General Trade Equilibrium of Integrated World Economy

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ABSTRACT

The general trade equilibrium is one of the most critical topics in international economics. This paper curries three related studies. The first one is to present Dixit and Norman's integrated world equilibrium (IWE) analytically by introducing a Dixit-Norman constant, which shows the structure of equalized factor prices. The second is to use the trade volume, defined with domestic factor endowments proposed by Helpman and Krugman (1985), to expose price-trade equilibrium. The optimality property of the equilibrium is that the trade volume reaches its maximum value. The last one displays the measurement of autarky prices and gains from trade. It illustrates that, if two countries have same technologies, free trade distributes gains from trade evenly to trade partners.

Keywords:
Factor content of trade, factor price equalization, General equilibrium of trade; Integrated World Equilibrium; IWE

JEL Classification Code: F10, F15

1. INTRODUCTION

The Heckscher-Ohlin model is ideal for exploring the general price-trade relationship among factor prices, commodity prices, production outputs, and trade volumes. Samuelson (1948) presented the famous theorem of factor price equalization. Immediately, he proposed an idea about the price-trade equilibrium that the equalized factor prices will not change when factors are mobilized across countries (see Samuelson 1949). It is the first effort to try to present the property and structure of equalized factor prices. Thirty years later, Dixit and Norman (1980) implemented the Integrated World Equilibrium (IWE) to illustrate the factor price equalization (the FPE), which fulfilled the factor mobility analysis perfectly. They proved that the world prices remain the same when the allocation of factor endowments changes within the FPE set in the IWE. Helpman and Krugman (1985) normalized the assumption of the integrated equilibrium. Deardorff (1994) illustrated the conditions of the FPE for many goods, many factors, and many countries by the IWE approach. He discussed the FPE for all possible allocations of factor endowments within lenses identified.

McKenzie (1955) proposed the cone of diversification of factor endowments, which is vital to understand FPE and trade from production supply constraints. Fisher (2011) proposed the concept of goods

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price diversification cone, which is the counterpart of the diversification cone of factor endowments. He also offered another brilliant idea of the intersection of goods price cones to illustrate the price-trade relationship when countries have different technologies.

Vanek (1968) variegated the preference taste on the Heckscher-Ohlin model by the share of GNP, which engaged prices with trade and consumption. It resulted in the HOV studies to convert the assumption of homothetic taste into consumption balance.

Woodland (2013, p.39) described the importance of the general equilibrium, "General equilibrium has not only been important for a whole range of economics analyses but especially so for the study of international trade." Deardorff (1982, p.685) said, "A trade equilibrium is somewhat more complicated."

The one focus of studies on the general equilibrium for constant returns and perfect competition is the social utility function and direct and indirect trade utility function (offer curve). It is difficult for those approaches to get a desired price-trade equilibrium either. It supplied a framework for solutions of equilibriums from consumption.

Helpman and Krugman (1985, p.23-24) proposed a unique idea of trade volume defined by domestic factor endowments and specified with world factor endowments. They derived an insight trade logic as "the differences in factor composition are the sole basis of trade." That moves the last enormous step toward general trade equilibrium after Dixit and Norman's integrated world equilibrium. This study extends their insights and methods to achieve the price-trade equilibrium within IWE.

The paper shows the optimality property of the equilibrium solution that the trade volume reached its maximum value when factor prices equalized. It illustrates that the world prices at equilibrium are the functions of the world factor endowments. The study proposes the calculation of autarky prices. the logic of calculating the world prices in the equilibrium solution can be used to calculate the autarky prices: autarky factor endowments determine the autarky prices. It also shows that the equalized factor prices ensure gains from trade for countries taking part in the trade.

This paper is organized into six sections. Section 2 names the Dixit-Norman constant, which shows why the world prices remain the same when the allocation of factor endowments changes within the FPE set. It derives the general trade equilibrium by using the trade volume defined with domestic factor endowments proposed by Helpman and Krugman (1985). The section also supplies another approach to confirm the trade equilibrium. Section 3 illustrates that the trade volume gets its maximum value at the price-trade equilibrium. It is also an independent way to reach the trade equilibrium. Section 4 proposes a way to measure autarky prices. The idea is that the autarky factor endowments determine autarky prices. It shows the measurement of gains from trade. Section 5 is the equilibrium result for the multiple-country economy. Section 6 is discussions related.

2. The General Trade Equilibrium With the IWE

2.1 *The Notation of the Heckscher-Ohlin Model*

We take the following typical assumptions of the Heckscher-Ohlin model in this study: (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are immobile across countries, but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals, and (7) full employment of factor resources. We denote the Heckscher-Ohlin model as follows. The production constraint of full employment of factor resources is

$$AX^h = V^h \qquad (h = H, F) \tag{2-1}$$

where A is the 2×2 matrix of direct factor inputs, X^h is the 2×1 vector of commodities of country h, V^h is the 2×1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(w/r), k = K, L, i = 1,2$. We assume that A is not singular. The zero-profit unit cost condition is

$$A'W^h = P^h \qquad (h = H, F) \tag{2-2}$$

where $W^h$is the 2×1 vector of factor prices, its elements are r rental for capital and w wage for labor, P^h is the 2×1 vector of commodity prices.

Factor prices will be equalized when prices and trade reach their equilibrium. We denote the world price equations as

$$A'W^* = P^* \tag{2-3}$$

The trade balance condition for the factor contents is

$$\frac{w^*}{r^*} = -\frac{F_K^H}{F_L^H} = -\frac{s^H K^W - K^H}{s^H L^W - L^H} \tag{2-4}$$

where F_L^H and F_K^H are factor contents of trade of country H, $K^W$and L^W are world factor endowments, s^H the share of the GNP in country H to the world GNP.

Embedded in the Heckscher-Ohlin system represented by (2-1), (2-3), and (2-4), there are seven equations with nine endogenous variables in the model which are p_1^* , p_2^* , w^* , r^* , x_1^H , x_2^H , x_1^F , x_2^F, and s^H . There are four exogenous variables K^H , L^H , K^F , and L^F . The system is not determined. By Walras' equilibrium, we can drop one of these market-clearing conditions, such as we can take one price as the numeraire to set its value to 1. That will leave only one uncertain condition for the equilibrium. If we result in that one, we will solve the equilibrium. Some optimality analyses can help with this; some economics principles or logic can help if the approaches are proper.

2.2 Trade Box on IWE Diagram and The Dixit-Norman Constant

The relative world commodity prices $\frac{p_1^*}{p_2^*}$ should lie between the rays of goods price diversification cone (see Fisher, 2011), in algebra, as,

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{2-5}$$

This condition can ensure that the factor prices are positive. We assume that industry 1 is factor intensity in capital in (2-5).

The range of the shares of GNP s^H, corresponding to the rays of the cone above, can be calculated as

$$s_b^H(p) = s\left(p\left(\frac{a_{K1}}{a_{K2}}, 1\right)\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^W} \tag{2-6}$$

$$s_a^H(p) = s\left(p\left(\frac{a_{L1}}{a_{L2}}, 1\right)\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^W} \tag{2-7}$$

These are just the range of s^H, Leamer (1984, p.9) first proposed, as

$$\frac{K^H}{K^W} > s^H > \frac{L^H}{L^W} \tag{2-8}$$

For convenience, we denote two parameters, which are the shares of the factor endowments in the home country to their world factor endowments respectively,

$$\lambda_L = \frac{L^H}{L^W} \tag{2-9}$$

$$\lambda_K = \frac{K^H}{K^W} \tag{2-10}$$

Figure 1 is an IWE diagram adding with a trade box. The dimensions of the diagram represent world factor endowments. The origin of the home country is the lower-left corner. It is the right-upper corner for the foreign country. ON and OM are the rays of the cone of factor diversifications. Any point within the parallelogram formed by ONO^*M is an available allocation of factor endowments of two countries. Helpman and Krugman (1985, p.15) call the parallelogram the FPE (Factor Price Equalization) set. Suppose that an allocation of the factor endowments is at point E, where the home country is capital abundant (we will use this assumption for all analyses of this study). Point C is the trade equilibrium point. It shows the sizes of the consumption of the two countries.

The trade box $EBDG$ is by the range of shares of GNP in (2-8). If a relative commodity price lies in the goods price diversification cone (2-5), the share of GNP by that price lies in the trade box.

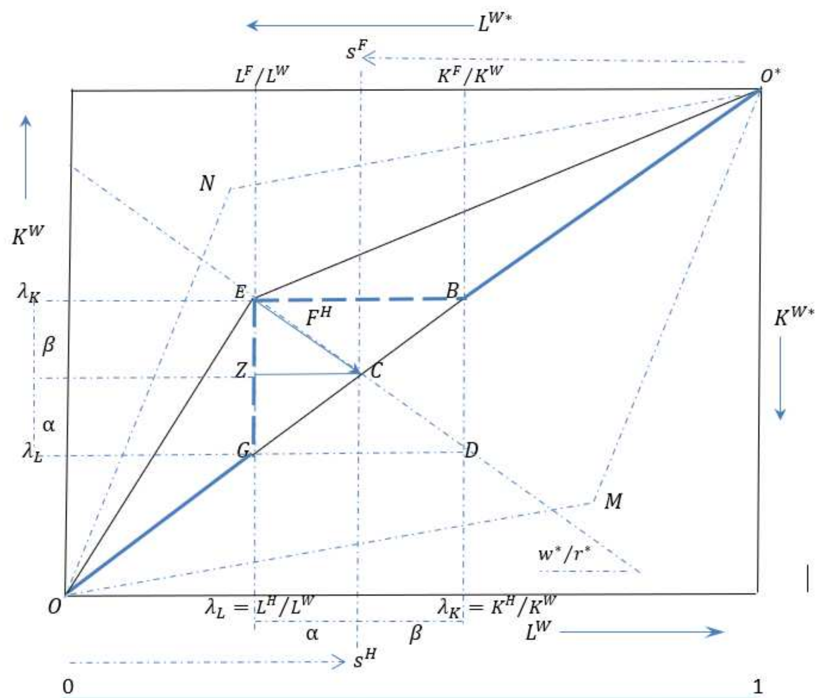

Figure 1 IWE Diagram with Trade Box

For a given allocation (or distribution) of factor endowments, E, its equilibrium point or the consumption point C needs to fall within the diagonal line $\overline{GB}$ of the trade box. The share of GNP s^H divides the trade box into two parts: α and β,

$$\alpha = s^H - \lambda_L \tag{2-11}$$

$$\beta = \lambda_K - s^H \tag{2-12}$$

When α increases, the home country's share of GNP increases, and the foreign country's share of GNP decreases, and vice versa.

We rewrite the trade balance of factor contents (2-4) as

$$\frac{w^*}{r^*} = \frac{(\lambda_K - s^H)}{(s^H - \lambda_L)}\frac{K^W}{L^W} = \frac{\beta}{\alpha}\frac{K^W}{L^W} \tag{2-13}$$

The trade competition between countries is that each country tends to maximize the factor price of its abundant factor and to maximize it income of export. That strategy will benefit a country to export less its product which is produced by using its abundant factor intensively and to import more the product which is produced by using its scarce factor intensively. It will make the country reach to bigger share of GNP and consume more of the two products. The trade is with trade-off. When country H export more, wage/rental ratio will be higher.

Dixit and Norman found the world prices will remain the same when a allocation or distribution of the world factor endowments change within the FPE set. Their finding implies that the world prices are constant within the FPE set. It means that $\frac{r^*}{w^*}$ is a constant. Introduce a constant

$$\varphi = \frac{(\lambda_K - s^H)}{(s^H - \lambda_L)} \tag{2-14}$$

Substituting it into (2-13) yields

$$\frac{w^*}{r^*} = \varphi\frac{K^W}{L^W} \tag{2-15}$$

If φ remains the same or is a constant, the world prices will stay the same when the allocation of the world factor endowments changes within the FPE set in the IWE. We call φ the Dixit-Norman constant to honor their contribution on the IWE and FPE property. Equation (2-15) interprets the factor price equalization in the IWE diagram analytically. Equation (2-15) reduces the mystery of the structures of world commodity prices and equalized factor prices.

The range of φ corresponding (2-5) or (2-8) is

$$\infty > \varphi > 0$$

The question now is what the value of the constant is.

2.3 General Trade Equilibrium by The Trade Volume Defined with Domestic Factor Endowments

Helpman and Krugman (1985, p.23) defined trade volume[1] by domestic factors constrained with world factor endowments. They illustrated that there are some variables (γ_L , γ_K) for all equal trade volumes lines, which satisfy the following relationships:

$$VT = \gamma_L L^H + \gamma_K K^H \tag{2-16}$$

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \tag{2-17}$$

The equal trade volume curves in the FPE set are straight lines, which are parallel to the diagonal line OO^* in the IWE diagram. (2-17) makes sure for it. The primary argument for the relationships above is that the trade volume is a linear function of K^H and L^H eventually (see Helpman and Krugman 1985, pp23, pp175).

[1] This paper uses the trade volume by factor content of trade.

The two equations also ensure that a higher difference in factor composition leads to a higher trade volume and that trade volume is zero if a factor endowment distribution allocates at the diagonal line OO^*. It is the first time to show that world factor endowments somehow relate to the equilibrium relationship. They showed that one of γ_L, γ_K is negative. If country H is capital abundant, its two variables are $\gamma_K > 0$ and $\gamma_L < 0$.

Vector V^H , the factor endowments in country H, can be written, by Figure 1, as

$$V^H = \begin{pmatrix} K^H \\ L^H \end{pmatrix} = \overrightarrow{OG} + \overrightarrow{EG} \tag{2-18}$$

$\overline{OG}$ stands for the part of the factor endowments that is under the proportion (composition) of world factor consumptions as

$$\overrightarrow{OG} = \begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} \tag{2-19}$$

$\overline{EG}$ is the excessive capital services, which is out of the proportion of world factor consumptions. We express it as

$$\overrightarrow{EG} = \begin{pmatrix} (\alpha+\beta)K^W \\ 0 \end{pmatrix} \tag{2-20}$$

Rewrite it as

$$V^H = \begin{pmatrix} K^H \\ L^H \end{pmatrix} = \begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} + \begin{pmatrix} (\alpha+\beta)K^W \\ 0 \end{pmatrix} \tag{2-21}$$

The trade volume (2-16) can be rewritten as a dot product of $V^H$and the pair of the variables $(\gamma_K^H \quad \gamma_L^H)$

$$VT^H = (\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} K^H \\ L^H \end{pmatrix} \tag{2-22}$$

where the two variables are marked with superscript h to say its country.
Substituting (2-21) into (2-22) yields

$$VT^H = (\gamma_K^H \quad \gamma_L^H)\cdot(\overrightarrow{OG} + \overrightarrow{EG}) = (\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} + (\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} (\alpha+\beta)K^W \\ 0 \end{pmatrix} \tag{2-23}$$

The first term on the right side above is zero by (2-17),

$$(\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} = 0 \tag{2-24}$$

Simplify (2-23) as

$$VT^H = (\alpha+\beta)K^W \gamma_K^H \tag{2-25}$$

The vertical line EG, in quantity as $(\alpha+\beta)K^W$, is the differences of factor composition described by Helpman and Krugman. It is just is the vertical boarder of trade box. Its value by free trade is trade volume. The trade volume by EG is

$$VT^H = (\alpha+\beta)K^W r^* \tag{2-26}$$

It implies

$$\gamma_K^H = r^* \tag{2-27}$$

The trade volume of factor content of trade, in country H, can be expressed also as

$$VT = 2F_K^H r^* = 2\beta K^W r^* \tag{2-28}$$

Substituting (2-28) into (2-26) yields

$$\beta = \alpha \tag{2-29}$$

It implies

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} \tag{2-30}$$

$$\varphi = 1 \tag{2-31}$$

It shows that the Dixit-Norman constant is 1. Write (2-30) and (2-17) together

$$\frac{w^*}{r^*} = -\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \tag{2-32}$$

Substituting (2-27) into (2-32) yields

$$\gamma_L^H = -w^* \tag{2-33}$$

Substituting (2-27) and (2-33) into (2-16) yields

$$VT = K^H r^* - L^H w^* \tag{2-34}$$

It is the price-trade equilibrium that Helpman and Krugman predicted.

Substituting (2-30) into (2-4) yields

$$s^H = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) = \frac{1}{2}(\lambda_K + \lambda_L) \tag{2-35}$$

Equation (2-34) shows that the difference between the total cost of the abundant factor and the total cost of the scarce factor of a country equals to its trade volume of factor contents. In other words, it shows that the monetary value of the differences in factor composition of a country is its trade volume.

With (2-35), we get the complete equilibrium solution of the Heckscher-Ohlin model as

$$s^h = \frac{1}{2}\left(\frac{K^h}{K^W} + \frac{L^h}{L^W}\right) \qquad (h = H, F) \tag{2-36}$$

$$r^* = L^W \tag{2-37}$$

$$w^* = K^W \tag{2-38}$$

$$p_1^* = a_{k1}L^W + a_{L1}K^W \tag{2-39}$$

$$p_2^* = a_{k2}L^W + a_{L2}K^W \tag{2-40}$$

$$F_K^h = s^h K^W - K^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W} \qquad (h = H, F) \tag{2-41}$$

$$F_L^h = s^h L^W - L^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W} \qquad (h = H, F) \tag{2-42}$$

In equation (2-38), I assume $w^* = K^W$ to drop one market condition. The factor content of trade (2-41) shows that when $\frac{K^H}{L^H} > \frac{K^W}{L^W}$, then[2] $F_K^H < 0$ and $F_L^H > 0$. It just tells the Heckscher-Ohlin theorem.

The trade volume (2-17) and (2-18) are full of economic logic. Helpman and Krugman (1985, p.25) summarized it as "This result is intuitively appealing for a model in which differences in factor composition are the sole basic for trade". Equation (2-17) specified the equal trade volume line that is orthogonal to anti-diagonal line, which is just wage-rental line $\frac{w^*}{r^*}$, most of studies refer it as the trade direction of factor content.

2.4 The ultimate proof of the general trade equilibrium

The Dixit-Norman constant φ should be 1 and it can only be one. Otherwise, it will cause a self-confliction inside the Hechsher-Ohlin model. We see now what will happen if $\varphi \neq 1$. Figure 2 presents the assumption that $\varphi > 1$, i.e.,

$$\frac{w^*}{r^*} > \frac{K^W}{L^W} \tag{2-43}$$

[2] I set negative sign as export.

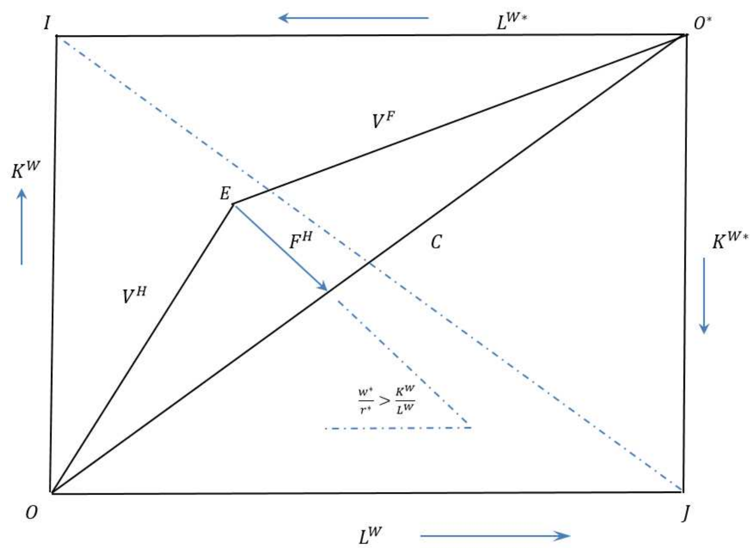

FIGURE 2
IWE Diagram, First country: Home, First factor: Capital

Figure 2 also shows country H being factor abundant

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{2-44}$$

By the original definition, the vector of world factor price and the vector of factor endowments as

$$W^* = \begin{bmatrix} r^* \\ w^* \end{bmatrix} \tag{2-45}$$

$$V^h = \begin{bmatrix} K^h \\ L^h \end{bmatrix} \qquad (h = H, F) \tag{2-46}$$

The equation (2-43) shows a logic pattern for relative factor prices as

$$\frac{the\ price\ \ of\ second\ factor\ (w^*\ )}{the\ price\ of\ first\ factor\ (\ r^*\ )} > \frac{the\ world\ factor\ endowment\ of\ the\ fisrt\ factor\ (K^W)}{the\ world\ factor\ endowments\ of\ the\ second\ factor\ \ (L^W)} \tag{2-47}$$

The first country in figure 2 is Home; the first factor is capital, which is the abundant factor of the first country. The statement of (2-47) should be always true.

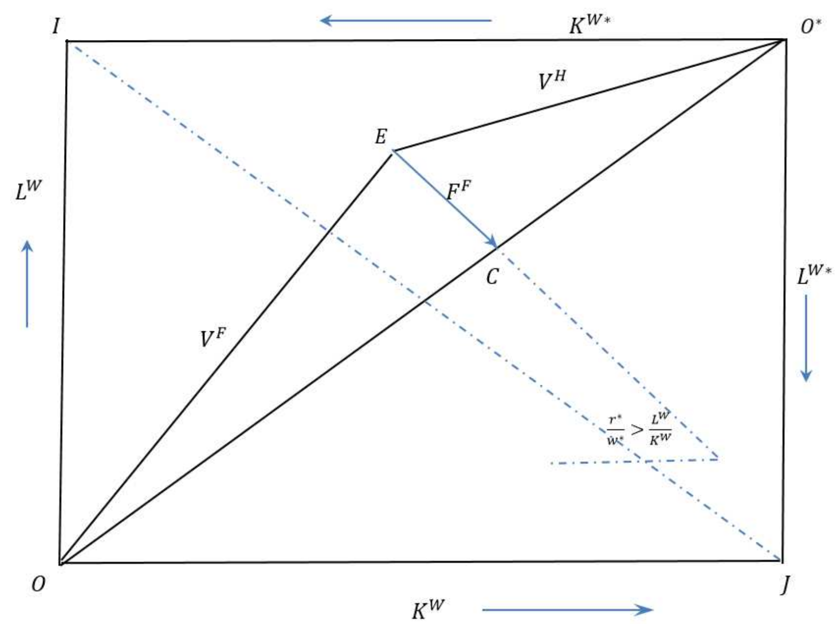

FIGURE 3
IWE Diagram: First Country: Foreign, First Factor: Labor

Figure 3 is difference from Figure 2. It stands for that country F is from origin O and country H is from origin O^*. It uses the horizontal axis for capital, and uses the vertical axis for labor. It uses

$$W^* = \begin{bmatrix} w^* \\ r^* \end{bmatrix} \tag{2-48}$$

$$V^h = \begin{bmatrix} L^h \\ K^h \end{bmatrix} \qquad (h = F, H) \tag{2-49}$$

It specifies that the first country is foreign; the first factor is labor. The foreign country is labor abundant as

$$\frac{L^F}{K^F} > \frac{L^H}{K^H} \tag{2-50}$$

By the logic pattern (2-47), the relative factor ratio for figure 3 is

$$\frac{the\ price\ of\ second\ factor\ (r^*)}{the\ price\ of\ first\ factor\ (w^*)} > \frac{the\ world\ factor\ endowment\ of\ the\ fisrt\ factor\ (L^W)}{the\ world\ factor\ endowments\ of\ the\ second\ factor\ \ (K^W)} \tag{2-51}$$

It implies

$$\frac{w^*}{r^*} < \frac{K^W}{L^W} \tag{2-52}$$

It conflicts with (2-43). Similarly, if assuming $\varphi < 1$, we also can see the conflict. The only available result is $\varphi = 1$. It is fixed within the model[3]. This is the third approach to proof the price-trade equilibrium.

The assumption of the identical homothetic taste is the source for this fixed solution. The two factors are equally important and no weighted priority in the model. The comparative advantage, the factor abundance, and the factor intensiveness are terms of relativeness, which are symmetrical somehow in the linear model.

3. OPTIMALITY PROPERTY OF THE GENERAL TRADE EQUILIBRIUM

[3] This argument made other analyses redundant. I do not like presenting it in my early version of my manuscript. However, it is one property of the Heckscher-Ohlin model. It provides the final argument to insistent the equilibrium solution of this paper.

Lionel McKenzie (1987, p.29) described the task of general equilibrium as

> "Walras set of major objectives of general equilibrium theory as they have remained ever since. First, it was necessary to prove in any model of general equilibrium that the equilibrium exists. Then its optimality properties should be demonstrated. Next, it should be shown how the equilibrium would be attained; that is, the stability of the equilibrium and its uniqueness should be studied. Finally, it should be shown how the equilibrium will change when conditions of demand, technology, or resources are varied."

What is the optimality property of the equilibrium above? This section shows that the trade volume reaches its maximum value at the equilibrium. It implies that both countries get their full benefits through free trade.

Triangle ΔEZC in figure 1 displays the trade flows of factor contents. The trade volume in country H is

$$VT = (\lambda_K - s^H)K^W r^* + (s^H - \lambda_L)L^W w^* \tag{3-1}$$

We assume, by (2-4),

$$r^* = (s^H - \lambda_L)L^W \tag{3-2}$$

It implies

$$w^* = (\lambda_K - s^H)K^W \tag{3-3}$$

Substituting them to (3-1) yields

$$VT = 2\left(\lambda_K - s^H\right)\left(s^H - \lambda_L\right)L^W K^W \tag{3-4}$$

It shows that VT is a quadratic function of s^H.

We introduce a utility function μ just as the trade volume,

$$\mu = 2(\lambda_K - s^H)(s^H - \lambda_L)L^W K^W \tag{3-5}$$

It reaches its maximum value as $\frac{1}{2}(\lambda_K - \lambda_L)L^W K^W$ when

$$s^H = \frac{1}{2}(\lambda_K + \lambda_L) = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{3-6}$$

Appendix A is the derivation of this solution. It confirms the equilibrium solution in the last subsection. It is another independent approach to reach equilibrium.

4. Autarky Price and Comparative Advantage

Leamer and Levinsohn (1995, p.1342) mentioned the importance of gains from trade as "Proofs of the static gains from trade fall into the unrefutable category yet these are some of the most important results in all of economics."

The general trade equilibrium above shows that world factor endowments determine world prices. We now apply it to evaluate the autarky prices of a country under an isolated market. The idea is that the autarky factor endowments decide its autarky prices. The IWE diagram itself supports this extension analytically. Consider the allocation of factor endowments, point *E,* in Figure 1. Assume that it moves closer to the origin O. The factor endowments of country H will shrink to exceedingly small; the factor endowments of country F will close to be world factor endowments. The autarky prices in country F are then world prices. Mathematically, when the allocation $V^H \to 0$, inside the IWE box, then $V^F \to V^W$ and the world relative factor price r^* will close to the relative autarky factor price of country H. We present the relative rental price as

$$r^* = \frac{L^W}{K^W} = \frac{L^H + L^F}{K^H + K^F} \tag{4-1}$$

Seeking the limit above yields

$$\lim_{\substack{L^H \to 0 \\ K^H \to 0}} \frac{L^H + L^F}{K^H + K^F} = \frac{L^F}{K^F} = r^{Fa} \tag{4-2}$$

At the same time, the world commodity prices will close to the autarky output prices of country F. We proved the autarky price measurement mathematically. Samuelson (1949) argued this idea. He mentioned that the autarky prices are the world prices if the country (or continent) is divided into two countries geographically (or artificially), supposing that all other things are unchanged. Now we know world prices; the calculation or the measurement of world prices can be used to calculate autarky prices.

We show another way to illustrate autarky prices.

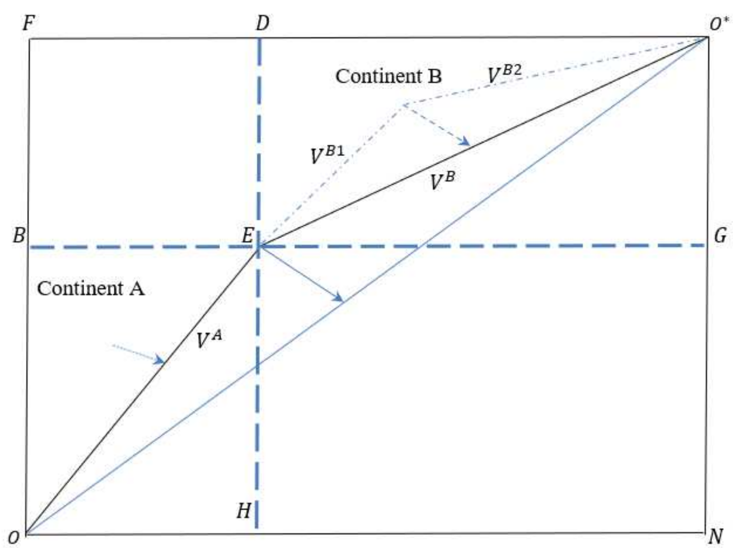

FIGURE 4
IWE Diagram of Two Continents Showing Autarky Prices

Suppose that there are two geographic continents: continent A and continent B, separated by an ocean. Continent A is a single country. Continent B is with two free-trade countries: B1 and B2. When transportation conditions are more available, two continents make free trade by no-cost shipping. We draw the scenario in figure 4. The rectangle $BEHO$ is the IWE diagram for continent A. The rectangle DO^*GE is the IWE diagram for continent B. The rectangle FO^*NO is the IWE diagram for the two-continent world. The continent prices for continent B can be decided with V^B by world prices (2-35) through (2-38), which can serve as the autarky price for continent B. The autarky prices of continent A can be decided by V^A too, even that it is a single country. We can figure out that a continent or a country's autarky prices by its factor endowments.

Helpman and Krugman (1985, p.16) proposed a clear-sighted conclusion about the factor price equalization (FPE) set in the IWE. They addressed "This FPE set is not empty because it always contains the diagonal OO^*. Since it is a convex symmetrical set around the diagonal, its boundaries defined the limits of dissimilarity in factor composition which is consistent with factor price equalization. Hence for sufficiently similar composition, there is a factor price equalization in the trading equilibrium". It normalized the FPE set. Without it, the nearby area to the diagonal line will not be valid for the FPE[4]. It can be used to derive autarky prices directly also.

Let us imagine an allocation of factor endowments, C, on the diagonal line OO^* in Figure 1. At this point, the factor compositions of the two countries are the same, and they equal to world factor composition as

$$\frac{L^H}{K^H} = \frac{L^F}{K^F} = \frac{L^W}{K^W} \tag{4-3}$$

At that moment, we know both countries' rental/wage ratios are the same. Otherwise, it will cause trade. It implies that the world rental/wage ratio equals the autarky rental/wage ratios of the two countries as

$$\frac{r^{aH}}{w^{aH}} = \frac{r^{aF}}{w^{aF}} = \frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{4-4}$$

where superscript ah writes down the autarky price of country h. At point C, the two countries' autarky prices are the same, and the autarky prices are world prices. We see that the logic of autarky prices formation is the same as world prices formation.

Based on the above discussion, we present the autarky prices of two countries as

$$r^{ah} = L^h \qquad (h = H, F) \tag{4-5}$$

$$w^{ah} = K^h \qquad (h = H, F) \tag{4-6}$$

$$p_1^{ah} = a_{k1}L^h \quad + a_{L1}K^h \qquad (h = H, F) \tag{4-7}$$

$$p_2^{ah} = a_{k2}L^h \ + a_{L2}K^h \qquad (h = H, F) \tag{4-8}$$

The gains from trade are measured by

$$W^{ah\prime}F^h > 0 \qquad (h = H, F) \tag{4-9}$$

$$P^{ah\prime}T^h > 0 \qquad (h = H, F) \tag{4-10}$$

We express the gains from trade for the home country as

$$(W^{ah})^{\prime} F^H > 0 \tag{4-11}$$

Write W^{aH} as

$$W^{ah} = \begin{bmatrix} L^H \\ K^H \end{bmatrix} \tag{4-12}$$

Substituting it into (4-11) yields,

$$[L^H \quad K^H] \begin{bmatrix} -\frac{1}{2}\frac{K^H L^W - K^W L^H}{L^W} \\ \frac{1}{2}\frac{K^H L^W - K^W L^H}{K^W} \end{bmatrix} > 0 \tag{4-13}$$

It can be rewritten to

$$\left(-\frac{L^H}{L^W} + \frac{K^H}{K^W}\right) \times \frac{1}{2}\left(K^H L^W - K^W L^H\right) > 0 \tag{4-14}$$

Or

[4] Mathematically, it makes sure that whole FPE set is on a plane. Otherwise, the FPE will be with a hole even a ditch along the diagonal line.

$$\frac{(K^H L^{hW} - K^{hW} L^H)^2}{2K^{hW} L^{hW}} > 0 \qquad (4\text{-}15)$$

As we assumed that country H being capital abundant, the above is valid.
Similarly, exercising gain from trade for country F yields

$$\frac{(K^F\ L^{hW} - K^{hW} L^F\ )^2}{2K^{hW} L^{hW}} > 0 \qquad (4\text{-}16)$$

It implies that the world prices at the equilibrium ensure the gains from trade for both countries. The gains from trade in quantitating are the same for the two countries[5]. It reflects that comparative advantage is "absolutely" relative between countries.

Corollary – Gain from trade
Free trade distributes gains from trade evenly to both trade partners.

This corollary is from the law of comparative advantage. It shows a feature of the law of comparative advantage

5. General equilibrium of trade of two factors, two commodities, and multiple countries

In an $2 \times 2 \times 2$ system by Figure 1, country H and country F are trade partners with each other. In a multi-country system, who is the trade partner with whom? Leamer (1984, preface page xiii) addressed this issue as "This theorem, in its most general form, states that a country's trade relations with the rest of the world depend on its endowments of productive factors...". The designated trade in this study is a transaction of goods between a country and its partner, the rest of the world. The trade relations are quite simple by this specification. It just likes the scenario of the $2 \times 2 \times 2$ system from the view of analyses.

Figure 5 draws an IWE diagram for three countries. The dimension box is world factor endowments. The vector $V^h(L^h\ , K^h)$ is the factor endowments of country h, h=1, 2, and 3. The origin of country 1 is arranged to start at the left-bottom corner. The origin of the rest of the world is from the upper-right corner. The vector of factor endowments of country 1 is V^1; and the vector of factor endowments of the rest of the world is $V^2 + V^3$.

[5] Remember that $K^H L^{hW} - K^{hW} L^H$ in (4-15) is trade volume for country H, similarly $K^{hW} L^F\ - K^F L^{hW}$ in (4-16) is the trade volume for country F. They are same in quantity.

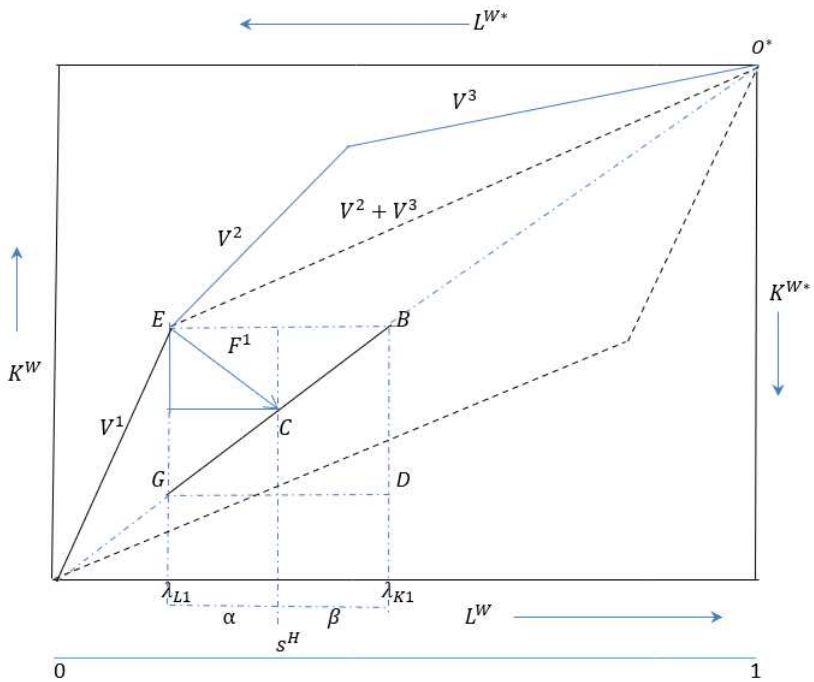

Figure 5 The Three-Country IWE Diagram

The system notation for the 2 x 2 x M model is as same as equations (2-1) and (2-2); the only difference is the country number. The country number now goes from 1 to M (In Figure 3, we present only three countries for illustration).

We now introduce two lists of parameters, which are the shares of factor endowments of country h to their world factor endowments, respectively as

$$0 \le \lambda_{Lh} \le 1, \quad 0 \le \lambda_{Kh} \le 1 \qquad (h = 1,2,\dots,M) \tag{5-1}$$

$$\sum_{h=1}^{M} \lambda_{Lh} = 1 \quad , \qquad \sum_{h=1}^{M} \lambda_{Kh} = 1 \tag{5-2}$$

The factor endowments of country h can be denoted as

$$L^h = \lambda_{Lh} L^W \qquad (h = 1,2,\dots,M) \tag{5-3}$$

$$K^h = \lambda_{Kh} K^W \qquad (h = 1,2,\dots,M) \tag{5-4}$$

The allocation of factor endowments of country 1 in Figure 5 is $E(\lambda_{L1}L^W, \lambda_{K1}K^W)$. It shows how country 1 trades with the rest of the world by factor endowments.

The factor contents of trade of country h are

$$F_K^h = s^h K^W - K^h = \left(\lambda_{Kh} - s^h\right) K^W \qquad (h = 1,2,\dots,M) \tag{5-5}$$

$$F_L^h = s^h L^W - L^h = \left(\lambda_{Lh} - s^h\right) L^W \qquad (h = 1,2,\dots,M) \tag{5-6}$$

The trade balance of factor contents for country h is

$$\frac{w^{*h}}{r^{*h}} = \frac{(s^h - \lambda_{Kh}) K^W}{(\lambda_{Kh} - s^h) L^W} \qquad (h = 1,2,\dots,M) \tag{5-7}$$

where r^{*h} is the equalized rental in country h, w^{*h} is the equalized wage in country h. Equation (5-7) displays the trade balance between country h and the rest of the world. Extending the result of the Dixit-Norman constant as 1 in the last section to the equation above, we have

$$\frac{(s^h - \lambda_{Kh})}{(\lambda_{Lh} - s^h)} = 1 \qquad (h = 1,2,\dots,M) \tag{5-8}$$

$$\frac{w^{*h}}{r^{*h}} = \frac{K^W}{L^W} \qquad (h = 1,2, \dots, M) \qquad (5\text{-}9)$$

This means that the relative factor price (rental-wage ratio) is the same for all countries.

$$\frac{w^{*h}}{r^{*h}} = \frac{K^W}{L^W} = \frac{w^*}{r^*} \qquad (5\text{-}10)$$

By assuming $w^* = L^W$ to drop one market-clearing condition by Walras's equilibrium, we obtain

$$s^h = \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} \qquad (h = 1,2, \dots, M) \qquad (5\text{-}11)$$

$$r^* = K^W \qquad (5\text{-}12)$$

$$w^* = L^W \qquad (5\text{-}13)$$

$$p_1^* = a_{k1} L^W + a_{L1} K^W \qquad (5\text{-}14)$$

$$p_2^* = a_{k2} L^W + a_{L2} K^W \qquad (5\text{-}15)$$

$$F_K^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W} \qquad (h = 1,2, \dots, M) \qquad (5\text{-}16)$$

$$F_L^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W} \qquad (h = 1,2, \dots, M) \qquad (5\text{-}17)$$

$$x_1^h = \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_1^W - x_1^h \qquad (h = 1,2, \dots, M) \qquad (5\text{-}18)$$

$$x_2^h = \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_2^W - x_1^h \qquad (h = 1,2, \dots, M) \qquad (5\text{-}19)$$

We see that

$$\sum_{h=1}^{H} s^h = \sum_{h=1}^{H} \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} = 1 \qquad (5\text{-}20)$$

Those are the equilibrium solution for the $2 \times 2 \times M$ model. We can demonstrate that all countries participating in trade gain from trade. It showed that world factor endowments determine world prices in the multi-country economy.

6. More Discussions of The General Trade Equilibrium

6.1 The chain of Inequalities and equalities of the Heckscher-Ohlin theory

At the equilibrium, the ratio of factor content of trade of a country equals its factor consumption ratio. We provide a chain to illustrate the Heckscher-Ohlin propositions,

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} = \frac{w^{Ha}}{r^{Ha}} >= -\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} = \frac{w^*}{r^*} = -\frac{F_K^H}{F_L^H} = \frac{K^H - F_K^H}{L^H - F_L^H} > \frac{K^F}{L^F} = \frac{w^{Fa}}{r^{Fa}} > \frac{a_{K2}}{a_{L2}} \qquad (6\text{-}1)$$

It presents the Heckscher-Ohlin theorem, the Leamer theorem, the factor price equalization theorem, the Dixit and Norman IWE prices, Helpman and Krugman trade volume, autarky prices, comparative advantages, and factor diversification cones, together. It is a comprehensive way to see the price-trade equilibrium from the one view of different views[6].

[6] The equilibrium solution of this paper is consistent with Lerner (1952)'s idea that revenue equals cost in each industry. However, it is not consistent with the isocost line. Appendix B show the detail about it.

The general trade equilibrium is a Pareto optimal solution since the trade box shows how social trade-off played. It is a balanced trade that the share of GNP of a country equals its share in world income. The equilibrium illustrates how free trade redistributes benefits into each country.

CONCLUSION

International trades promote world development. Trade creates value. This paper shows that free trade distributes trade gains evenly to trading partners when two countries have same technologies[7].

The paper presents the general trade equilibrium, and the world price structures of the Heckscher-Ohlin model. The equilibrium is consistent with Dixit and Norman's conclusion of the FPE set. The optimality of the solution is that the trade volume gets its maximum value at the equilibrium.

Dixit (2010) mentioned, "The Stolper-Samuelson and factor price equalization papers did not actually produce the Heckscher-Ohlin theorem, namely the prediction that the pattern of trade will correspond to relative factor abundance, although the idea was implicit there. As Jones (1983, 89) says, 'it was left to the next generation to explore this 2×2 model in more detail for the effect of differences in factor endowments and growth in endowments on trade and production patterns.' That, plus the Rybczynski theorem which arose independently, completed the famous four theorems." The equalized factor price at the equilibrium of this study presented the Heckscher-Ohlin theorem.

The study illustrates that world factor endowments, which equals to world factor consumption, decide world prices. Its first application is to find the measurement of autarky prices: the autarky factor endowments determine autarky prices. The autarky price is useful to show comparative advantages.

The Rybczynski trade effect and the Stolper-Samuelson trade effect are partial equilibrium analyses. The equilibrium solution provides a facility to do full trade effect analyses.

Trefler (1993) mentioned that the factor price equalization hypothesis and the HOV theorem hold in his equivalent-productivities system. Fisher (2011) also mentioned that factor price equalization and H-O theorem hold in the virtual endowment system. The structure of equalized factor prices provides the theoretical basis for further analyses of factor price none-equalization when countries have different productivities.

Appendix A

For the function

$$\mu = 2(s^H - \lambda_L)(\lambda_K - s^H)K^W L^W \tag{A-1}$$

to find its maximum or minimum value, we take differential of (A-1) with respective to s^H yields

[7] For trade among multiple countries, the trading partner is defined in section 5.

$$\frac{du}{ds^H} = 2\left(-2s^H + (\lambda_K + \lambda_L)\right)K^W L^W \quad \text{(A-2)}$$

Let it equal to 0, we get $s^H = \frac{1}{2}(\lambda_K + \lambda_L)$.

Take the second differential of (A-2) with respective to s^H yields

$$\frac{d}{ds^H}\left(\frac{du}{ds^H}\right) = -4K^W L^W \quad \text{(A-3)}$$

It is less than 0. By the secondary condition, μ is with its maximum value at $s^H = \frac{1}{2}(\lambda_K + \lambda_L)$.

Appendix B - Comparing with Lerner's Solution of Equalized Factor Prices

The equilibrium solution of this paper is consistent with Lerner (1952)'s idea that revenue equals cost in each industry.

We present the costs and revenues of industries in each country as

$$r^* K_1^h + w^* L_1^h = x_1^h p_1^* \qquad (h = H, F) \quad \text{(B-2)}$$

$$r^* K_2^h + w^* L_2^h = x_2^h p_2^* \qquad (h = H, F) \quad \text{(B-3)}$$

We can also express the cost and revenue of each industry for the world as

$$r^* K_1^W + w^* L_1^W = x_1^W p_1^* \quad \text{(B-4)}$$

$$r^* K_2^W + w^* L_2^W = x_2^W p_2^* \quad \text{(B-5)}$$

However, the equalized factor prices of this paper are not consistent with the isocost line analytically. Lerner isocost lines, technically, used the market clearing conditions twice by assuming

$$x_1 \ p_1^* = 1 \quad \text{(B-6)}$$

$$x_2 \ p_2^* = 1 \quad \text{(B-7)}$$

It means

$$x_1 \ p_1^* = x_2 \ p_2^* \quad \text{(B-8)}$$

However, no matter x_i is an individual country's output or the world output, this relationship does not hold. It conflicts with the commodity trade balance as

$$\frac{p_1^*}{p_2^*} = -\frac{x_1^H - s^H x_1^W}{x_2^H - s^H x_2^W} \quad \text{(B-9)}$$

Equations (B-6) and (B-7) implies that

$$w^* L_1 + r^* K_1 = 1 \quad \text{(B-10)}$$

$$w^* L_2 + r^* K_2 = 1 \quad \text{(B-11)}$$

Equations above are determinant since it is with two equations and two variables w^* and r^*. Its solution is not consistent with the trade balance of factor content (2-13) neither. Illustrating the factor price equalization geometrically by isocost line is insight in its original idea, which had taken important contribution on factor price equalization.

Reference

Deardorff, Alan V. (1982), General Validity of the Heckscher-Ohlin Theorem, The American Economic Review, Vol. 72, No. 4 (Sep. 1982), 683-694.

Deardorff, Alan V. (1994), The possibility of factor price equalization revisited, *Journal of International Economics,* XXXVI, 167-75.

Dixit, Avinish K. and Victor Norman (1980), *Theory of International Trade,* James Nisbet, Welwyn, and Cambridge University Press.

Dixit, A.K (2012) Paul Samuelson's Legacy, Annual Reviews of Economics, Vol. 4, 2012

Helpman, Elhanan (1984), The Factor Content of Foreign *Trade, Economic Journal,* XCIV, 84-94.

Helpman, Elhanan and Paul R. Krugman (1985), *Market Structure and Foreign Trade,* Cambridge, MIT Press.

Fisher, Eric E'O (2011), Heckscher-Ohlin Theory When Countries Have Different Technologies, International Review of Economics & Finance, Volume 20, Issue 2, April 1, 2011, pages 202-210.

Jones, Ronald (1965), The Structure of Simple General Equilibrium Models, Journal of Political Economy 73, 557-572.

Jones, RW. (1983). "International Trade Theory." In eds. EC Brown and RM Solow (1983), 69-103.

Leamer, Edward E. and James Levinsohn (1995), International trade theory: evidence, in Gene M Grossman and Kenneth S. Rogoff(eds), Handbook of international economics, vol. III. New York: Elsevier Science.

Lerner, Abba (1952), Factor Price and International Trade, Economica, n.s.19:1-15,

McKenzie, Lionel (1955), Equality of factor prices in world trade, Econometrica 23, 239-257.

McKenzie, Lionel (1987), General Equilibrium, The New Palgrave, A dictionary of economics, 1987, v. 2, pp 498-512.

Samuelson, Paul (1948), International trade and the equalization of factor prices. Economic Journal 58, 163–84.

Samuelson, Paul (1949), International factor-price equalization once again. Economic Journal 59, 181–97.

Samuelson, Paul (1953), Prices of factors and goods in general equilibrium. Review of Economic Studies 21, 1–20.

Samuelson, Paul (1971) Ohlin was right. Swedish Journal of Economics 73, 365–84.

Samuelson, Paul (1992), Factor-price equalization by trade in joint and non-joint production. Review of International Economics, v. 1, 1-9.

Vanek, Jaroslav (1968), The Factor Proportions Theory: The N-Factor Case, *Kyklos,* 21(23), 749-756.

Woodland, Alan (2013), General Equilibrium Trade Theory, Chapter 3, in Bernhofen, D., Falvey, R., Greenaway, D. and Kreickemeier U. (Eds.), Palgrave *Handbook of International Trade,* Palgrave Macmillan.