Public debt dynamics and nonlinear effects on economic growth : evidence from Rwanda

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Abstract

The purpose of this paper is to contribute to existing literature by investigating the nonlinear impact of public debt on economic growth, as well as the long and short run relationship between economic growth and its determinants in Rwanda. To this end, a quadratic polynomial function in debt and the autoregressive distributed lag (ARDL) bounds testing approach to co-integration have been employed for econometric analysis using time series data covering the period 1970-2018. Following many other empirical studies, this research assumed that at lower level, public debt may be growth-enhancing, while at higher level, it is deleterious to growth. Therefore, this study attempted to assess whether in the case of Rwanda, there exists a threshold level or a turning point above which the impact of public debt on economic growth shifts from positive to negative. The empirical results of this study strongly suggest the presence of a concave nonlinear or an inverted U-shape relationship between public debt and economic growth in Rwanda. The turning point above which additional public debt becomes harmful to growth has been evaluated at a public debt-to-GDP ratio equal to 50.2%. This finding provides an empirical support to the public debt convergence policy benchmark of 50% adopted in Rwanda as well in the East African Economic Community member countries. This result would be useful for policy makers in the design of a well-informed macroeconomic and public debt management strategy.

JEL Classification : C23, E62, F34, H63, O40.

Keywords: Economic growth, Public debt, Non-linear relationship, ARDL, Rwanda.

I. Introduction

Background

The growing sovereign debt of industrial countries in the aftermath of the global economic and financial crisis in 2008-2009 has revived the academic and policy debate on the impact of public debt on economic growth. The major question emerging was how to foster a country's economic growth while restoring fiscal sustainability. Empirical works have warned against the danger of excessive and persistent public indebtedness, pointing to the detrimental impact of debt on long-run economic growth and stability (Reinhardt and Rogoff, 2010; Cecchetti, Mohanty and Zampolli, 2010).

A key focus in recent literature on the debt-growth nexus has been the attempt to identify a non-linear relationship between the two variables and find a tipping point above which the impact of the public debt on economic growth switches from positive to negative. In this line of research, an inverted U-shaped relationship between public debt and growth is assumed implying that at lower levels debt has a positive effect on growth, while at higher levels a negative impact prevails. In other words, there is a level of public debt-to-GDP ratio, the threshold or turning point above which public debt is deleterious to economic growth.

Empirical studies have been conducted on the issue by Reinhardt and Rogoff (2010); Kumar and Woo (2010); Checherita-Westphal and Rother, (2010); Baum,Checherita-Westphal and Rother, (2013); Panizza and presbitero (2013); Chudik, Mohaddes, Hashem Pesaran and Raissi (2015); Eberhardt and Presbitero (2015); Gómez-Puig, and Sosvilla-Rivero (2015) and others. The results of these studies are mixed and inconclusive as they depend on the methodology used, the group of countries covered as well as the sample size and the time frame of the analysis; however, they have provided strong evidence that the relationship between public debt and growth is non-linear suggesting the existence of a threshold level above which additional public debt has an adverse effect on economic growth .In addition, they have also showed that due to intrinsic heterogeneities and different public debt-growth dynamics, there cannot be any universal threshold applicable across countries, suggesting that growth-enhancing debt thresholds are most likely to be country-specific.

While a large body of empirical studies has been devoted to the investigation on the public debt-growth nexus in developed countries and contributed to a better understanding of the relationship between the two variables, no such a systematic effort has been observed in carrying out similar studies related to developing countries, yet these countries have been facing challenging debt problems, resulting from the need to mobilize resources in order to promote economic growth and poverty reduction.

Following the debt crisis in the 1990s and its consequences on Governments' spending programs, it was obvious that the external debt burden of low-income countries could not be addressed through traditional debt relief mechanisms. It is in this context that the Heavily Indebted Poor Countries (HIPC) and Multilateral Debt Relief Initiative (MDRI) debt relief mechanisms were set

up by the international community in an attempt to reduce the external debt burden of these countries to sustainable levels so as to free additional resources for development purposes. However, ten years after the debt relief was granted, the external debt in certain low-income countries increased so rapidly that in mid 2010s, it was again a source of concern to policy makers, analysts and international financial institutions. For instance, the foreign debt of the Sub-Saharan African (SSA) countries that had declined from 38% of GDP in 2004 to 24% of GDP in 2006 as a result of the implementation of the HIPC and the MDRI debt relief initiatives, stood at 36.2 % of GDP in 2018, which is largely higher than the post-relief package level; as a consequence, the total public debt also increased from 30% to 56 % of GDP between 2006 and 2018 (International Monetary Fund,2019; World Bank, 2019).

As in other Sub-Saharan African countries, the public debt is a major challenge in Rwanda since the gap between public expenditures and Government revenues has been traditionally supplemented by domestic and foreign borrowing during the last decades in order to finance the national development strategy. The debt trajectory of Rwanda is similar to that of other SSA countries in recent years. Data show that the public debt has been on the rise since the country benefited from the HIPC and MDRI initiatives. While the public external debt had drastically dropped from 79% to 15% of GDP between 2004 and 2006 as a result of the debt relief, it had rebounded to 37.2% of GDP in 2018. Likewise, the total debt which had decreased from 91% to 24% of GDP between 2004 and 2006, has rebuilt to reach 53.1% of GDP in 2018 (IMF, 2019; Rwanda, MINECOFIN, 2019). One could say that the public debt in Rwanda as well as in the other SSA countries has been relatively low in the last years. For the case of Rwanda in particular, it has been consistently recognized through the debt sustainability analysis that Rwanda faces a low risk of external debt distress (International Monetary Fund, May, 2016); however, the observed upward trend of the public debt is a major source of concern because, as it has been pointed out in the literature, the trajectory of the public debt may be more important than the level of the debt itself (Chudik, Mohaddess, Pesaran and Raissi, 2013).

Higher public debt is a great concern, in particular for developing countries because it implies the diversion of government revenues to debt servicing obligations instead of allocating them to development projects; it is therefore important to have a good understanding of the relationship between public debt and other economic indicators to avoid fiscal imbalance that may lead to economic uncertainty and financial crisis and compromise economic growth.

Research problem and motivation

This study is motivated by several reasons. First, most of the empirical studies on nonlinear effects of public debt on growth have been carried out on industrial countries; some of them used samples combining industrial and developing countries and relied on panel data for different groups of countries; this study intends to add to the existing few studies investigating country specific cases in Africa. Second, in the case of Rwanda, the current policy benchmark for public debt-to-GDP ratio has been set at 50%, which is common to all East African Community country members. There are many reasons to believe that this common policy benchmark which

has likely been estimated with panel data from the group of countries, does not account for specific factors prevailing in each country such as the debt dynamics, stage of development, economic and institutional environments. Therefore, an endogenously determined debt threshold level using data on the Rwandan economy would provide a more accurate and reliable measure of the benchmark for policy purpose and new insights on how public debt affects economic growth. Along the lines of the above theoretical and empirical literature on the issue, this study is an attempt to ascertain the impact of public debt on growth in Rwanda and investigate the possibility of nonlinearities or threshold effects of public debt on economic growth.

Objective of the study

The objective of this study is to empirically investigate the possible nonlinear relationship between public debt and economic growth, that is to determine whether there exists a turning point or a threshold level above which the effect of public debt on growth switches from positive to negative in the case of Rwanda; to assess the long-run as well as the short-run relationship between the two variables, controlling for other growth determinants. The estimation of this relationship have been carried out using a novel methodology combining the autoregressive distributed lag (ARDL) bounds testing approach to cointegration developed by Pesaran, Shin and Smith (2001) and the quadratic polynomial function in debt as in the works of Blake, 2015; Ashfad and Padda ,2019; Sanusi, Hassan and Meyer, 2019; Bhatta and Mishra ,2020.

The remainder of the paper is organized as follows. Section 2 presents a selected review of the theoretical and empirical literature; section 3 highlights the trends of economic growth and public debt in Rwanda; section 4 outlines the methodology and the modelling strategy; section 5 describes the data used, while section 6 presents and discusses the empirical results; section 7 summarizes the conclusions and the policy implications of the study.

II. Literature review

The economic theory related to the impact of public debt on economic growth presents contrasted views. The conventional view asserts that in the short run, the output is demand-determined and the increase of public debt associated with a fiscal deficit has a positive effect on the disposable income, aggregate demand and overall output. This positive effect will be particularly large if the country's output is far from capacity (Elmendorf and Mankiw, 1999; Gomez-Puig and Sosvilla-Rivero, 2017). However, things are different in the long run. The decrease in public savings brought about by a higher budget deficit will not be fully compensated by an increase in private savings. As a consequence, national savings is expected to decrease, resulting in lower total investment; this will have a negative effect on GDP growth, as it leads to a smaller capital stock accumulation and lower labor productivity (Presbitero and Panizza, 2013). Furthermore, a growing public debt raises the returns on government securities market leading to increases of long-term interest rates which in turn raises the cost of capital and ultimately

crowds out private investments and reduces growth (Modigliani, 1961; Baldacci and Kumar, 2010).

Higher public debt can also be a drag on economic growth through the debt overhang effect linking external debt and investment. The debt overhang theory argues that if there is some likelihood that in the future, public debt will exceed the country's repayment ability, the expected costs of external debt servicing will depress economic growth. In this context indeed, the returns from investment in domestic economy will face a higher marginal tax induced by debt service payment to existing foreign creditors and consequently new investments by domestic and foreign investors will be discouraged (Krugman, 1988; Sachs, 1990; Karagol, 2002). The negative effect of public debt could even be much larger if higher public indebtedness increases uncertainty in economic policies or leads to expectations of future confiscation through inflation or financial repression (Barro, 1995; Cochrane, 2011).

Another strand of the theoretical literature distinguishes between the effects of "productive" and "unproductive" spending and "distortionary and non-distortionary" taxation on long-term growth (Semmler, Greiner, Diallo and Rajaram, 2007). This literature predicts that productive spending financed by non-distortionary taxes has a positive effect on long-term growth. Public debt can be seen as an alternative instrument for financing government expenditures without the need to raise existing taxes that may create growth-reducing distortions. When allocated to productive purposes such as education, health, roads, research and development, public debt will exhibit positive long-term effects on growth through its impact on the productivity of private sector. Therefore, a positive long run effect on growth might be expected when an increase of government's indebtedness is allocated to productive investments, while a negative effect would prevail if those resources are allocated to unproductive purposes (Devarajan, Swaroop and Zou, 1996; Zagler and Durnecker, 2003). On the other hand, development theory emphasizes that due to shortage of domestic savings in their early stage of development, developing and emerging countries need inflows of foreign resources to enhance capital formation and sustain economic growth (Chowdhury, 2001; Akram, 2016). In contrast to the above theories, the Ricardian equivalence proposition as advanced by Barro (1989) asserts that the change in public debt is neutral with regard to output. The hypothesis argues that when a fiscal stimulus takes place through an increase of budget deficit and acceleration of public indebtedness, the market players anticipate future periods of austerity and tax rises induced by the repayment of public debt. As a result, consumers and businesses increase their savings rate in order to have sufficient funds to offset future tax liabilities; this shift in spending behavior neutralizes the demand stimulating fiscal expansion.

Empirical studies investigating the public debt-growth nexus initially focused on the examination of external debt in emerging and developing countries (Pattilo, Poirson and Ricci, 2002; Schclarek, 2004). It is only recently when the economic and financial crisis reached a global scale in 2008 leading to unsustainable sovereign debt levels in industrial countries that empirical research paid more attention to the potential negative impact of total public debt on economic growth. Most

studies notably triggered by the paper of Reinhart and Rogoff (2010) consistently reported that the relationship between public debt and growth is negative, non-linear and characterized by the presence of a threshold level above which public debt has a detrimental effect on economic growth.

Pattilo, Poirson and Ricci (2002) used different methodologies (OLS, instrumental variables, fixed effects and system-GMM) to examine the relationship between external debt and growth and tested the non-linearity of the relationship between the two variables by means of quadratic debt terms, debt dummies and spline function. The authors used a large panel data set of 93 developing spanning the period 1969-1998. Their findings revealed a non-linear, Laffer-curve type relationship between the amount of external public debt and economic growth; they further estimated that for the considered panel of developing countries, the threshold level above which external debt has a harmful impact on growth is in the range of 35-40% of GDP.

Schclarek (2004) investigated both linear and non-linear relationship between external government debt and economic growth for a panel of 59 developing countries and 24 industrial countries for the period 1970-2002 using GMM dynamic panel data estimator. His findings revealed a significant inverse relationship between total external debt and economic growth for developing countries and this negative relationship was driven by the incidence of government external debt. Regarding the case of industrial countries, the findings did not support either any linear or robust non-linear effect in the relationship between government external debt and economic growth.

In their influential paper, Reinhart and Rogoff (2010) analyzed the impact of different levels of government debt (30% < ,30-60%,60-90%, > 90 %) on the long term real GDP growth for a sample of 20 advanced and 24 emerging countries over the period 1790-2009 using simple correlation. Their study identified the existence of a positive but weak impact of public indebtedness on long-term GDP growth rate before the debt reaches 90 % of GDP; above this threshold, the marginal effect of public debt on growth is negative and significant, meaning that the real economic growth reduction accelerates. Following the above paper by Reinhart and Rogoff, an increasing number of studies were carried out investigating the presence of a threshold level, seen as a turning point in the impact of growing public debt on economic growth. Kumar and Woo (2010) used a variety of methodologies (pooled OLS, fixed effects panel regression and system GMM dynamic panel regression) to examine the impact of public debt on long-run economic growth in 38 advanced and emerging countries over the period 1970-2007. Their empirical results revealed some evidence of non-linear relationship between the initial government debt and subsequent GDP growth, suggesting that in economies with a public debt ratio above 90% of GDP, the decline in economic growth is accelerated.

Checherita and Rother (2010) assessed the impact of public indebtedness on economic growth both in the short and long run for 12 Euro area member states over the period 1970-2008 using panel fixed-effects estimation technique. Using a quadratic model, their findings unveiled the existence of a concave (i.e. inverted U-shape) relationship between public debt and economic

growth with a threshold level of government debt-to-GDP ratio of about 90-100% beyond which the government debt has a deleterious effect on long-term growth. Likewise, Mecinger, Aristovnik and Verbic (2015) evaluated the impact of public indebtedness on economic growth in the short run for a panel of data set of 36 countries (including 24 developed economies and 12 emerging countries) using fixed effects panel regression (FE) and generalized method of moments (GMM). Their empirical results confirmed the general theoretical assumption that at low levels of public debt, the impact is positive, whereas beyond a certain turning point, a negative effect prevails, thus pointing to a non-linear and concave connection between public debt and growth. The estimated debt-to-GDP threshold beyond which the effect of accumulated public debt on economic growth turns to negative is roughly between 90% and 94% for developed economies and between 44% and 45 % for emerging countries.

In their study using data from a large sample of 118 developing, emerging and advanced economies over the period 1960-2012, Eberhardt and Pesibitero (2015) provided evidence that countries with higher average debt-to-GDP ratio are more likely to experience a negative effect on their long-run economic growth performance. However, the authors argue that the public debt-growth nexus differs significantly across countries and empirical results do not show the emergence of any common public debt threshold for all countries over time as was suggested by previous analyses. On the contrary, they point out that the relationship between public debt and growth is complex and the identification of a specific threshold that triggers an economic growth slowdown should take into account debt composition and a variety of country-specific characteristics such as macroeconomic stability and institutional frameworks. The latter conclusion was also stressed in the empirical findings by Chudik, Mohaddes, Pesaran and Raissi (2013) and Pescatori, Andri and Simon (2014) who did not find any universally applicable threshold in the relationship between public debt and economic growth. Furthermore, the authors unveiled a statistically significant and positive threshold effect in the case of countries with a rising debt-to-GDP ratio, suggesting that the debt trajectory may be more important than the level of the debt itself. Put differently, this study argues that only increasing and permanent debt levels will have a detrimental effect on growth, while temporary and declining levels will not compromise economic growth.

In the context of African economies, a few but growing number of empirical studies have explored the non-linear relationship between public debt and economic growth. These studies include: Lopes da Veiga, Ferreira-Lopes and Sequeira ,2014; Megersa ,2014; Baaziz, Guesmi, Heller and Lahiani,2015; Mupunga and Leroux (2015); Ndoricimpa ,2017; Eboreime and Sunday,2017; Sanusi, Hassan and Meyer ,2019; Mensah, Allotey, Sarpong-Kumankoma and Coffie,2019; Ndoricimpa ,2020.

Megersa (2014) employed a sample of 22 Sub-Saharan African countries to address the question of non-linearity in the long-term relationship between public debt and economic growth with the view to unveil the existence of a “Laffer curve” type relationship between the two variables over the period 1990-2011.The results of the study indicate that the contribution of public debt to

growth is positive at lower levels and negative at higher levels. For the sample of selected countries, the turning point above which the effect of additional public debt on growth shifts from positive to negative has been estimated at 45 % of GDP by means of a quadratic function.

Lopes da Veiga, Ferreira-Lopes and Sequeira (2014) used a panel data set from a larger sample of 52 African countries (including Rwanda) covering the period 1950-2012 to examine the implications of public debt on economic growth and inflation. Relying on the same methodology as in Reinhart and Rogoff (2010), the authors conducted a joint analysis of various predetermined public debt thresholds (30%<, 30-60%, 60-90% and > 90%) and the corresponding average growth rates through the sample period. The results of the analysis provide some evidence of a non-linear relationship between public debt and growth that may be described by an inverted U-shaped curve. For the overall sample, the study indicates that below a debt-to-GDP ratio of 60%, public debt is growth enhancing; beyond this threshold, the increase of public debt has a negative effect on economic growth. For the groups of North African and SADC countries, the highest economic growth rate is reached when the debt-to-GDP ratio is below 30%, while for the Sub-Saharan countries the turning point is estimated at 60%.

Mupunga and Le Roux (2015) estimated the optimal growth-maximizing public debt threshold in Zimbabwe for the period 1980-2012 using non-linear regression technique. In this regard, the authors applied a quadratic econometric model to test for the presence of a Laffer-curve type relationship between public debt and economic growth. The findings of the study confirmed the existence of an inverted U-shaped relationship between the two variables in Zimbabwe. The optimal growth-maximizing debt threshold level above which increase of public debt becomes a drag on growth was estimated at a public debt-to-GDP ratio ranging between 45 % and 50 %.

Eboreime and Sunday (2017) assessed the impact of government indebtedness on output growth in Nigeria over the period 1981-2015 with the view to validate the existence of threshold effects of public debt on economic growth. For their technical estimations, the authors relied on least squares, autoregressive distributed lag and optimization methods to estimate the growth-enhancing debt level in the Nigerian economy. The findings of the study point to different threshold levels depending on the selected public debt indicator. Thus, while the optimal public domestic debt-to-GDP ratio was found to be 13.6%, the public external debt is growth enhancing up to 50% of GDP; and there is supporting evidence that the optimal total debt-to-GDP threshold in Nigeria is 55.2%.

Sanusi, Hassan and Meyer (2019) investigated the non-linear effects of the public debt on growth in the Southern African Development Community (SADC) for the period 1998-2016.The authors used a combination of the autoregressive distributed lag (ARDL) bounds testing approach to cointegration and the quadratic polynomial function in debt to assess the long and short run dynamics and the nonlinear relationship between the two variables within a panel framework. The results of their study revealed a nonlinear relationship between the two variables and the existence of a threshold level of 57% of GDP above which public debt is harmful to growth in the long run in the SADC.

III. Economic growth and Debt trends in Rwanda

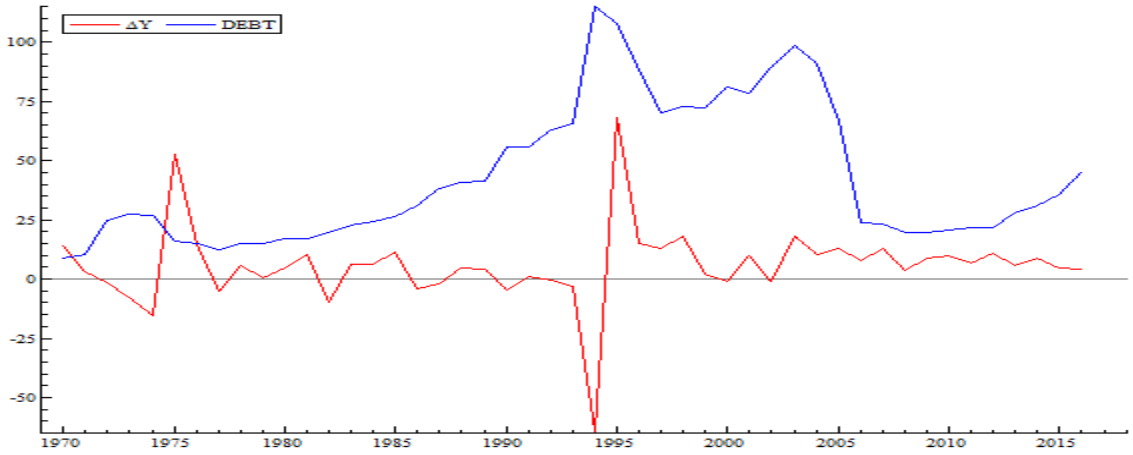
Economic growth

Over the years, like in many other developing countries, Rwandan government has failed to raise enough fiscal revenues to finance its rising expenditures and developmental projects leading to a situation of persistent budget deficit; consequently, it has relied on public domestic and external debt to bridge the gap. Borrowing is justified by the fact that if the amount of debt is used properly may lead to higher growth through capital accumulation and productivity growth and add to the capacity to service and repay the debt.

As a first step to understand the real nature of the relationship between economic growth and public debt in Rwanda, the trend of the annual real GDP growth and the debt-to-GDP ratio of the period 1970-2018 are depicted in Figure 1, while in Table 1, averages of the real GDP growth and different debt indicators in the sample period have been grouped in five sub-periods.

As can be seen in Figure 1 portraying the trends of the real GDP growth and the debt-to-GDP ratio, the Rwandan economic performance was mixed. The real GDP growth was characterized by notable fluctuations resulting from the combined effects of policy changes on the one hand and domestic and external shocks on the other hand.

Figure 1: Trends of economic growth and total debt-to GDP ratio



Since its accession to independence in 1962 until the early 1990s, Rwanda had an administered economy characterized by Government interventions. Many restrictions were imposed on trade and foreign exchange transactions and a fixed exchange regime was implemented.

Not only the Government owned and managed an important economic and financial portfolio, but it also determined the prices of goods and services. The financial system functioned according to the McKinnon-Shaw repression paradigm characterized by Government interference in the operations of the financial system through interest rate ceiling and direct credit control.

Table 1. Economic growth and other debt indicators

Periods / Variables	1970-1979	1980-1989	1990-1999	2000-2009	2010-2018
TD/GDP	17.1	27.8	76.6	60.1	37.03
Real GDP growth	6.06	3.08	4.45	8.27	7.27
DD/GDP	8.4	8.9	18.1	9.9	7.82
ED/GDP	8.7	18.9	58.5	50.2	29.21
DD/TD	49.1	32.0	23.6	20.82	21.28
ED/TD	50.9	68.0	76.4	79.18	78.72

Note: TD = Total debt; DD = Domestic debt; ED = External debt. Computed by the author.

This period also experienced two economic recessions in 1974 and 1982 reflecting the adverse effect of internal factors such as unfavorable weather conditions as well as the oil shocks in 1973-1974 and 1979-1980. It is in this context that as shown in Table 1, the average of real GDP growth rate was estimated at 6.06 % during the 1970s and at 3.08 % in the 1980s.

The period between the 1990 and 2009 was deeply marked by the genocide and the breakdown of the economic production and distribution system that led to the collapse and severe recession of the Rwandan economy at the end of the war in 1994. However, the period also witnessed important economic reforms that allowed Rwanda to not only restore financial stability and recover economic activities but also to gradually move from a state-controlled to a market oriented economy by the liberalization of the monetary and financial regimes. These reforms were implemented through successive adjustment and stabilization programs that focused on privatization of state-owned enterprises, domestic revenue mobilization, strengthening public finance management, enhancing the effectiveness of monetary policy, relaxing exchange restrictions and removing impediments to private sector development. It is worth mentioning that these reforms were supported by large inflows of foreign aid and international technical assistance. As a result of the implementation of sound and consistent economic policies, the national economy recovered to its pre-genocide level in 2002, while the overall GDP registered an average annual growth rate of more than 8% in the decade 2000-2009. During the subsequent period, 2010-2018, the Rwandan economy grew on average by 7.3% per year. This slight slowdown was due to reduction in public spending reflecting cuts and delays in budget support grants from development partners in 2012; it was also explained by the sharp drop in export earnings caused by the global decline in international commodity prices.

Rwanda's public debt profile

From the 1970s up to mid-1990s, the public debt stock in Rwanda exhibited a steady upward trajectory peaking in 1994 and declined thereafter to lower levels. As can be seen in figure 1 and Table 1, public indebtedness in Rwanda was low in the 1970s and 1980s with an average debt-to-GDP ratio equal to 17.1% and 27.8 % respectively. The situation has drastically changed in the 1990s and 2000s, where the debt-to-GDP ratio more than doubled. The Government debt reached its highest level in the 1990s with an average debt-to-GDP ratio of 76.6%. In this period, not only the public indebtedness exceeded 50% of GDP for the first time in 1990, but it also experienced its historical highest level in 1994 reaching 115% of GDP. Many factors explain such developments. Like many other Sub-Saharan African countries, Rwanda experienced an external debt crisis starting in the end of the 1980s; this debt was brought to unsustainable level in the early 1990s.The total debt worsened in the mid-1990s as this period covers the war period, 1990-1994, in which the pre-genocide regime had to borrow large amounts of money domestically as well as externally to finance the war efforts ; the upsurge of the public debt in this period was also aggravated by the successive devaluation of the national currency.

During the 2000s, the public debt has significantly declined dropping to the average of 60.1% of GDP in that period. This development reflected mainly the reduction of the external debt granted to Rwanda by the international community through the HIPC and MDRI debt relief initiatives. Indeed, the total external debt relief provided to Rwanda under the two initiatives amounted to US$ 1.4 billion for HIPC in 2005 and US$516 million for MDRI in 2006 (Cassimon, Essers and Verbeke, 2016) and reduced the relative weight of the external public debt from 79% to 15% of GDP between 2004 and 2006; as a consequence, the total public debt of Rwanda also declined from 91% to 24% of GDP in the same period.

The public debt during the sub-period, 2010-2018, recorded a further decline compared to previous periods, resulting notably from the effect of the previous debt relief, but significant changes in the trend of the public debt were observed within the sub-period reflecting policy changes on the part of the development partners vis-à-vis Rwanda. While the debt-to-GDP ratio was stabilized around 21 % during the first three years of the sub-period, 2010-2012, the trend accelerated in subsequent years as the average ratio increased to 40.1% between 2013 and 2018. This increase of public debt in the last six years of the sample period, 2013-2018, was due firstly to cuts in foreign aid provided to Rwanda in 2012, and secondly to the transformation of official transfers by international development partners from grants to loans that kicked off in 2014. These major changes in the nature of financial inflows explain the increase of public debt in 2013-2018, as the Rwandan authorities had to rely more on domestic as well as on foreign borrowings to compensate for cuts of grants in order to finance development projects. It is worth noting that, as shown in Table 1, the external debt has been predominating in the total Government indebtedness through the whole sample period and its relative weight is even higher in more recent years.

Regarding the nature of the relationship between economic growth and public debt, no obvious conclusion can be drawn on the basis of a visual inspection of the historical trends of the two variables depicted in Figure 1 and Table 1. For example, there is no evidence that higher indebtedness correlates systematically with lower economic growth or the reverse. It emerges instead that the correlation may be either positive or negative, meaning that the two variables may move in the same or in the opposite direction depending on the sub-periods of the whole sample. This ambiguous relationship between growth and public debt provides some preliminary evidence that there may be a non-linear relationship between growth and public debt, implying the existence of a threshold level of public debt around which the effect of public debt on growth may change. However, the identification of a non-linear relationship between public debt and growth and a threshold level of public debt cannot be inferred on the basis of a simple visual inspection and thus constitutes the subject of the empirical investigation of the study.

IV. Methodology and modelling strategy

This section discusses the methodological framework adopted for the empirical analysis of this research including the time series properties of the data, the specification of the model and the outline of the econometric approach.

Time series properties of the variables

To obtain reliable estimations in regression analysis with data based on time series, the variables must be stationary, because non-stationarity may cause spurious regression problems (Granger and Newbold, 1974). Gujarati and Porter (2009) also showed that the F-test, chi-square and t-test statistics of analyses performed with series containing unit roots become unreliable. Therefore, in order to avoid spurious regressions, the time series properties of the variables used in this research have been investigated. The Augmented Dickey-Fuller (ADF) [Dickey and Fuller, 1981] and KPSS (Kwiatkowski-Phillips-Schmidt-Shin, 1992) unit root tests were performed to check whether the variables are stationary or non-stationary; in case the variables were found to be non-stationary, their order of integration was tested. These tests may be complementary since the null hypothesis of the ADF test is the presence of unit root, while the KPSS test assumes stationarity of the variables (Chung and Chinn, 1997).

Basic growth model

The starting point of the investigation on the non-linear effects in the government debt-growth nexus in Rwanda is a general growth model describing the link between economic growth and public debt, while controlling for the other growth determinants. Following previous studies, this model takes the form of a neo-classical growth regression equation augmented with government debt variable:

$$\Delta Y_t = \beta_0 + \beta_1 DEBT_t + \beta_2 X_t + \varepsilon_t \qquad (1)$$

where ΔY_t is the annual real GDP growth rate and Δ is the first difference operator; $DEBT_t$ representing the government debt is defined as the ratio between the outstanding total public debt and the nominal GDP ; X_t is the vector of explanatory variables selected among the most commonly used in the growth literature ; β_1 and β_2 are the matrix of parameters of government debt and explanatory variables respectively; β_0 is a constant and ε_t is the i.i.d error term with mean zero and constant variance.

In growth theory, a common problem is the determination of the main sources of growth or the choice of the set of explanatory variables (X_t) to be included in equation (1). Neoclassical growth theory focuses on physical capital stock, labor force and technological progress as the main driving forces of growth (Solow, 1956), while the endogenous growth theory emphasizes the crucial role that human capital plays in economic growth (Lucas, 1988). In their augmented Solow growth model, Mankiw, Romer and Weil (1992) include human capital as a third factor along with labor and physical capital. Like many other studies, this paper follows these important contributions and relies on the growth determinants suggested by both theories. However, due to data constraint, this study did not use human capital and investment will substitute for physical capital stock in the empirical model.

In the empirical literature, Levine and Renelt (1992) and Sala-i-Martin (1997) argued that despite the existence of a large set of explanatory variables that can potentially be used in the growth regression, only a few of them may be significant; they further checked the robust regressors econometrically. As a result of Sala-i-Martin's test for robustness, the following explanatory variables have been identified as among the most important determinants of growth: investment, population growth or labor force, inflation rate, terms of trade and government expenditure. These variables have in common that they are systematically correlated with growth. Therefore, besides government debt, the empirical analysis of this research for the case of Rwanda relied on these results and used the following explanatory variables: investment (INV), terms of trade (TOT), labor force (LF) and government consumption expenditure (GOV).

Nonlinear model

Since this study seeks to investigate the existence of a non-linear relationship between government debt and economic growth, a model specification that accounts for the polynomial trend of the debt variable is considered. To this end, the quadratic polynomial function has been adopted to estimate the threshold level or the turning point above which the impact of public debt on economic growth shifts from positive to negative in the case of Rwanda. In recent studies, Checherita and Rother (2010); Alfonso and Alves (2014); Mecinger, Aristovnik and Verbic, (2015); Bilan and Ihtanov (2015); Swamy (2015); Afshaq and Padda (2019); Sanusi, Hassan and Meyer (2019), have also relied on the same methodology to capture the nonlinearity of the public debt effects on growth. In line with these works, the following quadratic polynomial function in debt has been used in the present study:

$$\Delta Y_t = \beta_0 + \beta_1 DEBT_t + \beta_2 DEBT^2_t + \beta_3 X_t + \varepsilon_t \quad (2)$$

in which the squared term of debt, $DEBT^2$, has been introduced as an additional regressor to capture the nonlinear relationship between economic growth and public debt.

Autoregressive distributed lag model (ARDL)

To estimate the long and short run dynamics between the variables of interest in equation (2), this study has adopted the autoregressive distributed lag (ARDL) bounds testing approach to cointegration developed by Pesaran, Shin and Smith (2001). To this effect, equation (2) is reformulated into a combined ARDL and quadratic polynomial function framework as follows:

$$\Delta Y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 DEBT_{t-1} + \beta_3 DEBT^2_{t-1} + \beta_4 X_{t-1} + \sum_{i=1}^{n} \eta_1 \Delta y_{t-k} + \sum_{i=1}^{n} \eta_2 \Delta DEBT_{t-k}$$

$$+ \sum_{i=1}^{n} \eta_3 \Delta DEBT^2_{t-k} + \sum_{i=1}^{n} \eta_4 \Delta X_{t-k} + \varepsilon_t \quad (3)$$

in which β_1, β_2, β_3 and β_4 are the long run parameters, while η_1, η_2, η_3 and η_4 are the short run dynamics coefficients of the model; n represents the number of lags of the first differenced variables. The above specification follows the works by Blake (2015); Sanusi, Hassan and Meyer (2019); Afshaq and Padda (2019); Bhatta and Mishra (2020) who used the same methodology combining the ARDL bounds testing approach to cointegration and the quadratic polynomial function to investigate the nonlinear effects of public debt on economic growth in Jamaica, Southern African Development Community, Pakistan and Nepal respectively.

The ARDL bounds testing approach to cointegration has been extensively employed in recent empirical analysis due to some significant advantages it presents over the two alternatives commonly used in the empirical literature, i.e. the univariate analysis proposed by Engle and Granger (1987) and the maximum likelihood procedure developed by Johansen (1988) and Johansen and Juselius (1990). First, the ARDL allows estimation of long run cointegration relationship between variables irrespective of whether the variables are purely I (0), I (1) or a combination of both. Second, unlike the conventional cointegration techniques, which are valid for large sample size, the ARDL bounds testing approach is suitable for small sample size (Pesaran, Shin and Smith, 2001; Ghatak and Siddiki, 2001). Third, the ARDL procedure captures simultaneously the long and short run effects of the independent variables on the dependent variable. Fourth, the ARDL approach provides unbiased estimates, valid and consistent t-test statistic even when some independent variables are endogenous (Chudik, Mohaddes and Raissi, 2015; Harris and Sollis, 2003).

In the ARDL framework, the test for the existence of a cointegration relationship between the variables is performed by testing the joint significance of the lagged level variables (y_{t-1}, $DEBT_{t-1}$,

$DEBT^2_{t-1}$ *and* X_{t-1}) in equation (3) by conducting the Wald coefficient restriction test (F-test); the null hypothesis of no cointegration is $H_0: \theta_1 = \theta_2 = \theta_3 = \theta_4 = 0$ against the alternative $H_1: \theta_1 = \theta_2 = \theta_3 = \theta_4 \neq 0$. The estimated F-test value is compared with the critical values tabulated in Pesaran, Shin and Smith (2001); Narayan (2005) also provided F-test critical values for small samples. For a given level of significance, if the estimated F-test is greater than the upper bound critical value, then the null hypothesis of no cointegration is rejected; conversely, if the computed F-test statistic is smaller than the lower bound critical value, then the null hypothesis is not rejected; on the other hand, if the computed F-test falls between the lower and upper bound values, then the result of the test is not conclusive. A significant F-test statistic for testing the joint significance of the lagged level variables indicates the existence of a long run relationship and on the basis of equation (3) the long run parameters capturing the long run effects of the explanatory variables on the dependent variable are normalized on β_1 and calculated as $\gamma_0 = -\theta_0/\theta_1$; $\gamma_2 = -\theta_2/\theta_1$; $\gamma_3 = -\theta_3/\theta_1$; $\gamma_4 = -\theta_4/\theta_1$ and the model with long run coefficients is derived as follows:

$$Y_t = \gamma_0 + \gamma_2 DEBT_t + \gamma_3 DEBT^2_t + \gamma_4 X_t + \eta_t \qquad (4)$$

Once the long run relationship is established between the dependent variable and the explanatory variables, the short run impact of the independent variables can be estimated by means of the corresponding ARDL-ECM model:

$$\Delta Y_t = \theta_0 + \sum_{i=1}^{n} \eta_1 \Delta Y_{t-k} + \sum_{i=1}^{n} \eta_2 \Delta DEBT_{t-k} + \sum_{i=1}^{n} \eta_3 \Delta DEBT^2_{t-k} + \sum_{i=1}^{n} \eta_4 \Delta X_{t-k} + \mu ECM_{t-1} + \varepsilon_t \qquad (5)$$

where μ is the coefficient of the error correction term which measures the speed of adjustment of the model towards the long run equilibrium, its value is expected to be negative and lie in the interval (0, -1). If on the contrary, the coefficient of the error correction term is positively signed, this suggests that the convergence to equilibrium does not occur; implying that exogenous shock leads to permanent deviation from equilibrium (Folarin and Asongu, 2017).

In order to determine the existence of nonlinearity in the relationship between public debt and economic growth, equation (4) is estimated and the coefficients of the linear and quadratic debt terms are assessed.If the coefficients of the linear and quadratic debt terms, that is γ_2 and γ_3, are significantly different from zero, it may be concluded that there exists a nonlinear relationship between public debt and growth and the nature of the nonlinearity is determined by the signs of the two coefficients. In the case γ_2 is negative and γ_3 is positive, then the relationship between the two variables follows a U-shaped pattern; if on the other hand, γ_2 turns out to be positive and γ_3 is negative, then the public debt-growth nexus may be described by an inverted U-shaped relationship.

In this study, the inverted U-shaped relationship is hypothesized, meaning that the linear term of government debt, $DEBT_t$, would have a positive sign to reflect the beneficial effects of low public debt on output, while the squared term of government debt, $DEBT_t^2$, is expected to have a negative sign and should measure the adverse impact associated with higher public debt. Since the squared term increases in value faster than the linear term, it implies that the presence of

negative effects of debt will eventually outweigh its positive effects. The peak of the quadratic function identifies the public debt threshold level or the turning point above which the marginal effect of additional public debt becomes negative.

To calculate the critical point corresponding to the growth-enhancing debt level, the first-order partial derivative of equation (4) is computed with respect to $DEBT_t$, and is set equal to zero:

$$\delta y_t / \Delta DEBT_t = \gamma_2 + 2\gamma_3 = 0 \qquad (6)$$

Solving the above equation for $DEBT_t$, the critical point of public debt above which the marginal impact of debt becomes negative is obtained as follows:

$$DEBT_t^* = -\gamma_2 / 2\gamma_3 \qquad (7)$$

V. Data description and sources

The basic data used in this study are time series of nominal GDP, total outstanding public debt, labor force (defined as the total active population aged between 15 and 64 years) , consumer price index (CPI), gross fixed capital formation (as a proxy for investment or physical capital), government consumption expenditure, imports and exports .The set of data spanning the period 1970-2018 was collected from the World Bank World Development Indicators (WDI) and from various documents published by the National Bank of Rwanda, the National Institute of Statistics of Rwanda (Statistical Bulletins and Annual Reports) and the Ministry of Finance and Economic Planning of Rwanda. The nominal gross domestic product valued in domestic currency (Rwandan francs) has been deflated by the consumer price index to obtain the real GDP and the base year of the CPI is 1990 = 100. From the above basic data, the following variables have been created for the empirical analysis: real GDP growth (ΔY_t); debt variables (DEBT and $DEBT^2$) , where DEBT is the public debt to nominal GDP ratio ; Investment (INV), represented by the ratio of the gross fixed capital formation to nominal GDP ; terms of trade (TOT) as a ratio between exports and imports to account for trade openness; labor force (LF) and government consumption expenditure (GOV) representing fiscal policy effects in the analysis.

Following a common practice in empirical literature, most of the variables have been expressed in their natural logarithm terms to ensure uniformity of scaling among the variables and mitigate the impact of heteroscedasticity; in addition, the log transformation of the variables allows the estimated coefficients to be interpreted as elasticities (Bashar, 2015; Idris, Bakar and Ahmad, 2018).The debt variables, DEBT and $DEBT^2$, have not been log-transformed since their transformation makes the regression model subject to perfect multi-collinearity.

VI. Empirical results and discussion

6.1. Time series properties

Before carrying out the empirical analysis, the unit root tests were conducted to check the order of integration of the variables (real GDP, public debt, investment, terms of trade, labor force and

government consumption expenditure) to avoid spurious regressions. In particular, when using the ARDL bounds testing approach to cointegration, this step is necessary to ensure that none of the selected variables is integrated of an order higher than one; indeed, in the presence of I (2) variables, the F-statistics computed by Pesaran, Shin and Smith (2001) and Narayan (2005) are no more valid since they are based on the assumption that the variables are I (0) or I (1).

Table 2: Results of Unit root tests: ADF and KPSS

Panel A	*ADF*			
Variables	*Level*		*First difference*	
	Constant	*Constant & trend*	*Constant*	*Constant & trend*
Real GDP growth	*-0.41026*	*-1.92994*	*-8.21649*	*-8.24332*
Debt	*-2.08641*	*-2.05321*	*-5.21096*	*-5.17285*
Debt^2	*-1.99366*	*-1.94580*	*-5.62158*	*-5.57130*
Investment	*-2.43056*	*-3.61779*	*-7.05290*	*-7.00147*
Terms of trade	*-2.68669*	*-259671*	*-9.01604*	*-8.99096*
Labour force	*-0.15724*	*-2.17424*	*-4.85622*	*-4.79960*
Government consumption expenditure	*-3.42693*	*-3.37973*	*-7.35066*	*-7.28240*
Critical values at 1% significance level	*-3.57444*	*-4.16114*	*-3.57772*	*-4.16575*

Panel B	*KPSS*			
Variables	*Level*		*First difference*	
	Constant	*Constant & trend*	*Constant*	*Constant & trend*
Real GDP growth	*0.729286*	*0.173117*	*0.205232*	*0.094556*
DEBT	*0.275925*	*0.165216*	*0.104008*	*0.070344*
DEBT2	*0.235629*	*0.160294*	*0.110992*	*0.086357*
Investment	*0.769180*	*0.113462*	*0.138775*	*0.100659*
Terms of trade	*0.317115*	*0.182804*	*0.226067*	*0.205972*
Labor force	*0.912858*	*0.087787*	*0.058823*	*0.059062*
Government consumption expenditure	*0.202662*	*0.086026*	*0.153502*	*0.135358*
Critical values at 1% significance level	*0.739000*	*0.216000*	*0.739000*	*0.216000*

The ADF and KPSS unit root tests have been performed with (i) an intercept and (ii) an intercept and a trend. As reported in Table 2, the results of the ADF (Panel A) and KPSS (Panel B) unit root tests show that the variables are either I (0) or I (1) at level, but they are all stationary at first difference; hence, none of them is integrated of an order higher than one. This outcome allows a valid use of the F-test proposed by Pesaran, Shin and Smith (2001) and Narayan (2005) for testing cointegration between the selected variables. In addition, the fact that the variables are integrated of different order at level makes the ARDL bounds testing approach appropriate for empirical estimation.

6.2. Results from ARDL bounds testing

Cointegration test

The ARDL bounds testing approach estimation process starts with the determination of the optimal lag length of the first differenced variables in equation (3). Given the relatively small sample size (49 observations) and the use of annual data in this study, the maximum number of lags has been set to 3 as suggested in Chudik,Mohaddes , Pesaran and Raissi (2015) and Afshaq and Padda (2019); this number of lags is long enough to capture the short run dynamics of the variables and ensure that there is no serial correlation in the residuals.

After the lag order of the model has been selected, the ARDL model of equation (3) has been estimated by OLS; thereafter, the Hendry's (1995) general to specific modelling procedure has been used to derive a parsimonious model on the basis of which the ARDL bounds testing for cointegration was carried out and the long run coefficients of the model were determined. It is worth noting that in the ARDL presentation, symmetry of lag lengths is not required, meaning that each variable may have different number of lag terms (Gomez-Puig and Sosvilla-Rivero, 2017). The final parsimonious model was an ARDL (1, 3, 1, 1, 1, 1).

The results of the estimated F-test by the bounds testing procedure to ascertain the existence of a cointegration relationship between the dependent and explanatory variables is reported in Table 3 in which the estimated value of the F-test is compared with the critical values tabulated by Pesaran, Shin and Smith (2001) and Narayan (2005).

As shown in Table 3 (Panel A), the null hypothesis of no cointegration is rejected at the conventional 5% significance level, since the value of the computed F-test from the parsimonious ARDL model, which is 4.5356, is greater than the upper bounds of the critical values tabulated by both Pesaran, Shin and Smith (2001) and Narayan (2005). This provides clear evidence for the existence of a long-run equilibrium relationship between real GDP, public debt, investment, terms of trade, labor force and government consumption expenditure.

The validity of the ARDL parsimonious model has been evaluated and according to the standard diagnostic tests reported in Table 3 (Panel B), the model is free from non-normality, serial correlation and heteroscedasticity. The goodness of fit of the model is also satisfactory, as more

than 90% of the variations of the dependent variable are explained by the selected independent variables.

Table 3. Results of bounds testing for cointegration

Panel A	*Calculated F-Test: 4.5346*		
Critical values at 5% significance level			
Pesaran ,Shin and Smith (2001)		*Narayan (2005)*	
Lower bound value I(0)	*Upper bound value I(1)*	*Lower bound value I(0)*	*Upper bound value I(1)*
2.32	*3.50*	*2.593*	*3.941*
Panel B	*Goodness of fit and Diagnostic tests*		
R-squared: 0.907223 *Adjusted R-squared: 0.848807* *Durbin-Watson statistic: 2.103074* *Normality test: F-statistic: 1.244126 (0.53836)* *Serial correlation test: F-statistic: 1.529843 (0.2362)* *Heteroscedasticity test :0.720583 (0.7567)*			

Since the cointegration relationship between growth and its determinants has been established, the next step is to estimate the long run coefficients of the model.

Long run relationship

The normalized long-run coefficients which were derived from the parsimonious ARDL model are reported in Table 4; these coefficients have been estimated by dividing the coefficients of the one period lagged independent variables of equation (3) by ($-\beta_1$), the coefficient of the one period lagged dependent variable to produce the coefficients of the long run model of equation (4).

Table 4: Long-run coefficients

Variables	*Coefficients*	*Std.Error*	*t-Statistic*	*Prob.*
C	*-6.3654*	*2.290313*	*-2.921430*	*0.0212*
DEBT	*0.9347*	*0.456951*	*2.441471*	*0.0507*
$DEBT^2$	*-0.9318*	*0.378335*	*-2.939629*	*0.0205*
lnINV	*0.8073*	*0.302929*	*3.180963*	*0.0128*
lnTOT	*0.2502*	*0.097737*	*3.055245*	*0.0164*
lnLF	*0.9462*	*0.286533*	*3.941506*	*0.0027*
lnGOV	*0.1326*	*0.165052*	*0.958651*	*0.4289*

As can be observed, the parameters of all the explanatory variables included in the long run model are statistically significant at the 5% significance level, except for the government consumption expenditure variable whose coefficient is not significant. Regarding the signs of the coefficients, the results indicate that they are generally in line with theoretical expectations.

Investment, terms of trade and labor force are positively associated with economic growth. A change of one percentage point in investment results in 0.81% percent increase of GDP growth in the long run, while one percent rise in terms of trade leads to 0.25 % increase of GDP growth in the long run. The positive and significant effect of investment on economic growth highlights the importance of physical capital accumulation in the production process; this result is consistent with the expectation of the study and is supported by other growth empirical studies including Megersa (2014); Pegkas (2018) and Ndoricimpa (2020).

Trade openness represented by the terms of trade in the model is growth has a positive impact on economic growth. This finding is line with theory as openness to trade increases access to free markets and contributes to exploitation of comparative advantages; it is further expected to enhance total factor productivity through the transfer of knowledge and efficiency gains in the allocation of resources. This result is also in accordance with recent empirical literature as shown by Afshaq and Padda (2020); Sanusi, Hassan and Meyer (2019) and Kummer-Noormamode (2018) who reported a positive relationship between trade openness and GDP economic growth.

Labor force enhances economic growth, as a change of one percentage point in the former variable induces an increase of around 0.95 % of real GDP growth. This outcome contradicts the neoclassical theory that predicts an inverse relationship between economic growth and population growth because the available capital must be spread over a larger population (Mankiw, Romer and Weil, 1992). In contrast, this result validates the findings of earlier empirical studies by Kuznets (1960); Kling and Pritchett (1995); it is also in line with the findings of the works of Bashar (2015) and Ndoricimpa (2017) who found a positive association between population and economic growth in the context of African economies. It has been suggested that Africa's youth population dividend is a major opportunity for the continent as the increasing working-age population is seen as an important determinant for economic growth in Africa (World Bank, 2019).

Government expenditure has a positive sign in the long run model, but it is not statistically significant; this result is similar to the findings by Sanusi, Hassan and Meyer (2019) showing that government expenditure does not affect economic growth in the long run in the Southern African Development Community. In fact, the impact of government expenditure on growth is mixed in the literature; some recent studies provided evidence of a positive impact (Pegkas, 2018; Eze, Nweke and Atuma, 2019) while others found an adverse effect (Mencinger, Aristovnik and Verbic, 2014; Idris, Bakar and Ahmad, 2018).

With regard to the public debt variable in the model, the empirical analysis focuses on the non-linearity of the relationship between this variable and the economic growth. As explained earlier

the nature of the non-linearity depends on the significance and the signs of the debt and debt squared terms in the model. As can be observed in Table 5, the debt term, Debt, and the squared debt term, $Debt^2$, are statistically significant; on the other hand, the debt term is positive, while the squared debt term is negative. These results point to the existence of a nonlinear relationship between public debt and economic growth that may be described by a concave or inverted U-shaped curve; they further suggest that there exists a turning point or a threshold level above which the impact of additional public debt on economic growth shifts from positive to negative. The turning point has been estimated by solving the partial derivative of growth in the long run model with respect to debt equated to zero as in equation (6); upon solving the equation, the point estimate of the debt threshold is equal to 50.2%, implying that beyond this level, public debt accumulation becomes deleterious to economic growth in Rwanda. This result is in the range of public debt threshold estimates in previous studies. Chudik, Mohaddes, Pesaran and Raissi (2017) found a public debt threshold in the range of 30-60% for developing countries; while Mensah,Allotey ,Sarpong-Kumankoma and Coffie (2019) estimated a public debt threshold level varying between 20% and 50% for a sample of African countries.

To sum up, the findings of this study are consistent with those of many other empirical studies that have confirmed in the context of African economies the existence of a nonlinear relationship between public debt and economic growth including Koffi (2019); Mensah, Allotey, Sarpong-Kumankoma and Coffie (2019); Ndoricimpa (2017); Mupunga and Le Roux (2015); Megersa (2014); Lopes da Veiga, Ferreira-Lopes and Sequeira (2014). More interestingly, the result of the present research corroborates the studies of Blake (2015), Sanusi, Hassan and Meyer (2019), Afshaq and Padda (2019), Bhatta and Mishra (2020) who found an inverted U-shaped relationship between public debt and economic growth using the same methodology that combines a quadratic polynomial function and the ARDL bounds testing approach to cointegration. Furthermore, this result confirms the general theoretical assumption that at low levels, the impact of public debt on growth is positive, while beyond a certain turning point, a negative effect on growth prevails (Elmendorf and Mankiw, 1999; Pattilo, Poirson and Ricci,2002). Finally, the results of this study provide an empirical support to the public debt-to-GDP ratio of 50% adopted by Rwanda and the other members of the East African Community as a convergence policy benchmark.

Short run dynamics

The dynamic model associated with the long run equilibrium model reported in Table (4) has been estimated and the coefficients as well as the error correction term of the short-run model are reported in Table 5. It is noteworthy that as in the long run, investment and terms of trade are statistically significant at the 1% significance level and have a positive sign, implying that these variables are also growth-enhancing in the short run.

Table 5: Error correction model and short-run coefficients

Panel A.				
Variables	*Coefficients*	*Std.Error*	*t-Statitstic*	*Prob.*
C	*0.229140*	*0.089705*	*2.554383*	*0.0148*
ΔlnY_{t-1}	*-0.187281*	*0.073529*	*-2.547043*	*0.0150*
ΔDEBT	*0.622798*	*0.324052*	*1.921905*	*0.0621*
$\Delta DEBT^2$	*-0.809069*	*0.233026*	*-3.472016*	*0.0013*
ΔlnINV	*0.509688*	*0.115567*	*4.410319*	*0.0001*
ΔlnTOT	*0.180905*	*0.051155*	*3.536373*	*0.0011*
$\Delta lnLF_{t-1}$	*1.161505*	*0.704388*	*1.648957*	*0.1074*
ΔlnGOV	*0.209345*	*0.080933*	*2.586648*	*0.0136*
ECM_{t-1}	*-0.304653*	*0.110854*	*-2.748233*	*0.0091*
Panel B				
Goodness of fit and Diagnostic tests *R-squared: 0.857012* *Adjusted R-Squared: 0.826909* *Durbin-Watson stat: 1.681232* *Normality test: 0.191583 (0.908653)* *Serial correlation test: F-test: 2.342643 (0.1106)* *Heteroscedasticity test : F-test : 0.820508 (0.5895)*				

In contrast to the results of long run estimates, population has a positive but insignificant impact on growth. This could be explained by the low quality of the Rwandan labor force in the short run; however, as training is provided to improve education and skills, the contribution of labor in economic growth would increase through higher labor productivity in the long run.

The short-run model also shows that contrary to the long run model, government consumption expenditure is significant and affects positively economic growth, suggesting that fiscal policy not only boosts economic growth in the short run, but would also be an effective policy instrument for stabilizing the economy and smoothing business cycles in Rwanda.

As expected, the coefficient of the error correction term is negative, less than unity and statistically significant at 1% significance level. This provides further support to the existence of a stable long run relationship between the dependent and the explanatory variables of the model as reported in Table 5 (Banerjee, Dolado and Mestre, 1998); it implies further that the short run model converges back to the long run equilibrium relationship after a disturbance resulting from any shock or policy effect. The coefficient of the error correction term represents the speed of adjustment to restore equilibrium in the dynamic model following a shock in the explanatory variables. The size of the parameter of the error correction term is equal to -0.30, suggesting that any deviation from the long run equilibrium caused by an external shock in the previous period

is corrected approximately by 30% within a year; the closer is the speed of adjustment to -1, the quicker will the economy return back to equilibrium path whenever a shock is experienced from any of the covariates included in the model.

Considering the debt variable, the empirical results show that as in the long run model, the parameters of the debt and squared debt terms are statistically significant and have positive and negative signs respectively. This outcome points to the existence of a nonlinear relationship between economic growth and public debt. Similar to the long run result, this nonlinearity in the short run may also be described by an inverted U-shaped relationship. Consequently, as in the long run, there exists also a turning point above which additional public debt is harmful to growth in the short run. The estimated turning point is a debt-to-GDP ratio equal to 38.5%. This contradicts the results by Sanusi, Hassan and Meyer (2019) who found that in the short run, public debt exerts neither linear nor nonlinear impact on economic growth in the Southern African Development Community; however, it is consistent with the results of Afshaq and Padda (2019) which confirmed in the case of Pakistan the existence of a nonlinear relationship between public debt and growth in the short run.

To assess the robustness of the short run analysis, the ARDL error-correction model has been subjected to the conventional diagnostic tests. As reported in Table 5 (Panel B), the short run model successfully passes the non-normality, autocorrelation and heteroscedasticity tests; on the other hand, the model fits reasonably well with an adjusted R^2 of 0.83.

Stability test

To complement the diagnostic tests and ascertain further the validity of the short run and long run models, it is now common practice in the empirical literature to test for the structural stability of the parameters using the cumulative sum of recursive residuals (CUSUM) and the cumulative sum of squares of residuals (CUSUMSQ) tests developed by Brown, Durbin and Evans (1975). In this regard, this study followed Dagher and Kovanen (2011) and Padhan (2011) and applied the CUSUM and CUSUMSQ to the residuals of the error correction model shown in Table 5.

Figure 2: CUSUM stability test

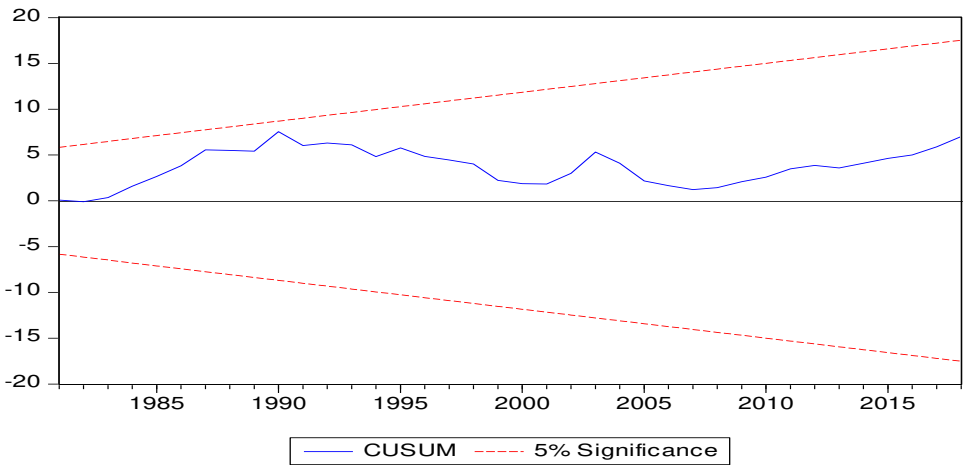
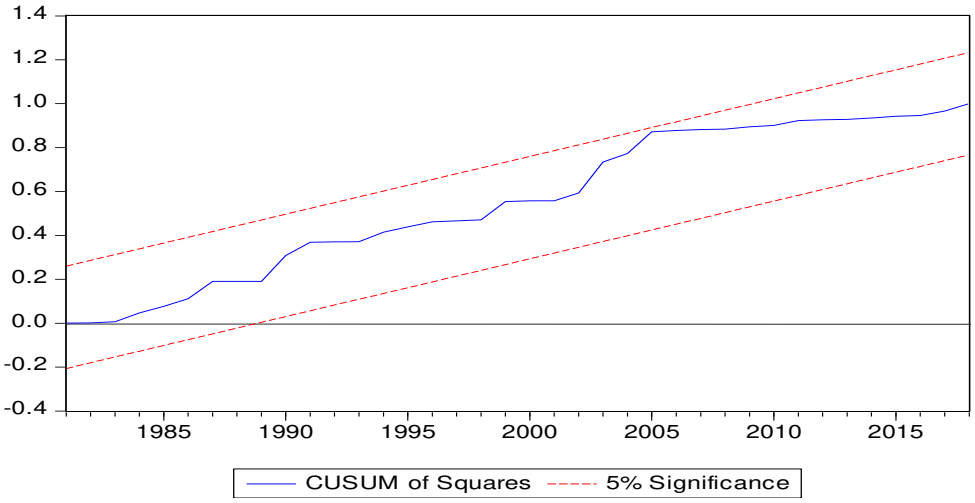


Figure 3: CUSUMSQ stability test



These tests which are based on the cumulative sums of the estimated residuals have been intensively used for testing linear regressions; they have further been extended to nonlinear regression functions such as the threshold autoregressive (TAR) and the quadratic polynomial function models (Kirsh and Kamgaing, 2011; Tenaw and Demeke ,2020). The wide spread use of the CUSUM tests in empirical literature was due to the fact that they are robust to the presence of endogenous regressors in both a stationary and cointegration environment (Carporale and Pittis, 2004).

In their application, the movement of the CUSUM and CUSUM squares of residuals observations are updated recursively and plotted in a band representing the bounds of the critical region. It is assumed that the estimated parameters are stable overtime if the plot of these statistics lies inside the critical bounds of 5% significance level.

Figures 2 and 3 report the results of the CUSUM and CUSUMSQ tests respectively. These results show that the plot of the CUSUM and CUSUMSQ test statistics stay within the critical bounds of 5% confidence interval, implying that the null hypothesis of structural stability of the parameters is not rejected. Therefore, this outcome indicates the absence of any instability in the regression coefficients and provides additional support to the robustness of the long and short run models.

VII. Conclusions and policy implications

The objective of this study was to investigate the dynamics and nonlinear effects of public debt on economic growth in Rwanda. The empirical analysis was performed by means of a novel methodology combining a quadratic polynomial function in debt and the ARDL bounds approach to cointegration using time series data spanning the period 1970-2018. The main results of the study reveal that the public debt exerts a nonlinear impact on economic growth in the long run as well as in the short run in Rwanda and this impact may be described by an inverted U-shaped relationship. More specifically, the estimated results indicate that there is a turning point or a

threshold level above which the effect of public debt on economic growth shifts from positive to negative.

The point estimate of the threshold level is 50.2% in the long run, while in the short run, the threshold level is 38.5%. The threshold level in the long run is quite close to the public debt-to-GDP ratio of 50% adopted by Rwanda and the other partners of the East African Economic Community as a convergence policy benchmark. The findings of the study are also in line with a growing number of empirical studies that provided evidence of a nonlinear relationship between public debt and economic growth in the context of African economies. Finally, this outcome confirms the general theoretical assumption that at low levels, public debt is growth enhancing, while at high level or beyond a certain turning point, public debt is growth detrimental.

Looking at the effects of control variables, the study revealed that physical investment, terms of trade and labor force are positively associated with economic growth in the long run, while government consumption expenditure did not play any role in economic growth in the sample period under review. In the short run, physical investment, terms of trade and government consumption expenditure are found to have a positive significant impact on economic growth, while labor force exerts no influence on output growth.

The results of this study have important policy implications. The first to be pointed out is that as highlighted in the study, to achieve a sustained growth rate in the long run, economic policy in Rwanda should focus on physical investment through capital accumulation to enhance the production capacity of the national economy; on the extension of trade openness to harness the potential of free market opportunities , transfer of technology and efficient gains in the allocation of resources ; on education and technical training to improve labor productivity.

The second implication is related to public debt. This study showed that the public-debt relationship has an inverted U-shaped form and the estimated threshold level is 50.2%. This implies that below the threshold level, public debt is growth enhancing in Rwanda, while beyond this turning point additional public indebtedness would have a negative impact on growth; if the government borrows, measures should be put in place to enforce fiscal discipline in public spending and ensure that borrowed funds are directed into development projects so as to generate the needed revenue to meet the country's debt service obligations. It follows that targeting a public debt higher than 50.2% of GDP would not be a wise policy option, while maintaining public indebtedness lower than the threshold level would benefit the county's economic performance. Furthermore, the estimated threshold provided an empirical support to the choice of the public debt-to-GDP ratio of 50% used as a convergence policy benchmark for fiscal policy.

The public debt threshold level should serve as a warning signal to policy makers regarding the degree of fiscal sustainability and economic stability risks and thereby call for the need to undertake in due time a stronger fiscal consolidation and debt stabilization. In addition, the estimated public debt threshold should be taken as a long term objective that may vary in the

short term; it should further be considered in combination with a judgement-based analysis on the developments of the other growth determinants as well as the institutional environment.

Finally, although this study used widely accepted econometric methodology in the empirical literature, the obtained results remain open to questions and debate. As the threshold estimates may vary depending on the data coverage (time dimension), modelling strategy, measure of public debt and selected control variables included in the model, the analysis could be extended further by employing other econometric approaches to investigate the presence of nonlinearities in the relationship between public debt and economic growth in Rwanda. Exploring the impact of the institutional environment and the channels through which public debt affects growth would also provide additional insights on the public debt-growth nexus in Rwanda.

REFERENCES

Afshaq, M. and Padda, H.U. (2019). "Estimating the optimal level of public debt for economic growth: An evidence from Pakistan". Quest Journal of Management and Social Sciences, 1:222-232.

Akram, N. (2016). "Public Debt and pro-poor economic growth". Economic Research, 29:746-757.

Alfonso, A. and Alves, J.R. (2015). "The Role of Government Debt in Economic Growth". Hacienda Pública Española IEF, 215: 9–26.

Baaziz Y., Guesmi K., Heller D. and Lahiani A. (2015). "Does public debt matter for economic growth? Evidence from South Africa". The Journal of Applied Business Research, 31: 2187-2196

Baldacci, E., and M. Kumar. (2010). "Fiscal Deficits, Public Debt and Sovereign Bond Yields," IMF Working Paper, WP/10/184.Washington: International Monetary Fund.

Barro R. (1989), "The Ricardian approach to budget deficits", Journal of Economic Perspectives, 3: 37-54.

Barro, R. (1995). "Inflation and Economic Growth," NBER Working Paper No. 5326 (Cambridge, Massachusetts: National Bureau of Economic Research).

Bashar, S. (2015). "Impact of government expenditure on economic growth, evidence from Ghana (1980-2013). Thesis. University of Ghana.

Baum, A., Checherita-Westphal, C., and Rother, P. (2013). "Debt and growth: New evidence for the euro area". Journal of International Money and Finance, 32: 809-821.

Bhatta, P, G. and Mishra, A. (2020). "Estimating optimum growth-maximizing public debt threshold for Nepal", Nepal Rastra Bank Review, 32: 1-28.

Bilan, I. and Ihnatov, I. (2015). "Public debt and economic growth: a two-sided story". International Journal of Economic Sciences, 4:24-39.

Blake, T. (2015). Investigating the impact of public debt on economic growth in Jamaica, Bank of Jamaica.

Caner, M., Grennes ,T., and Koehler-Geib, F. (2010). “Finding the Tipping Point-When Sovereign Debt Turns Bad,” World Bank Policy Research Working Paper 5391.

Caporale,G.G.and Pittis,N (2004).“Robustness of the CUSUM and CUSUMSQ of squares tests to serial correlation, endogeneity and lack of structural invariance. Some Monte Carlo evidence”, Economics Series No 157, Institute of Advanced Studies, Vienna.

Cassimon, D., Essers, D., and Verbeke, K. (2016). “The changing face of Rwanda’s public debt”. Working Paper, No 14, Institute of Development Policy and Management (IOB), University of Antwerp.

Cecchetti, S.G., Mohanty, M. S. and Zampolli, F. (2011). “The Real Effects of Debt”. BIS Working Papers No. 352, Basel: Bank for International Settlements.

Checherita, C. and Rother P. (2010). “The Impact of High and Growing Government Debt on Economic Growth: An Empirical Investigation for the Euro Area”. ECB Working Paper Series No. 1237, Frankfurt: ECB.

Chowdhury, A.R. (2001). “External debt and growth in developing countries: a sensitivity and causal analysis”. Discussion Paper, No.DP2001/95.Helsinki, Finland: United Nations University World Institute for Development Economic Research.

Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi. (2013). “Debt, Inflation and Growth: Robust Estimation of Long-Run Effects in Dynamic Panel Data Models”. Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute Working Paper No. 162.

Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi. (2015). “Is there a debt-threshold effect on output?” IMF Working Paper/2015/197. IMF.Washington, DC.

Chung,Y.W. and Chinn,M.D.(1997). “Further investigation of the uncertain unit root in GNP”, Journal of Business and Economic Statistics, 15: 68-73.

Clements, B., R. Bhattacharya and T. Q. Nguyen. (2003), “External debt, public investment, and growth in low-income countries”. IMF Working Paper/2003/249, IMF, Washington, DC.

Cochrane, J.H. (2011). “Understanding policy in the great recession: Some unpleasant fiscal arithmetic”. European Economic Review 55, 2-30.

Dagher, J. and Kovanen, A. (2011). “On the stability of money demand in Ghana: a bound testing approach”, IMF Working Paper /2011/273. IMF, Washington, DC.

Devarajan, S., V. Swaroop, and H. Zou. (1996). “The Composition of Public Expenditure and Economic Growth.” Journal of Monetary Economics 37: 313–44

Eberhardt, M., and Presbitero, A.F. (2015). “Public debt and growth: heterogeneity and non-linearity”. Journal of International Economics, 97: 45-58.

Eboreime,M I. and Sunday ,B.(2017). “Analysis of public debt-threshold effect on output in Nigeria”. Econworld2017@Paris Proceedings, Paris-France.

Egert, B. (2013). “The 90% Public Debt Threshold: The Rise and fall of a Stylized Fact”. CESifo Working Paper 4242. Munich: CESifo Group.

Elmendorf, D.W. and Mankiw G.N. (1998). “Government Debt”. NBER Working Paper 6470, Cambridge, MA: National Bureau of Economic Research.

Engel, R.F. and Granger, C.W. (1987). “Cointegration and error correction representation, estimation and testing”, Econometrica, 55: 251-276.

Eze, O.M., Nweke, A.M. and Atuma, E. (2019). “Public debts and Nigeria‘s economic growth”, Journal of Economics and Finance, 10:24-40.

Folarin,E.O.,and Asongu ,A.S. (2017). “Financial liberalization and long-run stability of money demand in Nigeria”, AGDI Working Paper, WP/17/018.

Ghatak, S. and Siddiki, J.U. (2001). “The use of the ARDL approach in estimating virtual exchange rate in India”, Journal of Applied Statistics, 28:573-583.

Gómez-Puig, M. and Sosvilla-Rivero, S. (2015), “On the bi-directional causal relationship between public debt and economic growth in EMU countries”, Research Institute of Applied Economics, Working Paper, No. 12: 1-32.

Gómez-Puig, M., Sosvilla-Rivero, S. (2017). “Public debt and economic growth: Further evidence for the euro area”. Research Institute of Applied Economics. Working Paper, No.15.

Gujarati,D.N., and Porter ,D.(2009). Basic econometrics. Mc Graw Hill International Edition ,New-York.

Harris, R. and Sollis, R. (2003). Applied time series and forecasting. Chichester: Wiley and Sons.

Hendry, D.F. (1995). Dynamic econometrics. Oxford: Oxford University Press.

Idris, M., Bakar, R. and Ahmad, T.S.T. (2018). “The effects of fiscal operations on economic growth and stability in Nigeria: empirical evidence based on time series data”, International Journal of Accounting and Economic Studies, 6, 36-47.

International Monetary Fund, World Economic Outlook Data Base, October 2019.

International Monetary Fund, Rwanda Fifth Review under the Policy Instrument, May, 2016

Jacobo, A. and I. Jalile (2017). “The relationship between debt and growth: A reality check for the pampas and beyond”, Paper presented at the Second World Congress of Comparative Economics, Saint Petersburg, June.

Johansen, S. (1988). “Statistical analysis of cointegration vectors», Journal of Dynamics and Control, 12: 231-54.

Johansen, S. and Juselius, K. (1990). “Maximum likelihood estimation and inference on cointegration-with application to the demand for money”, Oxford Bulletin of Economic and Statistics, 52: 169-210.

Karagol, E. (2002), “The causality analysis of external debt service and GNP: the case of Turkey”, Central Bank Review, Research and Monetary Policy Department, Central Bank of the Republic of Turkey, 2: 39-64.

Kirch, C. and Kamgaing, T. (2011). “Testing for parameter instability in non-linear autoregressive models”. https://doi.org/10.1111/j.1647-9892.2011.00764.x

Kling, J. and Pritchett, L. (1995). “Where in the world is population growth bad?” World Bank Policy Research Paper, No 1391. Washington D.C.

Koffi, S. (2019). “Nonlinear impact of public debt on economic growth: evidence from Sub-Saharan African Countries”, MPRA Paper No 96067.

Krugman, P., (1988). “Financing vs. Forgiving a Debt Overhang: Some Analytical Issues”. NBER Working Paper No. 2486, Cambridge, MA: National Bureau of Economic Research.

Kumar, M.S. and Woo, J. (2010). “Public Debt and Growth”. IMF Working Paper/2010/174, IMF, Washington, DC.

Kummer-Noormamode, S. (2018). “The relationship between public debt and economic growth: nonlinearity and country specificity”, MPRA Paper No 98075.

Kuznets, S. (1960). “Population change and aggregate output”. In “Demographic change in developed countries (pp.324-351)”. Columbia University Press.

Levine, R. and Renelt, D. (1992). “A Sensitivity Analysis of Cross-Country Growth Regressions,” American Economic Review, 82: 942–63.

Lopes da Veiga, J., Ferreira-Lopes, A. and Sequeira, T. (2014). “Public debt, economic growth and inflation in African economies”. MPRA Paper No 57377.

Lucas Jr.R.E. (1988). “On the mechanics of the economic development”, Journal of Monetary Economics, 22:3-42.

Mankiw, Gregory, David Romer, and David Weil. (1992). “A Contribution to the Empirics of Economic Growth,” Quarterly Journal of Economics, 107: 407-437.

Megersa, J. (2014). “The Laffer-curve and the Debt-growth Link in Low-income Sub-Saharan African Economies”. MPRA Paper No. 54362. Munich: University Library of Munich.

Mencinger, J., Aristovnik, A., & Verbic, M. (2014). “The impact of growing public debt on economic growth in the European Union”. Amfiteatru Economic, 16:403-414.

Mencinger, J., Aristovnik, A., & Verbic, M. (2015). “Revisiting the role of public debt in economic growth: The case of OECD countries”. Engineering Economics, 26: 61-66.

Mensah, L., Allotey, D., Sarpong-Kumankoma., E. and Coffie, W. (2019). “What debt threshold hampers economic growth in Africa”. International Journal of Development Issues, 19: 25-42.

Modigliani F. (1961). “Long-Run Implications of Alternative Fiscal Policies and the Burden of the National Debt”. Economic Journal, 71: 730-755

Mupunga, N. and Le Roux, P. (2015). “Estimating the optimal maximizing debt threshold for Zimbabwe”. South African Business Review, 19: 102-128.

Ndoricimpa, A. (2020). “Threshold effects of public debt on economic growth in Africa: a new evidence”. Journal of Economics and Development, 22:187-207.

Ndoricimpa, A. (2017). “Threshold effects of debt on economic growth in Africa”. African Development Review, 29:471-484.

Padhan, C. Purna (2011). “Stability of demand for money in India: evidence from monetary and liquidity aggregates”, International Journal of Economics and Finance, 3: 271-282.

Panizza, U. and Presbitero, A. F. (2013). “Public debt and economic growth in advanced economies: A survey”. Swiss Journal of Economics and Statistics, 149: 175–204

Panizza, U. and Presbitero, A. F. (2014). “Public debt and economic growth: is there a causal effect?” Journal of Macroeconomics, 41, 21-41.

Pattillo, C., Poirson, H. and Ricci, L. (2002). “External Debt and Growth”. IMF Working Paper 02/96, IMF, Washington, DC.

Pescatori, A., Sandri, D., Simon, J. (2014), “Debt and Growth: Is There a Magic Threshold?” IMF Working Paper no. 34, Washington, D.C.

Presbitero, A.F. (2012). “Total public debt and growth in developing countries”, European Journal of Development Research, 24: 606–626.

Reinhart, C. M., & Rogoff, K. S. (2010). “Growth in a time of debt”. American Economic Review, 100(2), 573-578.

Romer, P. (1990). “Endogenous technology change”. Journal of Political Economy, 98: N0 5, part 2, pp. S71-S101.

Rwanda, MINECOFIN (Ministry of Finance and Economic Planning), 2019.Updated Macroframework Public Dataset.

Sachs, J. (1989). “The debt overhang of developing countries”, In Ronald, F. (Ed.), Debt, Stabilization and Development: Essays in Memory of Carlos F. Diaz-Alejundro. Oxford: Blackwell.

Sala-i-Martin, X. (1997). “I just ran two million regressions”. American Economic Review, 87: 178–183.

Sanusi, K.A., Hassan, A.S. and Meyer, D.F. (2019). “Nonlinear effects of public debt on growth in Southern Africa Development Community (SADC) Countries”, International Journal of Economics and Management, 13: 193-202.

Schclarek, A. (2004). “Debt and Economic Growth in Developing and Industrial Countries”. Working Paper No. 34. Lund, Sweden: Department of Economics, School of Economics and Management, Lund University

Semmler, W.,Greiner ,A.,Diallo ,A. and Rajaram, A.(2007). “Fiscal policy, public expenditure composition, and growth”. Policy Research Working Paper, No 4405, World Bank.

Solow, R. M. (1956). A “Contribution to the theory of economic growth”. Quarterly Journal of Economics 70: 65-94.

Tenaw, D. and Demeke, H (2020).“Inflation threshold effects on growth in Ethiopia: evidence from food and non-food sectors”, Department of Economics, Dire Dawa University. Ethiopia.

World Bank (2019). International Debt Statistics.

World Bank (2019). Future drivers of growth in Rwanda: Innovation, Integration, Agglomeration and Competition.

Zagler, M. and Durnecker, G. (2003). “Fiscal policy and economic growth”. Journal of Economic Surveys, 17: 397-418.