# Economic Inequality and Heterogeneous Success Rates of Investment

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**Abstract**

Some investments succeed and others fail. Furthermore, the probability of success will differ among people who undertake investments. In this paper, we construct exogenous and endogenous growth models that show that this heterogeneity in success rates of investment can cause extreme economic inequality. A major implication of our models is that even if the success rates are only slightly heterogeneous, people with relatively higher success rates can accumulate a larger amount of capital than those with relatively lower success rates, and as a result, the latter cannot satisfy all of their optimality conditions leading to extremely high debt-to-consumption ratios and large indebtedness to the former group. I then modify the models to consider multilateral behavior and the necessity of government intervention to improve this situation by means of simultaneous heterogeneity. I find that to prevent such extreme economic inequality, it is indispensable for the government to intervene appropriately by transferring appropriate amounts of income from the former to the latter.

JEL Classification: D63, E22, H24

Keywords: Economic inequality; Success rate of investment; Heterogeneous ability

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# 1 INTRODUCTION

Some businesses succeed while others fail. What causes this difference is an important research subject in the field of business study, and indeed has been intensely studied (Lussier and Corman 1996; Everett and Watson 1998; Chen and Williams 1999; Lussier and Pfeifer 2001; Rogoff et al. 2004; Lussier and Halabi 2010; Marom and Lussier 2014; Olaison and Sørensen 2014). However, there is no standard theory of business success and failure (Lussier and Halabi 2010), and studies on this subject have mostly relied on various ad hoc factors that are thought to be related to success and failure. Importantly, many of these studies have commonly emphasized the importance of human factors (e.g., staffing, including education level, and the abilities of entrepreneurs).

The likelihood of success or failure of a business will be roughly equivalent to that of an investment because a new business (or new project) generally cannot start without investments. However, unlike the abundant research on business success and failure in the field of business study, success and failure in investing has been almost neglected in macroeconomic studies. A likely reason for this neglect is that in macroeconomic studies, the success rate of investments has been thought to be sufficiently high and furthermore almost identical among most people and most economies. Or, at least, the rate has been believed to be sufficiently within an allowable range to assume for simplicity that it is identical.

In actuality, however, success rates of investment are likely to be heterogeneous across people and economies. This is because the outcome of success or failure will be greatly influenced by people’s abilities, and abilities basically differ among people. The aforementioned literature in the field of business study on the success and failure of businesses strongly suggests that human factors play an important role. One of the important abilities related to the success rate of investments is the ability to anticipate and prepare for risks of investments before any investment is made.

Differences in success rates of investment may indeed be small, but the possibility remains that even a small difference can have a major impact on the economy, particularly on economic inequality. Given this possibility, this subject should not be ignored a priori, and the question of whether a small difference can have a major impact should be examined in adequate depth within the framework of macroeconomic studies. To the best of my knowledge, however, this possibility has not been examined. The purpose of this paper is thus to examine whether this possibility is truly important.

To examine the impact of heterogeneity among people, economic models that treat households as heterogeneous are needed. In this paper, I use modified versions of exogenous and endogenous growth models presented in Harashima (2010[1], 2013a[2], 2013b[3], 2014). In both exogenous and endogenous growth models, households are heterogeneous in their ability to make investments succeed, i.e., the success rates of investment are heterogeneous. The models in this paper show that small differences in success rates of investments can lead to extreme economic inequality and have a major impact on the economy. To prevent this dreadful state, appropriate interventions by governments (i.e., income transfers among households) are indispensable.

# 2 RISKS OF HETROGENITY IN INVESTMENT SKILL

## *2.1 Success rate of investment*

### 2.1.1 Investments in capital and technology

Investments can be divided broadly into two types: those in capital and those in technology. Here, I refer to "investments in capital" as those undertaken to accumulate the steady state level of capital for a given level of technology, and to "investments in technology" as those undertaken to create or utilize new technologies. In many cases, these two types of investments will be joined together and may not be easily discernable. However, in order to model both exogenous and endogenous growth, I assume that the two types of investments are clearly discernable.

### 2.1.2 Heterogeneity in success rate of investment

Investments have risks and are undertaken under uncertainty. The result (success or failure) of an investment can be known only in the future. They will be especially uncertain in the case of investments in technology, and the success rate of this type of investment seems to be quite low.

The success rate of investment in capital, on the other hand, does not seem to be as low as that in technology because investments in capital make use of existing technologies, and thus are not innovative activities. Rather, these investments may typically succeed. In macroeconomic studies, therefore, it is usually assumed that capital is steadily accumulated exactly as planned until it reaches the amount at steady state. In other words, it is assumed that investments in capital do not fail.

However, in actuality, not all investments in capital succeed. There are many examples where projects to expand a business in other areas have been abandoned prior

[1] Harashima (2010) is also available in Japanese as Harashima (2017).
[2] Harashima (2013a) is also available in Japanese as Harashima (2019).
[3] Harashima (2013b) is also available in Japanese as Harashima (2020a).

to completion or new facilities to increase production capacity have been shut down or demolished soon after their completion. The reasons for failure are various and include misjudging demand, cost overruns, and losing out in competition with rivals. The degree of risk for investments in capital may be far lower than for those in technology, but investments in capital still have some risks. If investments in capital fail, the amount invested is not accumulated as capital stock.

Although risks undoubtedly exist, why has the success rate of investment in capital been mostly ignored in macroeconomic models? One likely reason is that this rate is thought to be sufficiently high and almost identical across people and economies. And even if the success rate of investment is thought to be heterogeneous among people, this heterogeneity may have been considered quite small and therefore not to have a large economic impact. However, to the best of my knowledge, it has not been shown or proved, either theoretically or empirically, whether this heterogeneity in fact does not have a large economic impact. In fact, this topic has not even been examined. Until such an examination is conducted, the possibility that this heterogeneity would have a large impact on the economy should not be ignored.

### 2.1.3 Heterogeneity in ability to make investment succeed

Various factors, both external and internal, will cause investments in capital to fail. Here, I refer to "external factor" as an exogenous disturbance to economic activities such as the oil crisis in the 1970s or the COVID-19 pandemic in 2020 and 2021, and I refer to "internal factor" as the ability of people to anticipate and prepare for the risks of investment in capital, in other words, the ability to make the investment succeed. Many works in the field of business study have emphasized the importance of human factors in the success or failure of a business, which implies that the internal factor is important in the success or failure of investment in capital.

By their nature, external factors are common to all people. Or at least an apparently reasonable assumption is that failures of investments in capital caused by external factors occur to all people with equal probability. Internal factors, on the other hand, seem highly likely to be distributed heterogeneously across people because these factors are rooted in abilities. The relevant abilities to succeed in investing require a high level of intelligence and a wide range of applicable knowledge. In general, abilities, including the ability to make successful investments in capital, are distributed heterogeneously among people, often with substantial variation. Some people can more correctly anticipate and properly prepare for investment risks than others. They can more correctly judge future demand, more adequately control costs, and more often win out over rivals. In sum, they manage risks better.

The ability to make investments in capital succeed is often closely related to fluid intelligence. Many types of intelligence have been considered in the fields of psychology and psychometrics, and the importance of fluid intelligence, together with the contrasting concept of crystallized intelligence, has been particularly emphasized. Defined as the ability to solve novel problems by thinking logically without depending solely on previously acquired knowledge (Cattell 1963, 1971), fluid intelligence is the ability to deal with unexpected situations without relying solely on knowledge obtained from schooling or previous experience. With the help of fluid intelligence, people can adapt their thinking to new problems or situations. Clearly, fluid intelligence is a crucial requirement for managing risks properly and making investments in capital succeed.

Raven's Progressive Matrices test (Raven 1962; Raven and Court 1998) is regarded as the best test to measure fluid intelligence (Snow et al. 1984), and the results of the test indicate that fluid intelligence is distributed heterogeneously across people (Snow et al. 1984). One implication is that the ability to make investments in capital succeed is also highly likely to be distributed heterogeneously among people, perhaps greatly so.

## *2.2 Heterogeneous success rates under unilateral behavior*

In this section, I examine the effect of heterogeneous success rates of investment in capital by constructing exogenous and endogenous growth models in which people are heterogeneous in their ability to make investments in capital succeed. First, I examine the case that people behave unilaterally in the sense that they behave without regarding the optimality conditions of other people. Other cases, such as those including multilateral behavior and government intervention, are examined in Section 3.

### 2.2.1 Exogenous growth model

Under the supposition that technologies are given exogenously, I use an exogenous growth model that is constructed on the basis of the model shown in Harashima (2014). Because growth is exogenous in this model, investments consist only of those in capital and no investment in technology is undertaken.

#### 2.2.1.1 The model

Suppose that there is a country that consists of two groups of households, or economies, which I call Economy 1 and Economy 2. Within each economy, all households are identical and they assume the roles of consumer, laborer, and investor at the same time. The two economies are identical except for the ability of households to make investments in capital succeed. They are fully open to each other with goods, services, and capital freely transacted between the two economies, but there is no movement of labor between them. All variables are expressed in per capita terms.

The production function of Economy $i$ is

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \tag{1}$$

for $i$ = 1 or 2, where $y_{i,t}$ and $k_{i,t}$ are the production and capital, respectively, of Economy $i$ in period $t$; $A_t$ is the technology in period $t$; and $\alpha$ ( $0 < \alpha < 1$) is a constant and indicates the labor share. As for transactions between the two economies, the current account balance in Economy 1 in period $t$ is $\tau_t$ and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies. The economy with a current account surplus invests the surplus in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ are returns on investment,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \quad \text{and} \quad \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

represent income receipts or payments on the assets that one economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance of goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = \kappa\left(k_{1,t}, k_{2,t}\right) .$$

The government of the country can intervene in economic activities in both economies by transferring money between them. The amount transferred from households in Economy 1 to households in Economy 2 in period $t$ is $g_t$, and it is assumed that $g_t$ depends on capital such that

$$g_t = \bar{g}_t k_{1,t} \; .$$

The variable $\bar{g}_t$ is exogenous to households and firms and is appropriately adjusted by the government in every period so as to achieve sustainable heterogeneity (SH). Because $k_{1,t} = k_{2,t}$ and $\dot{k}_{1,t} = \dot{k}_{2,t}$,

$$g_t = \bar{g}_t k_{1,t} = \bar{g}_t k_{2,t} \; .$$

The amount of investment in capital is equal to the residual amount of income after subtracting consumption, the current account balance of goods and services, and net government transfers. Let $I_{i,t}$ be the amount of investment in capital in Economy $i$ in period $t$. Therefore, by equation (1),

$$I_{1,t} = A_t^{\alpha} k_{1,t}^{1-\alpha} - c_{1,t} + (1-\alpha) A_t^{\alpha} k_{1,t}^{-\alpha} \int_0^t \tau_s ds - \tau_t - \bar{g}_t k_{1,t} \tag{2}$$

and

$$I_{2,t} = A_t^{\alpha} k_{2,t}^{1-\alpha} - c_{2,t} - (1-\alpha) A_t^{\alpha} k_{2,t}^{-\alpha} \int_0^t \tau_s ds + \tau_t + \bar{g}_t k_{2,t} \; , \tag{3}$$

where $c_{i,\,t}$ is the per capita consumption of Economy $i$ in period $t$, and

$$(1-\alpha) A_t^{\alpha} k_{i,t}^{-\alpha} = \frac{\partial y_{i,t}}{\partial k_{i,t}} = r_t \tag{4}$$

is the real interest rate.

Let $\rho_i (0 < \rho_i < 1)$ be the success rate of investment in capital (i.e., the ratio of successful investments in capital to all investments in capital) of Economy $i$, and assume $\rho_1 > \rho_2$. Because some investments fail $(1 - \rho_i)$ and are not accumulated as capital, the budget constraint of a household in Economy 1 is, by equation (2),

$$\dot{k}_{1,t} = \rho_1 I_{1,t} = \rho_1 \left[ A_t^{\alpha} k_{1,t}^{1-\alpha} - c_{1,t} + (1-\alpha) A_t^{\alpha} k_{1,t}^{-\alpha} \int_0^t \tau_s ds - \tau_t - \bar{g}_t k_{1,t} \right] \tag{5}$$

and that in Economy 2 is, by equation (3),

$$\dot{k}_{2,t} = \rho_2 I_{2,t} = \rho_2 \left[ A_t^{\alpha} k_{2,t}^{1-\alpha} - c_{2,t} - (1-\alpha) A_t^{\alpha} k_{2,t}^{-\alpha} \int_0^t \tau_s ds + \tau_t + \bar{g}_t k_{2,t} \right]. \quad (6)$$

Each household in Economy $i$ maximizes its expected utility

$$E \int_0^{\infty} u_i(c_{i,t}) \exp(-\theta t) dt$$

subject to equation (5) for Economy 1 and to equation (6) for Economy 2, respectively, where $u_i$ is the utility function of Economy $i$, $\theta$ is the rate of time preference, and $E$ is the expectation operator.

#### 2.2.1.2 Optimal path

As the result of utility maximization behavior shown in the model in Section 2.2.1.1, the growth rate of consumption of households in Economy 1 is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1} \left\{ \rho_1 \left[ (1-\alpha) A_t^{\alpha} k_{1,t}^{-\alpha} - \alpha (1-\alpha) A_t^{\alpha} k_{1,t}^{-\alpha-1} \int_0^t \tau_s ds + (1-\alpha) A_t^{\alpha} k_{1,t}^{-\alpha} \frac{\partial \left( \int_0^t \tau_s ds \right)}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}} - k_{1,t} \bar{g}_t \right] - \theta \right\} \quad (7)$$

and that in Economy 2 is

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1} \left\{ \rho_2 \left[ (1-\alpha) A_t^{\alpha} k_{2,t}^{-\alpha} + \alpha (1-\alpha) A_t^{\alpha} k_{2,t}^{-\alpha-1} \int_0^t \tau_s ds - (1-\alpha) A_t^{\alpha} k_{2,t}^{-\alpha} \frac{\partial \left( \int_0^t \tau_s ds \right)}{\partial k_{2,t}} + \frac{\partial \tau_t}{\partial k_{2,t}} + k_{2,t} \bar{g}_t \right] - \theta \right\}. \quad (8)$$

where $\varepsilon$ is the degree of relative risk aversion.

#### 2.2.1.3 The consequence of unilateral behavior

As assumed above, households behave unilaterally. In this case, equations (7) and (8) indicate that, if $\bar{g}_t = 0$, the steady state such that

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{c}_{2,t}}{c_{2,t}} = 0 \tag{9}$$

is not necessarily naturally achieved even if the difference between $\rho_1$ and $\rho_2$ is very small. As Harashima (2014) shows, if households set $\tau_t$ without regard for the other economy's optimality conditions (i.e., households behave unilaterally), the ratio of Economy 2's debts to its consumption explodes to infinity, whereas all the optimality conditions of the households in Economy 1 are satisfied. Moreover, Economy 2's vast indebtedness (owed to Economy 1) leads to extreme inequality. It is not possible for households in Economy 2 to satisfy all of their optimality conditions unless the government intervenes appropriately, i.e., $\bar{g}_t \neq 0$. This is the same dreadful state as Becker (1980) describes in the case of heterogeneous time preferences.

### 2.2.2 Endogenous growth model

Next, suppose that technologies are endogenously generated. In this case, I use an endogenous growth model that is constructed on the basis of the model shown in Harashima (2010, 2013a, 2013b). The environment and setup of the model are identical to those of the exogenous growth model in Section 2.2.1 except that technologies are generated endogenously. Let $L_i$ be the population of Economy $i$, where $L_i$ is identical between the two economies and sufficiently large.

#### 2.2.2.1 The model

As shown in Harashima (2010, 2013a, 2013b), because of the substitution between production of capital and technology,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\varpi}{2mv}\frac{\partial\left(y_{1,t} + y_{2,t}\right)}{\partial A_t} = \frac{\partial y_{2,t}}{\partial k_{2,t}} \tag{10}$$

always holds, where $\varpi$, $m$, and $v$ are positive constants. Therefore, by equations (1) and (10),

$$A_t = (1-\alpha)^{-1}\frac{\varpi\alpha}{mv}k_{i,t} \tag{11}$$

and

$$r_t = \frac{\partial y_{i,t}}{\partial k_{i,t}} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} = r \ = \text{constant}. \tag{12}$$

Because growth is endogenous in this model, investments are undertaken not only in capital but also in technology. As a result, the amount of investment in capital is not the same as in the case of exogenous growth. Instead, it is equal to the residual income after subtracting consumption, investments in technology, the current account balance of goods and services, and net government transfers. Let $\rho_{T,i}\left(0 < \rho_{T,i} < 1\right)$ be the success rate of investment in the technologies of Economy $i$. Increases in $A_t$ are therefore determined by investments in technology multiplied by $\rho_{T,i}$. Harashima (2010, 2013a, 2013b) demonstrates that if $\rho_{T,i}L_i$ is sufficiently large, then by equations (1), (5), (6), and (11), the budget constraint of households in Economy 1 is approximately described as

$$\dot{k}_{1,t} = \rho_1 \left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} k_{1,t} - c_{1,t} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - \bar{g}_t k_{1,t}\right], \tag{13}$$

and in Economy 2 it is approximately described as

$$\dot{k}_{2,t} = \rho_2 \left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} k_{2,t} - c_{2,t} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds + \tau_t + \bar{g}_t k_{2,t}\right]. \tag{14}$$

Each household in Economy $i$ maximizes expected utility in the same manner as shown in our model in Section 2.2.1.1 subject to equation (13) for Economy 1 and equation (14) for Economy 2.

#### 2.2.2.2 Balanced growth path

As the result of utility maximization behavior under endogenous growth shown in Section 2.2.2.1, the growth rate of consumption of households in Economy 1 is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1} \left\{ \rho_1 \left[ \left(\frac{\varpi\alpha}{mv}\right) (1-\alpha)^{-\alpha} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{1-\alpha} \frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}} - k_{1,t}\bar{g}_t \right] - \theta \right\}, \tag{15}$$

and that in Economy 2 is

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\left\{\rho_2\left[\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{-\alpha} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}} + \frac{\partial \tau_t}{\partial k_{2,t}} + k_{2,t}\bar{g}_t\right] - \theta\right\}. \quad (16)$$

**2.2.2.3 The consequence of unilateral behavior**

Households behave unilaterally as assumed. In this case, equations (15) and (16) indicate that if $\bar{g}_t = 0$, a balanced growth path in which

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{c}_{2,t}}{c_{2,t}}$$

holds may not always be naturally achieved, even if the difference between $\rho_1$ and $\rho_2$ is very small—the same result as for the exogenous growth model in Section 2.2.1. If households behave unilaterally, it is not possible for households in Economy 2 to satisfy all of their optimality conditions even if all optimality conditions of the households in Economy 1 are satisfied, unless the government appropriately intervenes, i.e., $\bar{g}_t \neq 0$.

## *2.3 Heterogeneous abilities do matter*

Harashima (2010, 2013b) shows that even if productivity is distributed heterogeneously among people, economic inequality is not exacerbated, whereas heterogeneous preferences do exacerbate it. Productivity represents a type of human ability, and therefore productivity highlights a case in which heterogeneous abilities do not cause extreme economic inequality.

Harashima (2020b, 2020c, 2021) shows that heterogeneity in another type of ability—the ability to obtain persistent economic rents—can increase economic inequality to an extreme level. The model in this paper shows that heterogeneity in another type of ability also matters: heterogeneity in the success rates of investments. As indicated in Sections 2.1 and 2.2, because of heterogeneous success rates of investments, households with lower success rates will fall into extreme poverty unless the government appropriately intervenes.

# 3 SUSTAINABLE HETEROGENEITY

## *3.1 Sustainable heterogeneity*

Even if there are heterogeneities in preferences and abilities to obtain persistent rents across households, the dreadful state described by Becker (1980) of extreme economic inequality can be avoided if sustainable heterogeneity (SH), a state in which households are heterogeneous and all of their optimality conditions are satisfied, is achieved. The concept and nature of SH are described in Harashima (2010, 2013b, 2014). However, SH is not always naturally achieved, which was shown in Section 2 for the case in which households behave unilaterally.

Harashima (2010, 2013b, 2014) shows that if households behave multilaterally in the sense that they behave in a manner that gives sufficient regard to the optimality conditions of households in the other economy, SH is achieved. However, it seems unlikely that the voluntary behavior of all households is sufficiently multilateral to achieve this because humans seem to be inherently selfish to a greater or lesser extent. That is, SH still will not be naturally achieved under realistic conditions. Rather, SH needs to be achieved through some type of artificial or compulsory means, particularly by government intervention. Harashima (2013b, 2014) shows that even if households behave unilaterally, SH can be achieved if the government intervenes appropriately.

I now examine whether SH can be also achieved when the abilities with regard to the success rate of investment are heterogeneous and households behave multilaterally or the government intervenes appropriately.

## *3.2 SH in exogenous growth model*

First, I examine the case of exogenous growth using the same model and setup as used for the exogenous growth model in Section 2.2.1.

### 3.2.1 SH without government intervention

Suppose that the government does not intervene, i.e., $\bar{g}_t = 0$ for any *t*, but that household behavior is sufficiently multilateral. Because the behavior of households is multilateral, at steady state, equation (9) holds and then, by equations (7) and (8) for $\bar{g}_t = 0$,

$$\lim_{t\to\infty}\rho_1\left[(1-\alpha)A_t^{\alpha}k_{1,t}^{-\alpha} - \alpha(1-\alpha)A_t^{\alpha}k_{1,t}^{-\alpha-1}\int_0^t \tau_s ds + (1-\alpha)A_t^{\alpha}k_{1,t}^{-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}}\right] - \theta = 0 \qquad (17)$$

and

$$\lim_{t\to\infty}\rho_2\left[(1-\alpha)A_t^{\alpha}k_{2,t}^{-\alpha}+\alpha(1-\alpha)A_t^{\alpha}k_{2,t}^{-\alpha-1}\int_0^t\tau_s ds-(1-\alpha)A_t^{\alpha}k_{2,t}^{-\alpha}\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2,t}}+\frac{\partial\tau_t}{\partial k_{2,t}}\right]-\theta=0\,. \tag{18}$$

Hence, at steady state, by equation (17),

$$\rho_1\left[(1-\alpha)A_t^{\alpha}k_{1,t}^{-\alpha}[1+(1-\alpha)\Psi]-\Xi\right]-\theta=0\,, \tag{19}$$

where

$$\Psi=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{2,t}}$$

and

$$\Xi=\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}\ .$$

Because

$$\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,t}}{\tau_{i,t}}=0$$

at steady state, then $\Psi$ and $\Xi$ are constant at steady state. For $\Psi$ to be constant at steady state, it is necessary that $\lim_{t\to\infty}\tau_t=0$ and thus $\Xi=0$ at steady state. Therefore, by equation (19),

$$\rho_1\left[(1-\alpha)A_t^{\alpha}k_{1,t}^{-\alpha}[1+(1-\alpha)\Psi]\right]-\theta=0 \tag{20}$$

and similarly, by equation (18),

$$\rho_2\left[(1-\alpha)A_t^{\alpha}k_{2,t}^{-\alpha}[1-(1-\alpha)\Psi]\right]-\theta=0 \tag{21}$$

at steady state. Because $k_{1,t}=k_{2,t}$, then by equations (20) and (21),

$$\Psi = \frac{\rho_2 - \rho_1}{\rho_1 + \rho_2}(1 - \alpha)^{-1} \tag{22}$$

at steady state. Because $\rho_1 > \rho_2$, clearly $\Psi < 0$ by equation (22), such that

$$\Psi = \lim_{t \to \infty} \frac{\int_0^t \tau_s ds}{k_{1,t}} = \lim_{t \to \infty} \frac{\int_0^t \tau_s ds}{k_{2,t}} < 0 \ . \tag{23}$$

By inequality (23),

$$\int_0^t \tau_s ds < 0 \tag{24}$$

at steady state. That is, when SH is achieved by the multilateral behavior of households, Economy 1 possesses accumulated debts owed to Economy 2 at steady state, and Economy 1 must export goods and services to Economy 2 to service this debt.

Note that by equations (4), (20), (21), and (22), at SH,

$$r = \left(\frac{\rho_1 + \rho_2}{2\rho_1\rho_2}\right)\theta \ .$$

Because $0 < \rho_i < 1$ and $\frac{\rho_1+\rho_2}{2\rho_1\rho_2} > 1$, then,

$$r = \left(\frac{\rho_1 + \rho_2}{2\rho_1\rho_2}\right)\theta > \theta \ . \tag{25}$$

Inequality (25) indicates that the steady state for $\rho_i < 1$ and $\theta$ is equivalent to the steady state for $\rho_i = 1$ (a success rate of 100%) and $\left(\frac{\rho_1+\rho_2}{2\rho_1\rho_2}\right)\theta$, i.e., a higher value of $\theta$. This means that the steady state production and consumption in the case that $\rho_i < 1$ are lower than those in the usually assumed case that $\rho_i = 1$, if the rate of time preference ($\theta$) is the same in both cases.

### 3.2.2 SH with government intervention

Next, I examine the case that the government intervenes appropriately, i.e., that it adjusts $\bar{g}_t$ appropriately to achieve SH ($\bar{g}_t \neq 0$), under the assumption that households behave unilaterally. The government intervenes to make

$$\int_0^t \tau_s ds = 0 \tag{26}$$

at SH.

In this case, in accordance with equations (7) and (8), to achieve SH, i.e., to make equation (17) hold at steady state by satisfying equation (26), it is necessary to achieve

$$\rho_1\left[(1-\alpha)A_t^{\alpha}k_{1,t}^{-\alpha} - \bar{g}_t\right] - \theta = 0 \tag{27}$$

and

$$\rho_2\left[(1-\alpha)A_t^{\alpha}k_{2,t}^{-\alpha} + \bar{g}_t\right] - \theta = 0\ . \tag{28}$$

Hence, by equations (27) and (28),

$$\bar{g}_t = \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)(1-\alpha)A_t^{\alpha}k_{1,t}^{-\alpha}$$

and by equation (4),

$$\bar{g}_t = \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)r \tag{29}$$

at steady state, where *r* is the real interest rate at steady state.

Because $\rho_1 > \rho_2$, $\bar{g}_t$ in equation (29) is positive. That is, in order to satisfy equation (29) at steady state, the government should transfer money from households in Economy 1 to households in Economy 2 in every period.

## *3.3 SH in endogenous growth model*

I next examine the case of endogenous growth with the same model and setup as used for the endogenous growth model in Section 2.2.2

### 3.3.1 SH without government interventions

Again, I first examine the case of multilateral behavior of households without government intervention. In this case, to achieve SH, i.e., to make

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \text{constant}, \tag{30}$$

it is necessary to satisfy

$$\rho_1\left[\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{-\alpha}+\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}\right]$$
$$=\rho_2\left[\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{-\alpha}-\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}+\frac{\partial\tau_t}{\partial k_{2,t}}\right] \quad (31)$$

by equations (15) and (16) where $\bar{g}_t = 0$. Because $k_{1,t} = k_{2,t}$,

$$\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}=\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}-\frac{\partial\tau_t}{\partial k_{2,t}}\ , \quad (32)$$

and therefore, by equations (31) and (32), to fulfill equation (30) (i.e., to be on a balanced growth path), the following must hold:

$$\left(\frac{\rho_2-\rho_1}{\rho_1+\rho_2}\right)\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{-\alpha}=\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}\ . \quad (33)$$

Hence, by equations (15), (16), and (33), for $\bar{g}_t = 0$,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left[\frac{2\rho_1\rho_2}{\rho_1+\rho_2}\left(\frac{\varpi\alpha}{mv}\right)(1-\alpha)^{-\alpha}-\theta\right]\ . \quad (34)$$

Harashima (2010, 2013b, 2014) indicates that on the balanced growth path, equation (23) still holds, and therefore inequality (24) also holds. That is, assuming SH is achieved by the multilateral behavior of households, then on the balanced growth path Economy 1 carries accumulated debt owed to Economy 2, and Economy 1 must export goods and services to Economy 2 to service this debt.

Note that because $0 < \rho_i < 1$ and $\frac{2\rho_1\rho_2}{\rho_1+\rho_2} < 1$, the growth rate indicated by equation (34) is lower than that in the usually assumed case that $\rho_i = 1$ (i.e., a success rate of 100%).

### 3.3.2 SH with government intervention

Next, I examine the case that the government intervenes to make equation (26) hold on a balanced growth path. In this case, to achieve SH, which implies that equation (30) holds, it is necessary to satisfy

$$\rho_1 \left[ \left( \frac{\varpi \alpha}{m v} \right) (1-\alpha)^{-\alpha} - \bar{g}_t \right] = \rho_2 \left[ \left( \frac{\varpi \alpha}{m v} \right) (1-\alpha)^{-\alpha} + \bar{g}_t \right] \tag{35}$$

in accordance with equations (15) and (16). Hence, by equation (35), on a balanced growth path, the government transfers money from each household in Economy 1 to each household in Economy 2 in every period according to

$$\bar{g}_t = \left( \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \right) \left( \frac{\varpi \alpha}{m v} \right) (1-\alpha)^{-\alpha} \ . \tag{36}$$

By equations (12) and (36), it transfers money according to

$$\bar{g}_t = \left( \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \right) r \ . \tag{37}$$

Note that in the case of exogenous growth, equation (37) is exactly the same as equation (29).

Because $\rho_1 > \rho_2$, $\bar{g}_t$ in equation (37) is positive. That is, in order that society stays on a balanced growth path, the government should transfer money from households in Economy 1 to households in Economy and satisfy equation (37) in every period.

# 4 CONCLUDING REMARKS

Unlike studies on business success and failure in the field of business study, success and failure of investments in capital has been almost neglected in macroeconomic studies. A likely reason for this neglect is that the success rate of investment in capital has been thought to be sufficiently high and almost identical across people and economies, or at least it has been believed to be well enough within an allowable range to assume that it is so for simplicity in macroeconomic studies. However, if there is a possibility that a small difference in success rates of investments in capital can have a major impact on the economy, particularly regarding economic inequality, this subject should not be ignored a priori but instead be examined with adequate depth in macroeconomic studies. To the best of my knowledge, however, this possibility has not been examined.

It is highly likely that differences in success rates of investments in capital are caused by differences in certain types of people's abilities. One of the important abilities that is related to the success rate of investment in capital is the ability to correctly anticipate and properly prepare for the risks of investment in capital. In this paper, I have examined the effect upon economic inequality of heterogeneity in the success rate by use of exogenous and endogenous growth models that assume heterogeneity among households. These models are constructed on the basis of the models presented by Harashima (2010, 2013a, 2013b, 2014).

The exogenous and endogenous growth models in this paper both indicate that small differences in success rates of investment in capital can lead to extreme economic inequality. As in the case of heterogeneous preferences and persistent rent incomes, SH cannot be naturally achieved if success rates of investment in capital are heterogeneous, even if only slightly. If households behave unilaterally, it is not possible for SH to be achieved by households themselves. This means that heterogeneity in the ability to make investments in capital succeed does matter, whereas heterogeneity in abilities related to productivity does not matter because the latter form of heterogeneity naturally leads to the state of SH. However, in the case of heterogeneity in the ability to make investments in capital succeed, the resulting extreme economic inequality can be prevented through the achievement of SH. This must be accomplished through appropriate interventions by the government in the form of transferring necessary amounts of economic resources (money) from relatively more advantaged households to less advantaged households.

# References


Becker, Robert A. (1980) "On the Long-run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households," *The Quarterly Journal of Economics*, Vol. 95, No. 2, pp. 375–382.

Cattell, Raymond Bernard. (1963) "Theory of Fluid and Crystallized Intelligence: A Critical Experiment," *Journal of Educational Psychology*, Vol. 54, pp. 1-22.

Cattell, Raymond Bernard. (1971) *Abilities: Their structure, growth, and action*, Houghton Mifflin, Boston.

Chen, Jo-Hui and Martin Williams (1999) "The Determinants of Business Failures in the US Low-technology and High-technology Industries," *Applied Economics*, Vol. 31, No. 12, pp. 1551–1562.

Everett, Jim and John Watson (1998) "Small Business Failure and External Risk Factors," *Small Business Economics*, Vol. 11, pp. 371–390.

Harashima, Taiji (2010) "Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population," *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 24233.

Harashima, Taiji (2013a) "An Asymptotically Non-Scale Endogenous Growth Model," *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 44393.

Harashima, Taiji (2013b) "Sustainable Heterogeneity as the Unique Socially Optimal Allocation for Almost All Social Welfare Functions," *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 40938.

Harashima, Taiji (2014) "Sustainable Heterogeneity in Exogenous Growth Models: The Socially Optimal Distribution by Government's Intervention," *Theoretical and Practical Research in Economic Fields*, Vol. 5, No. 1, pp. 73-100.

Harashima, Taiji (2017) "Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population," in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 51, No. 1, pp. 31 – 80.（「持続可能な非均質性―均質ではない構成員からなる経済における不平等、経済成長及び社会的厚生―」『金沢星稜大学論集』第 51 巻第 1 号 31～80 頁）

Harashima, Taiji (2019) "An Asymptotically Non-Scale Endogenous Growth Model," in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 52, No. 2, pp. 71 – 86.（「漸近的に規模効果が消失する内生的経済成長モデル」『金沢星稜大学論集』第 52 巻第 2 号 71～86 頁）

Harashima, Taiji (2020a) "Sustainable Heterogeneity as the Unique Socially Optimal Allocation for Almost All Social Welfare Functions," in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 54, No. 1, pp. 71 – 95.（「殆ど全ての社会的厚生

関数に対して唯一の社会的に最適な配分をもたらすものとしての持続可能な非均質性」『金沢星稜大学論集』第 54 巻第 1 号 71～95 頁）
Harashima, Taiji (2020b) “Preventing Widening Inequality: Economic Rents and Sustainable Heterogeneity,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 103377.
Harashima, Taiji (2020c) “Economic Inequality Exacerbated by Economic Rents from Exploitative Contracting,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 104178.
Harashima, Taiji (2021) “Consequence of Heterogeneous Economic Rents under the MDC-based Procedure,” *Journal of Applied Economic Sciences*, Vol. 16, No. 2. pp. 185-190.
Lussier, Robert N. and Joel Corman (1996) “A Business Success Versus Failure Prediction Model for Entrepreneurs with 0-10 Employees,” *Journal of Small Business Strategy*, Vol. 7, No. 1, pp. 21-35.
Lussier, Robert N. and Claudia E. Halabi (2010) “A Three-Country Comparison of the Business Success versus Failure Prediction Model,” *Journal of Small Business Management*, Vol. 48, No. 3, pp. 360-377.
Lussier, Robert N. and Sanja Pfeifer (2001) “A Crossnational Prediction Model for Business Success,” *Journal of Small Business Management*, Vol. 39, No. 3, pp. 228-239.
Marom, Shaike and Robert N. Lussier, (2014) “A Business Success Versus Failure Prediction Model for Small Businesses in Israel,” *Business and Economic Research, Macrothink Institute*, Vol. 4, No. 2, pp. 63-81.
Olaison, Lena and Bent Meier Sørensen (2014) “The Abject of Entrepreneurship: Failure, Fiasco, Fraud,” *International Journal of Entrepreneurial Behavior & Research*, Vol. 20, No. 2, pp. 193-211.
Raven, John Carlyle (1962) *Advanced Progressive Matrices*: *Sets I and II,* H. K. Lewis, London.
Raven, J., John Carlyle Raven and J. H. Court (1998) *Manual for Raven's Progressive Matrices and Vocabulary Scales. Section 1: General overview*. Harcourt Assessment, San Antonio, TX.
Rogoff, Edward G., Myung-Soo Lee and Dong-Churl Suh (2004) “Who Done It?” Attributions by Entrepreneurs and Experts of the Factors that Cause and Impede Small Business Success,” *Journal of Small Business Management*, Vol. 42, No. 4, pp. 364-376.
Snow, Richard E., Patrick C. Kyllonen and Brachia Marshalek (1984) “The Topography of Ability and Learning Correlations,” in Sternberg, Robert J. ed. *Advances in the Psychology of Human Intelligence Vol. 2*, Erlbaum, Hillsdale, NJ.