Theoretical Economics and the Second-Order Economic Theory. What is it?

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ABSTRACT

We consider economic agents, agent's variables, agent's trades and deals with other agents and agent's expectations as ground for theoretical description of economic and financial processes. Macroeconomic and financial variables are composed by agent's variables. In turn, sums of agent's trade values or volumes determine evolution of agent's variables. In conclusion, agent's expectations govern agent's trade decisions. We consider that trinity - agent's variables, trades and expectations as simple bricks for theoretical description of economics. We note models that describe variables determined by sums of market trades during certain time interval Δ as the first-order economic theories. Most current economic models belong to the first-order economic theories. However, we show that these models are insufficient for adequate economic description. Trade decisions substantially depend on market price forecasting. We show that reasonable predictions of market price volatility equal descriptions of sums of ***squares*** of trade values and volumes during Δ. We call modeling variables composed by sums of squares of market trades as the second-order economic theories. If forecast of price probability uses *3-d* price statistical moment and price skewness then it equals description of sums of *3-d* power of market trades – the third-order economic theory. Exact prediction of market price probability equals description of sums of *n-th* power of market trades for all *n*. That limits accuracy of price probability forecasting and confines forecast validity of economic theories.

Keywords: theoretical economics, price probability, volatility, market trades, expectations

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1. Introduction

Studies in economic theory cover a broad research area and include, but not limit, topics in general equilibrium and market design, asset pricing and behavioral economics, macroeconomics and monetary economics, decision and game theory, networks and etc. However, the subject and problems of economic theory seems to be not considered as something important or self-sufficient issue of particular interest. Indeed, JEL Classification System of American Economic Association does not have any separate code or at least sub-code that defines studies in economic theory. The word "theory" can be found as component of monetary, value, decision, bargaining, matching, consumer, and contract studies. But theoretical investigations of general economic properties, elements, variables, processes, relations and approximations of economic system as a whole have no JEL code at all. Studies of Altruism have particular JEL code D64, but studies in theoretical economics have no. Of course lack of JEL codes doesn't prevent research of economic theory during last 300 years. "An Essay on Economic Theory" by Richard Cantillon (1755), "Researches into the Mathematical Principles of the Theory of Wealth" by A. Cournot (1838) and "Essentials Of Economic Theory" by J.B. Clark (1915) prove that economic theory is worthy some interest. These and other studies were complemented by enormous number of publications on different special economic models as General Equilibrium (Neumann, 1945; Vines and Wills, 2018), DSGE (Christiano, Eichenbaum and Trabandt, 2018), Economic Growth (Aghion and Durlauf, 2005), Game Theory (Samuelson, 2016), Behavioral Economics (Thaler, 2016), Maximum Principles in Economics (Samuelson, 1970) and other. These articles and references therein only illustrate the huge amount and diversity of publications on economic theory that should help develop and govern economic policy and investment strategies. Numerous models describe important relations that govern economic growth (Aghion and Durlauf, 2005), asset pricing and economic fluctuations (Cochrane, 2017), consumption (Friedman, 1957) and employment (Keynes, 1936).

However, there are left few unsolved "simple" general problems of theoretical economics that can be valuable for description and management of real economic processes.

A great number of economic variables underline complexity and diversity of economic and financial processes. Hence, it is important study general properties of variables that describe real economic state and evolution and consider laws that can describe these variables. It is useful consider general properties of economic and financial variables without specification of their particular importance. We don't study individual relations that define evolution of,

for example, consumption, investment or bank rates but consider all economic variables as equally important.
To do that one should simplify the problem and chose few issues, simple primary bricks of economics, those can serve as ground for successive approximations of real economic processes. As such primary elements we take economic agents, agent's economic and financial variables, trades and deals that agents perform with other agents and numerous agent's expectations that impact agents make transactions, trades and deals with other agents. All these issues are well known and are investigated worldwide for decades. We do not chose or highlight description of any specific or particular economic variable like consumption, demand or investment. We consider general properties of economic variables and trades and study how agent's variables and trades help describe macroeconomics and macro finance. We consider rules for aggregation of agent's variables, agent's trades and agent's expectations and study how these rules impact composition and evolution of macro variables. Models of aggregation of agent's variables introduce general tools for description of specific macroeconomic variables, market trades and expectations and give a general look on nature of business cycles (Olkhov, 2019a; 2020).
Almost all economic theories mentioned above describe macroeconomic and macro financial variables that are composed as sums, during certain time interval Δ, of market trade values and volumes. Consumption and investment, supply and demand, and etc., can be determined by sums of values or volumes of trades between agents during some interval Δ. We call theories that describe variables composed by sums of trade values and volumes as first-order economic theories. However, these first-order economic theories are insufficient for adequate description of economics.
Reasons to state this are simple. Agents make transactions under action of their expectations. Expectations can be induced by numerous factors that may have economic, political, technological, social or other origin. However, price of the trade and agent's assessment of future price play important role for agents expectations and trade decisions. Predictions of price probability at horizon T equal forecasting of price *n-th* statistical moments at same horizon for all n. The price volatility that depends on second price statistical moment plays the key role for trade decision. The price volatility is the first obstacle that makes first-order economic theories insufficient. Below we show that forecasting of the price volatility determined by *2-d* price statistical moment requires description of sums of squares of value and volume of trades during interval Δ. We call models that describe evolution of sums of squares of trade values and volumes as second-order economic theories. We argue that price

predictions that take into account *3-d* price statistical moment should describe sums of *3-d* power of trade values and volumes and equate third-order economic theories. Thus exact predictions of price probability at horizon T correspond development of economic theories that describe sums of *n-th* power of trade values and volumes during interval Δ for all n. Understanding of constrains between predictions of price probability and description of sums of *n-th* power of trade values and volumes may help develop successive approximations. However, no miracles and simple solutions of that problem exist.

We omit almost all technical details and math equations that form essence of our theoretical models but try in a most simple manner explain the necessity, imminency and meaning of the second-order economic theory.

2. Theoretical economics

Description of economics uses numerous different notions, variables and relations. We are delighted by Essay's of Richard Cantillon (1755) as absolutely contemporary research of economic theory. Due to efforts of M. Thornton and C. Saucier one can enjoy Cantillon's study and even its content indicates all main modern economic notions as international trade and business cycles, market prices, money and interest and etc. References above prove that these important economic variables and processes are under consideration till now.

Let us raise some questions. Are there any general relations and laws that govern evolution of economic variables? How one can describe factors that define the state and evolution of economic and financial variables? What are successive approximations that can describe economic evolution? Studies of these problems can help model economic relations between particular variables like supply and demand, investment and economic growth and etc. Below we consider these questions in a most simple manner. To do that, we go though conventional statements.

1. First, let us define simple elements of economic system. Let us consider all participants of economic relations as economic agents. Economic agent-based models are known for decades (Tesfatsion and Judd, 2006; Hamill and Gilbert, 2015). However, we use only conventional treatment of economic agents as primary units, simple bricks of economics. Different agents perform different economic roles and have different variables. Let us consider corporations and banks, industry plants and households, travel agencies and government entities and etc., as economic agents. Economic and financial variables of these agents – credits and investment, assets under management and turnover, profits and consumption, labor and taxes and etc., define origin of all macroeconomic and macro

financial variables. Aggregations of economic and financial variables of all agents of the economy (without duplication) define (Fox et al., 2017) macroeconomic and macro financial variables that describe economic state and evolution. Thus, description of agent's economic and financial variables helps model and forecast macroeconomic variables.

2. Agent's variables change due to transactions, trades and deals performed with other agents. Investment and credit deals, market trades with assets or currencies, transactions of commodities and energy between agents define economic activity and development. Leontief (1955; 1973) used aggregated transactions between productive sectors and industries as a tool for economy modeling. Trades between agents are the only direct ways that change their variables. Various deals, trades and transactions between agents are the only forms of economic processes. Consumption and investment, purchasing and taxes, international trade and labor appear as various forms of transactions and trades between agents. Thus, descriptions of trades, modeling relations that describe intensity of economic deals between agents help model evolution of agent's variables and therefor, model macroeconomic evolution as well.

3. Economic and financial transactions, deals and trades are performed by agents. In turn, agents take decisions to make a particular trade and deal. It is common that agents take trade decisions under their personal expectations. Agent's expectations are formed by economic and market environment, agent's projections, information on expectations of other agents and other "to-day" and "next-day" factors that impact market prices, economics and finance, inflation and currency rates and etc. Agent's expectations (Muth, 1961; Manski, 2017; Farmer, 2019) as drivers of agent's trade decisions and thus drivers of market activity and economic development are studied for decades and above references only indicate importance of expectations for adequate economic modeling.

We take these issues – macroeconomic and macro financial variables that design economic state and evolution, agent's economic and financial variables that compose macro-variables, trades and deals between agents that are the only way to change agent's variables and numerous agent's expectations that force agents to make deals – as the ground of theoretical economics. We consider descriptions of agent's variables, transactions and expectations as the key problems of theoretical economics.

3. First-order economic theory

Almost all current economic models can be treated as first-order economic theories. The meaning of such notion is follows. Modern economic models describe relations between macroeconomic and macro financial variables composed as sums of agent's economic variables. Macroeconomic consumption and investment, demand and supply, saving and credits are composed as sums of corresponding agent's variables (without duplication).

These agent's variables, in turn, are composed by sums of corresponding trades and deals between agents during some time interval *Δ*. For example, agent's investment made during time interval *Δ* are determined as sum of agent's investment deals made during that particular term *Δ*. Agent's consumption during *Δ* is determined as sum of all consumption deals of this agent during *Δ*. Leontief (1955; 1973) described input-output tables as "the inter-sectoral flows of goods and services" composed by sums of trades between productive sectors during some interval *Δ*. Such time interval *Δ* can be equal month, quarter, year and etc.

As we show below, time interval *Δ* that is used to smooth econometric time-series, determines sums of trades between agents or helps assess mean values of variables under consideration, and duration of interval *Δ* plays the key role for theoretical economics.

We underline that current economic models describe relations between variables composed as sums of trades between agents. For example, Keynes' General Theory (Keynes, 1936, Hicks, 1937; Burns, 1954) describes mutual relations between macroeconomic consumption, investment, saving, output. Leontief's model describes relations between input-output table's variables composed as sums of transactions between productive sectors. Behavioral (Thaler, 2016) and game models (Shubik, 2011) describe relations between macroeconomic variables using additional assumptions and special methods.

The common property of these models – economic variables under consideration are composed by sums of trades and transactions between agents during certain interval *Δ*.

However, economics and finance also depend on variables that can't be presented as sums of trades between agents. Price volatility is one of most influential factors that impact markets, macro finance and macroeconomic evolution. The problem of the price volatility predictions creates the next level of complexity of theoretical economics. We underline that impact of such a "particular" variable as the price volatility spreads from modeling of the financial market onto description of the entire macroeconomics. As a price one can consider price of commodities or other assets of market trades, bank rates as "price" of credits, return on invested capital as "price" of investment and etc. Price itself can be described by ratio of trade value to volume, or as ratio of sums of trade value to volume during *Δ*. To explain complexities generated by modeling the price volatility we, at first, discuss the problem of price probability forecast.

4. Price probability

Let us consider the market trades of some assets. Each market trade at moment t_i is described by trade value $C(t_i)$, trade volume $U(t_i)$ and price $p(t_i)$ determined by trivial relation:

$$C(t_i) = p(t_i)U(t_i) \tag{4.1}$$

Market trade time-series can define trade value $C(t_i)$, volume $U(t_i)$ and price $p(t_i)$ with high frequencies and, as usual, market price $p(t_i)$ at moment t_i is described as mean price $p(1)$ determined during some averaging time interval Δ – hour, day, week, etc. Definition of mean price $p(1)$ depends on choice of price probability measure during interval Δ. This "simple" issue hides a lot of complexities (Olkhov, 2021a-c). Indeed, usually it is assumed that price $p(t_i)$ time-series at moments t_i during Δ

$$\Delta = \left[t - \frac{\Delta}{2}, t + \frac{\Delta}{2}\right] \quad ; \quad t_i \in \Delta\,, \; i = 1, \ldots N \tag{4.2}$$

define frequency-based price probability $f(p_k)$ as follows:

$$f(p_k) = \frac{1}{N} m(p_k) \quad ; \quad E[p] = \frac{1}{N} \sum_k p_k m(p_k) = \frac{1}{N} \sum_{i=1}^{N} p(t_i) \tag{4.3}$$

Here $m(p_k)$ defines number of trades at price p_k. Definition of the price probability $f(p_k)$ for N trades performed during interval Δ is a usual and conventional way to treat the market price probability. However conventional issues are not always correct. Indeed, trade value $C(t_i)$ and volume $U(t_i)$ time-series during Δ also define frequency-based probabilities of trade value $\nu(C_k)$ and probability of trade volume $\mu(U_k)$:

$$\nu(C_k) = \frac{1}{N} m(C_k) \quad ; \quad \mu(U_k) = \frac{1}{N} m(U_k) \tag{4.4}$$

Here $m(C_k)$ defines number of trades with value C_k and $m(U_k)$ - number of trades with volume U_k. However, it is obvious that equation (4.1) prohibits independent definition of probabilities of the trade value $C(t_i)$, volume $U(t_i)$ and price $p(t_i)$. Given probabilities $\nu(C)$ and $\mu(U)$ of the trade value $C(t_i)$ and volume $U(t_i)$ (4.4) and (4.1) should define probability $\eta(p)$ of price $p(t_i)$ and it could be different from (4.3). To define the price probability $\eta(p)$ that match the equation (4.1) let us take *n-th* power of (4.1):

$$C^n(t_i) = p^n(t_i)U^n(t_i) \tag{4.5}$$

Then let us assume that time-series of *n-th* power of price $p^n(t_i)$ and *n-th* power of trade volume $U^n(t_i)$ are not correlated during Δ and obtain:

$$C_m(n) = E[C^n(t_i)] = E[p^n(t_i)U^n(t_i)] = E[p^n(t_i)]\, E[U^n(t_i)] = p(n)U_m(n) \tag{4.6}$$

$$C_m(n) = E[C^n(t_i)] = \sum_k C_k^n \; \nu(C_k) = \frac{1}{N} \sum_{i=1}^{N} C^n(t_i) \tag{4.7}$$

$$U_m(n) = E[U^n(t_i)] = \sum_k U_k^n \; \mu(C_k) = \frac{1}{N} \sum_{i=1}^{N} U^n(t_i) \tag{4.8}$$

We use $E[..]$ in (4.6-4.8) to note mathematical expectation. Relations (4.7; 4.8) define *n-th* statistical moments of trade value $C_m(n)$ and volume $U_m(n)$ according to their probability measures (4.4; 4.7; 4.8). Thus, (4.6) defines *n-th* statistical moments of price $p(n)$:

$$C_m(n) = p(n)U_m(n) \tag{4.9}$$

Let us mention that for $n=1$ (4.9) coincides with well-known Volume Weighted Average Price (VWAP) introduced more then 30 years ago (Berkowitz et.al. 1988; Duffie and Dworczak 2018). One can show that assumption of no correlations between time-series of *n-th* power of price $p^n(t_i)$ and trade volume $U^n(t_i)$ time-series does not mean statistical independence between price and trade volume random variables during Δ. In other words, time-series of *n-th* power of price $p^n(t_i)$ and time-series of *k-th* power of trade volume $U^k(t_i)$ for $n \neq k$ can correlate.

It is convenient describe random properties of price time-series during Δ by price characteristic function $F(x)$ and price probability measure $\eta(p)$ (Shephard 1991; Shiryaev 1999; Klyatskin 2005). Price characteristic function $F(x)$ and price probability $\eta(p)$ are determined by mutual Fourier transforms (4.10) (we omit 2π factors).

$$\eta(p) = \int dx\ F(x)\, exp -ipx \quad ; \quad F(x) = \int dp\ \eta(p)\, exp\, ipx \tag{4.10}$$

Price characteristic function $F(x)$ and price probability $\eta(p)$ define price *n-th* statistical moments $p(n)$ as:

$$p(n) = \int dp\ p^n\, \eta(p) = i^{-n} \frac{d^n}{dx^n}\, F(x)|_{x=0} \tag{4.11}$$

The set of price statistical moments $p(n)$ (4.9) for $n=1,2,\ldots$ defines Taylor series of price characteristic function $F(x)$:

$$F(x) = 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!}\, p(n)\, x^n \tag{4.12}$$

Thus, market trade statistical moments $C_m(n)$ (4.7) and $U_m(n)$ (4.8) define price statistical moments $p(n)$ and hence, define Taylor series of price characteristic function $F(x)$ (4.12). All statistical properties of price are described by (4.12) or by set of price statistical moments $p(n)$ (4.9). Taylor series of price characteristic function (4.12) do not allow directly derive price probability $\eta(p)$ via Fourier transform (4.10). However, (4.12) permits obtain successive approximations $F_k(x)$ of characteristic function $F(x)$ (4.12) taking into account finite number of price *n-th* statistical moments (Olkhov, 2021b; 2021c). One can take *k-th* approximations $F_k(x)$ of price characteristic function as:

$$F_k(x) = \exp\left\{\sum_{m=1}^{k} \frac{i^n}{m!}\, a_m\, x^m\right\} \quad ; \quad k = 1,2,.. \tag{4.13}$$

and define coefficients a_m within equation (4.11; 4.14):

$$p_k(n) = i^{-n} \frac{d^n}{dx^n} F_k(x)|_{x=0} = p(n) \;\; ; \; n \leq k \qquad (4.14)$$

For simple approximation $k=2$ obtain (Olkhov, 2021b; 2021c) Gaussian price characteristic function $F_2(x)$:

$$F_2(x) = \exp\left\{i\, p(1)x - \frac{a_2}{2} x^2\right\} \;\; ; \;\; a_2 = p(2) - p^2(1) = \sigma^2(p) \qquad (4.15)$$

Thus, price volatility $\sigma^2(p)$ (4.15) determines Gaussian approximation of the price characteristic function $F_2(x)$ (4.15) and corresponding (4.16) Gaussian price probability $\eta_2(p)$:

$$\eta_2(p) = \frac{1}{\sqrt{2\pi}\sigma(p)} \exp\left\{-\frac{(p-p(1))^2}{2\sigma^2(p)}\right\} \qquad (4.16)$$

Simplicity of Gaussian price probability approximation $\eta_2(p)$ (4.16) is complemented by complexity of price volatility $\sigma^2(p)$ (4.15) determined by *2-d* statistical moments of market trade value $C_m(2)$ and volume $U_m(2)$. Prediction of price volatility $\sigma^2(p)$ (4.15) depends on statistical moments of market trade value $C_m(2)$ and trade volume $U_m(2)$ (4.7; 4.8).

5. Second-order economic theory

Let us note that evolution of *n-th* statistical moments of trade value $C_m(n)$ and volume $U_m(n)$ (4.7; 4.8) are determined by sums of *n-th* power of trade value $C(n)$ and volume $U(n)$ (5.1).

$$C(n) = NC_m(n) = \sum_{i=1}^{N} C^n(t_i) \;\; ; \;\; U(n) = NU_m(n) = \sum_{i=1}^{N} U^n(t_i) \qquad (5.1)$$

As we discussed above, sums during interval Δ of market trades of the first degree $C(1)$ and $U(1)$ (5.1) define most macroeconomic variables. Macroeconomic investment and credits, demand and supply are composed as sums of agent's investment and credit, demand and supply. In turn, agent's investment and credit, demand and supply during Δ are determined by sums of investment and credit trades, sums of demand and supply deals performed by agents during Δ. Thus, sums of trades during interval Δ of the first power $C(1)$ and $U(1)$ (5.1) determine the origin of first order macroeconomic theory.

However, sums of ***squares*** of market trades $C(2)$ and $U(2)$ (5.1) during Δ define new set of macroeconomic variables of the second power. Aggregated by all economic agent sums of squares of investment define macroeconomic investment of the second order. The similar macroeconomic variables of the second order can be introduced for any macro variable of the first order. Evolution of macroeconomic variables of the second order depends on sums of squares of market trades. Actually, agents make decisions on market trades under action of their multiple expectations. These expectations can be created by state and forecasts of the first order variables and by second order variables and by other factors as well. As we show in (Olkhov, 2021a), assessment of average expectations should be made by weighting expectations by the value and volume of trades, made under these expectations. If so, the most influential expectations of the first order trades can be different from most influential expectations of the second order trades. We call approximation, that takes into account evolution of sums of the first and second order trades, expectations that govern first and second order trades and macro variables composed by first and second order trades as second-order economic theory. That is the next level of complexity of economic theory that complement conventional macroeconomic modeling. The first step in the development of second order economic theory is presented in (Olkhov, 2021a).

It is clear that prediction of price probability that takes into account *3-d* price statistical moment *p(3)* should model evolution of price skewness *Sk(p)*

$$Sk(p)\sigma^3(p) = E\left[\left(p - p(1)\right)^3\right] = p(3) - 3p(2)p(1) + 2p^3(1)$$

$$F_3(x) = \exp\left\{i\,p(1)x - \frac{\sigma^2(p)}{2}x^2 - i\frac{Sk(p)\sigma^3(p)}{6}x^3\right\}$$

Thus, approximation of price characteristic function $F_3(x)$ takes into account price skewness *Sk(p)* and *3-d* trade statistical moments $C_m(3)$ and $U_m(3)$ (4.9). Hence, prediction of price skewness *Sk(p)* equals forecasting of *C(3)* and *U(3)* (5.1) and that introduces the problem of third-order economic theory that should model evolution of sums of *3-d* power of trades. Attempt to predict price probability taking into account price kurtosis will require forecasting of *C(4)* and *U(4)* (5.1) and development of the *4-th* order economic theory. Each new level of accuracy of price probability forecasting increases complexity of economic theory. Exact forecasting of price probability at horizon *T* equals description of sums of *n-th* power of trades for all *n*. That seems impossible and hence, create an obstacle for precise forecasting of economic processes. However, understanding the nature of the problem can help develop successive approximations that can deliver reasonable predictions of economic evolution.

6. Conclusion

Theoretical models that describe economic evolution can’t neglect evolution of markets. However, projections of market trades can’t be made without price and price volatility forecasting and thus, should depend on modeling sums of squares of trades. Market trades are the origin of economic evolution and it is rather difficult describe economic processes neglecting market trade laws.

The choice of agent’s variables, trades between agents and agent’s expectations as primary bricks for development successive approximation of economic and financial processes gives opportunity use a wide range of theoretical methods and math. Economic evolution is impossible without risks. Up now risk grades are defined by letters and each rating company use its own risk letter codes (S&P, 2014, Moody's, 2018, Fitch, 2018). However almost 80 years ago Durand (1941) and then Myers and Forgy (1963) proposed numerical risk grades. Unified numerical risk grades contradict with current business models of rating agencies S&P, Fitch and Moody's but will be very beneficial for investors, economic agents and economic authorities. We propose move to numerical continuous risk grades and develop distributions of economic agents by their risk grades. Theory, based on distributions of economic agents by their risk grades gives a powerful tool for description of agent's variables, trades and expectations (Olkhov, 2016-2021a). Business cycles and price fluctuations, economic waves and option pricing can be described via unified theoretical approach. First steps and first results in development of the second-order economic theory confirm that theoretical economics – is worth studying.

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