Individual decision-making experiment with risk and intertemporal choice: a replication study

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February 2020

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Abstract: We replicate the experiment proposed by Lisa R. Anderson and Sarah L. Stafford (2009) by conducting it through the Instagram platform. The structure of questionnaire is the same as the one of the original experiment inasmuch subjects might choose between two options that differ in resolution timing. According to the experimental results, we show that the percentage of subjects choosing later option increases as the value of the later option increases, and the percentage of subjects choosing later option is smaller the longer the temporal delay between two options. We found that risk does not make subjects less patient and there are interactions between risk and the length temporal delay.

1. Introduction

When dealing with individual decision making, economists assume that subjects, or in general *decision-makers*, are rational and, under certain axioms, they make choices according to the alternative that gives them the largest payoff. To compute the payoff, when the alternative is not certain, subjects choose according to the *expected utility theory* because it is assumed that they can compute the exact expected value for each alternative. Thus, when facing a risk, subjects maximize the expected value of some potential payoffs and make their choice according to these computations. However, one may ask himself what happens in case of a subject is supposed to make a choice in which there are several trade-offs to be solved, in terms of both risk and time delay. In this case, when a choice implies a solution to an intertemporal and uncertain problem, another important theory employed in economic models is the Samuelson's *discounted utility theory*. Even though these theories are widely used in economic modelling, it is not sure that they apply also in practice. In fact, in experimental economics theory, several biases lead subjects to do not behave accordingly. There is experimental evidence on the fact that people face the *Allais paradox*, the *gain-loss asymmetry*, the *loss aversion*, the *framing effect*, the *preference reversal*, and other biases that make their behavior not consistent with the theoretical solution when they face risky choices and intertemporal decisions. It is important to study the behavior of decision-makers when they deal with risk and intertemporal uncertainty because experiments can

be useful to get evidence on whether agents apply the expected utility theory in evaluating a set of alternatives, and if they make their choice accordingly, in a rational way. According to *Von Neumann and Morgenstern's (1947)* expected utility theory, subjects are rational and always choose the alternative with the largest expected value which maximizes their utility function.[1] When the decision implies an intertemporal choice, Samuelson's discounted utility theory (1937) tells us that individuals choose the alternative with the largest discounted utility. In order test these theories, we replicate an experimental design by *Lisa R. Anderson and Sarah L. Stafford* (2009) and we conducted it on web without monetary incentives. As we expected, we find that many subjects do not appear to have consistent risk preferences, and this is in line with the work of Anderson and Stafford (2009) on which this experiment is based. This paper is structured as follows: *Section 1* deals with the review of the experimental design; *Section 2* shows the econometric analysis; Finally, in *Section 3,* we give some concluding remarks.

1. Experimental design

To replicate the experiment made by Anderson and Stafford (2009), we conducted an online experiment, via *Google Forms*, providing a questionnaire to a sample composed of 1262 subjects. The survey was published on social networks through three *Instagram* pages: "*Economia del suicidio*", "*Giurisprudenza del Suicidio*" and "*Ale.conomista*". The first two pages are the largest online Italian communities of economics and law students respectively, while the third is a personal blog page on economics. In the questionnaire, each participant is asked to answer 25 pair wise choice questions (participants have to report their preference between option A and option B). Each alternative gives a monetary reward that will be paid in the future, hence it does not give an immediate payoff to the subject. For instance, participants could choice between the two alternatives as follows: "*Option A: receive 20€ in 14 days; Option B: receive 22€ in 28 days".* In this way, we can observe the behavior of people when they deal with intertemporal choices. In option A the date of payment is always earlier than the one provided by option B. At the end of the experiment, one of the 25 scenarios is randomly selected, and subjects are rewarded according to the option chosen in the extracted scenario.[2] If a subject choose option A for the extracted scenario, the corresponding payoff will be paid on the date indicated in the same option. Especially, we offer a payment delayed by 14 days in option A, while option B will pay the reward in 14 + *n* days. On the factor *n*, hence on the time delay of the payment of option B, we base the three treatments that we run:

- Treatment 1: n=14 days;
- Treatment 2: n=28 days;

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[1] Von Neumann, J., & Morgenstern, O. (1947). *Theory of games and economic behavior*. Princeton, NJ: Princeton University Press.

[2] It is important to stress that in this experiment there are not monetary rewards.

- Treatment 3: n=56 days.

We are interested in grasping the risk preference of subjects and, in order to capture the presence of anomalous behavior such as framing effect or other inconsistent risk preferences, we insert a measure of uncertainty. To do so, we employ the *Ellsberg urn.*[3] The urn contains a known amount of balls of different colours, and each colour has attached a value. For instance, one can say that in the urn there are 50 red balls and 50 yellow balls, and if the subject draws a red ball, he would get a payoff equal to 20$, rather if he draws a yellow one, he would get a payoff equal to 24$. This happens for the risky options, which we insert only in some scenarios of each treatment. In the previous example, suppose that the urn is employed to make uncertain the value of option B, the expected value $E(B)$ of the risky option would be computed as:

$$E(B) = 20\$ * 0.5 + 24\$ * 0.5 = 22\$.$$

For each treatment (*n)* and period (*t)*, the expected value of option A is equal to 20$, and it is always lower than the option B expected value. In this case, one could have: *"Option A: receive 20€ in 14 days, Option B: receive 20€ if a pink ball is drawn (50 out of 100) or receive 24€ if a red ball is drawn (50 out of 100)".* Hence, according to the expected utility theory, the optimal choice would be always to select the option with the largest expected value, namely option B. For each risky option, subjects are told both the composition of the urn, that is the number of balls of each colour in the urn and the payment options associated with each coloured ball. A complete set of scenario options used in the experimental treatments is presented in Appendix.

The three treatments (*n)* are structured in the same way: in each of them, for one set of treatments, the payoffs are certain, and for other sets, the payoffs are probabilistic. The *t*=25 scenarios are divided, for each treatment, as shown in table 1.

Table 1: Option value for each scenario

Period	Value Option A	Value Option B
From *t*=1 to *t*=5	Certain	Certain
From *t*=6 to *t*=13	Certain	Uncertain
From *t*=14 to *t*=19	Uncertain	Certain
From *t*=20 to *t*=25	Uncertain	Uncertain

At the end of the questionnaire, subjects are supposed to fill seven sociodemographic questions. Among these, we ask participants whether they are smoker or not. We asked this information since also

[3] See *Ellsberg, Daniel, "Risk, Ambiguity, and the Savage Axioms." The Quarterly Journal of Economics, vol. 75, no. 4, 1961, pp. 643–669., www.jstor.org/stable/1884324.*

Anderson and Stafford (2009) did in their experiment, and they found a significant relationship between the status of being smoker and the need to get the sooner payoff. Moreover, differently from the Anderson and Stafford experiment, we do not modify the values of the options, but we keep values equal across the three treatments, maintaining the expected value of B always larger than the one of option A, and inserting the uncertainty of the payoff and the time delay in the way we already shown.

2. Econometric analysis

This section deals with the analysis of results of the experiment. We first present the results for certain scenarios, hence from $t=1$ to $t=5$. We consider the first five scenarios in which there are a certain payment of €20 in 2 weeks (option A) and a certain payment of €X in 2weeks + n days (option B). As one can see from the Figure 1, the percentage of subjects choosing later Option B increases as the value option B increases, while the share of option A choice decreases.

Figure 1: Percentage of subject's choices in risk-free scenarios
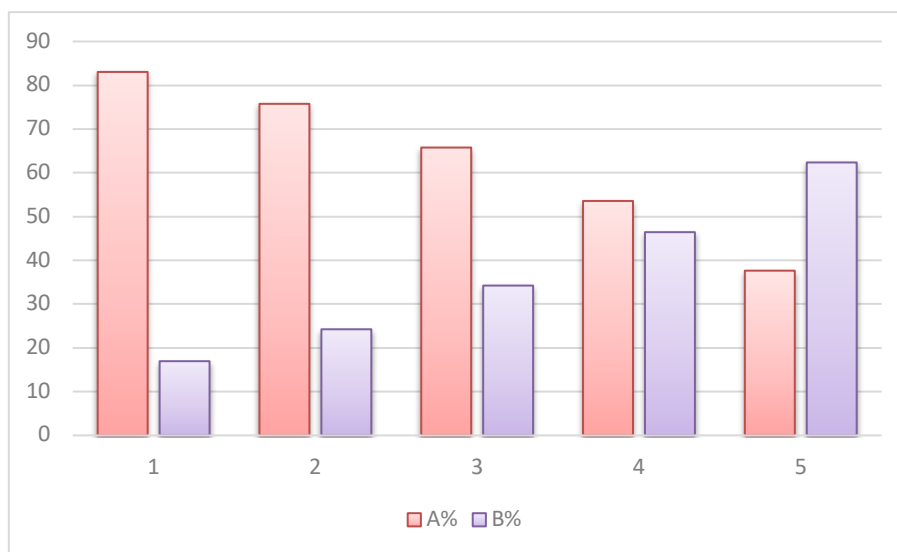
As we expected, when money paid by option B rise, then also the frequency of subjects that report to prefer option B respect to option A increases. However, the number of subjects choosing option B decreases when the time delay between the two options (A and B) increases. This result is visible in Table 2, which shows the frequency of subjects who have chosen option A or B in risk-free scenarios.

Table 2: Frequency of choices in risk-free scenarios (option A or B)

Treatment (time delay)	B value	Subjects choosing option A	Subjects choosing option B
14	22€	473 (79,76%)	120 (20,24%)
	24€	418 (70,49%)	175 (29,51%)
	26€	334 (56,32%)	259 (43,68%)
	28€	244 (41,15%)	349 (58,85%)
	30€	149 (25,13%)	444 (74,87%)
28	22€	224 (85,50%)	38 (14,50%)
	24€	206 (78,63%)	56 (21,37%)
	26€	187 (71,37%)	75 (28,63%)
	28€	160 (61,07%)	102 (38,93%)
	30€	120 (45,80%)	142 (54,20%)
56	22€	351 (86,24%)	56 (13,76%)
	24€	332 (81,57%)	75 (18,43%)
	26€	309 (75,92%)	98 (24,08%)
	28€	272 (66,83%)	135 (33,17%)
	30€	206 (50,61%)	201 (49,39%)

Shares of choices in parentheses. Sample size per treatment is: 593 in n=14; 262 in n=28; 407 in n=56.

In the case of certain scenarios, we show that there is a negative relation between the percentage of subject choosing option B and time delay. This result is visible in Figure 2, in which is clear that the percentage of subjects choosing Option B is smaller the longer the temporal delay between two options.

Figure 2: Percentage of choices in risk-free scenarios across all treatments
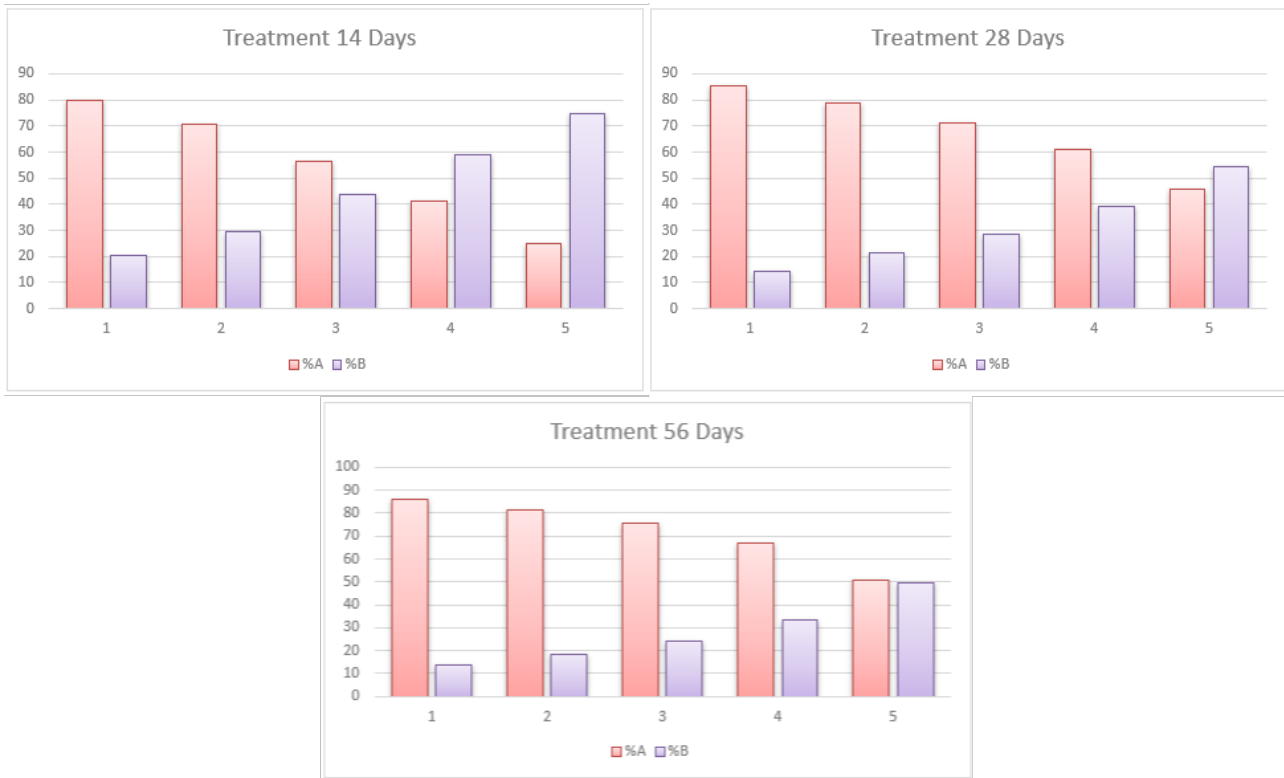
In **Table 3** we report the results of a random effects probit estimation. Results show that the effect of the Average Daily Interest Rate (ADIR) is negative and significant; that means the likelihood of choosing option A decreases when the ADIR increases.[4] Variables ADIR*28 and ADIR*56 are both positive and statistically significant, showing that the probability of choosing A increases with a time delay increase. The coefficient 28 Day Extension is positive but not significant, while the coefficient 56 Day Extension is positive and significant. This means that the longer the temporal extension, the fewer the share of subjects choosing the Option B. Finally, we can observe that demographic variables are not statistically significant.

Table 3: Random effects probit estimation of probability of choosing option A in risk-free scenarios

Explanatory Variable	Coefficient	Robust Standard error
Average daily interest rate (ADIR)	**-1.910*****	0.153
ADIR*28day extension	**0.418***	0.187
ADIR*56day extension	**0.438***	0.176
28 day extension	0.711	0.551
56-day extension	**1.126***	0.529
Gender	-0.0636	0.254
Smoker	0.0350	0.287
Constant	**4.569*****	**0.428**

* p<0.05, ** p<0.01, *** p<0.001

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[4] Specifically: $value\ B\ at\ switch = E(option\ A)(1 + (it))$, where i is the Average Daily Interest Rate (ADIR).

We now present the results for the *risky scenarios*. In scenarios 6 through 25, subjects have to choose between two options in which at least one is risky. We analyzed how the introduction of risk affects subjects in their choices conducting a random effects probit regression of the choice of option A. As shown in **Table 4,** in this regression we add a new variable for understanding some characteristics of risky option. For the coefficients ADIR, ADIR*28 Day Extension, and ADIR*56 Day Extension we have the same results achieved in the previous regression. On the other hand, the coefficient 28 Day Extension and 56 Day Extension are both significant. Consequently, the longer the temporal extension, the less likely participant are to choose option B. The coefficients of Risk in option A, Risk in option B, and Both options risky are negative and significant. This means that when the options become risky the probability of choosing A decreases. Finally, to explain the level of risk, we added a measure of the lottery volatility: option A Standard Deviation and Option B Standard Deviation. The higher standard deviation value, the higher the risk in the lottery. In our results, the coefficients are negative and significant. This indicates that higher levels of risk are associated with lower probability of choosing earlier Option A, regardless of the risk is presented in Option A or Option B. Also in this regression, demographic variables are not significant.

Table 4: Random effects probit estimation of probability of choosing option A (full data set)

Explanatory Variable	Coefficient	Robust Standard error
Average daily interest rate (ADIR)	**-0.722***	0.0213
ADIR*28 day extension	**0.126***	0.036
ADIR*56 day extension	**0.0713***	0.032
Risk in Option A	**-1.129***	0.0478
Risk in Option B	**-0.402***	0.0399
Both options risky	**-1.066***	0.0488
Option A standard deviation	**-0.0367***	0.0046
Option B standard deviation	**-0.0425***	0.0052
28-day extension	**0.352***	0.103
56-day extension	**0.548***	0.089
Gender	0.0392	0 .0742
Smoker	-0.0358	0.0803
Constant	**1.833***	.0852

* p<0.05, ** p<0.01, *** p<0.001

To study the interaction between risk and length of the temporal delay, we used random effects probit estimation adding interactions between Risk in Option A, Risk in Option B, and Both Options Risky

variables with 28 Day Extension and 56 Day Extension. The coefficients Risk in Option A*56 Day Extension and Risk in Option B*56 Day Extension are negative and significant (see **Table 5**). This means that when only Option A is risky, and the temporal length delay is 56 days, the probability of choosing the Option A decreases. Although, when only Option B is risky and the temporal length delay is 56 days, the probability of choosing the Option A decreases.

Table 5: Random effects probit estimation of probability of choosing option A with risk and time interactions (full dataset)

Explanatory Variable	Coefficient	Robust Standard error
Average daily interest rate (ADIR)	-0.708***	0 .022
ADIR*28 day extension	0.132**	0.041
ADIR*56 day extension	0.0152	0.037
Risk in Option A	-1.064***	0.063
Risk in Option A*28day extension	**0.00647**	0 .118
Risk in Option A*56day extension	**-0.232***	0 .105
Risk in Option B	-0.321***	0. .053
Risk in Option B*28day extension	**0.0105**	0.105
Risk in Option B*56day extension	**-0.294****	0.091
Both options risky	-1.024***	0 .066
Both options risky*28day extension	**0.0284**	0 .12
Both options risky*56day extension	**-0.174**	0 .109
Option A standard deviation	-0.0368***	0 .004
Option B standard deviation	-0.0425***	0 .005
28 day extension	0.331*	0.152
56 day extension	0.819***	0.141
Gender	0.0389	0.074
Smoker	-0.0359	0 .0803
Constant	**1.763***	**0.093**

* p<0.05, ** p<0.01, *** p<0.001

3. Conclusion

The main aim of our experiment has been to determine risky intertemporal decision making. We replicated the experiment proposed by Lisa R. Anderson and Sarah L. Stafford (2009) but we administered the questionnaire through Instagram platform, without monetary reward. In our questionnaire, subjects might choose between two different options; the latter could be represented by certain payments or involve lotteries. In addition, different resolution timing, presence and degree of risk, and payoff values characterize the two options. According to the experimental results, we can say that in the non-risk scenarios, the percentage of subjects choosing later Option B increases as the value option B increases. Furthermore, the percentage of subjects choosing Option B is smaller the longer the temporal delay between two options. As in the experiment proposed by Anderson and Stafford, the *Average Daily Interest Rate (ADIR)* is negative correlated with the choice of option A, and it is statistically significant; the higher the ADIR, the lower probability that Option A is selected.

Comparing our results with the authors' ones, we can make some concluding remarks: firstly, we observe that the variable *ADIR*28 Day Extension* is positive and statistically significant for us and negative and statistically significant for them. For the two authors, this means that the effect of ADIR is not always consistent. In our experiment, instead, the effect of ADIR is consistent because results show both *ADIR*28 Day Extension* and *ADIR*56 Day Extension* positive and statistically significant. Anderson and Stafford find that the variable *28 Day Extension* is positive and statistically significant, meaning that the longer the temporal extension, the less subjects choice Option B. In our results, this variable is positive but not significant; anyway, the *56 Day Extension* is positive and statistically significant and this means that the longer the temporal extension, the less subjects choice Option B. This could be caused by the fact that our subjects are more patient. Through a random effects probit regression of decision to choose Option A, we analysed the effects of risk introduction on subject's choice, using data of all treatments. The results for ADIR, ADIR*28 Day Extension and ADIR*56 Day Extension are the same as the previous. We obtained that *28 Day Extension* and *56 Day Extension* were both positive and statistically significant; also the two authors have obtained the same result. It means that the longer the temporal extension, the less subjects choose Option B, and this is consistent with the results we obtained in certain scenarios. Concerning *Risk in Option A, Risk in Option B* and *Both options risky* coefficients, Anderson and Stafford obtained positive and statistically significant coefficients and they justify this result stating that the Option B makes subjects more likely to pick the Option A (early option); this suggests that the presence of the risk makes subjects less patient. Otherwise, we obtained that these coefficients are negative and statistically significant and this shows that risk does not make subjects less patient, regardless of which option is risky. Anderson and Stafford also obtained that *Option A Standard Deviation* and *Option B Standard Deviation* are positive and significant, concluding that higher levels of risk are associated with a higher probability of choosing the earlier option. On the contrary, our results show that *Option A Standard Deviation* and *Option B Standard Deviation* are negative and significant,

implying that higher levels of risk are associated with a lower probability to choose earlier option, regardless of which option is risky. To conclude, we proposed the random effects probit with interactions to understand whether there are interactions between risk and the length of the temporal delay. Anderson and Stafford found that none of the coefficients of these interactions is significant. Otherwise, we obtained that *Risk in Option A*56 Day Extension* and *Risk in option B*56 Day Extension* coefficients are negative and significant, suggesting that there are interactions between risk and the length temporal delay. Specifically, when the option A is risky, the higher length temporal delay the less subjects choose Option A; when the option B is risky, the higher length temporal delay the less subjects choose Option A. Thus, there are evidences that the effect of risk on individual's choice depends on the time between the two payment options.

Appendix

Questionnaire administer and instructions:

Choose one of the two options given. In each of them there are two different days of payment. One of all scenarios will be selected at random by drawing. For instance, if you chose Option A for the scenarios selected by drawing, you will receive the equivalent award.

Remember: only one of 25 scenarios will be selected.

Scenario 1: please choose between the following options:

Option A: receive 20 euros in 14 days

Option B: receive 22 euros in 28 days

Scenario 2: please choose between the following options:

Option A: receive 20 euros in 14 days

Option B: receive 24 euros in 28 days

Scenario 3: please choose between the following options:

Option A: receive 20 euros in 14 days

Option B: receive 26 euros in 28 days

Scenario 4: please choose between the following options:

Option A: receive 20 euros in 14 days

Option B: receive 28 euros in 28 days

Scenario 5: please choose between the following options:

Option A: receive 20 euros in 14 days

Option B: receive 30 euros in 28 days

Scenario 6: please choose between the following options:

   Option A: receive 20 euros in 14 days

   Option B: receive in 28 days, 20 euros if a red ball is drawn (50 out of 100) or 24 euros if a navy ball is drawn (50 out of 100)

Scenario 7: please choose between the following options:

   Option A: receive 20 euros in 14 days

   Option B: receive in 28 days, 16 euros if a red ball is drawn (50 out of 100) or 28 euros if a yellow ball is drawn (50 out of 100).

Scenario 8: please choose between the following options:

   Option A: receive 20 euros in 14 days

   Option B: receive in 28 days, 22 euros if a purple ball is drawn (50 out of 100) or 26 euros if a black ball is drawn (50 out of 100).

Scenario 9: please choose between the following options:

   Option A: receive 20 euros in 14 days

   Option B: receive in 28 days, 24 euros if a navy ball is drawn (50 out of 100) or 28 euros if a yellow ball is drawn (50 out of 100).

Scenario 10: please choose between the following options:

   Option A: receive 20 euros in 14 days

   Option B: receive in 28 days, 20 euros if a red ball is drawn (75 out of 100) or 28 euros if a yellow ball is drawn (25 out of 100).

Scenario 11: please choose between the following options:

   Option A: receive 20 euros in 14 days

   Option B: receive in 28 days, 20 euros if a red ball is drawn (75 out of 100) or 36 euros if a

white ball is drawn (25 out of 100).

Scenario 12: please choose between the following options:

Option A: receive 20 euros in 14 days

Option B: receive in 28 days, 16 euros if a pink ball is drawn (25 out of 100), 20 euros if a red ball is drawn (25 out of 100) or 26 if a black ball is drawn (50 out of 100).

Scenario 13: please choose between the following options:

Option A: receive 20 euros in 14 days

Option B: receive in 28 days, 16 euros if a pink ball is drawn (25 out of 100), 28 euros if a yellow ball is drawn (25 out of 100) or 36 if a white ball is drawn (50 out of 100).

Scenario 14: please choose between the following options:

Option A: receive in 14 days, 18 euros if a gold ball is drawn (50 out of 100), 22 euros if a purple ball is drawn (50 out of 100)

Option B: receive 22 euros in 28 days.

Scenario 15: please choose between the following options:

Option A: receive in 14 days, 18 euros if a gold ball is drawn (50 out of 100), 22 euros if a purple ball is drawn (50 out of 100)

Option B: receive 24 euros in 28 days.

Scenario 16: please choose between the following options:

Option A: receive in 14 days, 18 euros if a gold ball is drawn (50 out of 100), 22 euros if a purple ball is drawn (50 out of 100)

Option B: receive 26 euros in 28 days.

Scenario 17: please choose between the following options:

Option A: receive in 14 days, 16 euros if a pink ball is drawn (75 out of 100), 32 euros if an orange ball is drawn (25 out of 100)

Option B: receive 24 euros in 28 days.

Scenario 18: please choose between the following options:

Option A: receive in 14 days, 12 euros if a clear ball is drawn (50 out of 100), 24 euros if a navy ball is drawn (25 out of 100) or 32 euros if an orange ball is drawn (25 out of 100)

Option B: receive 24 euros in 28 days.

Scenario 19: please choose between the following options:

Option A: receive in 14 days, 12 euros if a clear ball is drawn (25 out of 100), 20 euros if a red ball is drawn (25 out of 100) or 24 euros if a yellow ball is drawn (50 out of 100)

Option B: receive 22 euros in 28 days.

Scenario 20: please choose between the following options:

Option A: receive in 14 days, 18 euros if a gold ball is drawn (50 out of 100), 22 if a purple ball is drawn (50 out of 100)

Option B: receive in 28 days, 20 euros if a red ball is drawn (50 out of 100), 24 euros if a navy ball is drawn (50 out of 100).

Scenario 21: please choose between the following options:

Option A: receive in 14 days, 18 euros if a gold ball is drawn (50 out of 100), 22 euros if a purple ball is drawn (50 out of 100)

Option B: receive in 28 days, 22 euros if a purple ball is drawn (50 out of 100), 26 euros if a black ball is drawn (50 out of 100).

Scenario 22: please choose between the following options:

Option A: receive in 14 days, 18 euros if a gold ball is drawn (50 out of 100), 22 euros if a

purple ball is drawn (50 out of 100)

  Option B: receive in 28 days, 20 euros if a red ball is drawn (75 out of 100), 36 euros if a black ball is drawn (25 out of 100).

Scenario 23: please choose between the following options:

  Option A: receive in 14 days, 16 euros if a red ball is drawn (75 out of 100), 32 euros if a orange ball is drawn (25 out of 100)

  Option B: receive in 28 days, 20 euros if a red ball is drawn (50 out of 100), 24 euros if a navy ball is drawn (50 out of 100).

Scenario 24: please choose between the following options:

  Option A: receive in 14 days, 12 euros if a clear ball is drawn (25 out of 100), 20 euros if a red ball is drawn (25 out of 100) or 24 euros if a yellow ball I drawn (50 out of 100)

  Option B: receive in 28 days, 16 euros if a pink ball is drawn (25 out of 100), 20 euros if a red ball is drawn (25 out of 100) or 26 euros if a black ball is drawn (50 out of 100)

Scenario 25: please choose between the following options:

  Option A: receive in 14 days, 12 euros if a clear ball is drawn (50 out of 100), 24 euros if a navy ball is drawn (25 out of 100) or 32 euros if an orange ball I drawn (25 out of 100)

  Option B: receive in 28 days, 16 euros if a pink ball is drawn (50 out of 100), 28 euros if a yellow ball is drawn (25 out of 100) or 36 euros if a white ball is drawn (25 out of 100)

Are you:

  male

  female

Age:

  18-24

25-34

35-44

45-54

55-64

65+

Do you smoke? If you smoke, indicate the weekly frequency (e.g. If you smoke two packages a week, you should write "2". If you do not smoke, you should write "0")

_____

Occupation:

Self-employed

Employed

Housewife

Unemployed

Student

Other

Inhabitants in the town of residence:

Less of 5000

By 5.000 to 19.999

By 20.000 to 99.999

By 100.000 to 1.000.000

Over 1.000.000

What university did you go to/do you go to?

_____

What faculty did you go to/do you go to?

_____

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