Evidence of the middle-income trap in Pacific Alliance countries using Bayesian inference

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Slowdowns for extended periods after consistent economic expansion for middle-income countries is a well-documented phenomenon in the literature named the middle-income trap. Most of the possible causes share the same background: countries fall into the trap as a consequence of the mismatch between growth strategies and prevailing economic structure. The basis of this paper relies on influential work by Robertson & Ye (2013); however, we take into consideration a different approach when referring to a steady-state economy (or at least close to it). This different direction allows us to analyze specific groups (i.e. continents, trade blocs) if and only requirements are satisfied. We analyze time series corresponding to Pacific Alliance members in a purely Bayesian framework using the Full Bayesian Significance Test (Pereira & Stern, 1999; Diniz et al., 2011) so we unit root test time series corresponding to the subtracted values of each tested country and a reference steady-state economy separately. Peru is the only country that meet the two different criteria to be on the trap.

Keywords: Middle-Income Trap; Pacific Alliance; Unit Root; Full Bayesian Significance Test.

Mathematics Subject Classification: 62P20.

I. Introduction

For the last decade, *An East Asian Renaissance* has been reference literature to comprehend long-run patterns about economic growth especially in middle-income countries located in East Asian and Latin American regions. Among different topics related to economic growth, the *middle-income trap* has gained popularity in recent years. In general terms, this phenomenon is described as a trap of policy misdiagnosis principally when countries failed to match their growth strategies with prevailing economic structure. This trap was first described by Gill and Kharas (2007) mainly as an explanation why several middle-income countries struggled to grow after long periods of substantial economic expansion.

Why is the middle-income trap an important issue for economists? The distinction between low-income countries and middle-income countries is crucial since they require different growth strategies. Empirical evidence suggests that low-income countries rapidly grow boosted by cheap labor, reallocation of capital and workforce to different sectors that allow smooth absorption of domestic output to international markets. Once wages significantly rise and there is no room for lower prices, productivity starts playing a bigger role in economic growth. Up to this point, a different growth approach is indeed needed, one that could manage to respond effectively otherwise, a country could fall into the middle-income trap.

The neoclassical policy path for middle-income countries, especially in the Latin American region, has been a widespread market liberalization. Policymakers believed competition between firms generates incentives to invest and try to gain market share. Aghion et al. (2005) argue possible difficulties implementing this strategy straightforward especially in countries with considerable competition gap. Working on Schumpeterian basis they describe how greater competition promotes innovation only up to a certain level. Referring to a specific market, the technology gap between market leaders and followers is a key factor; this second group is in constant friction since they are all trying to absorb market share and somehow they push leaders to innovate especially when this gap is tight. But, what if a market has a wide gap? Certainly, leaders will have no incentive to innovate and productivity growth will shrink.

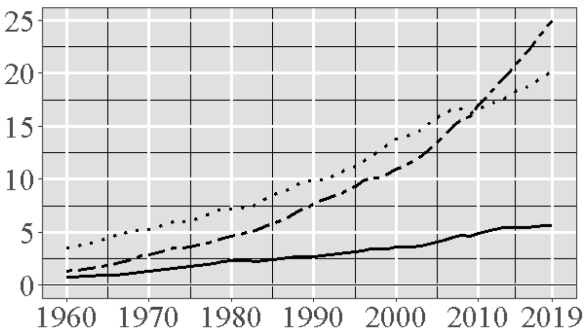

FIGURE 1. GDP (CONSTANT 2010 USD, TRILLIONS),
· · · = NORTH AMERICA, -·-· = EAST ASIA & PACIFIC,
—— = LATIN AMERICA & CARIBBEAN.

For instance, Latin America is a suitable example. This region has struggled to grow in a comparison to North American and East Asian countries as Figure 1 shows (i.e. 2000's).

Although there is an important presence of the biggest companies worldwide in Latin America, the lack of competition between mid-sized companies does not push leaders to innovate as Remes et al. (2019) suggest. This is not the only problem since mid-sized overcrowded markets drive intense competition which favors human capital. On the other hand, advanced economies that applied the same strategy had a weak market gap that held enough incentives for firms to invest and innovate continuously. This situation supports the idea that the neoclassical predominant path of market liberalization had weaknesses especially harmful in the long-run unless effective policies were boarded.

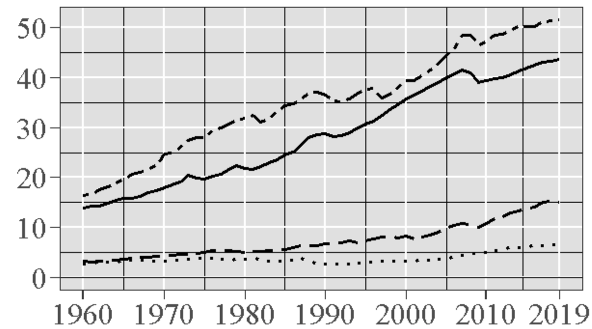

FIGURE 2. GDP per capita (CONSTANT 2010 USD, THOUSAND),
-·-· CANADA, —— = UNITED KINGOM,
— — = TURKEY, · · · = PERU.

Among the series presented in Figure 2, it is noticeable the growth difference between the two upper series (high-income countries) and the rest (middle-income countries). In the case of the middle-income countries, Turkey is considered an upper-middle income country (World Bank, 2014) while Peru has sluggish growth at first sight.

II. Methodology

First, we need to impose certain conditions for a country to suggest falling into the trap. In general terms, our experiment consists of evaluating convergence between two series: a reference economy and one from the Pacific Alliance. Therefore, we separately test the stationary of series containing subtracted values. This computation of the trap was first introduced by Robertson & Ye (2013). However, we consider a different approach to the reference economy that is fully described later.

Without loss of generality, consider the three different and possible outcomes from series containing subtracted values: a positive, negative or constant slope. In case the reference economy and a Pacific Alliance country share a significantly homogeneous growth rate over the time, the null hypothesis for unit root testing is expected to be rejected. This is because subtracted values from both series are significantly steady over time with a non-zero mean.

Two conditions are mandatory in case we want to suggest a country fall into the trap: (1) Consider a reference steady-state economy (or relatively close to it), then any country less productive (i.e. lower GDP per capita) will tent to grow either approximating or maintaining a constant distance to the closest steady-state economy. In case this distance fluctuates around a constant value we say that the less productive country has struggle converging to a reference steady-state economy then it is a candidate for the middle-income trap; (2) in case the first condition is met, then we need to check if the posterior sampling of the intercept (i.e. intercept of the regression) is mostly distributed in the negative real numbers space and only, in this case, we say that the country has difficulties approaching (from the bottom) to its closest steady-state economy and along with the first condition we conclude that country fell into the middle-income trap.

One may consistently argue about the first condition. Robertson & Ye (2013) reasonably consider the United States as the only reference economy when analyzing different countries. Nevertheless, evidence by Jones (2002) suggests that the United States economy is far from its steady-state due to rising educational attainment and research intensity. These two last-mentioned variables are capable to boost the United States growth rate far from an expected steady-state, as Jones (2002) highlights. As a counterpart, O'Neill (2015) analyze several countries for a decade using biophysical and social indicators. He concludes that no economy achieves a steady-state level, however, few countries approximate significantly: Colombia, Cuba, Kyrgyzstan, Romania, and South Africa. Among these countries, Colombia is an active Pacific Alliance member; therefore, we use Colombia as a reference economy.

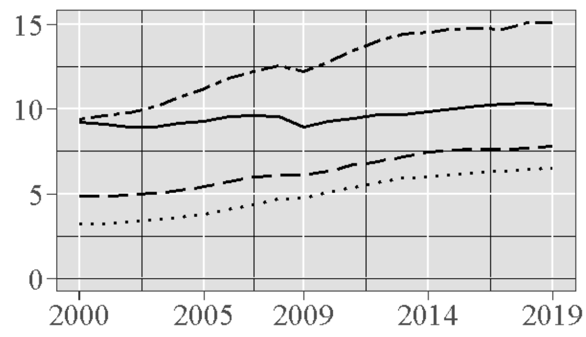

FIGURE 3. GDP per capita (CONSTANT 2010 USD, THOUSAND), -·-· CHILE, —— = MEXICO, --- = COLOMBIA, · · · = PERU

To summarize these ideas our interest relies on analyzing the long-run relationship between a tested and the reference economy. Let us say

$$x_{i,t} = \varphi_{i,t} - \varphi_{r,t},$$

where $\varphi_{i,t} \wedge \varphi_{r,t}$ correspond to adjusted per-capita GDP for the tested and reference economy respectively (series are plotted on Figure 3). Thus, $x_{i,t}$ is stationary as long as its parameters (i.e. mean and variance) do not change over time (i.e. as long as $t \to \infty$). If this condition is satisfied, then we say that both countries follow a similar growth path; in other words, the tested country is not converging to the reference steady-state economy.

Up to this point unit root testing to check for the first condition is straightforward, however, we must be aware that it is not a sufficient condition to be on the trap since we need to check for the posterior distribution of the intercept.

III. Econometric procedure

We will be unit root testing through a Bayesian framework using the Full Significance Test (FBST) developed by Pereira & Stern (1999) designed to deal efficiently with a sharp hypothesis. Making some transformation following Diniz et al. (2011) methodology we estimate the e-value in two steps: constrained optimization and then integration with the posterior density function; we describe this later.

Let us consider a vector of parameters denoted by θ within the following standard parametric space

$$\theta \in \omega \subset \mathbb{R}^m. \quad (1)$$

Besides, the sharp hypothesis subspace H from the whole parametric space is $\omega_H \subset \omega$.

In the FBST computation, the posterior probability density on the parameter space is used as an ordering system. Consider the posterior probability density $p(\theta|x)$, we define the relative surprise function described by Evans et al. (2006) like

$$\varphi(\theta) = \frac{p(\theta|x)}{r(\theta)}.$$

The former Bayesian significance test by Pereira and Stern (1999) is based on the highest posterior density principle. However, straightforward computation has issues especially with continuous posterior distributions. That's the key reason for using a relative surprise function and evaluating the highest relative surprise set (HRSS), see Madruga et al. (2003). We evaluate the ratio between the posterior and a reference density (for this case an improper) for every possible value of θ so that θ_i is strictly preferred over any other parameter if and only its ratio is greater than the others. In the appendix we show how Jacobians are canceled in the ratio so the least relative surprise estimate (LRSE) possesses the invariant property.

$$\varphi^* = \sup_{\theta \in \omega_H} \varphi(\theta). \quad (2)$$

The second statement indicates the supremum value according to the surprise function from the sharp hypothesis space.

Then using the surprise function and this value we need to compare and extract the following subset

$T(\varphi^*) = \{\theta \in \omega | \varphi(\theta) \geq \varphi^*\}$.

We have just defined the set of points from the parameter space we need to integrate from the posterior probability function

$W(\varphi^*) \propto \int_{T(\varphi^*)} p_m(\theta) d(\theta)$.

The integral for this case is the *e*-value which is briefly the supporting region of the hypothesis *H*. FBST test computation does not require the elimination of nuisance parameters, see Berger and Mortera (1994) for detailed discussion.

We use the former notation of the Augmented Dickey-Fuller Test. Let us start by the following AR(p) process plus a constant

$$x_t = \mu + \rho_0 x_{t-1} + \cdots + \rho_p x_{t-p} + \varepsilon_t, \quad (3)$$

where ε_t follows a Normal Distribution with zero mean and unknown variance. Notice that we don't consider a time trend. We focus on the long-run path of the series, we want to know if $x_{i,t}$ is smooth enough (i.e. bouncing around a constant value) to suggest a country cannot approach a steady-state economy for extended periods. More precisely, using the likelihood function we want to analyze the fit of a non-trend model (to meet the criteria described above) to any given sample of the data.

After a little calculation and subtracting x_{t-1} from both sides of (3) we have the standard model used in ADF test

$$\Delta x_t = \mu + \beta_0 x_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta x_{t-i} + \varepsilon_t, \quad (4)$$

where $\beta_0 = \rho_0 + \rho_1 + \cdots + \rho_p - 1$. In this case, an observed time series is considered stationary if we find enough statistical support against the null hypothesis (i.e. $\beta_0 = 0$). Once we set up the basics of the econometric model we simplify the presentation and further computation by using the following matrix notation

$\vec{x} = X\theta + \vec{\varepsilon}$;

$$\vec{x} = \begin{bmatrix} \Delta x_{p+1} \\ \Delta x_{p+2} \\ \vdots \\ \Delta x_{p+T} \end{bmatrix},$$

$$\theta = \begin{bmatrix} \mu \\ \beta_0 \\ \vdots \\ \beta_p \end{bmatrix},$$

$$X = \begin{bmatrix} 1 & x_p & \Delta x_p & \cdots & \Delta x_2 \\ 1 & x_{p+1} & \Delta x_{p+1} & \cdots & \Delta x_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & x_{T-1} & \Delta x_{T-1} & \cdots & \Delta x_{T-p+1} \end{bmatrix}.$$

Considering a Gaussian Likelihood function and a weakly informative prior $[f(\vec{\theta}, \sigma) \propto 1/\sigma]$, the joint posterior probability function is computed by the following expression

$$f(\theta, \sigma | \vec{x}) \propto \sigma^{-(T+1)} \exp\{-\tfrac{1}{2\sigma^2}[(\vec{x} - X\theta)'(\vec{x} - X\theta)]\}.$$

Once we have the posterior probability function, computing conditional posteriors is straightforward. In this case, we have

$$f(\theta | \vec{x}) \propto \exp\{-\tfrac{1}{2\sigma^2}[(\theta - \hat{\theta})' X' X (\theta - \hat{\theta})]\}, \quad (5)$$

$$f\left(\tfrac{1}{\sigma^2} \middle| \vec{x}\right) \propto \frac{1}{\sigma^{2\left(\frac{T+3}{2} - 1\right)}} \exp\{-\tfrac{1}{\sigma^2} * B\}. \quad (6)$$

So (5) ~ $N(\hat{\theta}; \sigma^2 (X'X)^{-1})$ and (6) ~ $\Gamma(\frac{T-p+3}{2}; B)$[2].

Sampling from the posterior distribution is possible using a Markov Chain Monte Carlo (MCMC) technique suitable when we deal with

[2] $B = 0.5 * [(\vec{x} - X\hat{\theta})'(\vec{x} - X\hat{\theta}) + (\theta - \hat{\theta})' X' X (\theta - \hat{\theta})]$.

posterior conditionals: Gibbs sampling. After setting up reasonably initial values for θ_0 we construct an algorithm in which the outcome of the Gamma posterior distribution is sourced to draw a sample of the Normal posterior distribution; this sample θ_1 is used as input to draw a new sample of the Gamma distribution repeating all the process (notice that we chain both conditionals through the precision). We use this chain to construct the posterior probability distribution of θ. For conditional posteriors we made 2000 repetitions.

Finally, the integration step allows us to obtain the *e*-value which is presented in the next section along with the standard ADF *p*-value. We also present the Bayes Factor alternative to the classical *t*-test (ratio between alternative and null hypothesis). For the prior election and its implication, we suggest Phillips (1990).

IV. Conclusions

TABLE 1 – UNIT ROOT TESTS OUTPUT

Series	*p*	ADF	*p*-value	*e*-value	*B*ayes *F*actor
Chile	3	-4.497	0.0015	0.0005	>10
Mexico	3	-3.146	0.0354	0.0015	>10
Peru	3	-3.876	0.0068	0.0006	>10

Although the concept has widely spread recently, multifactorial possible causes of the middle-income trap have made it difficult to find solid proof of the hypothesis, therefore statistical approximations have not been exempt from critics. In this paper, we use a very general definition of the-middle income trap: middle-income countries affected by persistent slowdowns that prevent them from approaching the closest steady-state economy. All tested countries satisfied the first condition primarily based on the *e*-value results, although classical p-value and Bayes Factor also supported this conclusion. Nevertheless, this was not the case for the second condition. Posterior probability distributions for Mexico, Chile and Peru (Figure 4, 5 and 6 respectively) gives us a different direction. Only Peru's intercept posterior density is mainly distributed in the negative real numbers space. While we cannot find evidence that Chile or Mexico fell into the trap, only Peru satisfied both conditions presented in the length of this paper; therefore, based on Bayesian analysis we concluded that Peru is the only country that meets all criteria to be on the middle-income trap.

The Bayesian Inference proposes a different approach to the problem. In this case, it concludes in the same direction as the classical *p*-value (rejecting the null hypothesis in this case). We think this is an example of how weakly-informative priors are applied in econometrics even though Schotman (1991) proposes a proper normal prior on the mean but it is a topic that goes beyond the purpose of this discussion. In the extension of this paper, other regions should be tested especially Asia.

Appendix

Proof of Invariant Principle: Let us start by the standard parametric space (1). If we consider the following one-to-one reparametrization $\vec{\eta} = f(\theta)$, we can express the Jacobian by

$$J_\theta(\vec{\eta}) = \frac{\partial f^{-1}(\vec{\eta})}{\partial \vec{\eta}}.$$

Now we notice that Jacobian factor cancels in the ratio that represents the M. Surprise estimator

$$\varphi(\vec{\eta}) = \frac{p(\vec{\eta})}{r(\vec{\eta})}.$$

$$\varphi(\vec{\eta}) = \frac{p(f^{-1}(\vec{\eta}))|J_\theta(\vec{\eta})|}{r(f^{-1}(\vec{\eta}))|J_\theta(\vec{\eta})|}.$$

$$\varphi(\vec{\eta}) = \frac{p(\vec{\theta})}{r(\vec{\theta})}.$$

Besides weighting each parameter so one could be strictly preferred over the other, we check that the presence of a reference function cancels the Jacobian factor. The reparametrization also affect the subset of the parametric space such as $\Omega_H = f(\omega_H)$, so considering the notation above we have that the supremum of both surprise functions produce the same output despite sharing different subsets of the whole parametric space ω; therefore, they share the same *e*-value from the integration step

$$\int_{\Omega_H} p(\vec{\eta})d\vec{\eta} = \int_{f^{-1}(\Omega_H)} p(f^{-1}(\vec{\eta}))|J_\theta(\vec{\eta})|d\vec{\eta},$$

$$\int_{\Omega_H} p(\vec{\eta})d\vec{\eta} = \int_{\omega_H} p(\theta)d\theta.$$

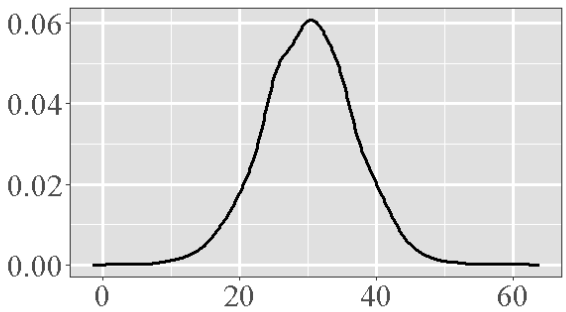

FIGURE 4. MEXICO - POSTERIOR DISTRIBUTION OF THE INTERCEPT IN (4).

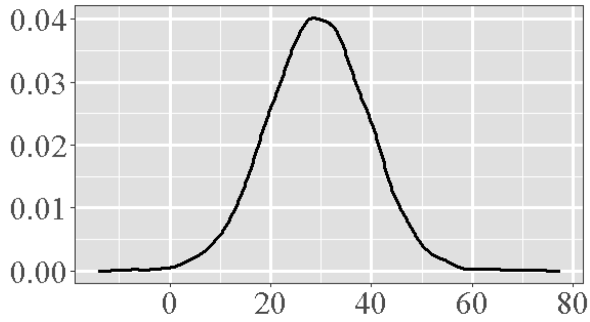

FIGURE 5. CHILE - POSTERIOR DISTRIBUTION OF THE INTERCEPT IN (4).

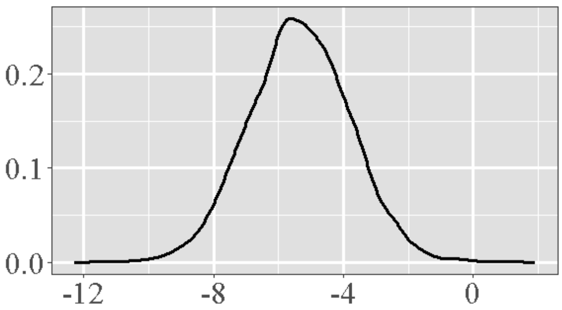

FIGURE 6. PERU - POSTERIOR DISTRIBUTION OF THE INTERCEPT IN (4).

Primary data source:

(A) DataBank, The World Bank.

Cyclical components of the time series were removed by The Hodrick-Prescott decomposition.

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