Using a two-level dynamic PDE model to synchronize the performance of technological equipment of a production line

Pihnastyi, Oleh and Ivanovska, Olha

National Technical University "Kharkiv Polytechnic Institute", Ukraine, National Technical University "Kharkiv Polytechnic Institute", Ukraine

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doctor of technical sciences Pihnastyi O.M., 0000-0002-5424-9843
National Technical University "Kharkiv Polytechnic Institute", Ukraine
pihnastyi@gmail.com
Ph.D Ivanovska O.V., 0000-0003-1530-259X
National Aerospace University "Kharkiv Aviation Institute", Ukraine
o.ivanovska@khai.edu

Annotation. The problem of designing a system for optimal operational control of random deviations in the productivity of technological equipment is considered. To construct an algorithm for optimal control of the performance of technological equipment, a PDE-model of a production flow line was used. The Lyapunov functions method was used to determine the value of the optimal control of the technological equipment performance. The production line is considered by a distributed dynamic system. Equations are written for the flow parameters of the production line in small perturbations. A criterion for the quality control of the performance of technological equipment of a production flow line has been introduced. Based on this criterion, the Lyapunov function is determined for the considered production system. Taking into account the form of the Lyapunov function, control actions are obtained that correct the performance of the technological equipment. The introduced control made it possible to ensure the synchronization of the productivity of the technological equipment of the production line and the asymptotic stability of the given planned state of the flow parameters of the production line for the steady and transient operation mode.
Keywords: production line, production control system, PDE-model, continuous production, synchronized production line, work in progress, distributed production system.

The problem formulation

The solution to the problem of synchronizing the performance of sequentially located equipment of the production line is directly related to the solution of the problem of stabilizing the parameters of the production line [1]. Solving the problem of stabilizing the parameters of the production line allows you to determine the control actions that ensure the asymptotic stability of a given planned state of the flow parameters [2]. The applied problems of stabilization of the parameters of the production line, along with the requirements for the asymptotic stability of the unperturbed state, contain wishes for the best quality of the transition process from the disturbed to the unperturbed state of the parameters at $t \to \infty$ [3].

For the problem of stabilization of the flow parameters of the production flow line [4], as well as for the general problem of stability, the research theory can be developed according to the first approximation [5] and conditions are obtained under which the problem of stabilization of the values of the production flow line parameters has a unique solution. The use of a two-level dynamic PDE-model [6] for stabilizing the performance of technological equipment for operations of a production line allows finding a solution to the problem of stabilizing the flow parameters of a production line, which provides asymptotic stability. The resulting control actions on the arising deviations of the flow parameters ensure the asymptotic stability of the steady-state of the flow line parameters provided that the control functions are defined and continuous in the region under consideration, are not constrained by any additional constraints.

The research purpose

The presence and relevance of the problems discussed above, associated with ensuring the continuity of production and synchronizing the productivity of individual technological operations of the production line, determined the purpose of the study. To construct optimal controls that ensure synchronization of the rhythm of technological operations of the production line, the class of PDE-models of the production line was used [6].

The research methods

To describe the state of the production flow line in this study, two flow parameters are used: the value of inter-operational backlogs before the technological operation $[\chi]_0(t,S)$; the rate of processing of parts in a technological operation $[\chi]_1(t,S)$ [7]. The coordinate S determines the technological position of the detail in the technological route production line at the time t [8].

The problem of synchronizing the technological equipment of the production line is the main task, the solution of which makes it possible to ensure the continuous production of products. Desynchronization of the rhythm of processing details in different technological operations leads to a stop in the production process. To ensure the continuous nature of production, it is necessary to increase the size of inter-operational backlogs, which ultimately leads to an increase in the total volume of work in progress. Another way to ensure synchronization of technological operations is to control the productivity of technological operations [9]. This is achieved directly by changing the mode of processing parts or using backup equipment [10]. Synchronization of the processing time of details in technological operations is necessary both for steady-state modes and for transient modes when it is necessary to perform the transition of the production system from one state to another state [11]. Such a transition is associated with an increase in production volumes, and, accordingly, the productivity of the production line. At the same time, for such a transition, the requirements are put forward that the transition should be carried out in the shortest possible time with a minimum deviation of the flow parameters from the required standard values [7].

Desynchronization of the performance of technological equipment can occur due to fluctuations in the flow parameters associated with the processing of details [9]. These fluctuations lead to the emergence of perturbations of the flow parameters, which can act both instantaneously and continuously. This means that the equations for determining the state of the flow parameters of the technological process differ from the true ones by some small correction terms. The state of the parameters of the production system will be described by the system of equations [2]:

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=0\,, \qquad (1)$$

$$\frac{\partial[\chi]_n(t,S)}{\partial t}+\frac{\partial[\chi]_{n+1}(t,S)}{\partial S}=nf(t,S)[\chi]_{n-1}(t,S)\,, \qquad n=1,2.3..... \qquad (2)$$

Let us supplement the system of equations (1), (2) with control functions $\Upsilon_n(t,S)$:

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=\Upsilon_0(t,S)\,, \qquad (3)$$

$$\frac{\partial[\chi]_n(t,S)}{\partial t}+\frac{\partial[\chi]_{n+1}(t,S)}{\partial S}-nf(t,S)[\chi]_{n-1}(t,S)=\Upsilon_n(t,S)\,, \quad n=1,2.3.... \qquad (4)$$

Let the system of equations (3), (4) correspond to the unperturbed solution [8]:

$$[\chi]_n=[\chi]_n^{*}(t,S)\,,$$
$$\Upsilon_n(t,S)=\Upsilon_n^{*}(t,S)\,, \qquad (5)$$

and the flow parameters of the production line receive at the current moment in time previously unknown random small perturbations $[y]_n$

$$[y]_n=[\chi]_n-[\chi]_n^{*}\,. \qquad (6)$$

To eliminate the arisen disturbances that lead to desynchronization of the performance of technological equipment, control actions are required $u_m=u_m(t,[y]_n)$. The linearized system of equations for small perturbations $[y]_n$ has the form:

$$\frac{\partial[y]_0}{\partial t}+\frac{\partial[y]_1}{\partial S}=\sum_{m=0}^{N_k}q_{0m}u_m\,, \qquad n=0,1,..N_n\,, \qquad (7)$$

$$\frac{\partial[y]_n}{\partial t}+\frac{\partial[y]_{n+1}}{\partial S}=nf(t,S)\big|_0[y]_{n-1}+$$

$$+\sum_{m=0}^{N_n}\left(n[\chi]_{n-1}(t,S)\frac{\partial f(t,S)}{\partial[\chi]_m(t,S)}\right)\Bigg|_0[y]_m+\sum_{m=0}^{N_n}q_{nm}u_m\ , \quad (8)$$

$$\sum_{m=0}^{N_n}q_{nm}u_m=Y_n(t,S)-Y_n^*(t,S)\ ,$$

$$\sum_{m=0}^{N_n}q_{nm}u_m<<Y_n^*(t,S)\ .$$

The system of equations will be used to construct optimal control over the performance of technological equipment, which will ensure synchronized operation of technological equipment on the production line. It is assumed that the dispatch service measures the current values of the flow parameters $[\chi]_0$, $[\chi]_1$. Based on this measurement, it generates a control $u_m=u_m(t,[y]_n)$, that affects the rate of processing of details and serves to ensure synchronization of the productivity of technological equipment. The functions $u_m=u_m(t,[y]_n)$ are assumed to satisfy the equalities

$$u_m(t,0)=0\ , \quad (9)$$

are defined and continuous in the considered scope, and are also not constrained by any additional inequalities.

Main results

The system of two-moment balance ratios (7), (8) for modeling the stabilization of deviations in technological equipment performance can be represented as

$$\frac{\partial[y]_0}{\partial t}+\frac{\partial[y]_1}{\partial S}=q_{01}u_1\ , \quad (10)$$

$$\frac{\partial[y]_1}{\partial t}+\frac{\partial[y]_1}{\partial S}B+[y]_1\cdot\frac{\partial B}{\partial S}=q_{11}\cdot u_1\ , \quad (11)$$

$$q_{01}=const_1\ , \qquad q_{11}=const_2\ ,$$

$$u_1(t,0)=0\ , \quad (12)$$

$$B=\frac{[\chi]_{1\psi}}{[\chi]_0}\bigg|_0\ . \quad (13)$$

It is believed that when the rprocesing tempo of details deviates $[\chi]_1$ from its undisturbed state $[\chi]_1^*(S)$ it is possible to compensate for the deviation by internal external elements by changing the performance of the equipment [12]. The term $q_{01}u_1$ n equation (10) determines the source of compensation for deviations of the backlog of parts due to changes in equipment productivity, maintains the value of inter-operational backlogs at the required level. Similarly, the term $q_{11}u_1$ determines the source of compensation for the deviation of the rate of processing parts $[\chi]_1$ from the planned value $[\chi]_1^*(S)$. Internal sources of compensation are determined by the following factors:

a) an increase in the coefficient of parallelism of operations;
b) the use of more productive workers;
c) the use of backup equipment;
d) overtime work on operations for which there is a backlog.

Let's choose the quality criterion of the transient process from the conditions that determine the minimum cost of technological resources that are required for solving the pointed problem

$$I = \int_{t_0}^{\infty} \frac{1}{S_d} \int_0^{S_d} \left(\alpha u_1{}^2\right) dS dt \,. \tag{14}$$

where the parameter α is the scale factor. The optimal control $u_1(t,S)$ will be searched in the form

$$u_1(t,S) = \theta_0 [y]_0 + \theta_1 [y]_1 \,, \tag{15}$$

The coefficients θ_0, θ_1 are unknown and to be determined. When calculating the control action $u_1(t,S)$ or the deviation of the tempo value $[y]_1$ from the undisturbed state $[\chi]_1^*(t,S)$ let's consider the balance equations of the model of the production process:

$$\frac{\partial \{y_0\}_0}{\partial t} = q_{01} \cdot \{u_1\}_0 \,, \tag{16}$$

$$\frac{\partial \{y_1\}_0}{\partial t} + \{y_1\}_0 \left.\frac{\partial B}{\partial S}\right|_0 = q_{11} \{u_1\}_0 \,, \tag{17}$$

$$u_1(t,0) = 0 \,. \tag{18}$$

The Lyapunov function $V^0\left(t,[y]_n\right)$ will be sought in the quadratic form in the quadratic form with coefficients constant in time c_0, c_1:

$$V^0([y]_n) = \frac{1}{S_d}\int_0^{S_d}\left(c_0[y]_0^{\,2} + c_1[y]_1^{\,2}\right)dS = c_0\{y_0\}_0^{\,2} + c_1\{y_1\}_0^{\,2}, \quad (19)$$

$$\frac{\partial V^0}{\partial t} = 0, \quad (20)$$

$$[y]_k = \{y_k\}_0 + \sum_{j=1}^{\infty}\{y_k\}_j \sin[k_j S] + \sum_{j=1}^{\infty}[y_k]_j \cos[k_j S], \quad k_j = \frac{2\pi j}{S_d},$$

$$[u]_n = \{u_n\}_0 + \sum_{j=1}^{\infty}\{u_n\}_j \cdot \sin[k_j S] + \sum_{j=1}^{\infty}[u_n]_j \cdot \cos[k_j S].$$

Let us obtain sufficient conditions for the solvability of the problem of synchronizing the productivity of the technological equipment of the production line and introduce the notation:

$$Q = \begin{vmatrix} q_{01} \\ q_{11} \end{vmatrix}, \quad (21)$$

$$P = \begin{vmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & -\left\{\dfrac{\partial B}{\partial S}\right\}_0 \end{vmatrix}, \quad (22)$$

taking into account which we consider the matrix

$$W = \begin{vmatrix} q_{01} & 0 \\ q_{11} & -\left.\dfrac{\partial B}{\partial S}\right|_0 q_{11} \end{vmatrix} = q_{11}\begin{vmatrix} q_{01} & 0 \\ 1 & -\left.\dfrac{\partial B}{\partial S}\right|_0 \end{vmatrix}. \quad (23)$$

The rank of the matrix $W = \{Q, PQ\}$ is equal to the rank of the system of balance equations in small perturbations. Consequently, system (16) - (18) is solvable relative to the optimal control functions of the production line parameters. Let us compose an expression $B[V^0]$ for the system of balance equations in small perturbations (10)

$$B[V^0] = \frac{\partial V^0}{\partial\{y_0\}_0}\frac{d\{y_0\}_0}{dt} + \frac{\partial V^0}{\partial\{y_1\}_0}\frac{d\{y_1\}_0}{dt} + \omega =$$

$$= 2c_0\{y_0\}_0 q_{01}\{u_1\}_0 + 2c_1\{y_1\}_0\left(q_{11}\{u_1\}_0 - \{y_1\}_0\left.\frac{\partial B}{\partial S}\right|_0\right) +$$

$$+ \alpha \cdot \{u_1\}_0^{\,2} = 0. \quad (24)$$

Differentiating $B[V^0]$ with respect to $\{u_1\}_0$, and equating the results to zero, we obtain the missing equation for determining the form of the Lyapunov function $V^0(t,[y]_n)$ and the optimal control action $\{u_1\}_0$:

$$\frac{\partial B[V^0,t]}{\partial\{u_1\}_0} = 2c_0\{y_0\}_0 q_{01} + 2c_1\{y_1\}_0 q_{11} + 2\alpha\{u_1\} = 0\,. \tag{25}$$

Let solve equation (25) with respect to the control action $\{u_1\}_0$

$$\{u_1\}_0 = -\left(\frac{c_0 q_{01}}{\alpha}\{y_0\}_0 + \frac{c_1 q_{11}}{\alpha}\{y_1\}_0\right). \tag{26}$$

Taking into account equality (26), expression (24) has the form

$$B[V^0] = 2c_1\{y_1\}_0\left(-\{y_1\}_0\left\{\frac{\partial B}{\partial S}\right\}_0\right) + \alpha\left(\frac{c_0 q_{01}}{\alpha}\{y_0\}_0 + \frac{c_1 q_{11}}{\alpha}\{y_1\}_0\right)^2 = 0\,. \tag{27}$$

We equate the factors at the products $\{y_1\}_0\{y_1\}_0$, $\{y_0\}_0\{y_0\}_0$, $\{y_0\}_0\{y_1\}_0$ to zero and determine the coefficients c_0, c_1:

$$c_0 = 0\,, \quad c_1 = \frac{2\alpha}{q_{11}^2}\left.\frac{\partial B}{\partial S}\right|_0. \tag{28}$$

Quadratic form (20) will definitely be positive if the coefficient c_1 is positive

$$c_1 = \frac{2\alpha}{q_{11}^2}\left.\frac{\partial B}{\partial S}\right|_0 > 0\,, \tag{29}$$

which gives the conditions, the fulfillment of which will provide asymptotic stability for the parameters of the production flow line

$$\left.\frac{\partial B}{\partial S}\right|_0 > 0\,, \qquad B = \left.\frac{[\chi]_{1\psi}}{[\chi]_0}\right|_0. \tag{30}$$

The existence of optimal control of small perturbations $[y]_n$ of flow parameters $[\chi]_n$ with the quality of operational control of the production process, expressed by the integral (15), is possible if condition (30) is met. In this case, the control action is determined as follows

$$\{u_1\}_0 = -\frac{2}{q_{11}} \frac{\partial B}{\partial S}\bigg|_0 \{y_1\}_0 . \tag{31}$$

Substituting the obtained expression (31) for the optimal control $\{u_1\}_0$ into equation (17), an equation is obtained in the following form

$$\frac{\partial \{y_1\}_0}{\partial t} = -3 \frac{\partial B}{\partial S}\bigg|_0 \{y_1\}_0 . \tag{32}$$

So, in a case arising of the deviations of the technological equipment performance, the optimal control will ensure reconstruction of the mode of the synchronization. The deviation in productivity $\{y_1\}_0$ will be eliminated due to condition (30). In this case, the control $q_{11}\{u_1\}_0$ is opposite in sign to the arising deviation $\{y_1\}_0$. The value of the period of damping of the disturbance is proportional to the value of the derivative of the function $\frac{\partial B}{\partial S}\bigg|_0$.

Conclusions and research prospects

This article proposes a method for constructing optimal controls for the performance of technological equipment to ensure synchronization of the flow of details along the technological route of the production line.

The applied problem of the operational control of the parameters of the production line is considered. Optimal control of the performance of technological equipment is obtained, which, along with the requirements for the asymptotic stability of a given planned unperturbed state, allows ensuring the best quality of the transient process. Requirements for the least expenditure of technological resources spent on the formation of control actions are expressed in the form of a condition for the minimum of the quality integral of the production system. Conditions are obtained under which the problem of optimal control of the parameters of a production line has a unique solution.

The prospect for further research is the development of algorithms for combined control of the productivity of technological equipment and interoperation backlogs of the production line for transient and stationary modes of operation.

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