House price convergence in the very long run: new evidence from Fourier quantile unit root test

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Abstract

We examine the house prices convergence across twelve OECD countries over the period 1905-2016. Using novel quantile unit root tests which allow for smooth breaks via a Fourier expansion series, we find that nine countries show the presence of relative house price convergence at all the quantiles. Focusing on several specific quantiles, eleven countries have significant convergence tendencies. Moreover, there are four definite patterns related to shocks on the relative house prices across quantiles.

Keywords: House prices; Convergence; Unit root; Quantile regression; Fourier expansion
JEL Classification: O18; R31

1 Introduction

The usual consideration of houses as the most important asset in homeowners' portfolios makes the issue of house price convergence a topic of profound interest among economists. House prices largely reflect country's distribution of wealth. Furthermore, relative house prices relate to labour mobility through housing affordability and relocation costs. According to the life-cycle theory of consumption developed by Modigliani and Brumberg (1954), an individual's consumption is determined by the entire lifetime expected income and the value of tangible and financial assets (Deaton, 1992). If such is the case, a housing market downturn can lead to slowing household consumption and hence an economic downturn. Housing, as a consumption good, has a lion's share of non-traded component and a tiny share of traded component. The non-traded component is likely to affect the house price convergence across different regions. The house prices at the country level therefore are expected to reflect a country's fundamentals such as per capita GDP and population. If the fundamentals converge among countries, house prices may also converge.

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Past empirical evidence on house price convergence however was mixed. An extensive studies examined club convergence of regional house prices using log t convergence test proposed by Phillips and Sul (2007) and found evidence of convergence among subgroups of states and cities to their common housing prices (see e.g. Kim and Rous, 2012; Montagnoli and Nagayasu, 2015; Holmes et al., 2019). Nevertheless, other studies presented no supporting evidence for regional house price convergence (see e.g. Holmes and Grimes, 2008; Awaworyi Churchill et al., 2018). The extant literature on house price convergence has mainly focused on the state or city level. There is however only a limited research on house price convergence at country level such as Tsai (2018). We fill this gap in the literature by exploring whether a unique long-run equilibrium exists for house prices where all OECD countries converge to.

The contribution of this paper is two-fold. First, to the best of our knowledge, this is the first study that uses long historical data for multiple countries to investigate convergence of house prices. The long data enable us to understand how evolution of house prices, in what are now the world's richest countries. Moreover, we are able to capture considerable variation in housing prices over time. Our second contribution is we employ a novel quantile unit root test developed by Bahmani-Oskooee et al. (2018). The test is appealing over conventional unit root and standard quantile unit root test for several reasons. First, regardless of whether house prices at a country level are above or below its steady state value, it may exhibit different behaviour to shocks. The quantile regression allows for different speed of adjustment at various quantiles of house prices distribution and capture its asymmetric behaviour. Second, to capture asymmetric behaviour, most unit root tests reply upon particular nonlinear models. In contrast, the quantile unit root test does not need to specify assumptions regarding the functional form of nonlinearities. Third, most of the OCED countries involved armed conflicts and global economic shocks, World Wars and Financial Crises during our long sample period, it is plausible to expect their house prices experienced structural breaks in some years. Our data series therefore may have outliers. The quantile regression enables us to control for non-normally distribution and for the presence of such outliers. Fourth, due to the low frequency of the annual data we used, a Fourier expansion allows us to capture structural breaks in the house prices series.

The remainder of this paper is organised as follows. Section 2 describes the data. Section 3 explores the econometric approaches we adopt in this study. Section 4 presents and discusses the empirical results. Section 5 concludes.

2 Data

We use a historical dataset[1] for twelve OECD countries spanning from 1905 to 2016 on house price index (nominal index, 1990=100) and consumer prices index (CPI) (1990=100) constructed by Jordà et al. (2019). To obtain the real house price index (RHP), we deflate the nominal house price using CPI (i.e. $\frac{\text{Nominal index}\times 100}{\text{CPI}}$). Our sample includes Australia, Belgium, Denmark, Finland, France, Germany, Netherlands, Norway, Sweden, Switzerland, United Kingdom and

[1]Available online at: http://www.macrohistory.net/data/

United States.[2] We select the average house price across all countries as a benchmark and take the natural logarithm of each country's real house price index divided by the mean value of all countries' house price indices.

The summary statistics are presented in Table 1. Over the sample period, Norway has the highest mean house price. Second is Netherlands, followed by Denmark, Sweden, Belgium, United States, Australia, Germany, Switzerland, France and United Kingdom. Finland has the lowest mean house price. To test the non-normality hypothesis of RHP series, we also report the Jarque and Bera (1980) test statistic. Our results provide a firm evidence of non-normal distribution for most of RHP series except, Switzerland and U.S. As argued in Koenker and Xiao (2004), the quantile autoregressive based unit root test has higher power than conventional unit root tests in the presence of non-normality. Therefore, we, in the present study, adopt quantile regression approach to test the convergence hypothesis.

Table 1: Descriptive statistics for real house prices index (1905-2016)

Country	Obs	Mean	Std. Dev.	Min	Max	J-B stat
Australia	112	71.16	57.92	21.65	247.8	44.98***
Belgium	112	88.05	51.96	14.25	217.22	33.59***
Denmark	112	93.82	49.5	35.61	237.94	17.21***
Finland	112	54.26	32.41	4.13	121.95	6.84***
France	112	62.67	48.81	7.63	182.55	17.07***
Germany	112	68.07	30.36	1.45	110.82	9.56***
Netherlands	112	103.04	62.12	39.8	265.67	38.13***
Norway	112	107.17	65.38	50.05	329.16	129.81***
Sweden	112	89.92	40.36	38.64	260.59	196.12***
Switzerland	112	65.91	18.33	34.01	116.01	3.92
United Kingdom	112	57.39	35.76	14.16	188.53	41.72***
United States	112	85.57	24.57	41.29	150.45	4.54

Note: *** denotes statistical significance at the 1% level.

3 Empirical Methodology

We attempt to examine the deterministic convergence hypothesis for RHP of each of the twelve OECD countries toward the group mean as benchmark. The RHP of country i will converge toward that of the benchmark if, and only if:

$$\lim_{n \to \infty} (Y_{i,t+n} - \lambda Y_{b,t+n} | \Omega_t) = 0 \tag{1}$$

where $Y_{i,t+n}$ and $Y_{b,t+n}$ stand for the natural logarithm of the RHP of country i and benchmark at time $t+n$; Ω_t represents the information set at time t. Given our long historical data, it is reasonable to expect the presence of structural breaks. To this end, we employ the most recent

[2]The data of house price index for Belgium, Germany and United Kingdom have some missing observations. We replace missing values using linear interpolation.

developed quantile unit root test by Bahmani-Oskooee et al. (2018) that allows for smooth breaks in the trend component.

Suppose the data generating process of a stochastic variable is:

$$Y_t = \alpha_1 + \alpha_2 t + \alpha_3 sin(\frac{2\pi kt}{T}) + \alpha_4 cos(\frac{2\pi kt}{T}) + o_t \tag{2}$$

where Y refers to the natural logarithm of relative real house prices (RRHP); α_1 is the intercept; t stands for a trend term; o_t represents the residuals of the regression; k denotes the number of frequencies of the Fourier function to capture the smooth breaks in the RRHP; α_3 and α_4 measure the amplitude and displacement of the frequency component respectively. The integer value of k is associated with transitory shocks and fractional value is related to permanent shocks.[3] We use the Becker et al. (2004) method to find the optimum frequency (k^*). Specifically, we set k at a value over the range [0.1, 5] that minimizes the sum of squared residuals (SSR) of ordinary least squares (OLS) estimation is applied to Equation (2). The null hypothesis of unit root in τ_{th} conditional quantile of the residuals ($\hat{o}_t$) from Equation (2) is tested by estimating the quantile regression below:

$$Q_{\hat{o}_t}(\tau|\xi_{t-1}) = \delta_0(\tau) + \delta_1(\tau)\hat{o}_{t-1} + \sum_{p=1}^{p=l} \delta_{1+p}(\tau)\Delta\hat{o}_{t-p} + \vartheta_t \tag{3}$$

where $Q_{\hat{o}_t}(\tau|\xi_{t-1})$ stands for τ_{th} quantile of $\hat{o}_t$ conditional on the past information set, ξ_{t-1}; $\delta_0(\tau)$ denotes τ_{th} quantile of ϑ_t and it measures the size of the observed shock that hits the real house prices within the τ_{th} quantile. Positive (negative) sign represents positive (negative) shock. Optimum lags (p^*) are selected by the Akaike's Information Criterion (AIC).

Although Equation (3) follows standard ADF test at each quantile, our focus is to estimate the vector δ. Following Bahmani-Oskoee et al. (2018), we test the unit root hypothesis within the τ_{th} quantile using the following *t ratio* statistic.

$$t_n(\tau_i) = \frac{\hat{f}(F^{-1}(\tau_i))}{\sqrt{\tau_i(1-\tau_i)}}(E'_{-1}P_x E_{-1})^{\frac{1}{2}}(\hat{\delta}_1(\tau_i) - 1) \tag{4}$$

where E_{-1} is the vector of lagged dependent variable ($\hat{o}_{t-1}$); P_x stands for the projection matrix onto the space orthogonal to $X = (1, \Delta\hat{o}_{t-1}, ..., \Delta\hat{o}_{t-k})$. We follow Koenker and Xiao's (2004) method to obtain a consistent estimator of $\hat{f}(F^{-1}(\tau_i))$.

$$\hat{f}(F^{-1}(\tau_i)) = \frac{(\tau_i - \tau_{i-1})}{X'(\Theta(\tau_i) - \Theta(\tau_{i-1}))} \tag{5}$$

[3]Many factors can lead to deviation of real house prices from its long run steady state. For instance, interest rates, consumer confidence, wars, geopolitical risk. Some of them have permanent effects, while others have transitory effects.

where $\Theta(\tau_i) = (\delta_0(\tau_i), \delta_1(\tau_i), \delta_2(\tau_i), ..., \delta_{1+p}(\tau_i))$ and $\tau_i \in [\underline{\mu}, \bar{\mu}]$. In the present study, we set $\underline{\mu} = 0.1$ and $\bar{\mu} = 0.9$. Bahmani-Oskooee et al. (2018) recommend the following quantile Kolmogorov-Smirnov (QKS) test statistic to test the unit root hypothesis over a range of quantiles.

$$QKS = \sup_{\tau_i \in [\underline{\mu}, \bar{\mu}]} |t_n(\tau)| \tag{6}$$

Since the limiting distribution of $t_n(\tau_i)$ and QKS test statistics are nonstandard and depend on nuisance parameters, we calculate the critical values using Bahmani-Oskooee et al. (2018) re-sampling procedures.

4 Empirical Results

As a benchmark exercise, we first use three traditional unit root tests, namely ADF (Dickey and Fuller, 1979), DF-GLS (Elliott et al., 1996) and KPSS (Kwiatkowski et al., 1992), to examine the stochastic properties of RRHP index (mean value of RHP index across countries as benchmark). The results are presented in Table 2. The results suggest that the unit root null hypothesis cannot be rejected for any of the countries by the ADF and DF-GLS tests. The KPSS test results indicate that the null of stationarity is rejected for all countries. Therefore, we conclude that all RRHP follow random walk processes over the sample period. Such finding, however, could suffer the issue of low power in the presence of structural breaks (e.g. Great Depression, World Wars) and/or non-normal distribution.

Table 2: Conventional unit root tests results (model with constant without trend)

Country	U.S. as benchmark country		
	ADF	DF-GLS	KPSS
Australia	-1.845 [0]	-0.174 [0]	1.161*** (9)
Belgium	-0.081 [11]	-0.528 [8]	0.974*** (8)
Denmark	-0.456 [2]	0.365 [2]	1.091*** (9)
Finland	-1.707 [1]	-1.136 [1]	0.990*** (8)
France	-0.854 [1]	-0.897 [1]	0.810*** (9)
Germany	-1.723 [2]	-1.712* [2]	0.782*** (9)
Netherlands	-0.416 [2]	-0.406 [2]	0.811*** (9)
Norway	0.607 [1]	0.630 [1]	0.677** (9)
Sweden	0.359 [5]	-0.196 [5]	0.709** (8)
Switzerland	-1.585 [1]	-1.018 [1]	1.086*** (8)
United Kingdom	0.242 [2]	0.743 [2]	1.116*** (9)
United States	-0.605 [4]	0.703 [4]	1.140*** (9)

Note: The numbers in the bracket and parenthesis indicate optimum lag length (determined using AIC criteria) and Bartlett (as suggested by Newey and West (1987)). *, **, *** denote statistical significance at the 10%, 5% and 1% levels respectively.

Figure 1: Actual data and estimated Fourier expansion series

Note: The abbreviations of countries are AUS for Australia, BEL for Belgium, DNK for Denmark, FIN for Finland, FRA for France, DEU for Germany, NLD for Netherlands, NOR for Norway, SWE for Sweden, CHE for Switzerland, GBR for the United Kingdom, and USA for the United States.

Next, we estimate the Fourier function represented in Equation (2). Figure 1 shows the time paths of RRHP and the estimated Fourier functions. On the whole, the estimated Fourier functions well fit the fluctuations of RRHP over time, though some series such as Belgium and Germany temporarily deviate from the estimated lines around 1920. From these plots in Figure

1, we should note that the RRHP series may have various types of unknown breaks. Therefore, our Fourier approximations seem to be supported by the data visualization.

Table 3a: Results of quantile unit root test with smooth breaks

Country	K*	F statistic	Fourier QKS statistic			
			Test statistic	10%	5%	1%
Australia	3.5	331.081***	5.124***	2.968	3.256	3.877
Belgium	0.1	163.854***	4.301***	2.792	3.140	3.896
Denmark	2.1	514.789***	4.000***	3.067	3.333	3.927
Finland	4.4	146.873***	3.985***	2.820	3.107	3.920
France	0.9	288.841***	2.153	2.884	3.211	3.961
Germany	0.1	85.209***	1.145	2.528	2.886	3.944
Netherlands	1.0	312.137***	2.840	2.973	3.281	4.010
Norway	0.3	403.606***	3.118*	3.022	3.330	4.008
Sweden	0.1	113.975***	4.882***	2.939	3.266	4.022
Switzerland	4.1	322.002***	5.183***	2.972	3.276	3.993
United Kingdom	0.1	1078.12***	5.614***	3.195	3.504	4.215
United States	2.1	585.961***	4.902***	2.917	3.211	4.008

Note: K* is optimum frequencies. The critical values of the F test and the Fourier QKS test are computed via Monte Carlo simulation with 5000 replications. *, and *** denote statistical significance at the 10% and 1% levels respectively.

Table 3a indicates the results of the Fourier QKS statistic, which tests the unit root null hypothesis at all the quantiles ranging from 0.1 to 0.9 against the stationarity alternative hypothesis. The test results show nine countries out of twelve significantly reject the null; in particular, except for Norway, other eight countries strongly support the stationarity, meaning that RRHP for each country converges to the cross-sectional country mean. K* indicates the optimum frequency for each series, which is between 0.1 and 4.4. As Bahmani-Oskooee et al. (2018) claimed, these optimum frequencies imply structural breaks rather than short-term business cycles. For example, Finland, which has the largest frequency of 4.4, shows at least a 25.5-year cycle of its data variation. Moreover, except for Netherlands, all these frequencies are fractional; therefore, the breaks permanently affect the movements of the relative real house prices. The F-test statistic (Becker et al., 2006), which tests the null of no sine and cosine terms in the model, also supports the inclusion of trigonometric functions because all the null hypotheses are rejected under the 1 percent significance level.[4]

[4]The critical values of the F test for our sample size are computed via Monte Carlo simulation with 10,000 replications. The 1% critical values are 4.871, 4.875, 4.967, 5.030, and 4.978 for frequencies of 1, 2, 3, 4, and 5, respectively.

Table 3b: Results of quantile unit root test with smooth breaks

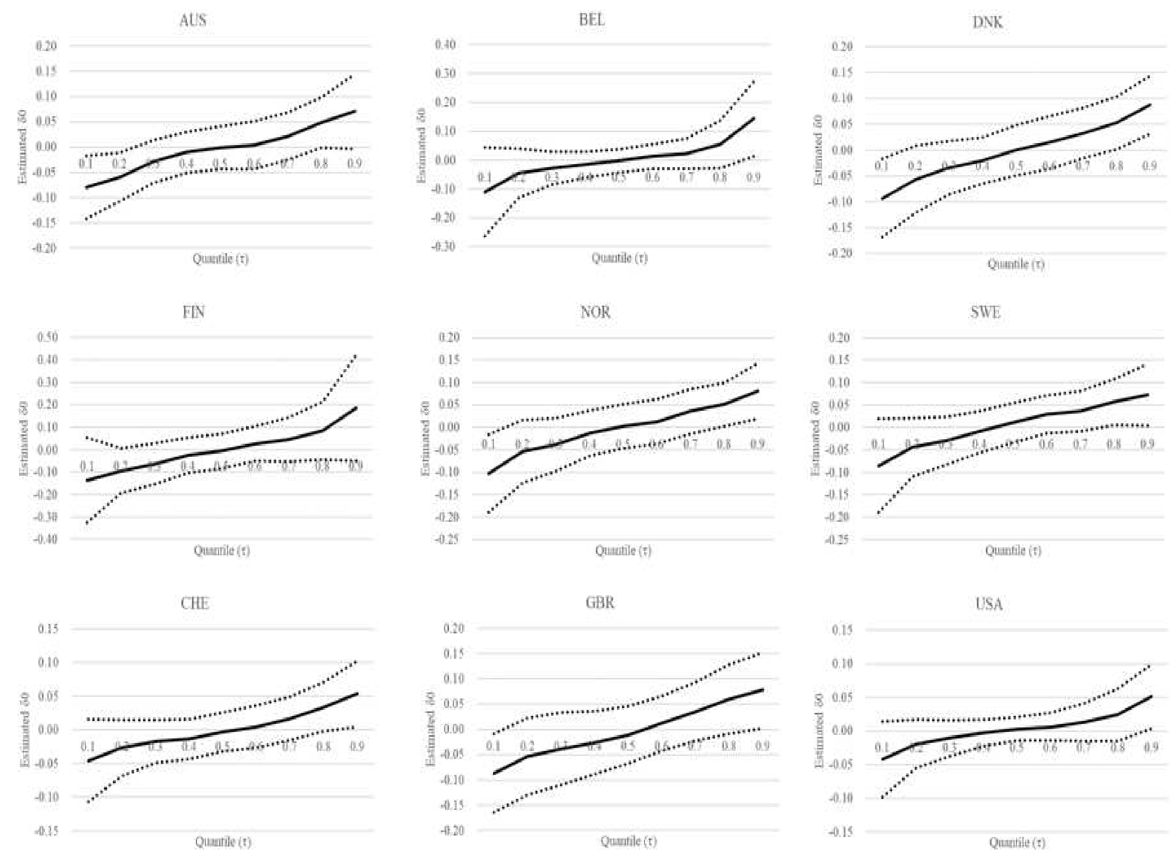
Country	p-value of $t_n(\tau)$								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Australia	0.340	0.136	0.005	0.000	0.000	0.000	0.000	0.051	0.487
Belgium	0.502	0.189	0.025	0.006	0.000	0.002	0.023	0.091	0.044
Denmark	0.002	0.005	0.006	0.014	0.103	0.048	0.011	0.054	0.020
Finland	0.480	0.088	0.002	0.002	0.003	0.001	0.024	0.021	0.036
France	0.444	0.420	0.141	0.133	0.327	0.274	0.057	0.056	0.135
Germany	0.924	0.955	0.837	0.755	0.951	0.923	0.977	0.712	0.139
Netherlands	0.024	0.075	0.058	0.019	0.040	0.015	0.009	0.149	0.087
Norway	0.110	0.094	0.059	0.006	0.020	0.035	0.068	0.152	0.382
Sweden	0.714	0.397	0.139	0.081	0.139	0.138	0.038	0.000	0.022
Switzerland	0.309	0.013	0.000	0.000	0.000	0.000	0.014	0.024	0.296
United Kingdom	0.062	0.000	0.000	0.000	0.000	0.010	0.004	0.053	0.109
United States	0.132	0.023	0.009	0.000	0.001	0.002	0.001	0.018	0.017

Table 3b displays the p-values of $t_n(\tau)$ tests for each quantile. Obviously, Denmark, Finland, Netherlands, the UK, and the US have strong tendencies of the stationarity, i.e., the mean convergence of RRHP, in all the quantiles with only a few exceptional 0.1- or 0.9-quantile cases. The converging trends of Australia, Belgium, Norway, and Switzerland are also comparable to those mentioned above. For each country, the null is rejected in seven or eight quantiles. On the other hand, France and Sweden show only two and four cases of relative price convergence, respectively. No case is observed in Germany. In sum, nine countries firmly support the presence of RRHP convergence, and two countries have weaker but significant converging tendencies. No house price convergence is observed in Germany.

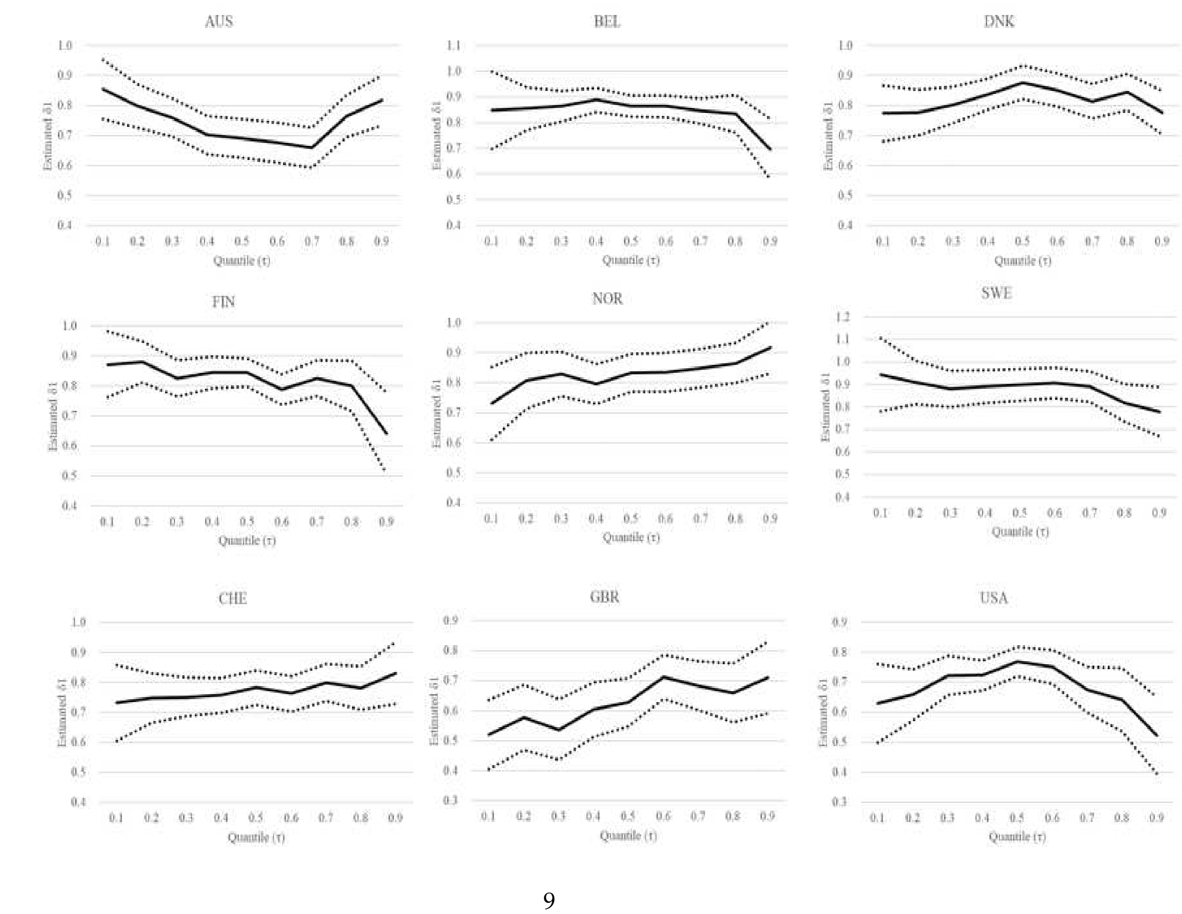
Figure 2 shows the estimated coefficients ($\delta_0(\tau)$ and $\delta_1(\tau)$) of Equation (3) for the selected nine RRHP, significant in the Fourier QKS test. In Panel A of Figure 2, all the estimated quantile intercepts $\delta_0(\tau)$ have upward trends across quantiles. This means that when an RRHP receives a negative shock, which makes its quantile lower, the intercept value correspondingly decreases. When an RRHP receives a positive shock, which makes its quantile higher, the intercept value correspondingly increases.

Figure 2: Selected estimated quantile intercepts ($\delta_0(\tau)$) and autoregressive coefficients ($\delta_1(\tau)$)

Panel A: Estimated quantile intercepts



Panel B: Estimated autoregressive coefficients



Panel B of Figure 2 observes four groups of the estimated autoregressive coefficients $\delta_1(\tau)$ in their shapes. First, Australia has a U-shaped curve, which means that when a negative shock on an RRHP occurs between its 0.1- and 0.4-quantiles, the impact of the shock is more persistent (the house price needs more time to converge to the cross-sectional mean) because the autoregressive coefficient becomes closer to one. When a positive shock occurs in more than its 0.7-quantile, the impact is also more persistent. However, in the middle of quantiles, any shock is transitory. Second, Belgium, Finland, and Sweden have downward trends in their estimated $\delta_1(\tau)$. In particular, their slopes are steeper at higher quantiles. This implies that a positive shock raising an RRHP level is more transitory, and it promotes convergence to the mean because the autoregressive coefficient becomes smaller. Third, Denmark and the US have concave curves. This is the opposite case to Australia. When a negative shock in lower quantiles or a positive shock in higher quantiles occurs, its impact becomes more short-lived. In middle quantiles, any shock is more persistent. Fourth, Norway Switzerland, and the UK show upward trends. If a relative house price rises, which means deviating from the cross-sectional mean, the tendency of deviation lasts longer. Interestingly, except for Norway's RRHP, all the other eight series strongly support convergence to the mean; moreover, there are four definite patterns related to shocks on RRHP across quantiles.

5 Concluding Remarks

This paper examines the house prices convergence across twelve OECD countries for 1905-2016. The novel quantile unit root tests allow us to consider smooth breaks in the relative house prices, expressed as a Fourier expansion series. As a result, we find evidence of convergence toward the cross-sectional mean in nine countries in the Fourier QKS test. Eight of their test results are firmly supportive. Moreover, Bahmani-Oskooee et al.'s (2018) *t ratio* test suggests that except for Germany, the convergence hypothesis holds in all the countries at some specific quantiles. In addition, among the nine countries that reject the unit root null in the Fourier QKS test, there are four definite patterns related to shocks on their relative house prices across quantiles.

Declarations of Interest: None

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