Profit-Sharing vs Price-Fixing Collusion with Heterogeneous Firms

Hattori, Keisuke

23 November 2021

Profit-Sharing vs Price-Fixing Collusion with Heterogeneous Firms*

Keisuke Hattori†

School of Business, Aoyama Gakuin University, JAPAN

Abstract

This paper compares the profitability and sustainability between profit-sharing collusion with side payments and price-fixing collusion without side payments in a two-firm repeated Bertrand game when firms differ in both cost and discount factor. Although profit-sharing collusion yields larger joint profits, bargaining over collusive agreements makes heterogeneous firms prefer different types of collusion: a low-cost (high cost) firm is more likely to adhere to profit-sharing (price-fixing) collusion. If both firms have the same discount factor, profit-sharing collusion is more sustainable. However, price-fixing collusion can be the only sustainable collusion if the efficient firm is more patient than the inefficient firm. Furthermore, we extend profit-sharing collusion by incorporating side payments with different enforcement procedures (i.e., different timing of side payments) and different purposes: to reach agreement and to make the agreement sustainable. Our results provide a theoretical rationale for why firms fail or succeed at reaching and sustaining some forms of collusion.

Keywords: Collusion; Asymmetric costs; Asymmetric discount factors; Side payments; Repeated game

JEL Classification: L41; L13; C73; C78;

*I would like to thank Amihai Glazer, Tadashi Sekiguchi, Takeharu Sogo, and Yusuke Zennyo for their helpful and constructive comments. This work was supported by JSPS KAKENHI Grant Numbers 19K01639.

†E-mail: hattori@busi.aoyama.ac.jp

1 Introduction

What types of collusion do heterogeneous firms agree to and sustain? This paper compares the profitability and sustainability between profit-sharing collusion with side payments and price-fixing (or uniform-pricing) collusion without side payments in a simple Bertrand duopoly model with heterogeneity in both costs and discount factors among firms. In particular, we ask whether some types of one-shot collusive agreements through Nash bargaining can be sustained in the long run by employing an infinitely repeated game with grim-trigger strategies.

It is well known that Bertrand competition with homogeneous products and different (constant) marginal costs results in an equilibrium where the low-cost firm charges a price equal to that of the high-cost firm (Blume, 2003). In this situation, two different types of collusion are possible, depending on the availability of interfirm side payments. One is profit-sharing collusion, in which an efficient firm charges its own monopoly price and makes side payments to an inefficient firm in exchange for the latter effectively withdrawing by charging arbitrarily higher prices. This mechanism of collusion is similar to bid rigging (cover pricing) and exclusive territories. The other is price-fixing collusion, in which firms only agree to set a common (uniform) price, with no side payments such as output quotas or other market-sharing provisions. The price matching guarantee is a typical example of this type of collusion. Compared to profit-sharing collusion, price-fixing collusion has the advantage of entailing less contracting and therefore being less likely to be detected by authorities, but the joint profits are lower than in the former because the existence of cost disparities leads to inefficiencies in the supply of goods. We consider collusive agreements through Nash bargaining on the amount of side payments in profit-sharing collusion and on the common price level in price-fixing collusion and examine the conditions under which the firms can comply with the existing agreements in the long run.

We establish the following results. There is a conflict of interest between firms regarding the preferred type of collusion: an efficient firm prefers profit-sharing collusion, while an inefficient firm prefers price-fixing collusion. If both firms have the same discount factor, then profit-sharing collusion is more sustainable than price-fixing collusion. However, price-fixing collusion can be the only sustainable form of collusion if the efficient firm is more patient than the inefficient firm.

Given that more productive firms may tend to be more patient than less productive firms, this paper offers a new rationale for why efficient profit-sharing collusion is less likely to occur between heterogeneous firms, which is not because of the greater risk of detection by antitrust authorities.

We further extend profit-sharing collusion by considering the following two points. First, we consider the effect of a different timing of side transfers on collusion sustainability. We find that the enforcement procedure in which side payments are made *before* price setting in each period does not affect their profitability, but it does affect their sustainability through changes in their deviation incentives: a low-cost (high-cost) firm's deviation incentives are eliminated (increased) by the procedure. Second, we consider the case in which side payments can be designed to achieve the long-term goal of sustaining collusion and to achieve the short-term goal of making collusion agreeable. We find that such collusion is necessarily more sustainable than price-fixing collusion. The existence of cost dispersion requires a greater average of the firms' discount factors to make such collusion sustainable than that under symmetric costs.

Our study contributes to the literature on collusion in the following ways: first, in a simple Bertrand duopoly framework with two types of firm heterogeneities, in costs and discount factors, we find that the association between costs and discount factors determines the type of sustainable collusion. Second, we elaborate on side payments in profit-sharing collusion by considering different enforcement procedures (i.e., distinguishing the timing of side transfers) and different objectives (i.e., distinguishing between those for reaching agreement and those for sustaining the agreement). Third, we show that heterogeneous firms may choose price-fixing collusion in the natural situation where inefficient firms are more myopic than efficient firms, for reasons different from the small risk of detection by antitrust authorities that is traditionally considered to be an advantage of uniform pricing.

The remainder of the paper is organized as follows. The next section provides an overview of the related literature. Section 3 presents the basic structure of the model. Section 4 analyzes one-shot agreement and the long-term sustainability of profit-sharing collusion, and Section 5 does the same for price-fixing collusion. Section 6 compares the profitability and sustainability of the two types of collusion and derives our main result that only price-fixing collusion can be sustainable if an inefficient firm is more myopic than an efficient firm. Section 7 extends the profit-sharing collusion by considering a different timing of side transfers and different purposes of the side payments and compares their sustainability with that of price-fixing collusion. Section 8 concludes the paper.

2 Related Literature

The relationship between firm heterogeneity and the feasibility of collusion has attracted the attention of economists and policy makers. The existing literature can be divided into two broad categories: static analysis, which focuses on how collusive agreement is reached, and dynamic analysis, which examines the sustainability of the agreement using a repeated-games approach. One of the important studies in the former is Osborne and Pitchik (1983) that uses the Nash bargaining solution (hereafter, NBS) to consider profit-sharing collusion in a price-setting duopoly model with heterogeneous capacity constraints. Regarding quantity-setting collusion among firms with heterogeneous costs, Schmalensee (1987) considers four different types for effecting collusion, including one in which firms divide the joint profit through Nash bargaining. By using the NBS to determine the distribution of joint profits (i.e., the side payments) or the common price, our study considers some types of one-shot collusive agreement with and without side payments in price-setting (Bertrand) duopoly with different marginal costs.

Various studies have examined the effects of heterogeneity among firms on the sustainability of collusive agreement in repeated-games (supergames) frameworks beginning with Friedman (1974). Some studies find that heterogeneous costs may hinder collusion.[1] For example, Bae (1987) considers the sustainability of tacit collusion and solves the market-sharing problem with using a price-setting supergame between firms with different costs. Subsequently, Harrington (1991), which is the most relevant to our paper, considers the sustainability of optimal collusive agreement for setting a common price and market-sharing rule by applying the NBS. These two studies do not tell us what type of collusion is preferred by and sustainable for different firms because they neither account for the heterogeneity of discount factors across firms nor distinguish between collusion with

[1] Some studies consider tacit collusion in Bertrand supergames with heterogeneous capacities rather than costs among firms (Davidson and Deneckere, 1984, 1990; Compte et al., 2002; Bos and Harrington, 2010; Garrod and Olczak, 2017, 2018; among many others). For surveys of theoretical and empirical studies on collusion in infinitely repeated games, see Ivaldi et al. (2003) and Feuerstein (2005).

side payments and collusion with market-sharing agreements.[2]

Another research stream has analyzed the effect of cost asymmetry on the sustainability of collusion in quantity-setting (Cournot) supergames. For example, Rothschild (1999) focuses on heterogeneous and convex production costs; Vasconcelos (2005) considers the effect of mergers on the sustainability of collusion with cost asymmetry that stems from the different shares of a specific asset that affects marginal costs; and Correia-Da-Silva and Phiho (2016) consider a profit-sharing rule that maximizes the sustainability of collusion, which bears some similarity to the profit-sharing collusion with the long-term goal of sustaining collusion in our study.[3] Since all of these studies assume convex (quadratic) production costs, even a high-cost firm can be assigned a positive production quota in joint-profit maximization and can also obtain positive profits in punishment (competition) phases. In contrast, in our simple Bertrand setting with asymmetric constant marginal costs, the high-cost firm receives a side payment from the low-cost firm as compensation for its de facto exit from the market, which achieves the maximization of the joint profit. In addition, in punishment phases, the high-cost firm cannot earn a positive profit, but its existence is a source of lower profits for the low-cost firm.[4]

There are few studies that consider heterogeneity in discount factors across firms. Using an n-firm price-setting model with heterogeneous discount factors (but homogeneous production costs), Harrington (1989) shows that price collusion can be sustainable if the firms' *average* discount factor exceeds $\frac{n-1}{n}$ and that the optimal collusive outcome requires a strict ordering of output quotas such that myopic firms obtain larger output quotas and profits. Obara and Zincenko (2017) further develop Harrington's (1989) results and provide a complete characterization of the possible collusion. As these studies demonstrate, the heterogeneity in discount factors can have a significant impact on the collusive allocation: a firm can benefit from having a smaller discount factor. In

[2]By considering optimal punishment schemes and allowing side payments, Miklós-Thal (2011) shows that cost asymmetry may facilitate collusion. Although our study only considers grim-trigger strategies as punishment schemes, it differs from Miklós-Thal (2011) by more precisely considering the side-payment procedures under asymmetric discount factors and asymmetric costs and comparing the sustainability of different types of collusion.

[3]By using a repeated Cournot model with heterogeneous production costs, Verboven (1997) shows that the allocation at which firms are indifferent between colluding and defecting is the enforceable collusive agreement.

[4]Brandao et al. (2014) examine the sustainability of collusion under cost heterogeneity between incumbents and entrants. They show that committing to a profit-sharing rule in which joint profits are divided through bargaining may make collusion sustainable before and after entry. Although their study differs from ours in several respects, both suggest the importance of sustaining the one-shot Nash bargaining allocation rule as a long-term collusive goal.

contrast, our study considers various types of collusion that clearly distinguish between differences in bargaining power due to differences in discount factors and those due to cost disparities.[5]

There are various empirical studies on the impact of cost heterogeneity on the sustainability of collusion in a repeated-games model. As analyzed in our price-fixing collusion, in reality, market share transfer (i.e., side payments) is often difficult to implement in a Bertrand environment with heterogeneous firms. Price cartels, especially in online and offline retail markets with price displays, cannot directly control where consumers purchase goods. In this regard, Clark and Houde (2013) provides empirical evidence of an intertemporal transfer mechanism employed in gasoline cartels in Canada that exploits delayed price adjustments: the cartel systematically allows the firms with the lowest costs to move last during price-increase phases, thereby giving them a transfer (i.e., a larger share of the market). Goto and Iizuka (2016) examine to what extent cost heterogeneity affects the sustainability of collusion by drawing data from a failed flu-shot cartel in Japan. In a recent contribution, Igami and Sugaya (2021) present a structural analysis that investigates the effect of a merger among firms with heterogeneous costs on the sustainability of vitamin cartels in the 1990s. Our simple theoretical model suggests the empirical importance not only of elucidating how collusion is sustainable among heterogeneous firms but also of identifying the types of collusion in which firms choose to engage.

We study the effect of different timings of side transfers in profit-sharing collusion and show that the procedure with side payments before pricing eliminates the incentive of the sender (i.e., the low-cost firm) to deviate from collusion, which is closely related to Mouraviev and Rey (2011) that explores the role of price/quantity leadership in facilitating collusion. They show that the deviations by the price-setting leader (first mover) can be more immediately punished without waiting for the next period, which reduces its incentive to deviate.

In sum, to the best of our knowledge, there is no other study that compares the sustainability of several collusive schemes using a simple Bertrand competition model with two types of heterogeneity in production costs and discount factors among firms, where the collusive agreements in the short

[5]Dal Bó (2007) considers how changes in the interest rate affect collusive prices and profits through their effect on firms' discount factors. Although his study does not include heterogeneity in production costs or in firms' discount factors, he shows that the interest rate faced by firms has a significant impact on the formation of collusive agreements. This is also important for our study, which examines how different interest rates faced by firms due to cost disparity affect the sustainability of price collusion.

run are considered separately from compliance with them (i.e., sustainability) in the long run.

3 The Model

Consider two firms (1 and 2) that engage in a standard Bertrand price competition with homogeneous products. Each firm $i \in \{1, 2\}$ has a constant marginal cost of production, c_i. Without loss of generality, we assume that firm 1 is more cost-efficient than firm 2:

$$c_1 = c - \varepsilon \quad \text{and} \quad c_2 = c + \varepsilon,$$

where $c > 0$ is a positive constant and $\varepsilon \in [0, c)$ represents the technology gap between firms. The market demand for each firm i is given by

$$q_i(p_i, p_j) = \begin{cases} \bar{a} - p_i & \text{if} \;\; p_i < p_j, \\ (\bar{a} - p_i)/2 & \text{if} \;\; p_i = p_j, \\ 0 & \text{if} \;\; p_i > p_j, \end{cases}$$

where $\bar{a} > 0$ represents a market size and p_i represents a price charged by firm i. The firm that charges the lowest price serves all market demand.

The profits of firm i are given by $\pi_i(p_i, p_j) = (p_i - c_i) \cdot q_i(p_i, p_j)$. We assume that the above one-shot price competition is infinitely repeated with perfect monitoring. We allow each firm i to have a different discount factor δ_i. All aspects of the environment are common knowledge.

Before we proceed to consider price collusion between firms, we derive the competitive and monopoly equilibria. In the case of competition (i.e., standard Bertrand competition), the equilibrium is characterized by

$$p_1^C = c_2, \quad \pi_1^C = (c_2 - c_1)(\bar{a} - c_2), \quad \pi_2^C = 0, \tag{1}$$

where the superscript C refers to the variable in the competition case. The more cost-effective firm (firm 1) chooses a price equal to c_2 and earns positive profits, while the less cost-effective firm (firm 2) earns zero profits (Blume, 2003). The outcome will, for all types of collusion that we study in

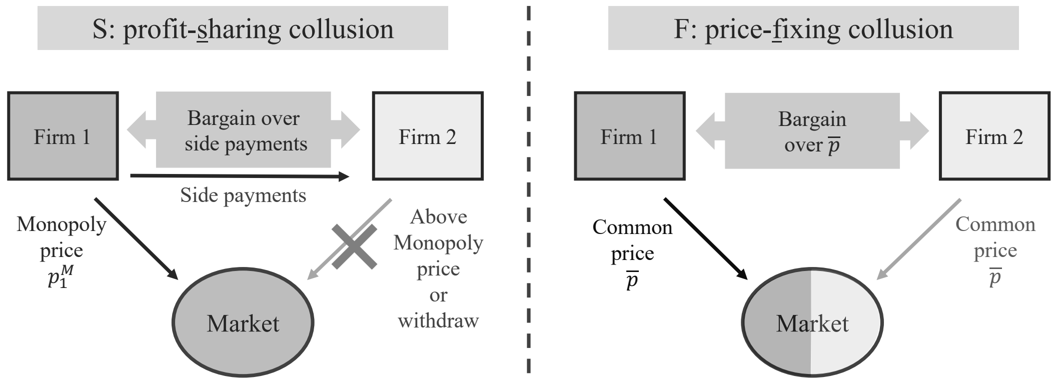

Figure 1: Profit-sharing and price-fixing collusions

this paper, not only be the disagreement point of one-shot Nash bargaining problems but also be the profits in the punishment phase of an infinitely repeated game.

If firm i monopolizes the market (i.e., if there are no competitors), then the outcome is

$$p_i^M = \mathrm{argmax}_p(\bar{a} - p)(p - c_i) = \frac{\bar{a} + c_i}{2}, \quad \pi_i^M = \left(\frac{\bar{a} - c_i}{2}\right)^2, \tag{2}$$

where the superscript M refers to the variable in the monopoly case.

For expositional reasons, we define $a \equiv \bar{a} - c > 0$.

Assumption 1. $a > 13\varepsilon$

This assumption requires market size, a, to be sufficiently large that any type of price collusion between the two firms will at least result in a larger profit for each firm (especially for firm 1) than if they were to compete. The implications of this assumption will be discussed later.

We consider two types of collusive agreement: profit-sharing and price-fixing agreements, which are visualized in Figure 1. Profit-sharing collusion (which we also call "Collusion S") is an agreement between firms under which the firm with inefficient production technology (firm 2) *de facto* exits the market, allowing the rival firm with efficient production technology (firm 1) to monopolize the market at the monopoly price of p_1^M, in exchange for receiving the agreed-upon amount of side payments from the rival.[6] In contrast, price-fixing collusion (which we also call "Collusion F")

[6]Such price collusion is similar to what is known as cover (or complementary) bidding in public procurement, under which cooperating bidders agree to submit higher priced bids to ensure the selection of the designated winner

is an agreement between firms only to set a certain common price without exchanging any side payments such as output quotas or other market-sharing provisions. We assume that there is no risk of detection (or equivalently, the probability of detection is the same) for the two types of collusion.

4 Collusion S: profit-sharing collusion with side payments

This section investigates profit-sharing collusion (Collusion S), in which a more cost-effective firm 1 charges its monopoly price p_1^M and firm 2 de facto withdraws from the market by charging a higher price than p_1^M in exchange for obtaining some side payments from firm 1. We first consider a one-shot profit-sharing agreement between firms that is represented as NBS, and then explore the conditions under which the agreement is sustainable in an infinitely repeated game with grim-trigger strategies.[7]

4.1 One-shot Nash bargaining solution

When side payments are possible, the firms can earn maximum joint profits at minimal cost and then redistribute the profits. The price pair $p_1^S = p_1^M$ and $p_2^S = p_2 > p_1^M$ yields the maximum attainable profits π_1^M, where the superscript S refers to the equilibrium variable under Collusion S. The amount of side payments from firm 1 to firm 2, denoted by τ, is determined by Nash bargaining with disagreement point (π_1^C, π_2^C). The Nash bargaining problem is given by[8]

$$\max_{\tau} \left(\left(\pi_1^M - \tau \right) - \pi_1^C \right) \left(\tau - \pi_2^C \right).$$

at higher prices. It also has similar characteristics to the type of price collusion called "segregation" or "exclusive territories", under which firm A (B) monopolizes market A (B), instead of competing in both markets. Note that the side payments need not to be monetary: for example, in-kind compensation or concessions made in one of the other markets if the same firms are active in more than one market (Ivaldi et al., 2003) In any case, joint profit maximization under Collusion S is not possible without making side payments to each other.

[7] Although side payments are generally prohibited by antitrust laws, there is evidence that illegal payments, monetary or otherwise, are nonetheless being made to establish collusion. For example, Clark and Houde (2013) empirically identify a transfer mechanism based on adjustment delays during price changes in gasoline cartels in Canada.

[8] The solution of the bargaining problem is equivalent to the solution of the bargaining problem of $\max_{\alpha_1}(\alpha_1\pi_1^M - \pi_1^C)((1-\alpha_1)\pi_2^M - \pi_2^C)$, where $\alpha_1 \in [0,1]$ represents firm 1's share of the joint profits. We adopt the formulation that the side payment is determined through bargaining because it is convenient to consider some extensions of the side payment agreement.

The first-order condition yields

$$\tau^S = \frac{\pi_1^M - \pi_1^C}{2}.$$

Thus, we characterize each firm's profits under the bargaining solution as

$$\pi_1^S = \frac{\pi_1^M + \pi_1^C}{2} = \frac{a^2 + 10a\varepsilon - 7\varepsilon^2}{8}, \tag{3}$$

$$\pi_2^S = \tau^S = \frac{(a - 3\varepsilon)^2}{8}. \tag{4}$$

We assume the procedure in which, in each period, firm 1 transfers τ^S amount of money to firm 2 only after both firms have confirmed that the agreed pricing p_1^S and p_2^S has been achieved. The timing at which side transfers take place matters in examining the sustainability of the collusive bargaining solution in the repeated-game setting, as will be discussed below in Section 7.1.

4.2 Sustainability

Now, we consider the sustainability of the above agreement of $\{p_1^S, p_2^S, \tau^S\}$ in an infinitely repeated game with grim-trigger strategies. We assume that each firm i maximizes its discounted stream of profits with discount factor δ_i. When both firms cooperatively maintain Collusion S, firms 1 and 2 obtain π_1^S and π_2^S given by (3) and (4); when firm 1 unilaterally deviates from the agreement, it obtains

$$\pi_{1D}^S = \pi_1^M = \left(\frac{a + \varepsilon}{2}\right)^2 \tag{5}$$

by not giving firm 2 side payments in the deviation period, but it obtains only the punishment profits π_1^C in the following periods; when firm 2 unilaterally deviates from the agreement, it sets its price slightly below p_1^M but cannot obtain side payments from firm 1, which yields the following deviation profits:

$$\pi_{2D}^S = \pi_2^M|_{p_2 = p_1^M} = (p_1^M - c_2)(\bar{a} - p_1^M) = \left(\frac{a + \varepsilon}{2}\right)\left(\frac{a - 2\varepsilon}{2}\right). \tag{6}$$

After the deviation period, firm 2 obtains only the punishment profits $\pi_2^C = 0$ in the following periods. Thus, Collusion S is sustainable (i.e., a subgame perfect Nash equilibrium) if the following incentive compatibility constraint is satisfied for $i = 1, 2$:

$$\frac{\pi_i^S}{1-\delta_i} > \pi_{iD}^S + \frac{\delta_i \pi_i^C}{1-\delta_i}. \tag{7}$$

Substituting (1), (3), (4), (5), and (6) into (7), we find that firm 1 cooperates if

$$\delta_1 > \delta_1^S \equiv 1/2, \tag{8}$$

and firm 2 cooperates if

$$\delta_2 > \delta_2^S \equiv \frac{a+5\varepsilon}{2(a+\varepsilon)}, \tag{9}$$

where δ_1^S and δ_2^S represent the critical discount factors for firms 1 and 2, respectively, to favor collusion over deviation. This implies that the high-cost firm, the recipient of the transfer, necessarily has stronger incentives to deviate from the profit-sharing collusion than the low-cost firm, the sender of the transfer.

From (8) and (9), we obtain

$$\delta_2^S - \delta_1^S = \frac{2\varepsilon}{a+\varepsilon} > 0, \quad \frac{d\delta_1^S}{d\varepsilon} = 0, \quad \text{and} \quad \frac{d\delta_2^S}{d\varepsilon} = \frac{2a}{(a+\varepsilon)^2} > 0.$$

Now we have the following lemma.

Lemma 1. *Consider the profit-sharing agreement given by* $\{p_1^S, p_2^S, \tau^S\}$*. Firm 1 cooperates if* $\delta_1 > \delta_1^S = 1/2$*, whereas firm 2 cooperates if* $\delta_2 > \delta_2^S$ *given in* (9)*, where* $\delta_1^S < \delta_2^S$ *holds for* $\varepsilon > 0$*. It holds that* $d\delta_1^S/d\varepsilon = 0$ *and* $d\delta_2^S/d\varepsilon > 0$*: the larger the difference in production cost, the more difficult it is for the inefficient firm to cooperate.*

The critical discount factor for the inefficient firm to comply with the profit-sharing agreement is larger than that for the efficient firm. Firm 1's immediate gains from unilateral deviation from Collusion S, τ^S, are equal to its future deviation losses, $\pi_1^S - \pi_1^C$, for all $\varepsilon > 0$, which leads to $\delta_1^S = 1/2$. In contrast, firm 2's immediate gains from unilateral deviation from Collusion S, $\pi_{2D}^S - \tau^S$, are always greater than its future deviation losses, τ^S. Therefore, firm 2's critical discount factor is larger than 1/2 for $\varepsilon > 0$. In addition, greater cost dispersion reduces τ^S, which will give firm 2 a larger incentive to deviate from cooperation.

There are two possible variants of profit-sharing collusion in which the efficient firm makes side payments to the inefficient firm to achieve the Pareto-optimal (joint) profits for them. First, if we consider an alternative procedure whereby the transfer of side payments in each period is preceded by collusive pricing, then the incentives for each firm to deviate from cooperation are very different from those presented here, which will be shown in Section 7.1.

Second, there could be a situation where the efficient firm increase the side payments in each period, with the aim of preventing the inefficient firm from deviating from cooperation. Such an increase in side payments has the effect of decreasing the critical discount factor necessary for the inefficient firm to adhere to the collusive agreement while simultaneously increasing that necessary for the efficient firm to adhere to it, which will also be shown in Section 7.2.

5 Collusion F: Price-fixing collusion without side payments

This section considers a more common type of price collusion, a price-fixing agreement (Collusion F), in which firms only agree to set a common price and not to engage in any side-payment schemes such as market-share transfers or monetary compensation.[9] As in the previous section, we first consider a one-shot collusive price-fixing agreement between firms that is represented as the NBS and then explore the conditions under which the agreement is sustainable in an infinitely repeated game with grim-trigger strategies.

5.1 One-shot Nash bargaining solution

In Collusion F, firms only agree to set a common (uniform) price, without any side payments such as output quotas or other market-sharing provisions. When coordinating on a common price, both firms cooperatively agree to charge the same price level $\bar{p}$ to maximize the product of their profit

[9] As mentioned by Clark and Houde (2013), transfer schemes based on unequal market divisions are difficult to implement, especially in markets with price displays, because the colluding firms do not have direct control over where consumers purchase the (homogeneous) goods.

gains. The Nash bargaining problem with disagreement point (π_1^C, π_2^C) is given by

$$\max_{\bar{p}} \ \Gamma \equiv \left(\pi_1\left(\bar{p}, \bar{p}\right) - \pi_1^C\right)\left(\pi_2\left(\bar{p}, \bar{p}\right) - \pi_2^C\right).$$

We have the following first-order condition:

$$\frac{\partial \Gamma}{\partial \bar{p}} = \underbrace{\frac{\partial \pi_1(\bar{p}, \bar{p})}{\partial \bar{p}}}_{-} \underbrace{(\pi_2(\bar{p}, \bar{p}) - 0)}_{+} + \underbrace{\frac{\partial \pi_2(\bar{p}, \bar{p})}{\partial \bar{p}}}_{+} \underbrace{(\pi_1(\bar{p}, \bar{p}) - \pi_1^C)}_{+} = 0.$$

Despite the simplicity of the model setup, it is difficult to derive the explicit solution for $\bar{p}$. However, we can easily identify the range of $\bar{p}$ that satisfies the above first-order condition. Since $\text{argmax}_{\bar{p}} \ \pi_1(\bar{p}, \bar{p}) = p_1^M < p_2^M = \text{argmax}_{\bar{p}} \ \pi_2(\bar{p}, \bar{p})$ holds for $c_1 < c_2$, the NBS of the common price $\bar{p} = p^F$ should lie between p_1^M and p_2^M (that is, $\partial \pi_1(\bar{p}, \bar{p})/\partial \bar{p} < 0$ and $\partial \pi_2(\bar{p}, \bar{p})/\partial \bar{p} > 0$ hold in the first-order condition). The superscript F indicates the variables under Collusion F.

The optimal collusive common price p^F is higher than the price that a low-cost firm would set if it were a monopolist, which is also shown by Harrington (1991). In his study, firms are assumed to be able to collude both on the common price and a market-sharing rule by which consumer demand is allocated between firms. In contrast, our collusion F is an agreement only to set a common price, so the market share is equally divided. This reflects the environment in retail markets with price labeling, where collusion cannot control where consumers buy goods.

Taking $p^F \in (p_1^M, p_2^M)$ into account, the profits of each firm in NBS should satisfy the following:

$$\pi_1^F \in \left(\underline{\pi}_1^F, \bar{\pi}_1^F\right), \quad \pi_2^F \in \left(\underline{\pi}_2^F, \bar{\pi}_2^F\right), \tag{10}$$

where

$$\underline{\pi}_i^F \equiv \pi_i(p_j^M, p_j^M) = \frac{\pi_i^M|_{p_i = p_j^M}}{2} \quad \text{and} \quad \bar{\pi}_i^F \equiv \pi_i(p_i^M, p_i^M) = \frac{\pi_i^M}{2} \tag{11}$$

represent the minimum and maximum attainable profits by Collusion F, respectively. Note that $\lim_{\varepsilon \to 0} \ p^F = p_1^M = p_2^M$ and $\lim_{\varepsilon \to 0} \ \pi_i^F = \pi_1^M/2 = \pi_2^M/2$. In addition, each firm's profits under Collusion F must be higher than its profits under competition (or at the disagreement point). Thus, both firms will benefit from price-fixing collusion because

$$\underline{\pi}_1^F - \pi_1^C = \frac{1}{8}(a - 13\varepsilon)(a - \varepsilon) > 0,$$
$$\underline{\pi}_2^F - \pi_2^C = \frac{1}{8}(a - 3\varepsilon)(a + \varepsilon) > 0,$$

where the inequalities are assured by Assumption 1.

5.2 Sustainability

Now, we consider the sustainability of Collusion F in an infinitely repeated game. When both firms cooperatively maintain Collusion F, firms 1 and 2 obtain π_1^F and π_2^F; when firm 1 unilaterally deviates from cooperation, it obtains

$$\pi_{1D}^F = \pi_1^M = \left(\frac{a+\varepsilon}{2}\right)^2$$

by charging $p_1 = p_1^M < p^F$ in the deviation period, but it obtains only π_1^C in the following punishment periods; when firm 2 unilaterally deviates from cooperation, it obtains

$$\pi_{2D}^F = \pi_2^M|_{p_2 = p^F}$$

by charging a price (slightly below) p^F in the deviation period because firm 2's optimal (monopoly) price is above p^F. After the deviation period, it obtains punishment profits ($\pi_2^C = 0$). Therefore, we have the following incentive compatibility constraint for firm i in Collusion F:

$$\frac{\pi_i^F}{1-\delta_i} > \pi_{iD}^F + \frac{\delta_i \pi_i^C}{1-\delta_i}.$$

Since $\pi_1^F > \underline{\pi}_1^F = \pi_1(p_2^M, p_2^M) = (\pi_1^M|_{p_1 = p^F})/2$, the sufficient condition for firm 1 to cooperate is as follows:

$$\delta_1 > \bar{\delta}_1^F \equiv \frac{\pi_1^M - \underline{\pi}_1^F}{\pi_1^M - \pi_1^C} = \frac{(a+\varepsilon)^2 + 4\varepsilon^2}{2(a-3\varepsilon)^2}. \tag{12}$$

Similarly, the necessary condition for firm 1 to cooperate is obtained by

$$\delta_1 > \underline{\delta}_1^F \equiv \frac{\pi_1^M - \bar{\pi}_1^F}{\pi_1^M - \pi_1^C} = \frac{(a+\varepsilon)^2}{2(a-3\varepsilon)^2}. \tag{13}$$

Hereafter, we define the critical discount factor for firm 1 to adhere to Collusion F as $\delta_1^F \in [\underline{\delta}_1^F, \bar{\delta}_1^F]$.

Since $\pi_{2D}^F = 2\pi_2^F$ and $\pi_2^C = 0$ hold for $\varepsilon > 0$, we find that firm 2 cooperates if

$$\delta_2 > \delta_2^F \equiv 1/2.$$

Now, we have the following lemma:

Lemma 2. *Consider the price-fixing agreement p^F. Firm 1 cooperates if $\delta_1 > \delta_1^F \in [\underline{\delta}_1^F, \bar{\delta}_1^F]$ given in (12) and (13), whereas firm 2 cooperates if $\delta_2 > 1/2$, where $\delta_2^F < \delta_1^F$ holds for $\varepsilon > 0$. It holds that $d\delta_1^F/d\varepsilon > 0$ and $d\delta_2^F/d\varepsilon = 0$: the larger the difference in production costs, the more difficult it is for the efficient firm to cooperate..*

In contrast to Collusion S, the critical discount factor for the efficient firm to adhere to the price-fixing agreement is larger than that for the inefficient firm. This is because the larger the cost dispersion is, the larger the inefficiency of the collusion (i.e., $p_1^M - p^F$) and thus the greater the incentive for firm 1 to deviate from Collusion F. In contrast, since $p^F < p_2^M$, the deviation price for firm 2 is (slightly below) p^F, firm 2's immediate deviation gains, $\pi_2^M|_{p_2=p^F} - \pi_2^F = \pi_2^F$, are equal to its future deviation losses, π_2^F, which leads to $\delta_2^F = 1/2$, which is independent of ε.

6 Comparisons

This section compares Collusion S and F in terms of their profitability and sustainability. First, we have the following proposition on the comparison of the profitability.

Proposition 1.

Suppose the Assumption 1 holds. Then, for any $\varepsilon \in (0, c)$,

(a) $\pi_1^S > \pi_1^F$ *and* $\pi_2^S < \pi_2^F$,

(b) $\dfrac{d(\pi_1^S - \pi_1^F)}{d\varepsilon} > 0$ *and* $\dfrac{d(\pi_2^F - \pi_2^S)}{d\varepsilon} > 0.$

Proof:

(a) From (3), (10), and (11), we have

$$\pi_1^S = \frac{\pi_1^M + \pi_1^C}{2} > \frac{\pi_1^M}{2} = \bar{\pi}_1^F.$$

Additionally, from (4), (10), and (11), we have

$$\underline{\pi}_2^F - \pi_2^S = \frac{(a - 3\varepsilon)\varepsilon}{2} > 0.$$

Thus, we have that $\pi_1^S > \pi_1^F$ and $\pi_2^S < \pi_2^F$ necessarily hold for any $\varepsilon > 0$.

(b) From (3), (4), (10), and (11), we have

$$\frac{d\left(\pi_1^S - \bar{\pi}_1^F\right)}{d\varepsilon} = a - 2\varepsilon > 0, \qquad \frac{d\left(\pi_1^S - \underline{\pi}_1^F\right)}{d\varepsilon} = a - \varepsilon > 0,$$
$$\frac{d\left(\bar{\pi}_2^F - \pi_2^S\right)}{d\varepsilon} = \frac{a - 4\varepsilon}{2} > 0, \qquad \frac{d\left(\underline{\pi}_2^F - \pi_2^S\right)}{d\varepsilon} = \frac{a - 6\varepsilon}{2} > 0. \quad \square$$

The proposition indicates that (a) the more cost-efficient firm prefers Collusion S, whereas the less cost-efficient firm prefers Collusion F, and (b) this tendency will be stronger when the cost dispersion between firms is larger. We illustrate the intuition behind this proposition in Figure 2 that depicts NBSs for Collusion S and F in the case of $\bar{a} = 20$, $c = 2$, and $\varepsilon = 1$. Under Collusion S, the efficient production allocation yields the largest possible joint profits and the feasible locus of profit pairs obtainable through agreement is represented by the area below the bold line of $\pi_1 + \pi_2 = \pi_1^M$. Given that the disagreement point is $d(\pi_1^C, \pi_2^C)$ and the curve tangent to the line is the Nash product, the point $S(\pi_1^S, \pi_2^S)$ is shown to be the NBS for Collusion S. In contrast, the pairs of profits achievable through a price-fixing agreement are represented by the shaded areas (surrounded by the dashed curve).[10] The greater the cost disparity, the more the northeast vertex of this region moves inward, away from the line of $\pi_1 + \pi_2 = \pi_1^M$. The NBS of

[10]To understand the shape of the set of possible profit pairs obtainable through a price-fixing agreement in this diagram, the following explanation is helpful. When $p^F = c_2$, we have $\pi_1^F = .5\pi_1^C$ and $\pi_2^F = 0$, which are depicted by the intersection of the dashed curve and vertical axis (not the origin). As p^F increases, the profits of both firms increase, and π_1^F reaches its maximum value when $p^F = p_1^M$. In the range $p_1^M < p^F < p_2^M$, π_1^F is decreasing and π_2^F is increasing in p^F, and π_2^F reaches its maximum value when $p^F = p_2^M$. From the point, as the common price rises, the profits of both firms decrease, reaching the origin when $p^F = a$.

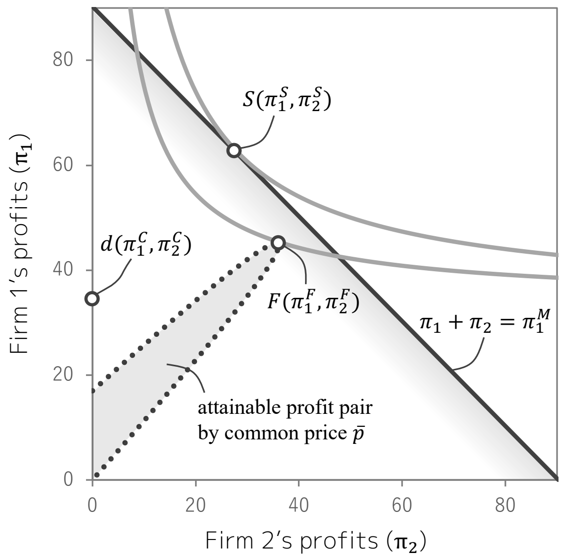

Figure 2: NBS for Collusion S and F: The case of $\bar{a} = 20$, $c = 2$, and $\varepsilon = 1$

Collusion F is illustrated by the point $F(\pi_1^F, \pi_2^F)$, the tangent between this region and the Nash product. The NBS of $S(\pi_1^S, \pi_2^S)$ is necessarily located to the northwest of the NBS of $F(\pi_1^F, \pi_2^F)$, which represents a conflict of interest with firm 1 preferring Collusion S and firm 2 preferring Collusion F. The greater the cost dispersion between the firms, the larger firm 1's profit is under competition, so the disagreement point moves upward on the vertical line, which makes firm 1 prefer Collusion S more and firm 2 prefer Collusion F more.

Next, we have the following proposition on the comparison of the sustainability of two collusive agreements.

Proposition 2.

Suppose that Assumption 1 holds. Then, for any $\varepsilon \in (0, c)$,

(*a*) $\delta_1^S = \delta_2^F < \delta_2^S < \delta_1^F$,

(*b*) $\dfrac{d(\delta_2^S - \delta_2^F)}{d\varepsilon} > 0$ *and* $\dfrac{d(\delta_1^F - \delta_2^S)}{d\varepsilon} > 0$.

Proof:

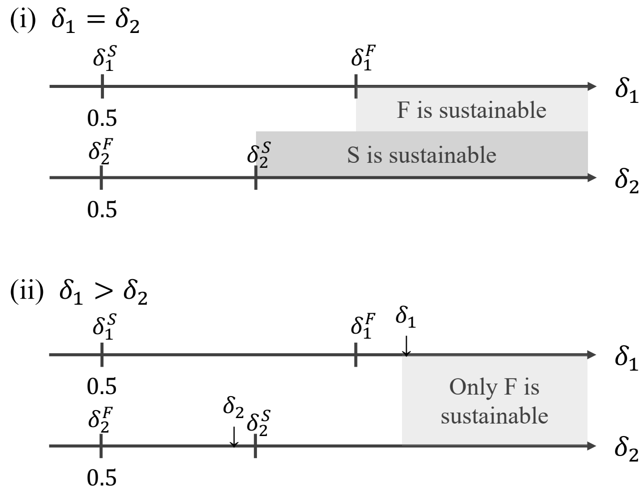

Figure 3: Comparison of sustainability

From Lemmas 1 and 2, we have $\delta_2^S = \delta_1^F = 1/2$, $\delta_2^S > \delta_1^S$, and

$$\underline{\delta}_1^F - \delta_2^S = \frac{2\varepsilon[(a-2\varepsilon)(a+8\varepsilon)+5\varepsilon^2]}{(a-3\varepsilon)^2(a+\varepsilon)} > 0,$$

implying that $\delta_2^S = \delta_1^F = 1/2 < \delta_2^S < \delta_1^F$ holds for $\varepsilon > 0$. In addition, we have

$$\frac{d(\underline{\delta}_1^F - \delta_2^S)}{d\varepsilon} = \frac{2a\left\{(a+11\varepsilon)(a-\varepsilon)^2+6a^2\varepsilon+18\varepsilon^3\right\}}{(a-3\varepsilon)^3(a+\varepsilon)^2} > 0,$$

which indicates that the difference between δ_1^F and δ_2^S increases as the cost dispersion increases. From Lemmas 1 and 2, we already know that $d\delta_2^S/d\varepsilon > 0$ and $d\delta_1^F/d\varepsilon = 0$, which proves $d(\delta_2^S - \delta_2^F)/d\varepsilon > 0$. $\square$

Proposition 2 shows the order of the critical discount factor of each firm for Collusion S and F. What is important here is that the critical discount factor required for firm 1 to adhere to Collusion F (δ_1^F) is the largest among them. This is because firm 1 can earn the largest possible profits of π_1^M from its deviation from Collusion F, while the profit from adhering to it is fairly small because agreement F requires firm 1 to set prices that are too high in terms of maximizing its own profit for half of the market share.

Table 1: Numerical examples with $\bar{a} = 20$ and $c = 2$

Cost dispersion (ε)	Critical discount factor				Price	
	δ_1^S	δ_2^S	δ_1^F	δ_2^F	p^S	p^F
(i) $\varepsilon = 0.2$	0.5	0.52	0.55	0.5	10.9	11.00
($c_1 = 1.8$, $c_2 = 2.2$)						
(ii) $\varepsilon = 0.5$	0.5	0.55	0.63	0.5	10.75	10.95
($c_1 = 1.5$, $c_2 = 2.5$)						
(iii) $\varepsilon = 1.0$	0.5	0.61	0.80	0.5	10.5	10.74
($c_1 = 1.0$, $c_2 = 3.0$)						

Then, we have the following corollary showing the relationship between different discount factors between firms and the sustainability of the two types of collusion.

Corollary.

(a) *Suppose that* $\delta_1 = \delta_2 = \hat{\delta}$. *If* $\hat{\delta} < \delta_2^S$, *then there is no sustainable collusion. If* $\hat{\delta} \in [\delta_2^S, \delta_1^F)$, *then only Collusion* S *is sustainable. Otherwise, if* $\hat{\delta} \geq \delta_1^F$, *then both types of collusion are sustainable.*

(b) *Only Collusion* F *is sustainable if and only if* $\delta_1 > \delta_2$ *such that* $\delta_1 > \delta_1^F$ *and* $\delta_2 \in [0.5, \delta_2^S)$.

Figure 3 illustrates the comparison of sustainability between the two types of collusion. If the discount factor is the same for both firms, as shown in Figure 3-(i), then Collusion S is more sustainable than Collusion F. However, as shown in Corollary (b) and Figure 3-(ii), if the inefficient firm is more short-sighted than the efficient firm (i.e., $\delta_1 > \delta_2$), then there can be a situation where only Collusion F is sustainable. The results are important, as discussed in the following subsection, to indicate possible reasons why heterogeneous firms simply agree to commit to the same price, even in situations where there is little likelihood of detection by antitrust authorities for profit-sharing collusion with side payments.

Table 1 provides a numerical example with $\bar{a} = 20$ and $c = 2$. We can see from the table that $\delta_1 \geq 0.63$ and $\delta_2 \in [0.5, 0.55)$ make Collusion F the only sustainable collusion when $\varepsilon = 0.5$, and $\delta_1 > 0.80$ and $\delta_2 \in [0.5, 0.61)$ do the same when $\varepsilon = 1.0$.

6.1 Discussion

Thus far, we have shown that although profit-sharing collusion supported by side payments can generate greater joint profits than fixed-pricing collusion can, there may be a case in which price-fixing collusion would be the only sustainable form of collusion if high-productivity firms are also more far sighted than low-productivity firms. In what follows, we will discuss the significance of the results and the validity of the condition.

There are two possible reasons that justify inefficient firms valuing future profits less (or, equivalently, having a smaller discount factor) than efficient firms. The first reason is that cost dispersion is the cause of the different time discount rates. For example, firms with efficient production technology can raise funds at lower borrowing rates (e.g., through preferential borrowing rates from banks or lower rates on corporate bonds) than firms with inefficient production technology, which increases (decreases) the discount factor for efficient (inefficient) firms. Another example concerns the relationship between CEO turnover and firm performance. A CEO who performs poorly is more likely to be replaced than one who performs well (Weisbach, 1988; Hermalin and Weisbach, 2001; Huson et al., 2001; and Brickley, 2003, among many others). In other words, good performance through efficient management increases a CEO's discount factor by increasing her expected tenure.

The second reason is that different time discounts are the cause of the cost dispersion. For example, far-sighted managers will be more willing to make investments that will lower production costs in the long run (e.g., engaging in cost-reducing R&D and installing more efficient production equipment) than myopic managers. As a result, firms led by managers with a larger (smaller) discount factor will have lower (higher) production costs. These two reasons justify the situation where firms' production costs and discount factors are inversely correlated, although the direction of causality is different.[11]

Collusive firms are less likely to engage in profit-sharing collusion with side payments due to the higher risk of being detected by competition authorities. Our results show that price-fixing collusion can be more sustainable than profit-sharing collusion, even if profit-sharing collusion does not contain such a risk of detection (or contains the same risk as price-fixing collusion). Thus, the

[11] For other reasons regarding the possibility of firms having different discount factors (e.g., imperfections in capital markets, agency problems, and various horizons of investors), see Harrington (1989) and Obara and Zincenko (2017).

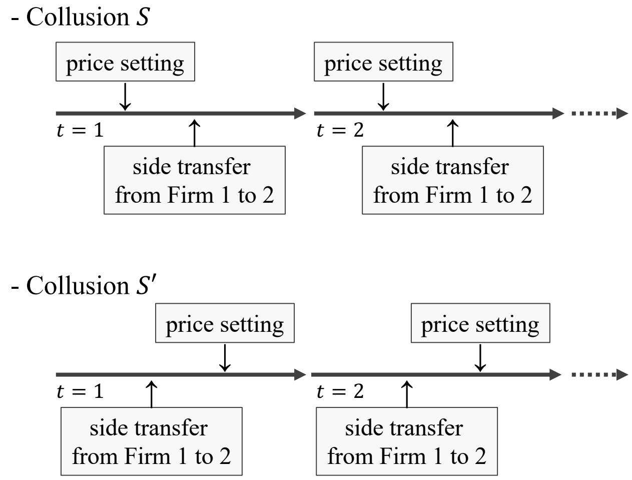

Figure 4: Timeline for Collusion S and S'

present model provides possible reasons why firms fail to reach and sustain price-fixing agreements and why firms sometimes succeed at doing so.

7 Extensions

This section considers two variants of profit-sharing collusion. First, we investigate a different enforcement procedure for exchanging side payments. Second, we consider the case in which side payments are used not only to reach one-shot agreement but also to sustain collusion.

7.1 Collusion S': Making side payments before price setting

In Section 4, we considered a procedure for Collusion S in which firm 1 confirms cooperative pricing by firm 2 and then pays side payments to firm 2 in return. Here we consider an alternative enforcement procedure (we call it Collusion S') whereby the transfer of side payments is preceded by collusive pricing, which is illustrated by the timeline in Figure 4.

Under Collusion S', each firm's collusive profits under the NBS are the same as in Collusion S, which are expressed by (3) and (4). However, the deviation incentives for each firm are different. Suppose that firm 1 deviates from cooperation by not granting firm 2 side payments. Then, firm 2 will not take the cooperative action of setting its price above p_1^M. In other words, in the procedure of Collusion S', firm 1's deviation is immediately punished within a period, and therefore, firm 1 never deviates from cooperation for a non-negative discount factor, which means that $\delta_1^{S'} = 0$.

In contrast, the deviation incentives for firm 2 are quite different. When firm 2 unilaterally deviates from the cooperation, it obtains

$$\pi_{2D}^{S'} = \pi_{2D}^{S} + \tau^S = \frac{1}{8}(a - 3\varepsilon)(3a - \varepsilon)$$

by receiving side payments from firm 1 and undercutting p_1^M to capture the total demand. Therefore, firm 2 has a greater deviation incentive under Collusion S' than it has under Collusion S.

The critical discount factor required for firm 2 to adhere to Collusion S' is obtained by

$$\delta_2^{S'} = \frac{2(a + \varepsilon)}{3a - \varepsilon},$$

which has the following properties:

$$\lim_{\varepsilon \to 0} \delta_2^{S'} = 2/3 \text{ and } \frac{d\delta_2^{S'}}{d\varepsilon} = \frac{8a}{(3a + \varepsilon)^2} > 0.$$

Thus, $\delta_2^{S'}$ is increasing in ε and is at least greater than 2/3.

By comparing $\delta_2^{S'}$ with δ_2^{S}, we have

$$\begin{aligned} \delta_2^{S'} - \delta_2^{S} &= \frac{(a - 3\varepsilon)^2}{2(3a - \varepsilon)(a + \varepsilon)} > 0, \\ \frac{d\left(\delta_2^{S'} - \delta_2^{S}\right)}{d\varepsilon} &= -\frac{2a(a - 3\varepsilon)(5a + \varepsilon)}{(3a - \varepsilon)^2(a + \varepsilon)^2} < 0, \end{aligned}$$

which indicates that $\delta_2^{S} < \delta_2^{S'}$ and the difference shrinks as ε increases. Greater cost dispersion lowers the amount of side payment that firm 1 pays to firm 2, which reduces the difference between $\delta_2^{S'}$ and δ_2^{S}.

In sum, whether side payments are made before or after price setting in each period (i.e., Collusion S' or S) does not affect their profitability (i.e., $\pi_i^S = \pi_i^{S'}$ for $i = 1, 2$), but it does affect their sustainability, through changes in their deviation incentives. In particular, we find that $\delta_1^{S'} = 0 < \delta_1^{S} = 1/2 < \delta_2^{S} < \delta_2^{S'}$.

The remaining issue is to compare the sustainability of Collusion S' with that of Collusion F.

Proposition 3.

Suppose that Assumption 1 holds. Then,

(*a*) $\delta_1^{S'} < \delta_2^F < \delta_1^F < \delta_2^{S'}$ *holds for* $\varepsilon \leq 0.040 \times a$,

(*b*) $\delta_1^{S'} < \delta_2^F < \delta_2^{S'} < \delta_1^F$ *holds for* $\varepsilon \geq 0.041 \times a$.

Proof: We can easily confirm that $\delta_1^{S'} < \delta_2^F < \delta_1^F$ from Proposition 2 and $\delta_2^F < \delta_2^{S'}$ from $\delta_2^{S'} > 2/3$. By comparing δ_1^F and $\delta_2^{S'}$, we have

$$\delta_2^{S'} - \underline{\delta}_1^F \gtreqless 0 \quad \Leftrightarrow \quad \varepsilon \lesseqgtr \bar{\varepsilon} \equiv \frac{(13 - 2\sqrt{33})a}{37} \approx 0.041a.$$

From Assumption 1, we have $\varepsilon < a/13 \approx 0.077a$, the sufficient condition for $\delta_2^{S'} < \delta_1^F$ is $\varepsilon \in [\bar{\varepsilon}, a/13)$ and the necessary condition for $\delta_2^{S'} > \delta_1^F$ is $\varepsilon \in (0, \bar{\varepsilon})$. In addition, we have

$$\delta_2^{S'} - \bar{\delta}_1^F \gtreqless 0 \quad \Leftrightarrow \quad \varepsilon \lesseqgtr \tilde{\varepsilon} \approx 0.040a,$$

which shows that the necessary condition for $\delta_2^{S'} < \delta_1^F$ is $\varepsilon \in [\hat{\varepsilon}, a/13)$ and the sufficient condition for $\delta_2^{S'} > \delta_1^F$ is $\varepsilon \in (0, \hat{\varepsilon})$. $\square$

Proposition 3 shows that if the discount factors for both firms are the same, or firm 2 is more myopic, then Collusion F is more sustainable than Collusion S' except for the case of large cost dispersion, which is illustrated in Figure 5-(i). Given the natural situation in which a less efficient firm has a smaller discount factor than a more efficient firm, an ex ante side payment exchange that is contingent on later collusive price setting makes the sustainability of Collusion S' even more difficult by providing the inefficient firm with further incentives to deviate from cooperation. However, if the efficient firm is much more myopic than the inefficient firm, say $\delta_1 < \delta_1^S = 1/2 < \delta_2^S < \delta_2$ holds, then Collusion S' will be the only sustainable collusion because it discourages the efficient firm from deviating from the collusion. The result indicates the importance of coordination for a specific enforcement procedure (i.e., the timing of transfers) between firms to make profit-sharing collusion with side payments sustainable.

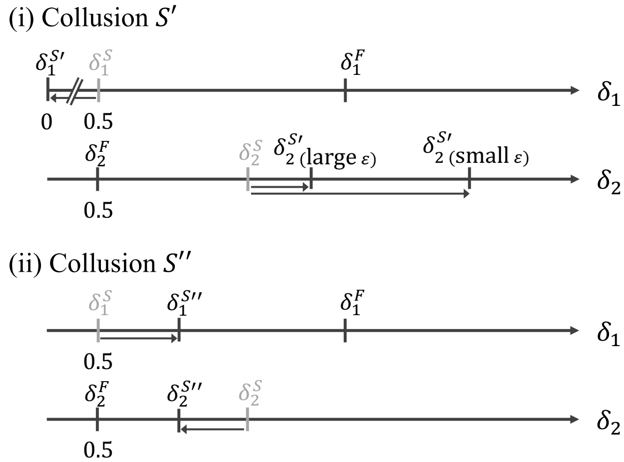

Figure 5: Critical discount factors for Collusions S, S', S'', and F

Note that in Collusion S', the side transfer precedes price setting, implying that the low-cost firm (the sender of the side transfer) is the first mover in each period. We show that the first mover's deviation incentives are completely eliminated because its deviation of not paying side payments is immediately punished by the high-cost firm (the recipient of the side transfer) in price setting, which corresponds to the mechanism proposed by Mouraviev and Rey (2011), who show that price leadership facilitates collusion by making it easier to punish deviations by the price-setting leader.

7.2 Collusion S'': With side payments to sustain collusion

In Collusion S, the side payments made by firm 1 to firm 2 are for the sole purpose of building a one-shot collusive agreement. However, to achieve long-term sustainability of cooperation, it should be possible for firm 1 to prevent firm 2 from deviating from cooperation by increasing the amount of side payments in each period, although this would also increase the incentive for firm 1 to deviate from cooperation.[12] We call such collusion with increased side payments to sustain cooperation Collusion S''.[13]

[12] If firm 1's discount factor is smaller than firm 2's, and Collusion S is not sustainable because firm 1's incentive compatibility constraint is binding, then the case of $\Theta < 0$ is also possible. However, we exclude the case of $\Theta < 0$ because in that case it is possible to satisfy firm 1's incentive compatibility constraint with the combined use of $\Theta > 0$ and the option of exchanging side payments before price setting as in Collusion S'.

[13] Our model setting here corresponds to the market-sharing rule in Harrington (1989). In contrast to his work, we distinguish between side payments required for agreement in one-shot collusive bargaining (in Collusion S) and the side payments required for the sustainability of that agreement (in Collusion S''). This is because to determine

Now, consider that firm 1 adds $\Theta \geq 0$ amount of money (or resources) to the side payment τ^S determined by Nash bargaining in Collusion S. We assume that in one-shot bargaining, side payments are made after price setting as in Collusion S. Then, in an infinitely repeated game with grim-trigger strategies, we have incentive compatible constraints for each firm to adhere to Collusion S'' as

$$\pi_1^M + \frac{\delta_1 \, \pi_1^C}{1-\delta_1} < \frac{\pi_1^S - \Theta}{1-\delta_1} \quad \text{and} \quad \pi_2^M|_{p_2=p_1^M} < \frac{\pi_2^S + \Theta}{1-\delta_2}.$$

Thus, we have the critical discount factors as

$$\delta_1^{S''}(\Theta) = \frac{1}{2} + \frac{4\Theta}{(a-3\varepsilon)^2} \quad \text{and} \quad \delta_2^{S''}(\Theta) = \frac{(a+5\varepsilon) - \frac{8\Theta}{a-3\varepsilon}}{2(a+\varepsilon)},$$

where $d\delta_1^{S''}/d\Theta > 0$ and $d\delta_2^{S''}/d\Theta < 0$.

Now, suppose both firms have the same discount factor $\delta_1 = \delta_2 = \hat{\delta}$. Then, we determine that the amount of $\hat{\Theta}$ that minimizes the critical discount factor required for both firms to adhere to Collusion S'' as $\delta_1^{S''}(\hat{\Theta}) = \delta_2^{S''}(\hat{\Theta})$, where

$$\hat{\Theta} = \frac{(a-3\varepsilon)^2}{4(a-\varepsilon)}\varepsilon.$$

As illustrated in Figure 5-(ii), side payments equal to $\tau^S + \hat{\Theta}$ can increase the sustainability of Collusion S'' the most under symmetric discount factors.

Next, we compare Collusion S'' with Collusion F. In Corollary, we have shown that if $\delta_1 > \delta_2$, then it is possible that Collusion F would be sustainable but Collusion S would not be despite that the latter yields greater (the maximum) joint profits than the former. However, Collusion S'', in which side payments also serve to sustain long-term cooperation, is necessarily more sustainable

the amount of side payments needed to guarantee the sustainability of collusion, information about the long-term discount factors of both firms is needed, which is difficult to know at the beginning of long-lasting coordination for the following reasons. First, firms will have a strategic incentive to signal that their discount factor is low in collusive bargaining because a firm with a smaller discount factor will receive more side payments (or pay less side payments) if the sustainability of the collusion is taken into account in the bargaining. Second, a firm's discount factor is relatively more volatile than its cost of production due to changes in market conditions, such as changes in interest rates. Note that since we consider heterogeneity not only in discount factors but also in marginal costs, it is necessary to distinguish whether bargaining power in collusive bargaining reflects only differences in marginal costs (as in Collusion S) or also differences in discount factors (as in Collusion S'').

than Collusion F. The result is summarized by the following proposition.

Proposition 4.

There always exists a side payment that makes Collusion S'' more sustainable than Collusion F and S.

Proof: First, Collusion S'' includes Collusion S as a special case of $\Theta = 0$, and thus Collusion S'' is more sustainable than Collusion S.

Second, Collusion F is sustainable if and only if $\delta_1 > \delta_1^F$ and $\delta_2 > \delta_2^F = 1/2$ simultaneously hold. Because $\delta_1^{S''}(0) < \delta_1^F$ holds, firm 1's deviation incentives are smaller under Collusion S'' than those under Collusion F even without any incremental side payments. In contrast, $\delta_2^F = 1/2 < \delta_2^S = \delta_2^{S''}(0)$ means that firm 2's deviation incentives under Collusion S'' would exceed those under Collusion F without an increase in side payments. We have

$$\delta_2^{S''}(\Theta^+) \leq \delta_2^F = 1/2 \quad \Leftrightarrow \quad \Theta^+ \geq \frac{1}{2}(a - 3\varepsilon)\varepsilon,$$

which indicates that by adding the amount of Θ^+ to the side payments, the critical discount factor required to prevent firm 2 deviating from Collusion S'' is lowered to at least the same as that for Collusion F. In addition, when the side payments are increased by Θ^+, the critical discount factor for firm 1 to adhere to Collusion S'' becomes

$$\delta_1^{S''}(\Theta^+) = \frac{a + \varepsilon}{2(a - 3\varepsilon)} < \underline{\delta}_1^F,$$

where the last inequality comes from

$$\underline{\delta}_1^F - \delta_1^{S''}(\Theta^+) = \frac{2\varepsilon(a + \varepsilon)}{(a - 3\varepsilon)^2} > 0.$$

This proves that if Collusion F is sustainable, then Collusion S'' is always sustainable when choosing an appropriate $\Theta \geq \Theta^+$. $\square$

The proposition shows that profit-sharing collusion, where side payments can play a role not only in encouraging the inefficient firm to agree to cooperate in the short run but also in encouraging it to maintain cooperation in the long run, will always be more sustainable than price-fixing collusion. Moreover, the combined use of the option of exchanging side payments before setting the price in each period can further increase its sustainability in more general situations, including cases in which the discount factor for the efficient firm is lower than 1/2.

Finally, we propose the proposition on the conditions for each firm's discount factor to make Collusion S'' sustainable.

Proposition 5. *Collusion S'' can be sustainable if*

$$\frac{\delta_1}{1-\delta_2} \geq \Lambda(\varepsilon) \equiv \frac{a+\varepsilon}{a-3\varepsilon}. \tag{14}$$

Proof: Suppose that firm 2's incentive compatibility constraint is binding when $\Theta = 0$ (i.e., $\delta_2 < \delta_2^{S''}(0)$). Then, to satisfy the incentive compatibility constraint of firm 2, we need to have at least the amount of $\Theta^\dagger$ such that

$$\delta_2^{S''}(\tilde{\Theta}) \leq \delta_2 \;\Leftrightarrow\; \tilde{\Theta} \geq \frac{a(a-3\varepsilon)(1-2\delta_2)+(5-2\delta_2)\varepsilon}{8}.$$

Firm 1's incentive compatibility constraint should be also satisfied if firm 1 grants an additional $\tilde{\Theta}$ to firm 2. Thus, δ_1 should satisfy

$$\delta_1 \geq \delta_1^{S''}(\tilde{\Theta}) \;\Leftrightarrow\; \delta_1 \geq \frac{(a+\varepsilon)(1-\delta_2)}{a-3\varepsilon},$$

which reduces to (14) in the proposition. □

Proposition 5 shows the condition for each firm's discount factor to make Collusion S'' sustainable by increasing the amount of side payments. If there is no cost disparity between firms, the condition reduces to $\frac{\delta_1+\delta_2}{2} \geq \frac{1}{2}$: collusion can be sustainable if their average discount factor exceeds $\frac{n-1}{n} = \frac{1}{2}$, which corresponds to the result first shown by Harrington (1989). In addition, we have $\Lambda'(\varepsilon) > 0$, implying that the greater the cost disparity, the larger the average discount factors for firms required for collusion to be sustained. Figure 6 illustrates condition (14) in the case of $\bar{a} = 20$ and $c = 2$. If $\varepsilon = 0$, then Collusion S'' can be sustainable for situations where the average discount

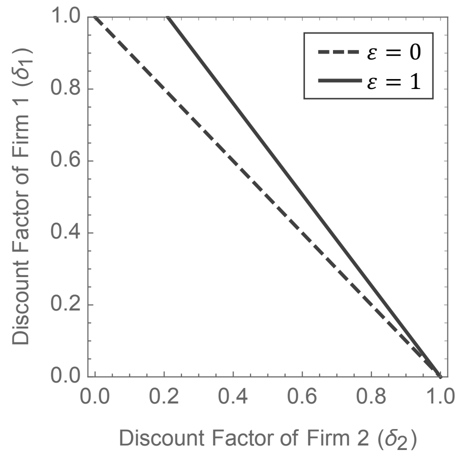

Figure 6: The condition for Collusion S'' to be sustainable: the case of $\bar{a} = 20$ and $c = 2$

factor is greater than 1/2, which is represented by the area above the dashed line. If $\varepsilon = 1$, the area that guarantees the sustainability of collusion (the area above the solid line) will be smaller. Thus, we find that even if side payments were feasible to sustain collusion, cost heterogeneity would hinder collusion.

There are two caveats regarding side payments for the purpose of maintaining long-term cooperation. First, we assumed equal bargaining power in all other types of collusion (S, F, and S'), which means that all differences in the allocation of bargaining solutions reflect only differences in the profits of the disagreement point (π_1^C, π_2^C). However, the increase in side payments Θ is the minimum amount necessary for the firm that binds its incentive compatibility constraint, which implicitly assumes full bargaining power of the firm without a binding incentive compatibility constraint in determining Θ. Second, to determine the incremental amount of the side payments, firm 1 needs to know the discount factor of firm 2. Otherwise, firm 2 will be tempted to pretend to be myopic to obtain a larger side payment, which is also mentioned by Harrington (1989). Therefore, in the main body, we adopt an approach that distinguishes between examining the short-term (one-shot) bargaining over collusive agreement on side payments or the common price level and examining the sustainability of the agreement, which enables us to develop a simple characterization of the equilibrium by eliminating the effect of differences in bargaining power due to differences in their discount factors on short-term agreements.

8 Concluding Remarks

The main obstacle for successful collusion is to coordinate the interests of different firms, and for this purpose, it is essential to analyze a specific collusion arrangements. Considering Nash bargaining for the explicit agreement and infinitely repeated interaction with the grim-trigger strategy for the sustainability of the agreement, we study the coordination problem in a situation where two firms with different production costs and different time discount rates compete on the price of a homogeneous product. In particular, we compare the profitability and sustainability of profit-sharing collusion with side payments, which allows an efficient firm to specialize in production and sales, and price-fixing collusion, which forces firms to agree only to set a common price.

Our results provide a theoretical rationale for why firms fail to reach and sustain some forms of collusion and why firms sometimes succeed in doin so. Although profit-sharing collusion with side payments is the most effective way for firms with asymmetric costs to collude, inefficient firms may prefer price-fixing collusion because common pricing makes the profit distribution more equitable for each firm. If the discount factors of both firms are equal, or if the efficient firm is more myopic, then adjusting the side-payment enforcement procedure will always make profit-sharing collusion more sustainable than price-fixing collusion. However, if the inefficient firm is more myopic, then price-fixing collusion could be the only sustainable collusion. The results can provide a new reason for why price-fixing collusion is more likely to occur in some industries, beyond its lower risk of detection by competition authorities.

Our results have important policy implications for antitrust authorities to detect collusion. It is well known that price-fixing collusion in which firms agree only on a common price makes it difficult for antitrust authorities to determine whether it is the result of price competition. Our results suggest that even if antitrust authorities were not capable of detecting profit-sharing collusion with side payments, heterogenous firms may endogenously opt for price-fixing collusion. Compared to profit-sharing collusion, price-fixing collusion undermines not only consumer surplus due to higher collusive prices but also producers' surplus due to inefficient production allocation. As this paper has shown, it is important for the authorities to know in which environment Collusion F is likely to be selected, which can be an important guideline for detecting collusion.

A possible next step in this line of research would be to compare the profitability and sustainability of collusion by accounting gor heterogeneity in detection risk through, for example, leniency programs such as in Motta and Polo (2003) or buyer detection such as in Harrington and Chen (2006). In that case, the heterogeneity of risk attitudes among firms, in addition to that of costs and discount factors, will have a significant impact on firm preferences for different types of collusion. It is also worthwhile to test whether actual collusion choices support our theoretical results by empirically revealing the relationship between the heterogeneity of firms' discount factors, which reflect differences in the interest rates they face and differences in CEO tenure, and firms' cost disparities.

References

[1] Bae, H. (1987). A price-setting supergame between two heterogeneous firms. European Economic Review, 31(6), 1159–1171.

[2] Blume, A. (2003). Bertrand without fudge. Economics Letters, 78, 167–168.

[3] Bos, I., & Harrington, Jr, J.E. (2010). Endogenous cartel formation with heterogeneous firms. RAND Journal of Economics, 41(1), 92–117.

[4] Brandão, A., Pinho, J., & Vasconcelos, H. (2014). Asymmetric collusion with growing demand. Journal of Industry, Competition and Trade, 14(4), 429–472.

[5] Brickley, J.A. (2003). Empirical research on CEO turnover and firm-performance: A discussion. Journal of Accounting and Economics, 36(1-3), 227–233.

[6] Clark, R., & Houde, J.F. (2013). Collusion with asymmetric retailers: Evidence from a gasoline price-fixing case. American Economic Journal: Microeconomics, 5(3), 97–123.

[7] Compte, O., Jenny, F., & Rey, P. (2002). Capacity constraints, mergers and collusion. European Economic Review, 46(1), 1–29.

[8] Correia-da-Silva, J., & Pinho, J. (2016). The profit-sharing rule that maximizes sustainability of cartel agreements. Journal of Dynamics & Games, 3(2), 143–151.

[9] Dal Bó, P. (2007). Tacit collusion under interest rate fluctuations. RAND Journal of Economics, 38(2), 533–540.

[10] Davidson, C., & Deneckere, R. (1984). Horizontal mergers and collusive behavior. International Journal of Industrial Organization, 2(2), 117–132.

[11] Davidson, C., & Deneckere, R. (1990). Excess capacity and collusion. International Economic Review, 521–541.

[12] Feuerstein, S. (2005). Collusion in industrial economics—a survey. Journal of Industry, Competition and Trade, 5(3), 163–198.

[13] Friedman, J.W. (1971). A non-cooperative equilibrium for supergames. Review of Economic Studies, 38(1), 1–12.

[14] Garrod, L., & Olczak, M. (2017). Collusion under imperfect monitoring with asymmetric firms. Journal of Industrial Economics, 65(3), 654–682.

[15] Garrod, L., & Olczak, M. (2018). Explicit vs tacit collusion: The effects of firm numbers and asymmetries. International Journal of Industrial Organization, 56, 1–25.

[16] Goto, U., & Iizuka, T. (2016). Cartel sustainability in retail markets: Evidence from a health service sector. International Journal of Industrial Organization, 49, 36–58.

[17] Harrington Jr, J.E. (1989). Collusion among asymmetric firms: The case of different discount factors. International Journal of Industrial Organization, 7(2), 289–307.

[18] Harrington Jr, J.E. (1991). The determination of price and output quotas in a heterogeneous cartel. International Economic Review, 767–792.

[19] Harrington Jr, J.E., & Chen, J. (2006). Cartel pricing dynamics with cost variability and endogenous buyer detection. International Journal of Industrial Organization, 24(6), 1185–1212.

[20] Hermalin, B.E., & Weisbach, M.S. (1988). The determinants of board composition. RAND Journal of Economics, 589–606.

[21] Huson, M.R., Parrino, R., & Starks, L. T. (2001). Internal monitoring mechanisms and CEO turnover: A long-term perspective. Journal of Finance, 56(6), 2265–2297.

[22] Igami, M., & Sugaya, T. (2021). Measuring the Incentive to Collude: The Vitamin Cartels, 1990–99. Review of Economic Studies, forthcoming.

[23] Ivaldi, M., Jullien, B., Rey, P., Seabright, P., & Tirole, J. (2003) The economics of tacit collusion. `http://idei.fr/doc/wp/2003/tacitcollusion.pdf`

[24] Miklfós-Thal, J. (2011). Optimal collusion under cost asymmetry. Economic Theory, 46(1), 99–125.

[25] Motta, M., & Polo, M. (2003). Leniency programs and cartel prosecution. International Journal of Industrial Organization, 21(3), 347–379.

[26] Mouraviev, I., & Rey, P. (2011). Collusion and leadership. International Journal of Industrial Organization, 29(6), 705–717.

[27] Obara, I., & Zincenko, F. (2017). Collusion and heterogeneity of firms. RAND Journal of Economics, 48(1), 230–249.

[28] Osborne, M.J., & Pitchik, C. (1983). Profit-sharing in a collusive industry. European Economic Review, 22(1), 59–74.

[29] Rothschild, R. (1999). Cartel stability when costs are heterogeneous. International Journal of Industrial Organization, 17(5), 717–734.

[30] Schmalensee, R. (1987). Competitive advantage and collusive optima. International Journal of Industrial Organization, 5(4), 351–367.

[31] Vasconcelos, H. (2005). Tacit collusion, cost asymmetries, and mergers. RAND Journal of Economics, 39–62.

[32] Verboven, F. (1997). Collusive behavior with heterogeneous firms. Journal of Economic Behavior & Organization, 33(1), 121–136.

[33] Weisbach, M.S. (1988). Outside directors and CEO turnover. Journal of Financial Economics, 20, 431–460.