Deterministic chaos within the transfer space - An unstable fixed point as a narrow ford to complexity through chaos

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The complete reinvestment of the net profit of a previous production cycle as substrate of the next cycle of a single party may result in deterministic chaos. The dynamics of such a feedback loop is controlled by the size relation of a benefit factor (serves also as complexity factor) and a cost factor. An increasing benefit factor or decreasing cost factor trigger bifurcation and deterministic chaos at certain size relations. In deterministic chaos the size of the net profit of the reinvestment is no longer reliable. Thus, a limit to the evolution of complexity via an increasing benefit factor and complete reinvestment would be expected.

Chaos already starts when benefit exceeds cost; a sink. In a source cost exceeds benefit. Both conditions met, source and sink form an ensemble, peacefully transfer substrate when in contact, and produce superadditivity. At low substrate concentrations a sink has to pass through the region of chaos to become a source. To suppress chaotic behaviour, an ensemble could become active when on both sides benefit still exceeds cost; two sinks. The emerging superadditivity supports such a behaviour. In addition, the mathematical analysis of my model identifies a unique substrate concentration leading to an unstable fixed point. Notably, this concentration is independent of an increasing benefit factor and thus does not collide with evolution towards complexity. Moreover, this concentration is a turning point as the result of a further complete reinvestment no longer grows. This limit guides the ensemble through chaos towards complexity and division of labour by a sink and a source.

source, sink, ensemble, net profit, benefit factor, cost factor, superadditivity, subadditivity, deterministic chaos, stable fixed point, unstable fixed point, bifurcation, evolution of complexity, division of labour

Introduction

The features of the transfer space have been recently described again in detail (1). A short abstract: I observe the transfer space of an ensemble and its constituents (source and sink) with respect to the quality of their net profit (np>0 or np=0 or np<0). Net profit is the difference of benefit (b) and cost (c) on basis of the same units (b-c). The net profit of the ensemble is the sum of the net profits of source and sink. Transfer of substate from source to sink will result in superadditivity or subadditivity due to the non-linear nature of the benefit function. The cost function is linear and a fixed cost is usually absent. In the absence of transfer there is only simple additivity. Both parties may differ with respect to affinity or maximal reaction velocity for the same substrate. In addition, the substrate has a benefit and a cost aspect expressed in a benefit factor (bf) and cost factor (cf) for source and sink. They may also be of different size in source and sink. The benefit factor bf may serve as a complexity factor in addition. This is primarily understood as increasing efficiency through architectonical and structural complexity and hierarchy of ensembles (2).

Structure of the investigation

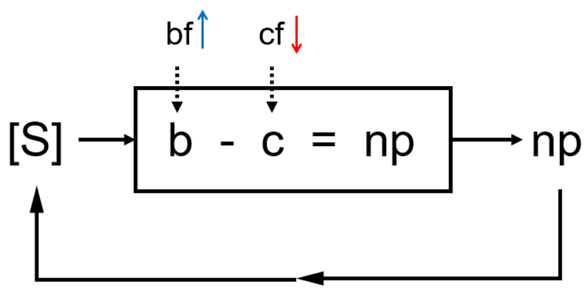
In the beginning I observe a single party of an ensemble. The size of the net profit there depends besides the biochemical basics (Km, Vmax, and substrate concentration [S]) on size and relationship of the benefit factor and the cost factor. I investigate what happens to the net profit of a feedback loop if the benefit factor or the cost factor are continuously changed in a single party. Then I observe different strategies to find a path to increased complexity (increased benefit factor) by mutation and selection with improved net profit as assessment standard. Is it possible to evolve a sink at low external substrate concentration to a source with higher substrate concentration? Finally, I briefly investigate the interplay of source and sink to further control the emerging dynamics.

Feedback loops

In biology and in economy the net profit of a past period could be the start substrate or start capital for the following period (figure 1). Therefore, I no longer observe all possible concentrations within a single party and a single transfer in parallel. Now I want to observe the dynamics of a feedback loop where the net profit of the previous period is fed back as substrate into the same party to give rise to the net profit of the present period and so forth.

I look *e.g.* at a single organism consuming glucose to acquire glucose. This organism could be a single cell foraging for food or a person earning money to buy food. This feedback loop (figure 1) may lead to growth, an equilibrium or starvation. The crucial quantity is the size of the net profit (np) of this action. This is determined by the Michaelis-Menten constant (Km, 0.5mM), the maximal reaction velocity (Vmax, 5µmol/min), the benefit factor (bf, b*min/µmol), the cost factor (cf, c/mM), and the initial substrate concentration [S] (mM).

Figure 1



$b = (\mathrm{Vmax} * \frac{[\mathrm{S}]}{\mathrm{Km}+[\mathrm{S}]}) * \mathrm{bf}$ $\quad$ $c = [S] * cf$ $\quad$ $np = f([S]) = b\text{-}c$

Figure 1
Here we look at the feedback loop of the net profit production of a single party. The benefit function (b) is set according to Km and Vmax. The cost function is c. The consumption of a start substrate [S] leads to a net profit (np=b-c). The net profit is used in a feed-back loop as substrate within 1050 such cycles. The effect of an increasing benefit factor (bf, blue arrow) or a decreasing cost factor (cf, red arrow) on the last 250 cycles will be observed.

Net profit here is the difference between a non-linear benefit function and a linear cost function. Both functions are monotonously increasing. I observe the effect of an increase of the benefit factor or a decrease of the cost factor on the long-term behaviour of a feedback loop. Km and Vmax remain unchanged. The substrate concentration is variable but is the result of the feedback loop. The increase of a benefit factor and the decrease of a cost factor have here both the same effect. The reason is that the point of equilibrium b-c=0 shifts to higher substrate concentrations and the area of benefit domination (b>c) is increased. An increased benefit domination is positively correlated to net profit (figure 2).

Figure 2

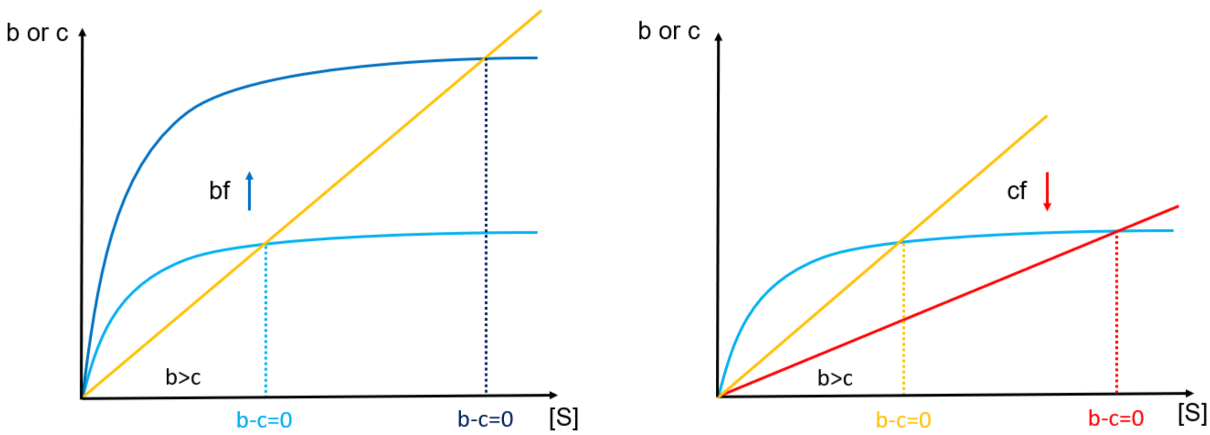


Figure 2
Left: When the benefit factor bf of a saturating benefit function is increased from a start value (light blue) to a higher value (dark blue) the point b-c=0 shifts to a higher substrate concentration (dotted lines) in the case of a linear, constant cost function (orange). The total net profit b-c, the area between the orange line and the dark blue curve, is increased compared to the start.

Right: When the cost factor cf of a linear cost function is decreased from a start value (orange) to a smaller value (red) the point b-c=0 shifts to a higher substrate concentration (dotted lines) in the case of a constant, saturating benefit function (light blue). The total net profit b-c, the area between the red line and the blue curve, is increased compared to the start. A fix cost is not included.

Results

Emergence of deterministic chaos on the level of a single party

The following results are obtained in analogy to the logistic map. For figure 3 (right part, the Feigenbaum diagram) the benefit factor (bf, b*min/μM) is varied from zero to ten at fixed Km (0.5mM), Vmax (5μM/min), and cost factor cf=5/3 (c/mM) values. The feedback loop is started at several different substrate concentrations (not 0.3mM; to be explained in the discussion). The result - the net profit - is calculated and used in the next cycle as substrate. However, in case the net profit drops to or below zero, the feedback loop is terminated (figure 3, bf=8/3). The reason is that at this point the behaviour of the system will change. As long as net profit is positive, I observe a sink (b-c>0). Beyond about bf=8/3 net profits, depending on the orbit, sooner or later become negative, a source (b-c<0).

The feedback loop is performed 1050 times. The results of the first 800 iterations are omitted. The results of the next 250 iterations are printed (y-axis, np). The np value for any substrate start value (0.15mM, 0.25mM, 0.35mM, 0.45mM, 0.55mM, 0.65mM, 0.75mM, 0.85mM, 0.95mM) between bf=0 and bf=8/30 are either zero, converge to zero with increasing bf or are negative values and set to zero (inset figure 3). Positive values at low bf, if observed, are very small.

Only between bf=8/30 and bf=8/3 always positive, large values are observed. Beyond bf=8/3 sooner or later np values are negative and therefore the whole result is set to zero, too. This is observed up to bf=10. The behaviour resulting in large values is similar to the overall observations with the quadratic iterator. Starting at bf=8/30 attractive and stable fixed points are observed with increasing bf. At higher bf values unstable fixed points (bifurcations) are observed. After that deterministic chaos with windows of stability follows.

Figure 3

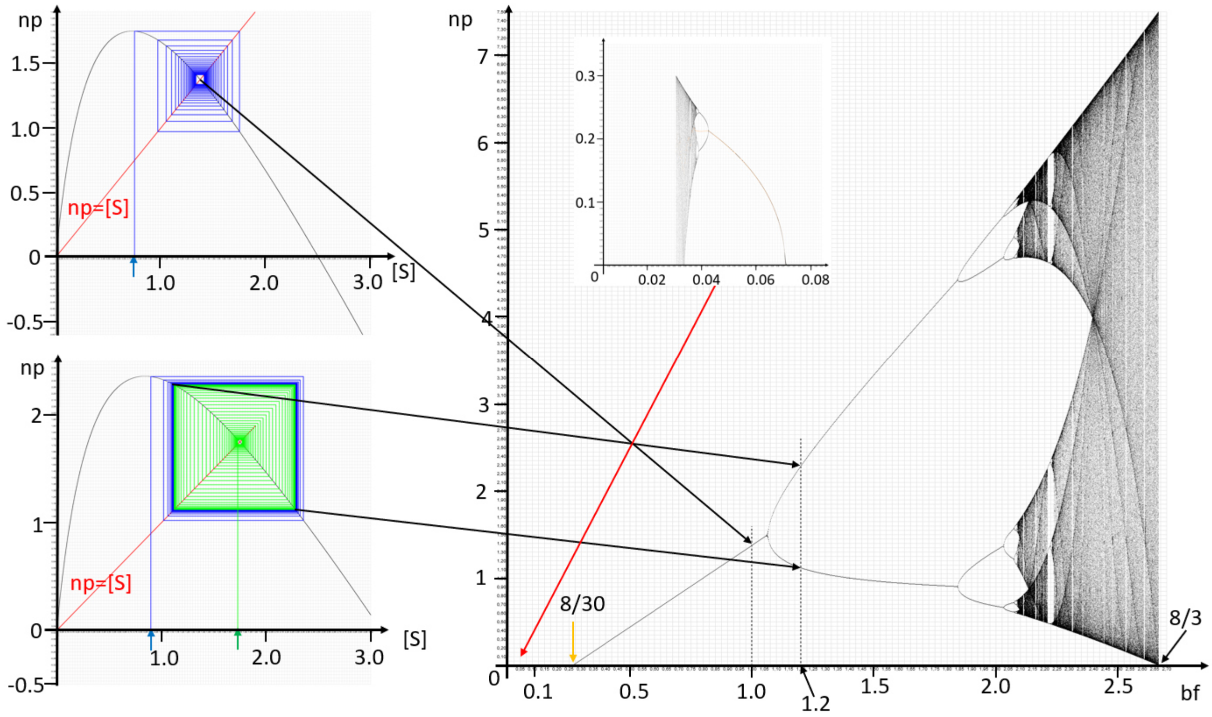


Figure 3

Left: Two cobweb-diagrams are displayed ([S] invested on the x-axes, np received on the y-axes). The upper diagram shows at bf=1 the inwards spiral (blue) to the attractive, stable fixed point (dotted grey line at bf=1, right). The lower diagram shows the situation after the first bifurcation (right, grey dotted line at bf=1.2). Two start concentrations are observed. The green orbit spirals outwards, away from an unstable fixed point towards a periodic cycle. The blue orbit spirals inwards to the same attractive, stable cycle. The result oscillates between an upper and lower value (bifurcation of np). Right: The feedback loop examined with a continuously increasing benefit factor (the step size is 0.0016875bf - one pixel is one step) results in a Feigenbaum diagram (bf on the x-axis, np on the y-axis). The net profit becomes positive at bf=8/30 (orange arrow). The grey dotted lines at bf=1 and bf=1.2, the limit at bf=8/3, and the orange arrow reappear in figure 5. The red arrow points to the location of the inset.

Net profit will also increase when cf is decreasing (figure 2, right). Therefore, a similar behaviour appears when net profit is observed while cf is decreasing. This, however, is limited especially in the presence of a fix cost. The cost factor in figure 4 is decreased from ten to zero at two different benefit factors. The first benefit factor is before the first bifurcation in figure 3 (bf=1). The other benefit factor is after the first bifurcation but before the second bifurcation (bf=1.2) of figure 3.

Figure 4

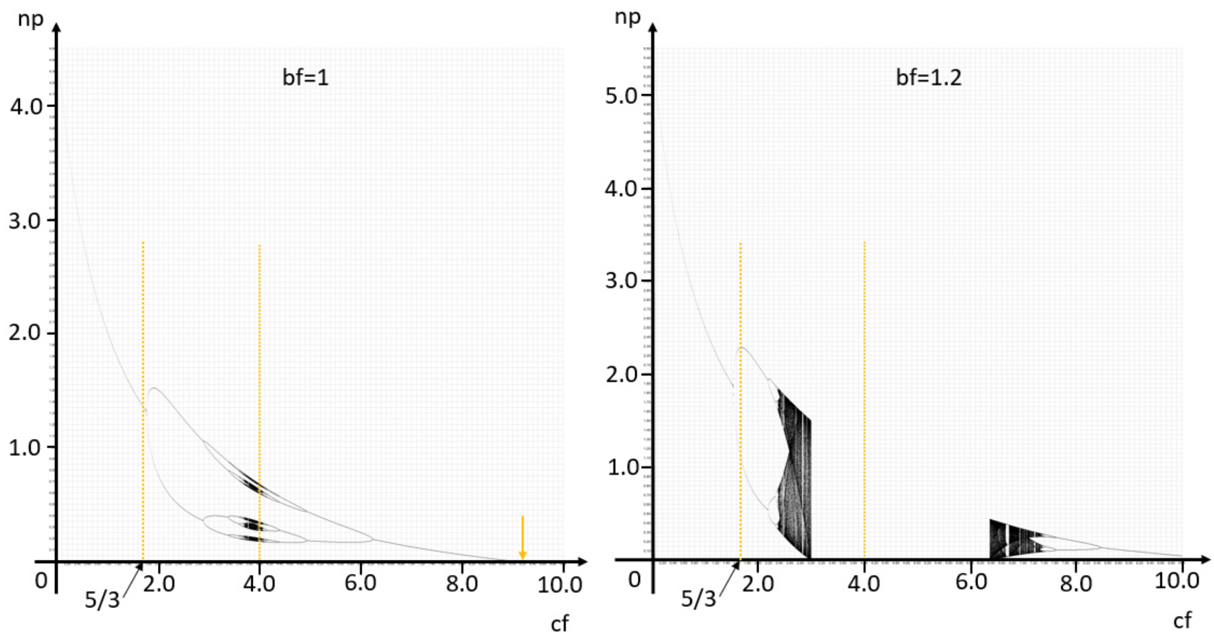


Figure 4

Here, the feedback loop is examined according to a continuously falling cost factor (from 10 to zero). This is observed at two different benefit factors (left bf=1, right bf=1.2), the net profit on the y-axis, the cost factor (cf) on the x-axis. The orange dotted lines mark two special cost factors (5/3 and 4) appearing also in the next figure. The orange arrow marks cf where np is zero and reappears also in figure 5.

The overall picture of both results is also known from other iterated functions. In case the cost factor is high, only a low net profit is achievable. Observing the graph (figure 4 left, bf=1) from right (high cf) to left (low cf) stable fixed points, bifurcations and partial deterministic chaos appear and disappear again. At bf1.2 (figure 4, right) the complete deterministic chaos is followed by a change of behaviour to a source (b-c<0 for some values) and then at much lower cf values the sequence is mirrored. After the area of the behavioural change, deterministic chaos reappears followed by unstable fixed points, bifurcation, and stable fixed points. This result leads to the question what pattern bf and cf follow.

I assume benefit factors and cost factors to be independent from each other. Therefore, I now observe the outcome of net profit when both, bf and cf, are varied between zero and ten. This results in a two-dimensional surface. When the last 250 iterations contain a negative net profit value the area is red. If only a single positive net profit appears (stable fixed point) the area is coloured in green. If the values oscillate or show chaotic behaviour with values above zero, they are coloured in blue (figure 5).

Figure 5

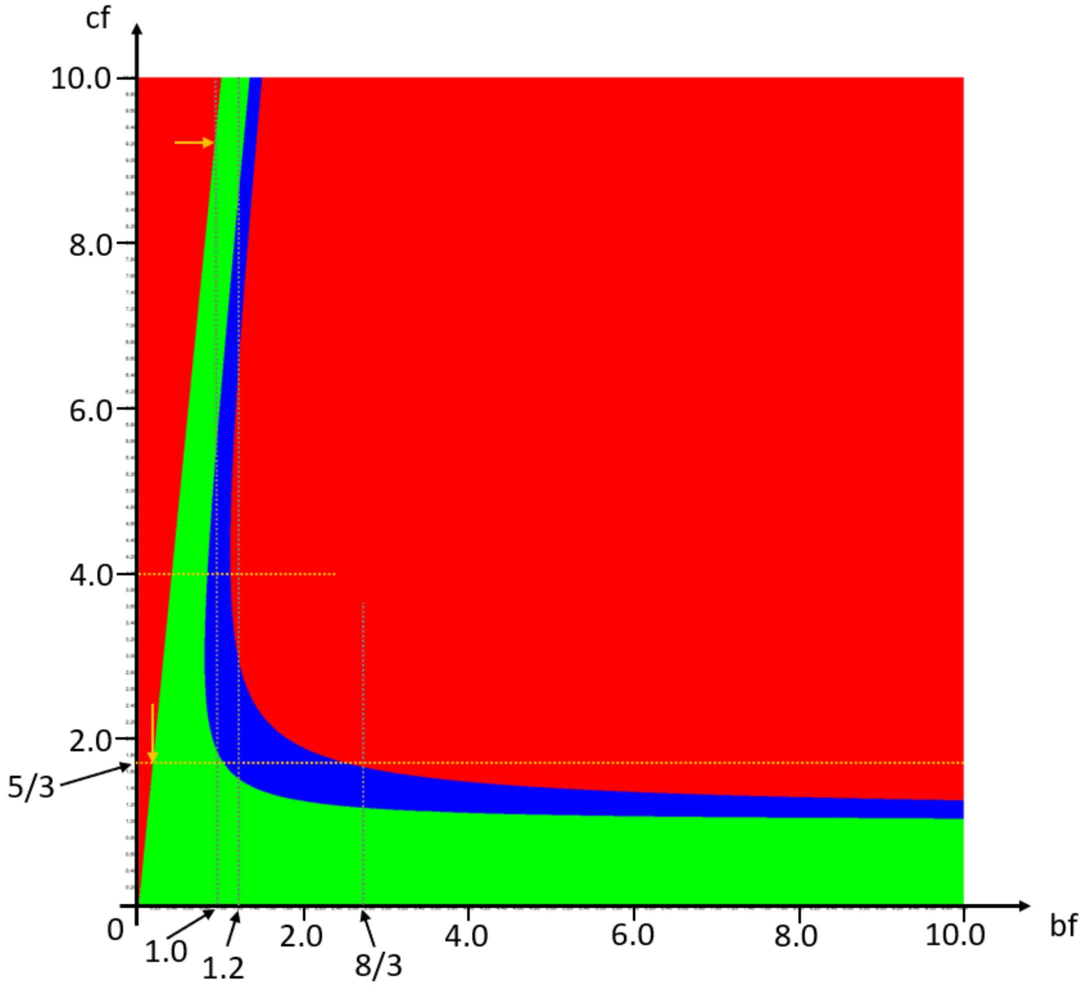


Figure 5

The benefit factor is on the x-axis and the cost factor is on the y-axis depicted. Within the large red area, the last 250 iterations contain a negative np value. In the blue area bifurcation (unstable fixed points) or deterministic chaos are observed and in the green area the last 250 iterations are a stable fixed point and a positive net profit. The grey and orange dotted lines are connected to figure 3 and figure 4 and follow a fixed cf (orange) or bf (grey) value. Orange arrows mark related positions in figure 3 and figure 4. The colour coding here has no connections to the colour coding used in my older papers. The result of the inset in figure 3 appears in the triangular shaped red area. As the values are very small, they are omitted to give a clear picture.

The orange cuts of figure 5: At a constant cf=5/3 and an increasing bf, the following observations are made: Starting at zero and increasing bf, net profit is chaotic or converges to zero (purposely depicted as red triangular shaped area only; figure 3, inset). At bf=8/30 the green area is entered (orange arrow). Between bf=1.0 and bf=1.2 the results switch from stable fixed points to bifurcation, the initial part of the blue area. Finally, at bf=8/3 the results leave the chaotic part of the blue area and enter the red area where negative net profits (b-c<0) appear within the last 250 iterations.

The grey cuts of figure 5: At about cf=9.2 and bf=1.0 (orange arrow) net profit starts to become positive and at bf=1.2 stable fixed points (green) are reached but they are only of a very small net profit (figure 4). At cf=4.0 and bf=1 the results are in the blue area (bifurcation and chaos) while at bf=1.2 they are already in the red area. It should be clear that the net profit in direction of larger cf is decreasing and net profit is increasing in direction of increasing bf – a third dimension. Within the green and blue area net profit is positive. In the red area a first negative net profit value appears and the iteration stops because this is the start of a behavioural change.

In the green and blue area of figure 5 sink behaviour is observed; b-c>0. All net profits - the substrate of the next cycle - are positive. Although, they may no longer be predictable. A source behaviour will start only if the concentration of substrate is high enough to meet the condition that the cost of the substrate exceeds the benefit of the substrate (b-c<0). This starts to begin in the large red area. Substrates which will result in a negative net profit are given away – a source.

It seems that there are three paths to complexity. Two green branches are obvious, it is the complexity of a sink: First, when the cost factor is very low, the benefit factor can increase unhindered by orders of magnitude. Second, the benefit factor can slowly grow controlled by an even stronger increasing cost factor. The area of similar sized cost and benefit factors and the evolutionary development of a source behaviour (giving) seems to be excluded by a blue wall of bifurcation and deterministic chaos, where increased net profit is no longer reliable. The large red area of a source is not a homogeneous area without any structure (figure 6).

An increasing complexity (bf) might confer additional cost (cf). The idea of cf as a function of bf is not considered.

Figure 6

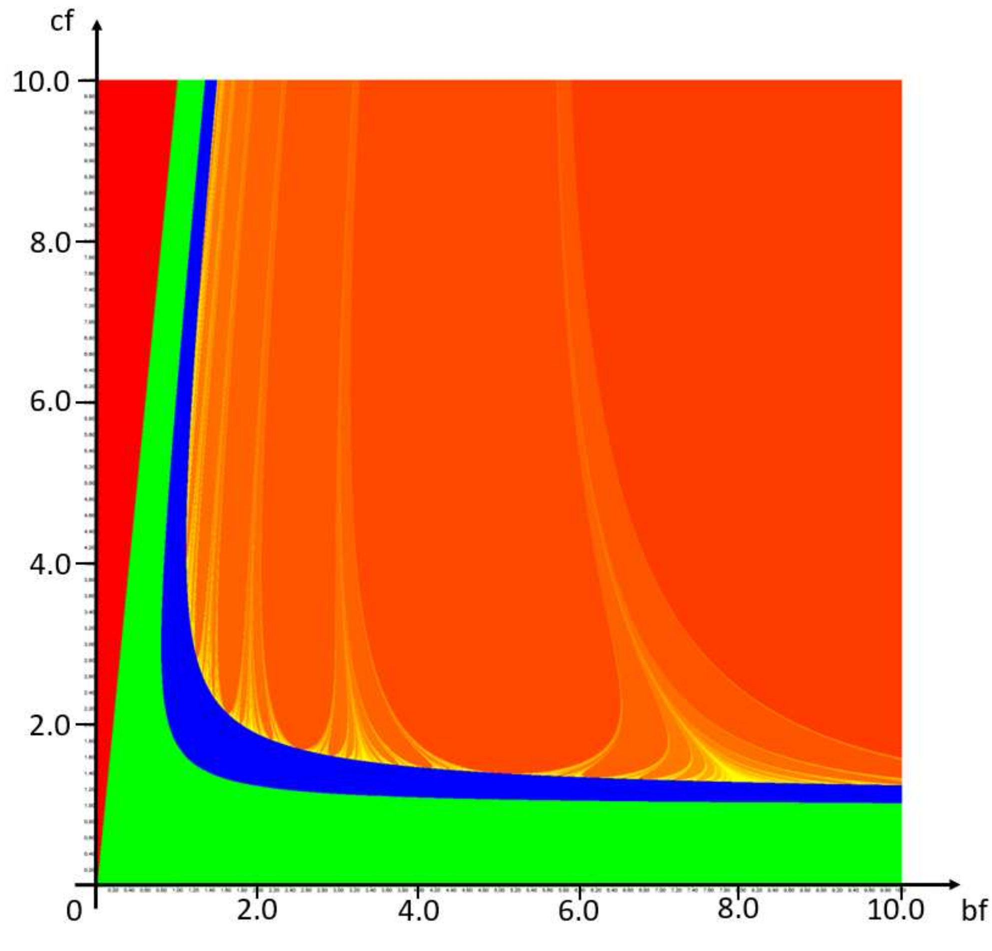


Figure 6

The triangular shaped red region for small bf values is not further investigated. The large red area in figure 5 is further differentiated. A pixel is coloured in red when the first iteration of the last 250 is negative (a fast escape velocity to a source behaviour) and in the brightest yellow when the 23^{rd} iteration is negative (a slow escape velocity to go from sink to source). The start concentration of the single orbit is [S]=0.01nM.

Evolution of a sink into a source at low substrate concentrations

Substrate is usually scarce in nature and therefore the world should be full of sinks. A single sink is a simple layout. Two parties are a more complex layout when they function as a source and a sink in an ensemble.

An easy way to convert a sink into a source in the absence of high substrate concentrations is an increase of the cost factor, an increase of Km, or a decrease of Vmax. This will move the point b-c=0 to even lower substrate concentrations so that substrate is easily given; a source. The result of such unfavourable biochemical changes is a decrease in net profit of the affected party. This is not a good evolutionary move for a party as long as the ensemble is not yet fully integrated.

To evolve two independent sinks into an ensemble of source and sink is a step upwards in complexity. This can be achieved by increasing the complexity factor (bf, figure 2, left). A decrease of the cost factor (cf) is limited, especially as there will be usually a fix cost. Complexity is not directly measurable by a system. An indicator of increased complexity could be increased net profit. Different strategies to reach increased net profit and guide the system towards increased complexity are conceivable.

In the following section four behavioural strategies (1 to 4) to evolve a sink into a source are investigated. They range from complete reinvestment to the reinvestment of only a fixed amount. In a process of mutation and selection the evaluation depends always on measured net profit. Increased complexity (bf) can only prevail when net profit is increased.

1. A reinvestment strategy where the single party always completely reinvests the net profit as substrate and the resulting net profit is

measured and reinvested again ($[S]_{n-1}$ produces np and np becomes $[S]_n$). A mutated bf will prevail if np is increased.

2. The single party reinvests the latest net profit but judges the performance according to the average net profit of the last 20 net profits. This will result in an average net profit as measure of success. A mutated bf will prevail if the average np is increased.
3. The single party chooses any fixed concentration and obtains a net profit. This net profit is used to regenerate the start concentration and is the measure of the evolutionary success of a mutated benefit factor. The rest of the net profit (substrate) is discarded.
4. This strategy compares the present net profit $(b\text{-}c)_n$ and the previous net profit $(b\text{-}c)_{n-1}$ of a start concentration. When $(b\text{-}c)_n/(b\text{-}c)_{n-1}>1$ the reinvested amount of substrate is increased by this factor. When $(b\text{-}c)_n/(b\text{-}c)_{n-1}<1$ the reinvested amount of substrate is decreased by this factor. Finally, the reinvested substrate stays unchanged when $(b\text{-}c)_n/(b\text{-}c)_{n-1}=1$. That concentration is the limit. The success of a changed complexity (changed bf) is measured as achieved net profit. The surplus is discarded. Only a mutated bf with increased net profit will survive.

Strategy 3 and 4 are basically already on the way to an ensemble of source and sink. A fixed or limited concentration requires that substrate is discarded when there is substrate in surplus of the concentration limit. This surplus could be released into the environment, but it could also be given to a neighbour. If this neighbour is genetically related, the advantage of entanglement starts (3).

Strategy 1:

100 individuals with an initial bf=0.2, Km=0.5, Vmax=5, cf=5/3 and initial substrate concentration [S]=0.1 were randomly mutated between -0.0005 and +0.0005 according to bf. The production cycle is started and the resulting net profit is calculated. In addition, the average net profit and the average bf is calculated for the graph in figure 7. The size of the individual net profit ranks the 100 parties. The top five individuals reproduce once and the bottom five individuals die, bf is mutated randomly and the next production cycle starts. The result is shown in figure 7.

Figure 7

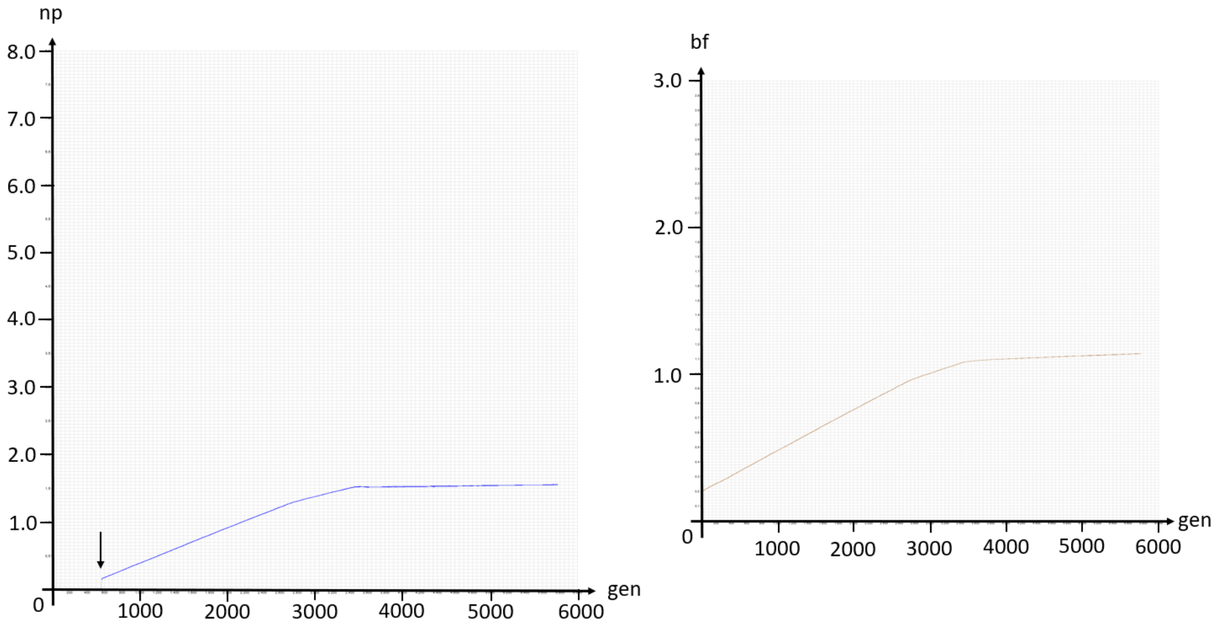


Figure 7

Left: The size of the average net profit of 100 individuals is shown (y-axis, np) over a time span of 6000 generations (x-axis, gen). The increase in net profit is the result of random mutations in the values of bf. After about 3500 generations the average net profit levels off at about 1.5 (np). The black arrow indicates a sudden jump in net profit after about 560 generations when bf has grown beyond 8/30. Right: Here, the size of the benefit factor (y-axis, bf) in dependence of the generation time (x-axis, gen) is observed. The benefit factor does not exceed a value of about 1.15.

The single party is only able to increase its complexity to bf=1.15. This is just behind the first bifurcation (see also figure 5). In bifurcations the resulting net profit does no longer continuously increase; it alternates between a larger value and a smaller value. It is surprising that the increase in bf does not immediately stop at the first bifurcation. A more intensive selection process (10 individuals die) moves further into the zone of bifurcation (not shown) but bf also stops to grow. As the system starts with bf=0.2, there is for many generations no visible growth in net profit. There, the average substrate concentration (net profit) is decreasing from the start values and bf grows very slowly. After bf has grown to over 8/30 net profit explodes (arrow in figure 7). However, the growth of bf finally stops. The growth of the complexity factor bf stops because the growth in net profit stops. The strategy of complete reinvestment of the net profit does not penetrate far into the area of bifurcation. A large accumulation of complexity is not possible with this strategy.

Strategy 2

100 individuals with an initial bf=0.5, Km=0.5, Vmax=5, cf=5/3 and initial substrate concentration [S]=0.8 were randomly mutated between -0.0005 and +0.0005 according to bf over 10 000 generations. The individuals alternate between 20 production cycles and a mutation cycle. For every individual the average np of the 20 production cycles is determined. According to np the ten best individuals reproduce, the ten worst die. The average np and bf of all 100 individuals is calculated and is displayed with generation time in figure 8.

Figure 8

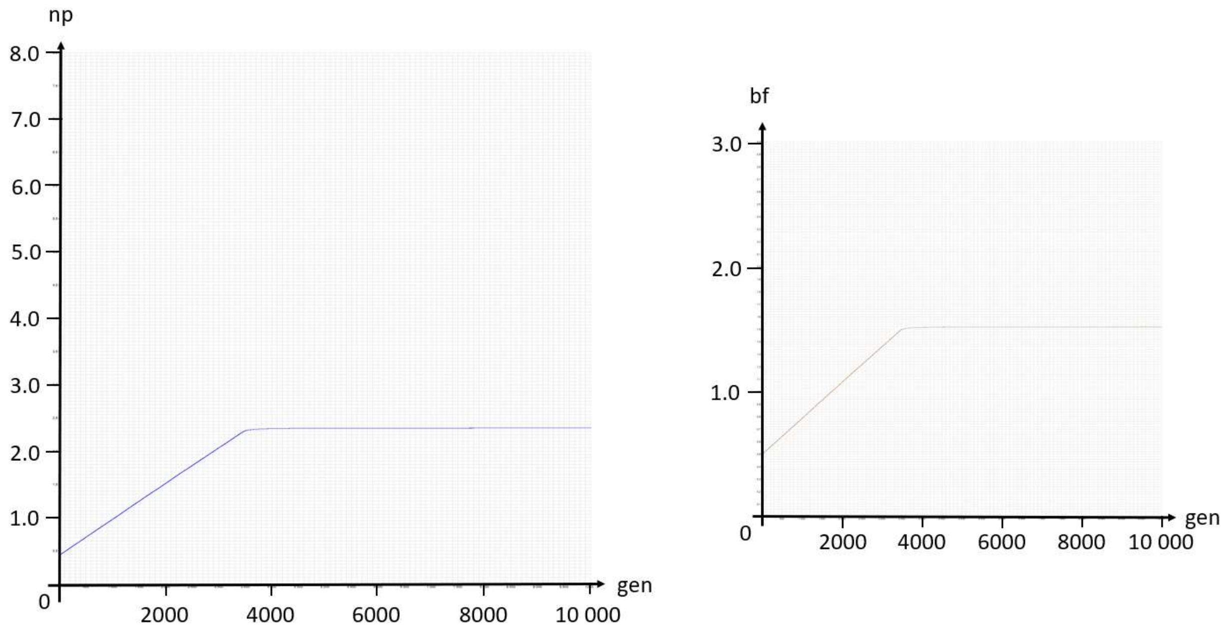


Figure 8

Left: The size of the average net profit of 100 individuals after 20 production cycles is shown (y-axis, np) over a time span of 10 000 generations (x-axis, gen). The increase in net profit is the result of random mutations in the values of bf. After about 3500 generations the average net profit levels off at about 2.5 (np). Right: Here, the size of the benefit factor (y-axis, bf) in dependence of the generation time (x-axis, gen) is observed. The benefit factor does not exceed a value of about 1.5.

The start concentration (0.8mM) is not visible in figure 8 as the system adjusts within a few generations. With other start concentrations and other initial bf values the figures vary only slightly. Again, the system is not able to accumulate large complexity (bf). Within the first bifurcation the evolution of bf (complexity) comes to a halt.

Strategy 3

This strategy uses a (any) fixed substrate concentration. In the shown example (figure 9) the substrate concentration is 0.2mM and start bf is set to 0.4. As long as bf>28/75 the outcome is larger than the investment. At bf=28/75 the investment is replaced. Below this value the strategy would not work. The population is set to 100 individuals. According to the net profit the worst ten individuals die and the top ten individuals reproduce once to replace the dead.

Figure 9

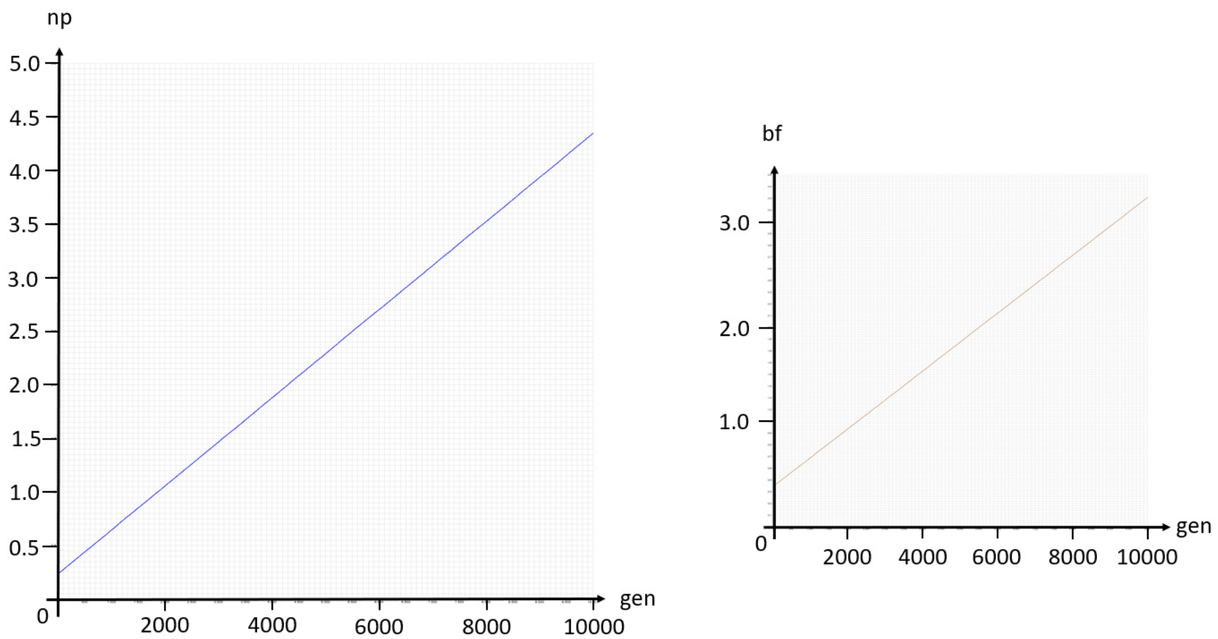


Figure 9

Net profit (left) and benefit factor (right) grow continuously over the generations.

This, however is a too simple strategy. As soon as a competing strategy will use a slightly higher fixed concentration (*e.g.* figure 11), the competitor will prevail. Large start concentrations found in the outside world will be rare. A reinvestment strategy will produce this on its own. But reinvestment is coupled to chaos. Is there a reinvestment strategy avoiding chaos?

Strategy 4

The longer the reinvestment lasts, the more chaos will be observed. The shortest reinvestment is a two-step procedure. As soon as the net profit no longer grows (np_{n-1} equals np_n), the substrate concentration is kept constant. This should be detectable by a simple algorithm comparing the past net profit and the present net profit. The search strategy for the optimal start concentration is:

$$[S]_n = [S]_{n-1} * f(f([S]_{n-1})) / f([S]_{n-1})$$

simultaneously the benefit factor is mutated (figure 10). Again, the population is set to 100 individuals. According to the net profit the worst ten individuals die and the top ten reproduce once to replace the dead.

Figure 10

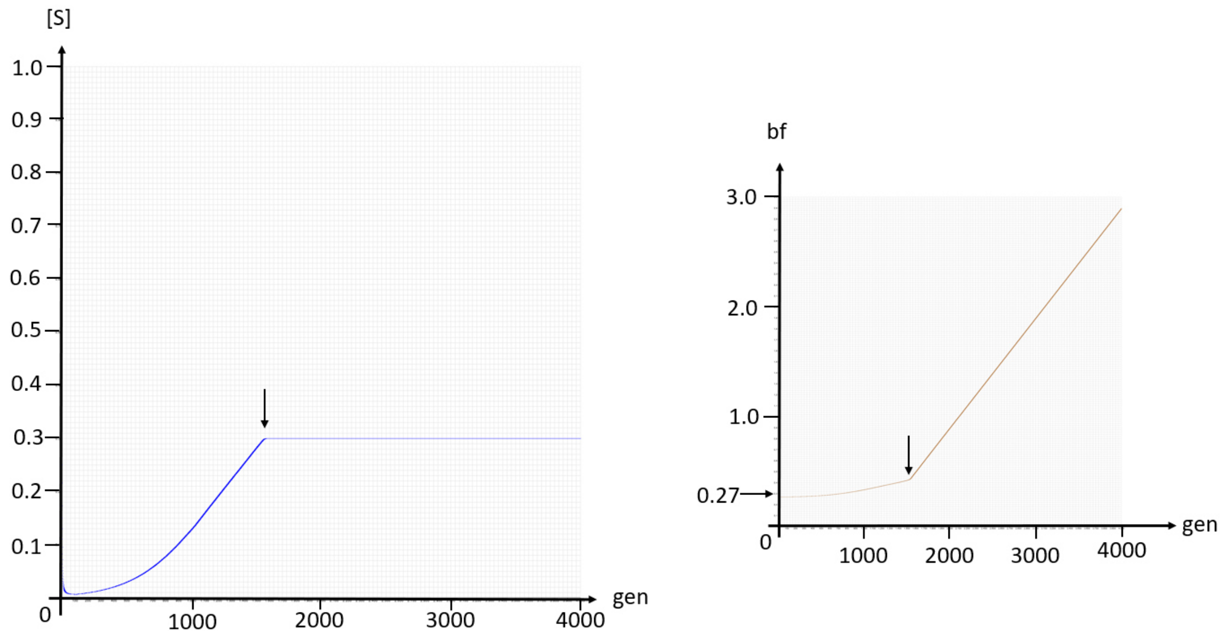


Figure 10

The average substrate concentration [S] or benefit factor (bf) of 100 individuals is shown over a time span of 4000 generations. In less than 1600 generations the optimal start concentration is reached (black arrow, left). At 0.3mM the net profits of the previous and the actual substrate concentration are identical $(b\text{-}c)_n/(b\text{-}c)_{n-1}=1$. When the optimal start concentration is found, bf dramatically increases at that fixed concentration (black arrow, right).

At *e.g.* a low start concentration bf is slowly growing over many generations. When $f(f([S]_n))/f([S]_{n-1})=1$, a stable concentration is achieved (0.3mM). When this concentration is reached the increase in bf (complexity) drastically increases. The net profit (np=f([S]=b-c) is produced according to a saturating benefit function (Michaelis-Menten) and a linear cost function: $[S]_n$ = bf * Vmax* $[S]_{n-1}/([S]_{n-1}$+Km) - cf * $[S]_{n-1}$. The biochemical values used are: Km=0.5, Vmax=5 und cf=5/3. For figure 10 and 11 bf=0.27; there bf is mutated in step sizes of +/- 0.001.

The substrate concentration [S] to start with and the size of the benefit factor to start with are very critical. Three different behaviours are observed with *e.g.* bf=0.5:

a.) [S] < 0.3: The two-step procedure leads to increasing start values and finally to a stable start concentration of 0.3mM.

b.) 0.3 < [S] < 0.4375: The two-step procedure leads to a slowly falling start value and finally to a stable start concentration of 0.3mM (not shown).

c.) [S] > 0.4375: The values increase steadily and finally become negative ([S] > 1.0). The sink has changed into a source. Here, the start concentration is to the right of the unstable fixed point (see discussion). To avoid this the search strategy for the optimal start concentration should search for the lowest possible start concentration.

Finally, there is an upper limit where the values immediately, after the first iteration, become zero or negative above that limit.

bf	0.27	0.28	0.29	0.30	0.35	0.40	0.50
[S] upper limit	0.31	0.34	0.37	0.40	0.55	0.70	1.00

In figure 11 the optimal start concentration (0.3mM) is used right from the very beginning and only as a single investment – no reinvestment of the result. The net profit and the benefit factor increase in a linear fashion.

The benefit factor reaches a value of bf=3 within 3000 generations. This is not directly comparable to other strategies due to differently sized mutational steps. In figure 3 this bf value is beyond the values that would cause chaos. Within the limits of the set values of Km, Vmax and cf, this fixed substrate concentration of 0.3mM leads through chaos. A sink keeping a special fixed concentration can pass through the area of chaos and increase complexity.

Figure 11

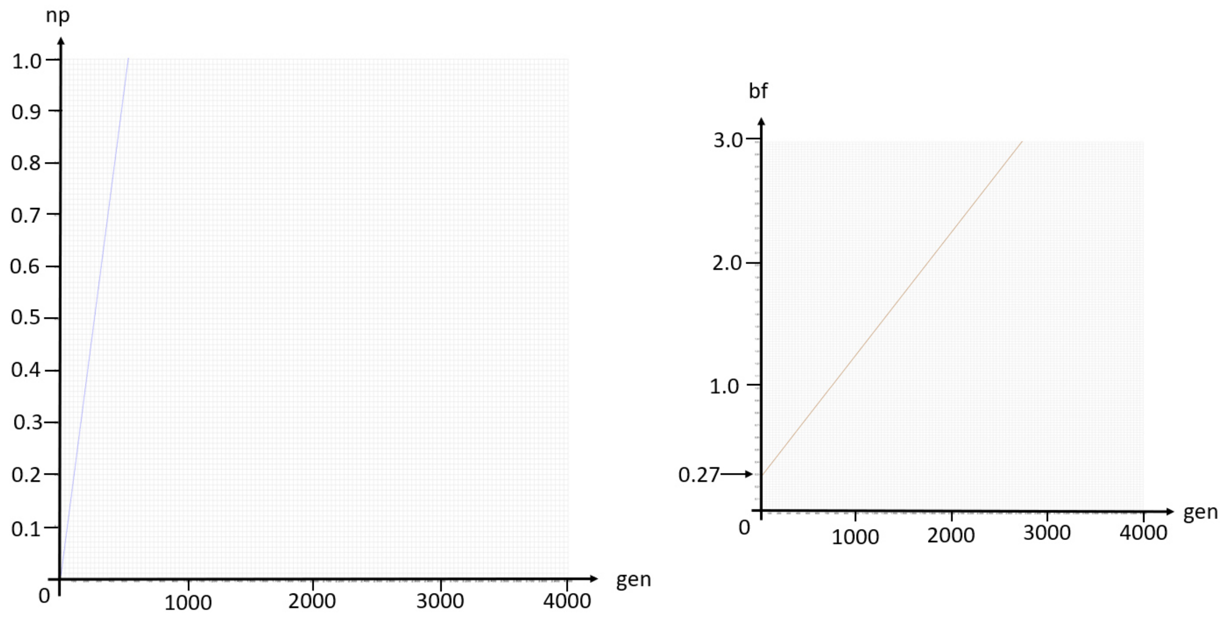


Figure 11

Increasing net profit (np, left) guides the evolution of the benefit factor (start value bf=0.27, right) through the region of bifurcation and in less than 3000 generations. The fixed substrate concentration of 0.3mM has the feature that it leads to an unstable fixed point (see discussion).

An ensemble can suppress further deterministic chaos

In the following section I compare an active, symmetric ensemble of a source and a sink with the same two identical parties acting on their own, which is basically in my definition an inactive, symmetric ensemble. Every party of the inactive ensembles will completely reinvest the net profit of each cycle as substrate of the next cycle. To demonstrate my idea this will happen at a high benefit factor causing deterministic chaos. The ensemble of a source and a sink will be an integrated ensemble. The source will give the excess net profit (beyond 0.3mM) to the sink where the substrate is used once and the product (net profit) is consumed completely. Therefore, sink will need new (excess) substrate from the source. Although 0.3mM is not the substrate concentration for the maximum net profit (0.36mM), it is very close in a broad peak. Therefore, I do not change the variables.

The transfer space has 4 different compartments according to the size relation of benefit and cost. Area I: source b-c<0 and sink b-c>0 results in a peaceful transfer if the ensemble is active. Area II: source b-c<0 and sink b-c<0 will result in a transfer only by force and deception to solve the conflict which party has to carry the encumbrance. Area III: source b-c>0 and sink b-c>0 will result in a transfer only by force and deception to solve the conflict which party has access to the benefit. Area III is here the area of interest. Finally, area IV: source b-c>0 and sink b-c<0 will result in a transfer only by force and deception through an irrational master.

In area III (figure 12 left) the inactive ensemble (bf=2.5) produces chaos in both parties not acting as source and sink. On the right side of figure 12 the development of net profit over 500 production steps is shown. The net profit of the ensemble is also chaotic. Although not inside quadrant I, the ensemble becomes active (figure 13) and the production of net profit is constant and high; an active ensemble supresses chaos.

Figure 12

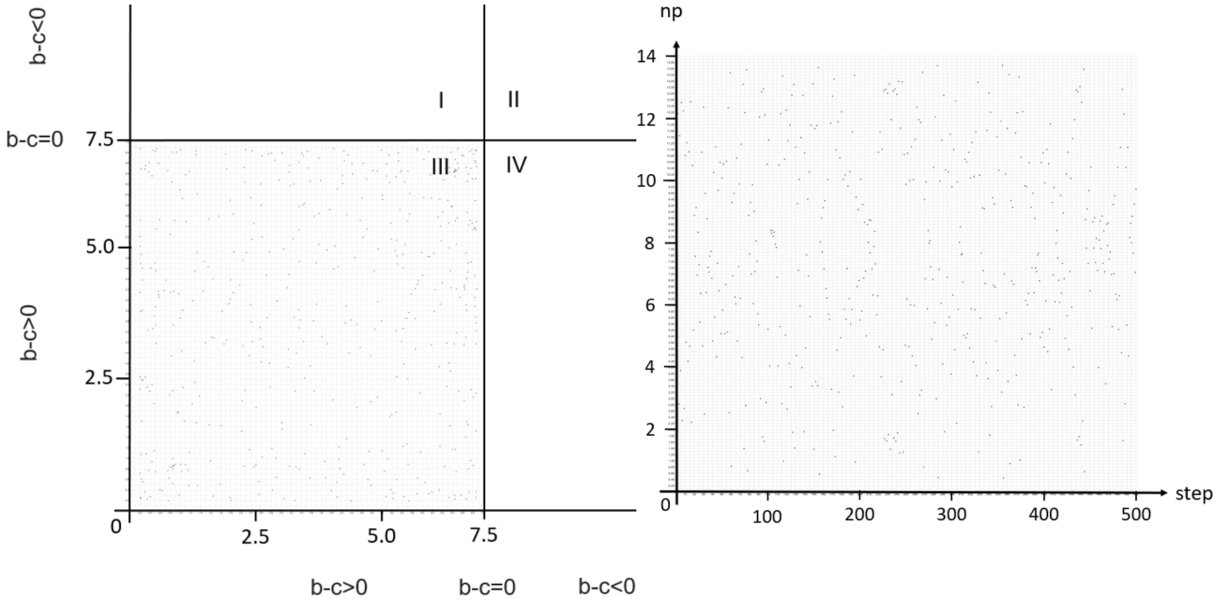


Figure 12

Two parties of an inactive ensemble are depicted. Left: The concentrations in quadrant III of the transfer space range from near zero to 7mM in both parties. The concentration pairs of 500 production steps are scattered all over quadrant III. Right: The chaotic concentrations during 500 steps result in an average of np=7.588.

Figure 13

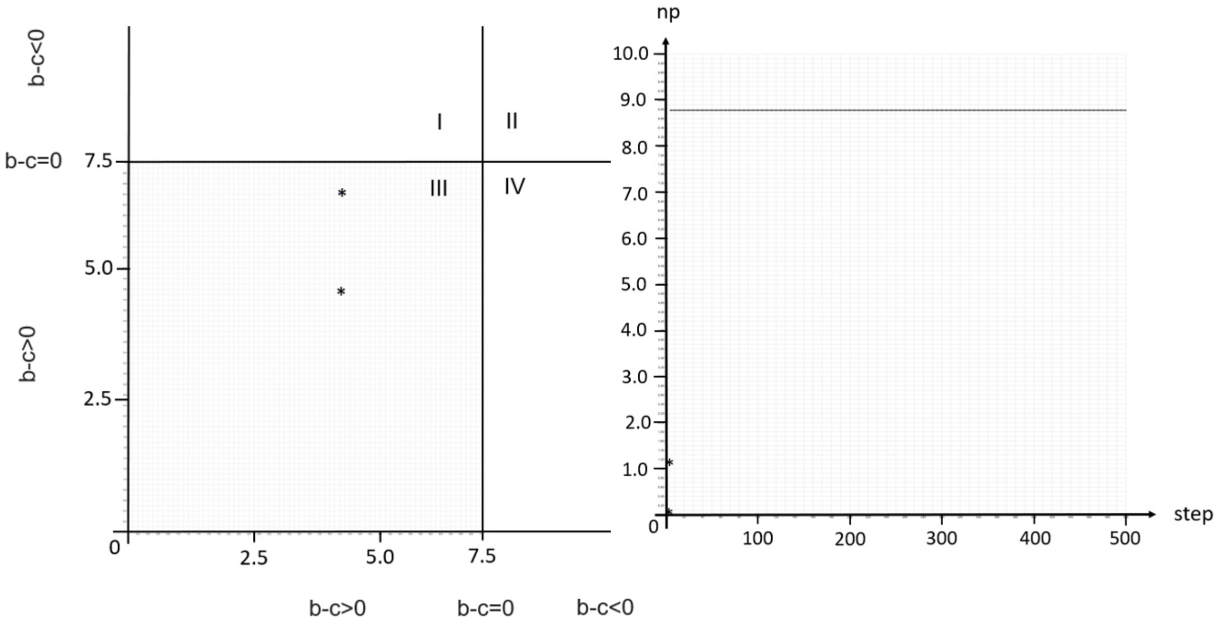


Figure 13

Similar to figure 12. Left: The ensemble alternates between two values (asterisks, production and consumption phase). Right: the net profit (np, 8.8mM) output is constant over 500 production steps after a few start values. The total average is np=8.737.

Discussion

Biologic life began as a single celled replication unit. The transfer space is a model for at least two entities. Biologists are convinced that organisms become more complex in the process of evolution. The step from a single-celled organism to a two-celled organism is such a step of increased complexity. I interpret the benefit factor as a complexity factor (2). Is my model able to understand the evolution from a single cell to an ensemble of two cells? Multicellularity usually starts with identical cells not separated after division. Those cells are genetically entangled (3). But this is not a prerequisite. Organisms can form intimate and helpful interactions in the complete absence of any genetic relation. This has happened often in the history of life. One of the most important cases is known as endosymbiotic theory (4). Any kind of genetically based altruism does not seem necessary! Increased complexity is also observable in economics and society. From cells to human societies division of labour and architectural hierarchy are observable. To me this is the essence of complexity. A feature of complexity is non-linear behaviour. Non-linear behaviour in my model has the primary root in the non-linearity of the benefit function.

The model is not restricted to cells. Molecules similar to RNA, believed to be on the basis of biochemical evolution, combine the features of a source and a sink. Increasing complexity on this level is also included.

Emergence of deterministic chaos on the level of a single party

Single parties of an ensemble show the behaviour of the quadratic iterator: $x_n = a*x_{n-1}*(1-x_{n-1})$, when “a” approaches a value of 4. This similarity is a surprize to me as there is no quadratic term involved in my model. An increase of bf or a decrease of cf (figures 3, 4, and 5) results in a comparable behaviour. Deterministic chaos starts in an area where benefit still exceeds cost (b-c>0). This is the general condition in a sink; the benefit of a substrate is larger than its cost and therefore substrate is taken but not given. To become a source the substrate has to become a burden (b-c<0). The combination of a source and a sink will form an ensemble. To reach this goal the biochemistry (Vmax increase, Km decrease, substrate concentration [S] increase) could change. However, there will be limitations to a change in Km and Vmax by mutations. A further possibility is the increase of the benefit factor. I do not investigate in detail the decrease of the cost factor as there is a natural end at zero or even earlier when there is a fix cost.

The benefit factor is also a complexity factor. Complexity has many aspects including hierarchical organization and structure. This is an architectonical feature on the level of molecules, cells, organisms, and groups of organisms. To become a source to a second party, the sink has to go through an area where increased net profit is no longer a reliable compass to measure progress, as bifurcation and deterministic chaos appear (figure 5). Are there strategies to pass through this area?

Evolution of a sink into a source at low substrate concentrations

Strategy 1

This strategy chooses a complete and continuous reinvestment of the net profit. The single party is only able to increase its complexity to bf=1.15 (figure 6). This is just behind the first bifurcation. In bifurcations the resulting net profit does no longer continuously increase; it alternates between a larger value and a smaller value. A large accumulation of complexity is not possible. With an even stronger selection (10 individuals die) the net profit starts to bifurcate, too. This is very surprising, as there is a direct competition. The rest of the observations are very similar to the data shown.

Strategy 2

A further advanced strategy would rely on an average of values to avoid errors by fluctuations. In figure 8 an average of 20 steps is used. This also ends the increase of bf within the first bifurcation. But even if an average of all 1050 iterations would be used, the drop of the average within the windows of stability would hinder the further increase of bf (figure 14).

Figure 14

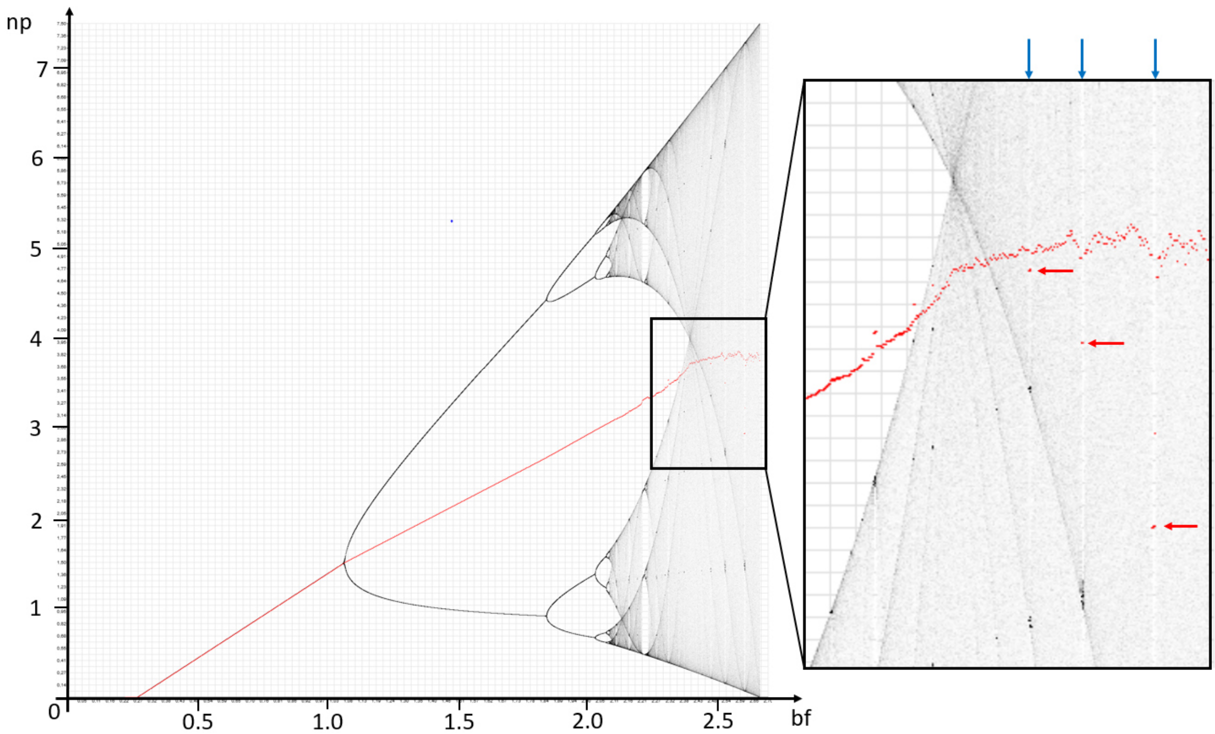


Figure 14

The average value of all 1050 iterations is displayed (red) within the Feigenbaum diagram (grey). Enlarged section: blue arrows mark the windows of stability; red arrows mark average values within those windows.

Strategy 3

To reinvest always the same amount of substrate is very successful to avoid deterministic chaos in net profit and to maintain a linear growth of the benefit factor (figure 9). However, any slightly higher substrate concentration will outcompete the organism with the lower substrate concentration. Very low substrate concentrations may not produce enough net profit to replace the start concentration.

Substrate concentrations in the real world are usually very small. Therefore, the system would have to cope with a small net profit. Only reinvestment starting at low concentrations can produce larger concentrations. But it seems to be a reasonable strategy to give parts of the net profit, although still beneficial (b-c>0), in case the full reinvestment would lead to a substrate concentration where b-c≤0. The sink would act as a source when the limiting concentration would be reached.

Strategy 4

This strategy results in a safe passage through chaos towards complexity. Figure 10 and figure 11 show that there is an optimal fixed concentration. This concentration has a very special feature. The net profit coming from this concentration reinvested as substrate will result in the same net profit, again. The second reinvestment is no further improvement and should not be made. The special start concentration is optimal, but not necessarily the maximum gain. Why is just this concentration observed and is there a general rule?

A fixed point [S] of a function f is characterized by f([S]) = [S].

$$\frac{V_{max} \cdot [S]}{K_m + [S]} \cdot b_f - [S] \cdot c_f = [S]$$

$$\iff \quad \frac{V_{max}}{K_m + [S]} \cdot b_f - c_f = 1 \qquad (\text{for}[S] > 0)$$

$$\iff \quad [S] = \frac{b_f \cdot V_{max}}{1 + c_f} - K_m$$

Let us ask now, which start concentrations [S] lead to this fixed point?

$$f([S]) = \frac{b_f \cdot V_{max}}{1+c_f} - K_m$$

$$\Longleftrightarrow \qquad b_f \cdot \frac{[S]}{[S]+K_m} \cdot V_{max} - c_f \cdot [S] = \frac{b_f \cdot V_{max}}{1+c_f} - K_m$$

$$\Longleftrightarrow b_f \cdot [S] \cdot V_{max} - c_f \cdot [S] \cdot ([S]+K_m) = \left(\frac{b_f \cdot V_{max}}{1+c_f} - K_m\right) \cdot ([S]+K_m)$$

$$\Longleftrightarrow -c_f \cdot [S]^2 - c_f \cdot K_m \cdot [S] + b_f \cdot V_{max} \cdot [S] = \frac{b_f \cdot V_{max} \cdot [S]}{1+c_f} + \frac{b_f \cdot V_{max} \cdot K_m}{1+c_f} - K_m \cdot [S] - K_m^2$$

$$\Longleftrightarrow -c_f \cdot [S]^2 + (b_f \cdot V_{max} - c_f \cdot K_m - \frac{b_f \cdot V_{max}}{1+c_f} + K_m) \cdot [S] - \frac{b_f \cdot V_{max} \cdot K_m}{1+c_f} + K_m^2 = 0$$

$$\Longleftrightarrow -c_f \cdot [S]^2 + \frac{b_f \cdot V_{max} \cdot c_f - c_f \cdot K_m - c_f^2 K_m + K_m + K_m c_f}{1+c_f} \cdot [S] + \frac{K_m^2 + K_m^2 c_f - b_f V_{max} K_m}{1+c_f} = 0$$

$$\Longleftrightarrow [S]^2 - \frac{b_f V_{max} c_f - c_f^2 K_m + K_m}{(1+c_f) \cdot c_f} \cdot [S] - \frac{K_m^2 + K_m^2 c_f - b_f V_{max} K_m}{(1+c_f) \cdot c_f} = 0$$

$$[S]_{1/2} = \frac{b_f V_{max} c_f - c_f^2 K_m + K_m}{2(1+c_f)c_f} \pm$$
$$\sqrt{\frac{b_f^2 V_{max}^2 c_f^2 + c_f^4 K_m^2 + K_m^2 - 2 b_f V_{max} c_f^3 K_m + 2 b_f V_{max} c_f K_m - 2 c_f^2 K_m^2}{4(1+c_f)^2 c_f^2} + \frac{K_m^2 + K_m^2 c_f - b_f V_{max} K_m}{(1+c_f)c_f}}$$
$$= \frac{b_f V_{max} c_f - c_f^2 K_m + K_m}{2(1+c_f)c_f} \pm \frac{K_m + 2K_m c_f + K_m c_f^2 - V_{max} c_f b_f}{2(1+c_f)c_f}$$

$$[S]_1 = \frac{-K_m(1+c_f) + V_{max} b_f}{1+c_f} = \frac{V_{max} b_f}{1+c_f} - K_m$$

$$[S]_2 = \frac{K_m}{c_f}$$

The solution $[S]_1$ is obvious, the fixed point will always lead back to itself. However, this fixed point (substrate concentration [S]) will change when bf changes in the course of increasing complexity. The solution $[S]_2$ is surprising as it is independent of bf and Vmax. The substrate concentration 0.3mM in my standard case (Km=0.5, cf=5/3) is such an unstable fixed point. This concentration will not change over a wide range of benefit factors (Figure 15). In case the organism is keeping this concentration as limit or fixed value, chaos will not be observed. Complexity can increase unhindered at the concentration Km/cf.

Figure 15

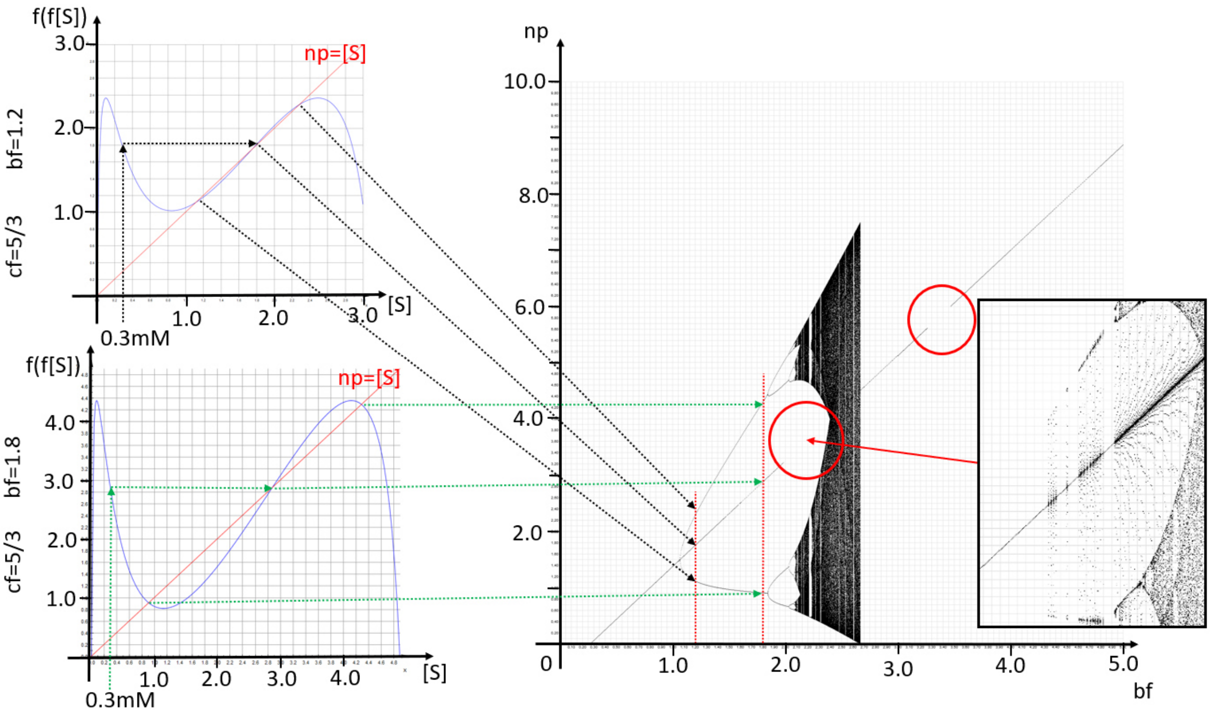


Figure 15

With the set values of cf=5/3 and Km 0.5 the start concentration of 0.3mM (Km/cf) will always reach the unstable fixed point (left side, bf1.2 and bf1.8). The straight line of the initial stable fixed point goes on but is then an unstable fixed point. This line is still present in the area where the behavioural change starts (beyond bf=8/3). The interruptions (red circles) are due to a butterfly effect caused by the graphics program (inset on the right with less iterations). Besides 0.3mM start concentrations from 0.1mM to 0.9mM were used. They all produce chaos and finally negative net profit values.

Minding the concentration Km/cf a sink may become a source in area III. Km/cf is a new type of border useful to avoid the loss of orientation by deterministic chaos. In addition, this concentration is a turning point in reinvestment. All concentrations smaller than Km/cf will, when their result is reinvested, result in a larger net profit than the prior net profit. All concentrations larger than Km/cf, when their result is reinvested, become smaller. When Km/cf is invested, the result of this investment reinvested is of identical size. Km/cf is, as a final investment, optimal and therefore a reasonable limit. This limit may not lead to the maximum net profit. However, there are conditions where Km/cf results in maximum net profit.

Maximum net profit is a central goal in evolution. Evolution by mutation and selection will always look for the maximum net profit (aquila non captat muscas). Net profit is benefit minus cost (b-c) and is a function of the substrate concentration. The local maxima and minima of a function can be determined by the first derivative:

$$f([S]) = \frac{V_{max} \cdot [S]}{K_m + [S]} \cdot b_f - [S] \cdot c_f$$
$$f'([S]) = \frac{K_m}{([S] + K_m)^2} \cdot V_{max} \cdot b_f - c_f$$

In the point of maximum net profit, the first derivative of the function is zero; f´([S]=0 (and f´´<0) The substrate concentration leading to this maximum net profit is:

$$[S]_{max} = \sqrt{\frac{K_m \cdot V_{max} \cdot b_f}{c_f}} - K_m$$

Are there conditions where the substrate concentration leading to the maximum net profit is identical to the substrate concentration leading to the unstable fixed point where the reinvestment no longer grows (Km/cf)?

$$\sqrt{\frac{K_m \cdot V_{max} \cdot b_f}{c_f}} - K_m = \frac{K_m}{c_f}$$

This equation can be solved for Vmax or Km or cf:

$$V_{max} = \frac{K_m}{b_f} \cdot \frac{(c_f + 1)^2}{c_f}$$

$$K_m = V_{max} \cdot b_f \cdot \frac{c_f}{(c_f + 1)^2}$$

$$c_{f1/2} = \frac{V_{max} \cdot b_f}{2K_m} - 1 \pm \frac{\sqrt{V_{max}^2 b_f^2 - 4V_{max} b_f K_m}}{2K_m}$$

From this calculation it is clear what size relations Km, Vmax and cf have to have so that the concentration Km/cf reaching the unstable fixed point also results in maximum net profit.

An ensemble, once formed, can suppress further deterministic chaos

Once an ensemble has formed it becomes resilient to chaos. The chaos producing overflow of net profit is buffered by a superadditivity creating sink. In figure 12 and 13 the bf value is set to 2.5. The inactive ensemble (figure 12) produces in its separate entities according to the benefit and cost function. Both parties are basically a sink. The net profit values within the transfer space (figure 12 left) and over 500 production steps (figure 12 right) show a chaotic distribution. In figure 13, the active ensemble, no chaos is observed. A sink (b-c>0) has been transformed into a source although not yet b-c<0. This is reasonable to optimize net profit. In addition, the average np of the active ensemble is better. The system decides in favour of more net profit and less benefit. Where are the borders to decide in favour of different behaviours?

Borders within my model

As long as my model is used to observe and compare benefit and cost, the borders within the model are clear. I observe a region where b-c>0; this is benefit domination. A sink takes by free will a beneficial substrate and can produce a positive net profit. And I observe a region where b-c<0; this is cost domination. A source gives by free will a detrimental substrate or will produce a negative net profit. The net profit is either larger or smaller than zero. Between these two areas there is a clear border where b-c=0. This is equivalence of benefit and cost. The optimal benefit is extracted.

No valuable benefit is wasted. In addition, there is no cost in excess, no cost is a burden. To simply observe whether benefit is larger or smaller than cost is a qualitative approach.

When I start to look at the amount of net profit, I follow a quantitative approach. In figure 6 I observe fuzziness. The net profit of a single start concentration but of different bf and cf values is not simultaneously negative. Some bf or cf values lead quickly to negative net profit values and some slowly. This could be interpreted as an escape velocity into a source behaviour. Sinks convert fast or more slowly into a source. This is the side where taking (sink behaviour) ends. The transition is gradual (figure 15).

Figure 15

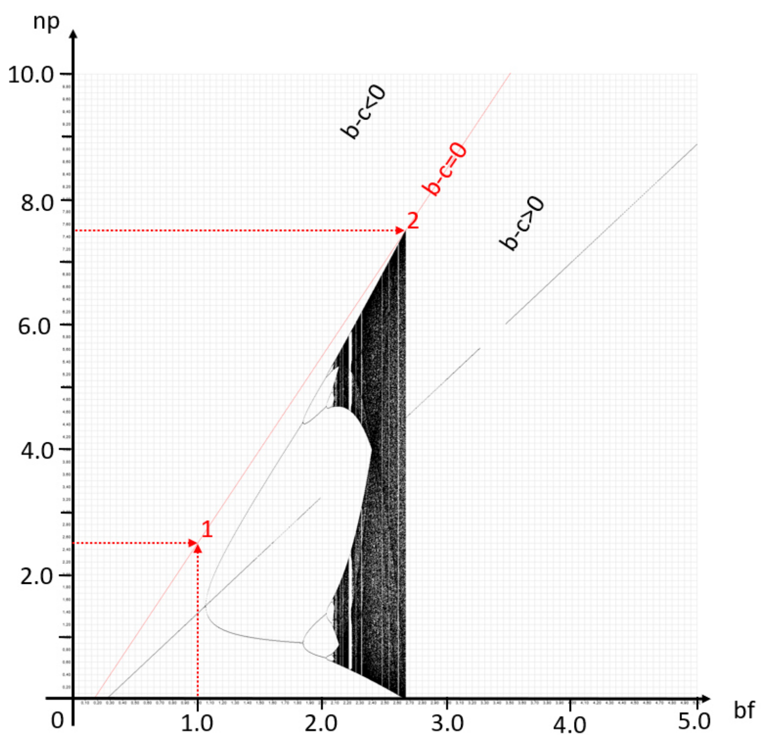


Figure 15

The red line (b-c=0) separates benefit domination (b-c>0) and cost domination (b-c<0). Chaos starts in benefit domination. Red arrows 1: the location of my standard example, b-c=0 at 2.5mM and bf=1. Red arrow 2: at 7.5mM and bf=8/3 more and more results induce a change of behaviour through cost domination. The sink slowly becomes a source when large net profits are used as substrate – the result will be a negative np.

In figure 10 I look at the other side when giving according to the net profit might start. It is reasonable to start to give parts of the net profit while approaching Km/cf. The closer the party is to Km/cf, the more of the resulting net profit will be given. The start of giving is also fuzzy as the amount depends on the size of the net profit and the distance to Km/cf.

The equilibrium of benefit and cost results in a clear limit; b-c=0. However, net profit has a maximum which can never be identical to this limit. This maximum is located within the region of b-c>0, a sink. I argue here that net profit growth is already declining in the region of sink behaviour. It could be reasonable for a sink (b-c>0) to give to optimize net profit size. This would be the start of a source behaviour - to give - for a sink. According to net profit quantity a sink will become a source at free will although, regarding to benefit quality, force would be necessary.

Furthermore, there seems to be a conflict of goals (Zielkonflikt). Either you optimize benefit with a clear border (b-c=0) or you maximize net profit with fuzzy borders. Incompatible decisions between maximum net profit and optimal benefit have to be made.

A problem with my model

Here I want to mention a problem. In the early exploration of my model, I decided to give certain units to bf and cf for very practical reasons – to get rid of them in “b” and “c”. Now it would be better if cf would be a dimensionless factor to obtain a concentration when dividing Km by cf. Here I would really need some input! The calculations work out numerically, the units however do not fit.

Final remarks

Finally, I want to discuss two points on extreme ends of the spectrum of time and size:

1. The technological and general progress of civilisation results in increased complexity. Net profit from complexity does not solely refer to the “in and out” of money. Money as universal medium of exchange can connect different categories of phenomena, thus creating mixed feedback loops. Money has the ability to spread chaotic behaviour. As demonstrated, chaos may emerge from complete reinvestment. In chaos an increased complexity may result in a decreased outcome, thus destroying the compass. But there is an alternative strategy to full reinvestment. However, an unstable fixed point is a delicate and fragile place. The narrow ford towards complexity has a steep abyss to the right and left and needs a cautious balance.
2. Where does the initial complexity come from? In my model, at very low complexity, net profit and increasing complexity (bf) behave antagonistically (figure 3, inset). I think at the lowest level in evolution - the transition from chemistry to biochemistry and then to biology - the driving force to increase complexity does not come from the aim to increase net profit but from spontaneous self-organization.

   It could well be that on every level of organization, from an ensemble of molecules to an ensemble of cells, organisms and groups of organisms, we observe the self-similarity of an even larger scale fractal object – life itself. To increase complexity the system depends at first on the forces of self-organization. This is depicted on the left side of the graph (bifurcation diagram of figure 3, inset). Here, increased complexity is not connected to increased net profit. Then a region follows where increased complexity is connected to increased net profit (in my model in a linear fashion, figure 3). In the right part of the Feigenbaum diagram the connection of net profit and complexity becomes chaotic (starting with bifurcations, figure 3). A solution to this problem is suggested; Km/cf.

In addition, there could be a second way to further complexity. As there is no direction in chaos, the system does not know where it stands. The chaos at high bf (right side of figure 3) and the chaos at very low bf (left side of the inset of figure 3) can´t be discriminated. Yes, there is a range difference, but there will be slices of similar size. A new round of self-organization out of chaos could emerge. In biology there is the idea of major transitions in evolution (5). My interpretation now would be that a transition happens when the system no longer follows the narrow road of the unstable fixed point within the chaotic zone to increased complexity but develops out of chaos by self-organization a new stable perspective.

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