_t, which in turn grows at the - possibly negative - constant net rate n. The complementary fraction $s_{At} = 1 - s_{Yt}$ of the labour force denotes the R&D labour share. Total R&D employment is then $L_{At} = s_{At} L_t$.

I assume that the manufacturing total factor productivity, A_t, grows according to the following general function:

$$\frac{A_t - A_{t-1}}{A_{t-1}} \equiv g_{At} = F(g_{\text{sem}t}, g_{\text{end}t}), \tag{2}$$

where

$$g_{\text{sem}t} = (A_{t-1})^{\varphi - 1} (s_{At} L_{t-1})^{\lambda_1^A}$$

is the semi-endgenous growth rate (Jones, 1995b) and

$$g_{\text{end}t} = s_{At}^{\lambda_2^A}$$

[4] Chu and Wang (2020) analyse a similar example.

is the fully endogenous growth rate without scale effects (Smulders and Van de Klundert, 1995).

Following Jones (1995b), in the steady state:

$$\begin{aligned} g_{\text{sem}} &= \frac{\lambda_1^A n}{1-\varphi}, \text{ if } n \geq 0 \text{ and} \\ g_{\text{sem}} &= 0, \text{ if } n < 0 \ . \end{aligned}$$

3 Both Growth Engines Essential

In this section, I will show that if the semi-endogenous R&D-driven growth mechanism must be at work for the economy to grow, then the steady state growth rate will be semi-endogenous.

Let us make the following two assumptions:

Assumption 1. *Function F is non-negative, continuous and strictly increasing in both its arguments for all possible* $(g_{\text{sem}t}, g_{\text{endt}}) \in$ [5]

This assumption is very general and natural. It is positing that given the R&D fraction of the labour force, growth will be higher in a more populated economy. Assumption 1 captures well the focus on population size that semi-endogenous growth theory has inherited from Romer's fully endogenous growth framework. Humans produce ideas, and these are non-rival in the production of new ideas. Hence, the economy's scale matters, even though the "strong scale effect" (Jones, 2005) is absent.

At the same time, assumption 1 shows that given the scale of the economy, the higher the fraction of it engaged in R&D, the higher the resulting growth rate. This way, it captures the fully endogenous growth element, inherited from Romer (1990), but purged of the strong scale effect.

Assumption 2. *Function F is zero if one of* $g_{\text{sem}t}$ *is zero, that is:* $F(0, g_{\text{endt}}) = F(0,0) = 0$ for all $g_{\text{end}t} \in R_{+.}$

Notice that assumption two posits that the semi-endogenous growth mechanism is essential: no growth will occur if it is missing. The economy's scale cannot tend to zero while still creating a significant growth rate of ideas. It leaves the door open to the fully endogenous growth mechanism to be or not to be essential. That is, a large economy with a negligible fraction of people or GDP involved in R&D may or may not generate insignificant growth.

As Jones (1995b), I am not describing economy exchanging ideas with the rest of the world because the imported innovation will spur growth. I am describing the growth rate of the whole world. Viewed this way, a negligible number in the function's variable would reflect quite a complex world, undoubtedly unable to display any meaningful growth performance. Hence we can consider Assumption 2 as quite natural too.

The two general assumptions above are enough for us to prove our main result:

Proposition 1. *The steady-state growth rate, g_A, if it exists, is always semi-endogenous.*

[5] With R^2_{++}, I mean the set of ordered pairs of strictly positive real numbers. Instead, R^2_{+} means the set of ordered pairs of non-negative real numbers.

Proof. In a steady state, the growth rate and R&D fraction of the labour force is constant, that is: $s_{At} = s_A$ and $g_{At} = g_A$. Hence we can rewrite (2) as:

$$g_A = F\left[(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}, s_A^{\lambda_2^A}\right] \tag{3}$$

If population is non-increasing, that is $n \leq 0$, the semi-endogenous growth part will tend to zero. In fact, $(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}$ tends to zero as $L_{t-1} \to 0$ or if $A_{t-1} \to \infty$. If population is increasing, that is $n > 0$, the growth rate function $F\left[(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}, s_A^{\lambda_2^A}\right]$ is a positive constant if and only if both of its arguments are positive constants. Since $s_A^{\lambda_2^A}$ is by construction a positive, only $(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}$ has to be a positive constant. But this happens if and only if the growth rate of $(A_{t-1})^{\varphi-1}$ is equal to the growth rate of $(L_{t-1})^{\lambda_1^A}$, that is if and only if $g_{\text{sem}} = \frac{\lambda_1^A n}{1-\varphi}$. **QED**

Proposition 1 showed that if the fully endogenous growth mechanism and the semi-endogenous growth mechanism are essential to the growth process, the only steady-state possible is the semi-endogenous steady state.

Remark. No CES function F allows the essentiality of only one of its inputs. If the elasticity of substitutions is less than or equal to one, both inputs are essential, while if it is higher than one, no input is essential. Hence the results of Proposition 1 would be impossible to obtain in the special cases analyzed by Cozzi (2017b).

3.1 A Simple Microfoundation of Semi-endogenous Growth

This section loosens the previous section condition by requiring that only the semi-endogenous be essential. In particular, I will make the following general assumption:

Assumption 3. *For each positive level of the stock of ideas, A_t, fewer researchers always imply fewer new ideas.*

We can now prove the following:

Lemma 1. *Under Assumptions 1 and 3, semi-endogenous growth is essential.*

Proof.

Suppose that the semi-endogenous growth mechanism were not essential, and that growth were only driven by the fully endogenous growth model without scale effect, that is:

$$g_{\text{endt}} = F\left(0, s_A^{\lambda_2^A}\right) = F\left[0, \left(\frac{L_{At}}{L_t}\right)^{\lambda_2^A}\right]. \tag{4}$$

Let population drop in half. Now, the number of researchers, following population, will be halved:

$$L_{At'} = 0.5 L_{At},\ t > t\prime.$$

Since

$$L_{t'} = 0.5 L_t,\ t > t\prime,$$

eq. (4) implies that

$$
\begin{aligned}
A_{t'} - A_{t'-1} &= A_{t'-1} g_{\mathrm{endt}} = A_{t'-1} F\left[0, \left(\frac{L_{At'}}{L_{t'}}\right)^{\lambda_2^A}\right] = A_{t'-1} F\left[0, \left(\frac{0.5 L_{At}}{0.5 L_t}\right)^{\lambda_2^A}\right] \\
&= A_{t'-1} F\left[0, \left(\frac{L_{At}}{L_t}\right)^{\lambda_2^A}\right] = A_t - A_{t-1},
\end{aligned}
$$

that is the same flow of new ideas will be produced. This violates our previous Assumption 3.

Lemma 1 provides a simple rule for the essentiality of semi-endogenous growth om a world where the only two possible growth regimes are the semi-endogenous and the fully endogenous. Given Proposition 1, if Assumption 3 is satisfied the steady-state growth rate will be semi-endogenous.

3.2 A Romer (1990) Example

A large class of growth models satisfy our assumed technology represented by function $F(\cdot, \cdot)$. While we used a constant labour share assumption for ease of exposition, we could well adapt our result to a fully microfounded case. For example, let us assume a general Romer (1990) economy, in which the introduction of new varieties (horizontal innovation) drives growth.

Each household optimizes its percapita consumption, c_t, according to

$$
\max \int_0^\infty e^{-(\rho - n)t} \frac{c_t^{1-\varepsilon} - 1}{1 - \varepsilon} dt, \text{ with } \varepsilon > 0,
$$

where ρ is the subjective rate of time preferences, ε is the CRRA, and $\rho > n$.

Letting r_t denote the real interest rate, the Euler equation is

$$
\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\varepsilon},
$$

which in equilibrium implies:

$$
r_t = \rho + \varepsilon g_{At}.
$$

In a steady-state $r = \rho + \varepsilon g_A$.

Final good is produced in a perfectly competitive industry according to

$$
Y_t = L_{Yt}^{1-\alpha} \int_0^{A_t} x_{it}^\alpha di,
$$

where A_t is the mass of intermediate product varieties: each intermediate product $i \in [0, A_t]$ is used in production in amount x_{it}.

Under perfect competition, real wage equals the marginal product of labour:

$$
w_t = \frac{\partial Y_t}{\partial L} = (1 - \alpha) \frac{Y_t}{L}
$$

and the real price of each intermediate good equals its marginal product

$$p_t = \frac{\partial Y_t}{\partial x_{it}} = \alpha L^{1-\alpha} x_{it}^{\alpha-1},$$

which can be interpreted as an inverse demand function for intermediate good $i \in [0, A_t]$.

This demand function is taken as given by its monopolist producer, i.e. by the patent holder of its blueprint.

Each intermediate good is produced by a monopoly, which maximizes profits

$$\Pi_{it} = p_{it} x_{it} - x_{it},$$

where $p_{it} = \frac{\partial Y_t}{\partial x_{it}} = \alpha L_{1t}^{1-\alpha} x_{it}^{\alpha-1}$. Equilibrium production is:

$$x_{it} = L_{Yt} \alpha^{\frac{2}{1-\alpha}} = x_t,$$

symmetric, as are the maximized profits:

$$\Pi_{it} = \frac{1-\alpha}{\alpha} L_{Yt} \alpha^{\frac{2}{1-\alpha}} \equiv \Pi_t.$$

In a balanced-growth path they grow at rate n, and the firm present discounted value, V_t, becomes

$$V_t = \frac{(1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}}{(r-n)} (1-s_A) L_t,$$

which equals the new variety patent value.

In symmetric equilibrium,

$$Y_t = A_t L_{Yt}^{1-\alpha} x^{\alpha}$$

and the real wage is

$$w_t = \frac{\partial Y_t}{\partial L_{Yt}} = (1-\alpha)\, A_t L_{Yt}^{-\alpha} x_t^{\alpha} = (1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}} A_t. \tag{5}$$

We will assume that varieties evolve according to:

$$\dot{A}_t = A_t F(g_{\mathrm{sem}t}, g_{\mathrm{end}t}), \tag{6}$$

where

$$\begin{aligned} g_{\mathrm{sem}t} &= (A_t)^{\varphi-1} (s_{At} L_t)^{\lambda_1^A}, \text{ and} \\ g_{\mathrm{end}t} &= s_{At}^{\lambda_2^A}, \end{aligned}$$

and $F(g_{\mathrm{sem}t}, g_{\mathrm{end}t})$ has the previously stated properties. We will here add constant returns to scale to facilitate equilibrium computation.

Let us assume an R&D subsidy rate $\sigma \in [0, 1[$ financed with lump-sum taxes. Free entry (zero profit) into R&D implies:

$$\dot{A}_t V_t = w_t L_{At} (1-\sigma),$$

which in steady-state becomes:

$$A_t F(g_{\text{sem}t}, g_{\text{end}t}) \frac{(1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}}{r-n}(1-s_A)L_t = w_t s_A L_t(1-\sigma). \tag{7}$$

Let us focus on a steady-state. Using (5) simplifies eq. (7) to

$$F(g_{\text{sem}t}, g_{\text{end}}) = \frac{s_A(r-n)(1-\sigma)}{\alpha(1-s_A)}. \tag{8}$$

Condition (8) holds only if $g_{\text{sem}t}$ is constant, that is if $\frac{dg_{\text{sem}\,t}}{dt} = 0$, which only happens if $g_A = g_{\text{sem}} = \frac{\lambda_1^A n}{1-\varphi}$.

The Euler equation implies $r = \rho + \varepsilon g_{\text{sem}}$. Remembering that

$$g_{\text{sem}} = g_A = F(g_{\text{sem}}, g_{\text{end}}),$$

equation (8) allows us to solve for the steady-state R&D share of the labour force:

$$s_A = \frac{\alpha g_{\text{sem}}}{(\rho-n)(1-\sigma) + (\varepsilon+\alpha)\, g_{\text{sem}}}.$$

Notice that all macroeconomic variables are obtained independently of the characteristics of function $F(\cdot,\cdot)$. Comparative statics implies that s_A decreases with impatience the parameter, ρ, and increases with the R&D subsidy rate, σ.

4 What if Semi-Endogenous Growth were not Essential?

In this section, we will explore the case in which at least semi-endogenous growth is not essential. This means, by Corollary 1, that our previous Assumption 3 is violated. Therefore, we will keep postulating Assumption 1, but we will drop both assumptions 2 and 3. In particular, Assumption 2 will be replaced by the following:

Assumption 4. *Function F is positive only if the fully endogenous growth mechanism is positive, that is:* $F(0, g_{\text{endt}}) > 0$, if and only if $g_{\text{end}t} > 0$.

Remark. Notice that Assumption 4 leaves the door open for the fully endogenous growth mechanism to be or not to be essential.

This result follows:

Proposition 2. *Under assumptions 1 and 4, the steady-state growth rate, g_A, if it exists, is fully endogenous if and only if*

$$n \leq \frac{F\left(0, s_A^{\lambda_2^A}\right)(1-\varphi)}{\lambda_1^A} \equiv \bar{n}. \tag{9}$$

Proof. In a steady state, the growth rate and R&D fraction of the labour force is constant, that is: $s_{At} = s_A$ and $g_{At} = g_A$. Hence we can rewrite (2) as:

$$g_A = F\left[(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}, s_A^{\lambda_2^A}\right] \tag{10}$$

If population is non-increasing, that is $n \leq 0$, the semi-endogenous growth part will tend to zero. In fact, $(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}$ tends to zero as $L_{t-1} \to 0$ or if $A_{t-1} \to \infty$. Consequently:

$$g_A \to F\left(0, s_A^{\lambda_2^A}\right) > 0. \tag{11}$$

Moreover, by Assumption 1, $F\left(0, s_A^{\lambda_2^A}\right)$ will increase in $s_A^{\lambda_2^A}$, which means that the long-run growth rate is fully endogenous.

If, instead, population is increasing, that is $n > 0$, two cases are possible:

(A) Condition (9) is satisfied, that is: $F\left(0, s_A^{\lambda_2^A}\right) \geq \frac{\lambda_1^A n}{1-\varphi} \equiv g_{\text{sem}}$;

(B) Condition (9) is not satisfied, that is: $F\left(0, s_A^{\lambda_2^A}\right) < \frac{\lambda_1^A n}{1-\varphi} \equiv g_{\text{sem}}$.

In case (A), by Assumption 1, it must be:

$$g_A = F\left[(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}, s_A^{\lambda_2^A}\right] \geq F\left(0, s_A^{\lambda_2^A}\right) > \frac{\lambda_1^A n}{1-\varphi}. \tag{12}$$

Consequently, $(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}$ will tend to zero and g_A will tend to $F\left(0, s_A^{\lambda_2^A}\right)$. This means that the steady-state growth rate is fully endogenous.

In case (B), $g_A = F\left(0, s_A^{\lambda_2^A}\right) < \frac{\lambda_1^A n}{1-\varphi}$ implies that $(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}$ will tend to infinity, which is inconsistent with a constant level of g_A. A constant level of g_A is achieved if and only if $(A_{t-1})^{\varphi-1}(s_A L_{t-1})^{\lambda_1^A}$ is a positive constant, that is if and only if $g_A = \frac{\lambda_1^A n}{1-\varphi}$. **QED**

Proposition 2 has shown that if the semi-endogenous growth mechanism is not essential to the growth process, the steady-state growth rate will be fully endogenous, depending on the population growth rate. Notice that this does not put any constraint on the essentiality of the fully endogenous growth part, which would be impossible with a CES aggregator like in Cozzi (2017a and b).

In light of Proposition 2, we can now generalize Cozzi's (2017a and b) result on the endogeneity of the threshold population growth rate, $\bar{n}$, below which the fully endogenous growth mechanism will dominate the steady state. In fact, $\bar{n}$ is a function of the R&D share of GDP, s_A, defined by this equation:

$$\bar{n}\left(s_A^{\lambda_2^A}\right) = \frac{F\left(0, s_A^{\lambda_2^A}\right)(1-\varphi)}{\lambda_1^A}. \tag{13}$$

Consequently, the higher s_A the higher the population growth rate threshold $\bar{n}$, and the more likely a fully endogenous steady-state growth rate.

4.1 A Romer Example

We could here apply our theory to the simple model solved in Section 3.2, obtaining the same results if $n \geq \bar{n}$. If instead $n < \bar{n}$, equation (8) in steady-state will become

$$F(0, s_A^{\lambda_2^A}) = \frac{s_A(r-n)}{\alpha(1-s_A)}, \tag{14}$$

which we can implicitly solve for the steady-state R&D share of the labour force:

$$\frac{\alpha}{\frac{(\rho-n)(1-\sigma)}{F\left(0, s_A^{\lambda_2^A}\right)} + (\varepsilon+\alpha)} - s_A = 0.$$

Unlike in the case of Section 3.2, s_A is not independent of the characteristics of function $F(\cdot,\cdot)$. We can still analyse the comparative statics. For example, an increase in the subjective rate of interest, ρ, will imply

$$\frac{\partial s_A^{\lambda_2^A}}{\partial \rho} = \frac{\frac{\alpha}{\left(\frac{(\rho-n)(1-\sigma)}{F\left(0, s_A^{\lambda_2^A}\right)} + (\varepsilon+\alpha)\right)^2} \frac{1-\sigma}{\frac{(\rho-n)(1-\sigma)}{F\left(0, s_A^{\lambda_2^A}\right)}}}{\frac{\alpha}{\left(\frac{(\rho-n)(1-\sigma)}{F\left(0, s_A^{\lambda_2^A}\right)} + (\varepsilon+\alpha)\right)^2} F_2\left(0, s_A^{\lambda_2^A}\right) - 1} < 0$$

which, if $F_2\left(0, s_A^{\lambda_2^A}\right) < 0$, implies that s_A declines with impatience. Under the same condition, we can prove that

$$\frac{\partial s_A^{\lambda_2^A}}{\partial \sigma} = -\frac{\frac{\alpha}{\left(\frac{(\rho-n)(1-\sigma)}{F\left(0, s_A^{\lambda_2^A}\right)} + (\varepsilon+\alpha)\right)^2} \frac{\rho-n}{\frac{(\rho-n)(1-\sigma)}{F\left(0, s_A^{\lambda_2^A}\right)}}}{\frac{\alpha}{\left(\frac{(\rho-n)(1-\sigma)}{F\left(0, s_A^{\lambda_2^A}\right)} + (\varepsilon+\alpha)\right)^2} F_2\left(0, s_A^{\lambda_2^A}\right) - 1} > 0$$

meaning that that s_A increases with the R&D subsidy rate. While qualitatively the results are similar to those of Section 3.2, it is useful to notice that quantitatively speaking, the specific function form of $F(\cdot,\cdot)$ will matter for s_A.

5 Conclusions

In a world where the only two growth processes are either semi-endogenous or fully endogenous, is the stock of innovation always decreases as the aggregate R&D labour decreases, the steady-state will be semi-endogenous. This paper has proved that even if the semi-endogenous and fully endogenous innovation mechanisms both capture essential macroeconomic growth processes, then the semi-endogenous growth will dictate the economy's steady-state growth rate. It is important to stress that this does not mean that the fully endogenous growth model's logic without scale effect would not play any role in the growth process. It can play a significant role in the transition. Moreover, evidence of convergence towards semi-endogenous steady-state reveals, according to the theory sketched in the paper, that its fully-endogenous component could be an essential part of it.

It is helpful to notice that, empirically speaking, all combinations of growth frameworks studied in this paper predict that ideas are getting harder to find. However, in the cases of Section 4, the long-run consequences for growth are not necessarily semi-endogenous. In fact, I have also proved that if the semi-endogenous growth part is not essential, the growth rate will be fully endogenous if the population growth rate is not too large. This result is independent of whether or not the fully endogenous growth mechanism is an essential part of the growth mechanism.

There is only a limitation of the current paper: I have assumed that the true growth process has to consist of a combination of the two prevailing growth paradigm: fully endogenous and semi-endogenous. Further research shall establish what could happen if neither of them, nor their combination, dictates the economic growth rate.

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