Allocation of time in ideal family: golden ratio as a means of survival in preindustrial societies and its applications in modern family

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Allocation of Time in Ideal Family: golden ratio as a means of survival in preindustrial societies and its applications in modern family.

The paper analyzes the model of the preindustrial family where the hunter and the housewife share the quarry and leisure. The model discovers multiple equilibria in marriage markets, where mating of unlikes results in unequal allocation of leisure time, while mating of likes equalizes leisure time of spouses, but the allocation of homework time stays unfair for both inferior and superior partners. There is a unique equilibrium solution when the hunter fairly supplies both leisure and consumption in exchange for housewife's attractiveness and home productivity. The proportions of the allocation of time in this ideal family match with the properties of golden ratio. However, golden ratio leaves for spouses only six hours and a half for common leisure. This result corresponds to field studies of natural sleep in African and Latin American preindustrial societies and to the historical analysis of sleeping habits before the industrial revolution in Europe, when people went to sleep after sunset and awakened before sunrise, breaking the sleep at midnight for household activities, praying, and conceiving. The correspondence between the model and results of applied and historical studies provides a basis for the hypothesis, that in preindustrial societies the family was a means of survival, and leisure was limited by the vital need in

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sleeping time. The need in six hours and a half is also confirmed by actual statistics of sleeping time in France. In general, the model of ideal family challenges modern trends in allocation of time, but its analytics discovers the difference between economic viability and feasibility, when the mating of likes gets an additional time with respect to limits of working hours and raises the total leisure time to current leisure habits of working spouses.

Key words: golden ratio, ideal family, marriage markets, mating of likes, gravitation

JEL Classification: D11,D13,D82,D83.

Introduction

The analysis of consumption-leisure choice on commodity market where the farmer allocates his working time between production and delivery, while the consumer optimizes his labor-search-leisure trade-off, resulted in the proof of the invisible hand. The uniformed farmer delivers goods to the meeting point on his production possibility frontier where the uninformed consumer stops the search. Pursuing his own interests, the producer unintentionally optimizes customer's consumption-leisure choice (Malakhov 2021a,b).

The exchange in marriage markets between hunters and housewives, where unequal partner's attractiveness or the mating of unlikes results in local equilibria, while the equal partner's attractiveness or the mating of likes comes to the general equilibrium and separates happy families from unhappy ones, described by corner solutions (Malakhov 2021c). The presentation of mutual attractiveness like the gravitation between men and women confirms the common belief that wealthy men and beautiful women really attract each other due to strong gravitational fields of both parts in transaction but their alliance represents the corner solution and fails due to disproportional allocation of time.

The general idea of the hunter-housewife model is to analyze the exchange of time between both parts in transaction for the given quantity of quarry. However, the analysis of the workings of invisible hand in marriage markets is largely based on imputed values of prices, costs, and wages, which reduce the reliability of the analysis of allocation of time in the family. This paper successfully escapes from imputed values and discovers some properties of the equilibrium mating of both likes and unlikes that haven't been observed before. Even equilibrium solutions can hide features of unfair trade of time. And the only one mating of likes results in the fair trade and therefore can be regarded as the perfect equilibrium. This is the model of ideal family. But this model also has some exceptional properties.

The paper is organized as follows:

Part I presents the basic model of exchange between hunter and housewife and its specific properties with respect to the exchange on commodity markets.

Part II develops the hypothesis of the gravitation or the mutual attractiveness between men and women and describes its effects in the allocation of time.

Part III exhibits multiple equilibria of exchange in marriage markets where mating of both likes and unlikes should be questioned, because the exchange of time stays unfair.

Parts IV and V present the model of perfect equilibrium and formulate the hypothesis that ideal family was a means of survival in preindustrial societies.

Part VI demonstrates the implicit role of the ideal family in modern trends of allocation of time.

Part I. Basic model of exchange in marriage markets

The 'labor-search-leisure' model discovered the equilibrium solution for the exchange between uninformed producer and uninformed buyer, where the producer who is working at his production possibility frontier, unintentionally optimizes the customer's consumption-leisure choice. This solution results in model where the farmer allocates his time between production and delivery, and the customer allocates his time between labor, search, and leisure (Malakhov 2021a,b). This illustration of the workings of invisible hand on commodity market gave an idea to find the same mechanism in marriage markets. And that mechanism was described by the model of the hunter, who allocated his working time between hunting T_f and men's homework T_d, and the housewife, who allocated her time between women's homework L, a part of men's work S, and leisure H (Malakhov 2021c).

There are three important differences between the commodity market model and the marriage market model. On commodity market, producers and buyers have different time horizon. In the family, the hunter and the housewife share the quarry Q and leisure H, and they have the same time horizon T. Then, on commodity market the farmer is unaware of customer's wage rate, while in marriage market the hunter knows in advance the unit attractiveness of the bride w. He can directly get this information during the presentation, or indirectly, with the help of bride price – dowry tradeoff. We can get the historical illustration for that. The Babylonian Marriage Market, described by Herodotus almost five centuries BC, was organized as the bride price – dowry auction, which started with the sale of the most beautiful maiden and progressed to the least. Swains paid the bride price for attractive maidens and got dowry for unattractive ones.

This example gives an idea that some beauty with 'zero bride price – zero dowry' existed. This assumption results in the concept of the equilibrium unit attractiveness w_e. This is the first step to escape from imputed values, which dominated in the previous paper and made its conclusions less plausible. We don't know the real unit attractiveness, especially when it represents a set of many variables – income, beauty, diligence, etc. But the idea of the equilibrium unit attractiveness provides an efficient tool. While we cannot take negative values for unattractive women, we take the equilibrium unit attractiveness to be equal to one, or $w_e=1$. So, unattractive brides get $w<1$ values, while attractive brides get $w>1$ values.

Then, on commodity market, the consumers are paying the $QP=wL$ value under price dispersion with respect to their willingness to pay. But the equilibrium price is unique, because it equalizes the lowest willingness to pay of consumers with zero search costs and the willingness to accept of consumers with positive search costs. Other words, buying the quantity demanded at low price, the consumer cannot resell it at the price greater than the lowest willingness to pay at the zero search level.

This arbitrage doesn't work in marriage market, and both dowry and bride price prove that. As a customer, the woman 'buys' the quarry Q. But at the same time, she is 'selling' her total attractiveness $w(L+S)$. But this imputed value is not perfectly competitive. In the local marriage market, an unattractive woman cannot 'sell' her total attractiveness at the price greater than the price of an attractive woman. But the attractiveness of beautiful brides differs as well as their willingness to accept or to sell. The existence of equilibrium unit attractiveness w_e gives an idea that the equilibrium price P_e also exists. But it means that the willingness to accept of unattractive women is lower than the equilibrium price, while the willingness to accept of attractive brides is greater than the equilibrium price. As a result, we get some multiple equilibria solution, where for any marriage the price of the quarry is equal to the willingness to accept of a woman:

$$(1) \quad w(L+S) = QP$$

Eq.1 gives us the solution for the maximization of women's consumption-leisure utility $U(Q;H)$ with the help of the marginal rate of substitution of leisure for consumption:

$$(2) \quad MRS\ (H\ for\ Q) = -\frac{\partial Q}{\partial H} = \frac{MU_H}{MU_Q} = \frac{w}{P} = \frac{Q}{L+S}$$

The bride has her preferences, how much quarry and how much leisure she can get with respect to her unit attractiveness.[1] The hunter evaluates the unit attractiveness of the bride and makes the decision, how much quarry he should get with respect now to his attractiveness *v*, or his productivity on hunting, and how much time he should spend on the homework.

Making his choice, he cuts the time on hunting T_f in favor of homework T_d. And his best guess results in the housewife's optimal consumption-leisure choice, when the allocation of his time T_d/T_f is equal to her marginal rate of substitution of leisure for consumption:

$$(3) \quad \frac{T_d}{T_f} = \frac{Q}{L+S}$$

Eq.3 appears as the result of the formal logic. When we don't know real values, the confirmation of the optimal consumption-leisure choice can be done only with the help of the geometrical normal from the origin of time horizon of the family. And this normal provides the following result:

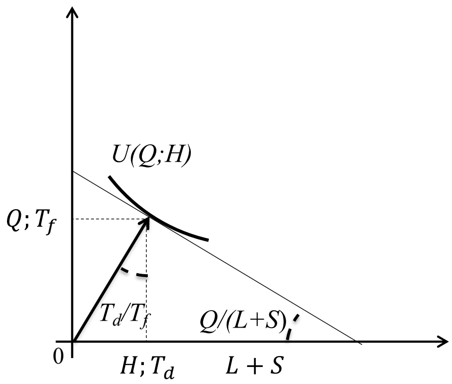

Fig.1. Optimal choice in family

Hunter's costs both on hunting and homework are constant to scale with respect to the given quarry Q:

$$(4) \quad AC_f = MC_f; AC_d = MC_d$$

[1]The study of labor-search-leisure trade-off (Malakhov 2021a) paid attention to the difference between explicit trade units, rabbits or partridges in the case of hunting, and analytical implicit units of consumption or portions when, for example, one rabbit can be equal to four portions or units of consumption (S.M.)

where AC_f – average costs of hunting; AC_d – average costs of men's homework; MC_f – marginal costs of hunting; MC_d – marginal costs of men's homework.

So, the T_d/T_f ratio represents the marginal costs' ratio, or the rate of transformation of hunting for homework RPD_{FD} for any hunter's productivity v:

$$(5)\quad \frac{T_d}{T_f} = \frac{TC_d}{TC_f} = \frac{AC_d}{AC_f} = \frac{MC_d}{MC_f} = RPD_{FD}$$

where TC_f – total costs of hunting; TC_d – total costs on men's homework.

As a result, the optimal choice gets the following properties:

$$(6)\quad MRS\ (H\ for\ Q) = -\frac{\partial Q}{\partial H} = \frac{MU_H}{MU_Q} = \frac{Q}{L+S} = \frac{T_d}{T_f} = \frac{MC_d}{MC_f} = RPT_{FD}$$

Eq.6 is very important for family decision-making. From the very beginning it separates equilibrium solutions from corner solutions. When Eq.6 doesn't hold, like it takes place in marriage of very lucky hunter with very beautiful lady, either the marriage needs the change in preferences or the utility falls, and it is better for an individual to stay alone.

Coming back to equilibrium solutions, provided by Eq.6, we see that the hunter can 'sell' the quarry at any imputed price P. But for any price, including the equilibrium one, we get from Eq.5 the following rule:

$$(7)\quad P = MC_f + MC_d = MC_f(1 + \frac{T_d}{T_f})$$

If the hunter's choice is optimal, Eq.7 results in the following consideration:

$$(8)\quad P = MC_f\left(1 + \frac{T_d}{T_f}\right) = MC_f\left(1 + \frac{Q}{L+S}\right)$$

Eq.8 is true for any woman's unit attractiveness w. As a result, both parts enter the marriage market with some imputed prices of the quarry:

$$(9)\quad P_w = w\frac{L+S}{Q}; P_h = MC_f\left(1 + \frac{Q}{L+S}\right)$$

The transaction takes place, if the price of the quarry P_w, imputed by wife, is equal to its price P_h, imputed by husband:

$$(10)\quad w\frac{L+S}{Q} = MC_f\left(1+\frac{Q}{L+S}\right)$$

This consideration holds for any content of w value. Indeed, the housewife's contribution might be different – not only beauty and homework skills, but also fruits and vegetables from the plant, or even berries and mushrooms from the forest. But here it becomes unimportant because we're going to escape from imputed values:

$$(11)\quad \frac{w}{MC_f} = \frac{1+\frac{Q}{L+S}}{\frac{L+S}{Q}} = \frac{Q}{L+S} + \left(\frac{Q}{L+S}\right)^2$$

Really, the *w/MC$_f$* ratio loses its monetary meaning. The 'money/time' value with respect to 'money/quantity' value produces the 'quantity/time' ratio, which corresponds by its physical order to the relative value of the marginal rate of substitution of leisure for consumption. And the square of the marginal rate of substitution, as we will see later, has no physical order at all.[2]

The need in imputed values becomes unimportant when we evaluate the total attractiveness of both man and woman, now with the help of the hypothesis of gravitation between them.

Part II. Gravitation between man and woman

The idea of the gravitation has come from the simple reasoning, that when the hunter supplies not only the quarry, but also leisure, there should be the transformation rate of hunter's working time into the time of housewife's leisure:

$$(12)\quad H = \delta(T_f + T_d)$$

Here the δ value looks like the rate of transformation of hunter's working time into housewife's leisure. However, the formal logic of the optimal choice at Fig.1 exhibits the true δ value:

[2] For example, the value w can represent housewife's skills in production of hides and skins a day. If we take the other quantity like berries and mushrooms, we should also consider her as a 'hunter', the option that will be examined later in the analysis of modern family (S.M).

(13.1) $H = Q\dfrac{Q}{L+S}$

(13.2) $\delta(T_f + T_d) = \dfrac{Q^2}{L+S}$

(13.3) $\delta = \dfrac{Q}{T_f + T_d}\dfrac{Q}{L+S}$

(13.4) $\delta_i = \dfrac{Q}{T_f + T_d}\dfrac{q_i}{L_i + S_i}$

Eq.13.3 and Eq.13.4 look like specific forms of Newtonian law of universal gravitation. Here 'masses of particles' are presented by quantities, and the 'distance between particles' is presented by the time that both parts in transaction spend either to meet each other on commodity market or to share quarry and leisure in marriage market. On commodity market the gravitational force represents the product of seller's gravitational field or his productivity and buyer's gravitational field or his purchasing power. In marriage markets gravitational force is equal to the product of total attractiveness of partners. As a result, Eq.13.3 represents the gravitational force or the mutual interest in monogamy. And Eq.13.4 gives us the understanding of the mutual interest between a hunter and one of his wives in polygamy, where both the quarry and homemaking time are distributed between many women.

The hypothesis of gravitation discovers the implicit consumption-leisure utility function of the hunter $U_h(Q;H)$. Indeed, if there is the trade-off between the quarry and leisure time, the hunter optimizes it when the marginal rate of substitution of his leisure to consumption MRS_h *(H for Q)* is equal to his gravitational field δ_h like the housewife's gravitational field δ_w also is equal to the marginal rate of substitution of her leisure to consumption MRS_w *(H for Q):*

(14.1) $MRS_h(H\ for\ Q) = \delta_h = \dfrac{Q}{T_f + T_d}$

(14.2) $MRS_w(H\ for\ Q) = \delta_w = \dfrac{Q}{L+S}$

Eq.13.3 provides very important implications for the allocation of time in the family, especially with respect to hunter's leisure time H_h and housewife's leisure time H_w. Here gravitational fields or total attractiveness start to work. We see that spouses spend leisure together for the given time horizon T only when their gravitational fields are equal:

$$(15.1) \quad \frac{Q}{T_f + T_d} = \frac{Q}{L + S}; T_f + T_d = L + S; H_h = H_w$$

$$(15.2) \quad \frac{Q}{T_f + T_d} > \frac{Q}{L + S}; T_f + T_d < L + S; H_h > H_w$$

$$(15.3) \quad \frac{Q}{T_f + T_d} < \frac{Q}{L + S}; T_f + T_d > L + S; H_h < H_w$$

Eq. 15.2 and 15.3 tell us that if one partner is more attractive than another, he automatically gets more leisure time. However, it seems that Eq.15.2 and 15.3 represent corner solutions with respect to Eq.15.1, which equalizes marginal rates of substitution of both spouses and looks like a unique equilibrium solution. But it is not so. The picture of family equilibrium exhibits a variety of solutions, which can be considered as multiple equilibria.

Part III. Multiple family equilibria

First, we rearrange Eq.11 in a quadratic equation:

$$(16) \quad \left(\frac{Q}{L + S}\right)^2 + \frac{Q}{L + S} - \frac{w}{MC_f} = 0$$

Then we can rearrange Eq.13.1 into the following form:

$$(17) \quad H = Q\frac{Q}{L + S} \to \frac{H}{L + S} = \left(\frac{Q}{L + S}\right)^2$$

Eq.17 shows us that the square of the marginal rate of substitution really loses its physical order.

$$(18) \quad \frac{w}{MC_f} = \frac{H}{L + S} + \frac{Q}{L + S}$$

However, here the MC_f value hides husband's unit attractiveness or his productivity on hunting. Really, if his unit attractiveness v is equal to his productivity, we get the value of MC_f equal to one:

$$(19.1) \quad v = \frac{Q}{T_f}$$

$$(19.2) \quad MC_f = \frac{TC_f}{Q} = \frac{vT_f}{Q} = \frac{Q}{T_f}\frac{T_f}{Q} = 1$$

Later we will use different $v=Q/T_f$ values, but now Eq.19.2 gives us the final form of Eq.10:

$$(20) \quad \left(\frac{Q}{L+S}\right)^2 + \frac{Q}{L+S} - w = 0$$

We see that the w value really loses its original meaning and represents the relative unit attractiveness.

Eq.20 closes the set of equation, which provide efficient tools for the analysis of equilibrium solutions in family, both for mating of likes and unlikes.

We start with the illustration for the set of Eq.15, which represents the mating of unlikes. We can calculate the allocation of time of both inferior ($w<1$) and superior ($w>1$) women with respect to the unit male attractiveness $v=Q/T_f=1$ within 24-hours' time horizon (Table 1):

Table 1. Equilibrium Mating of Unlikes

w	Q/(L+S)	Hw/(L+S)	Tf	Td	Q	P	L+S	Hw	Hh	δ_h	δ_w
0,9	0,57	0,33	10,35	5,92	10,35	1,57	18,08	5,92	7,73	0,64	0,57
1,1	0,66	0,44	11,05	7,31	11,05	1,66	16,69	7,31	5,64	0,60	0,66

We see that the inferior woman really gets less leisure time than her husband ($H_w<H_h$), while the superior woman gets more leisure time ($H_w>H_h$). And their gravitation fields δ_w and δ_h exhibit these inequalities, produced by Eq.15.2 and 15.3, that can be confirmed by ratios of attractiveness to imputed price of the quarry:

$$(21) \quad \delta_h = \frac{Q}{T_f + T_d} = \frac{v}{P} \neq \delta_w = \frac{Q}{L+S} = \frac{w}{P}$$

These inequalities also can be derived analytically, if we suppose that a superior mate starts his activity later than the inferior one and gets extra leisure time. In this way partners stop their activity at the same time and start to enjoy leisure together.

We see that the gravitational field of superior mate is greater than the gravitational field of inferior mate. It means that either a man looks more attractive in eyes of his wife, than she looks in his eyes, or vice versa. And we don't know how and with whom the superior mate spends extra leisure time.

However, Eq.6 and 10 hold, and prices of the quarry, imputed by both parts in transaction are equal even despite unequal gravitational fields. While imputed prices of both parts are equal, the transaction takes place. It means, that ***both parts voluntarily accept the inequality in attractiveness***. The inferior wife can be so attracted by strong gravitational field of her superior mate that she can voluntarily put up with his extra leisure time, and vice versa.

This consideration gives an idea that here we get some specific family solutions, when the equilibrium takes place, but its stability should be questioned. And we can consider solutions for the mating of unlikes as second-order local equilibria. We need such a differentiation because mating of likes also produces the equilibrium solutions, which are not undoubtful, and they can be considered as first-order local equilibria.

If we take unit male attractiveness equal to unit female attractiveness, or $v=Q/TC_f=w$, we get the following results (Table 2):

Table 2. Equilibrium Mating of Likes

w=v	Q/(L+S)	Hw/(L+S)	Tf	Td	Q	P	L+S	Hw	Hh	δ_{wh}
0,9	0,57	0,33	11,50	6,58	10,35	1,57	18,08	5,92	5,92	0,57
1,0	0,62	0,38	10,73	6,63	10,73	1,62	11,10	6,63	6,63	0,62
1,1	0,66	0,44	6,86	4,54	11,05	1,66	16,69	7,31	7,31	0,66

We see that under the assumption of equal unit attractiveness Eq.6, 10, and 20 produce equal gravitational fields $\delta_w = \delta_h = \delta_{wh}$ and leisure time $H_w=H_h$ for both partners. In addition, if we separate utilities of mates, we get the equality of their marginal rates of substitution of leisure for consumption. The mating of likes seems to be the equilibrium solution at any level of superiority rate $v=w$:

$$(22) \quad \delta_h = \frac{Q}{T_f + T_d} = \frac{v}{P} = \delta_w = \frac{Q}{L+S} = \frac{w}{P}$$

The mathematical confirmation of mating of likes really proves the reliability of the hunter-housewife model. And we see that this confirmation really doesn't need imputed values. However, the stability of such theoretical equilibria can also be questioned.

We see that in the inferior family ($v=w<1$) the men's homework T_d is greater than women's leisure H_w. It means that at home the hunter provides not only the time of housewife's leisure, but also the time for the part of women's work. In some sense, the hunter becomes not only the suppler but also the sponsor. The equilibrium holds, but the trade looks unfair. Nobody knows how long this sponsorship will continue.

The equilibrium of the superior family ($v=w>1$) also looks like unfair trade. Here the hunter's homework time T_d is less than leisure of his wife H_w. It means that his wife gets some leisure time by her own efforts, due to her own superior attractiveness or home productivity. Despite his superior productivity on hunting, the husband can supply only the quantity demanded, but not the leisure demanded by his attractive wife. As a result, the wife becomes less dependent from her husband.

In general, we see that gravitational fields or mutual attractiveness rises with the superiority rate $v=w$, but the economic interdependence of partners falls. Superior mates increase their individual utilities, but they become more independent. And real life sometimes challenges equal attractiveness.

In addition, here the risk of corner solution for superior partners we have talked about in Introduction appears. For example, very productive hunter can marry very beautiful woman ($v=w=2.5$), but at the equilibrium his time on homework becomes greater than the time on hunting, or $T_f<T_d$ (Table 3):

Table 3. Equilibrium Mating of Superior Likes

w=v	Q/(L+S)	Hw/(L+S)	Tf	Td	Q	P	L+S	Hw	Hh	δ_{wh}
2,5	1,16	1,34	4,75	5,50	11.87	2,16	10,25	13,75	13,75	1,16

This result can look plausible from some sentimental point of view but not from the economic one. The hunter cannot change his preferences and become a 'housewife'. If he prefers hunting, the ratio $T_d/T_f <1$; it cannot be equalized with attractive woman's optimal choice, which produces the MRS_w *(H for Q)>1*. So, Eq.3 and 6 don't hold; the corner solution appears, and it destructs the family.

However, real life can smooth the corner solution. Lucky hunter spends more time in the forest; he brings quarry at the level above the quantity Q demanded by his wife; while there also a need in men's homework that lucky hunter cannot do because he is busy, he hires a servant; the hunter feeds the servant by extra quarry, and the servant makes men's homework. But the mathematics of hunter's choice also leaves him leisure time less than one demanded by his very attractive wife. And she begins to spend her extra leisure time with the supplier of men's homework, i.e., with the servant.

The example of mating of superior likes also exhibits the loss of mates' economic interdependence. If we separate woman's physical attractiveness from her productivity, on hides and skins, for example, and analyze this kind of mating from the modern point of view, when women have an option to work and to get money, we see that the industrious woman can stay alone. She earns enough money by her 'market work' L; she spends time S on men's 'non-market work', which is equal here to 5.50 hours; and she finally gets enough leisure of 8.25 hours. It means that even in traditional family, where industrious woman has an option to make hides and skins for her less industrious neighbors, she becomes less dependent from her husband and exhibits the unfair trade.

However, real life usually hides this unfair trade. From the analytical point of view, inferior husband starts hunting earlier, while the superior husband starts later. But in real life the allocation of time of likes looks as follows: both man and woman start their activity at the same time; the husband comes back from hunting when his wife continues to do women's homework; the hunter starts his homework, and both mates happily finish their activity at the same time and start to enjoy leisure together. This is the way the everyday allocation of time hides the unfair trade.

There is only one mating of likes, when the trade is fair. This is the case of the ideal family.

Part IV. Perfect equilibrium of ideal family

Really, one result from the set of equilibria for mating of likes produces an undoubtful solution. All equilibrium equations hold; both unit and mutual attractiveness as well as the common leisure time are equal, and the hunter really supplies for the housewife both consumption and leisure, where women's leisure time is equal to men's homework, or $H_w=T_d$.

This is really a fair trade, confirmed on the analytical level by the equality of marginal rates of substitution of leisure for consumption MRS_h *(H for Q)*= MRS_w *(H for Q)*.

This fair trade takes place when both male and female unit attractiveness are equal to one, or $v=w=1$.

This solution has some exceptional properties. Table 2 presents round-off values, but if we calculate them more precisely, we get the following result:

$$(23) \quad \frac{T_d}{T_f} = \frac{Q}{L+S} = 0{,}61803398\ldots = \frac{1}{\varphi} = \Phi$$

or ***both rate of transformation of hunting into homework and the marginal rate of substitution of leisure for consumption are equal to golden ratio conjugate***.

Really, if we solve Eq.20 with respect to the value $w=1$, we get:

(24.1) $$\left(\frac{Q}{L+S}\right)^2 + \frac{Q}{L+S} - 1 = 0$$

(24.2) $$\frac{Q}{L+S} = \frac{-1+\sqrt{5}}{2} = 0{,}61803398\ldots = \frac{1}{\varphi} = \Phi$$

Golden ratio immediately equalizes mates' marginal rates of substitution of leisure for consumption, i.e., their gravitational fields, with respect to the value *v=1*:

(25.1) $$MRS_h(H\ for\ Q) = \frac{Q}{T_f + T_d} = \frac{Q}{T_f}\frac{1}{1+\frac{T_d}{T_f}} = v\frac{1}{1+\frac{T_d}{T_f}} = \frac{1}{1+\frac{T_d}{T_f}}$$

(25.2) $$MRS_h(H\ for\ Q) = \frac{Q}{T_f + T_d} = \frac{1}{1+\frac{T_d}{T_f}} = \frac{1}{1+\Phi} = \frac{1}{\varphi} = \Phi$$

(25.3) $$\delta_h = \frac{Q}{T_f + T_d} = MRS_h(H\ for\ Q) = \delta_w = \frac{Q}{L+S} = MRS_w(H\ for\ Q)$$

In addition, the relative price *P* of the unit of consumption of quarry becomes equal to golden ratio itself and really becomes an equilibrium price P_e with respect to female equilibrium attractiveness w_e:

(26) $$MRS_w\ (H\ for\ Q) = \frac{Q}{L+S} = \frac{w_e}{P} = \frac{1}{P} = \Phi \rightarrow P_e = 1{,}61803398\ldots = \varphi$$

These magic values only strengthen the perfect allocation of time in the family under unit *v=w=1* assumption. But this assumption doesn't come from nowhere. We started the discussion by the assumption that in marriage markets the equilibrium female attractiveness exists, and it corresponds to zero values of both dowry and bride price. The same happens with the male attractiveness. Here we can address to Adam Smith's idea about values of profits, rent, and wages, which were natural for some local markets (Smith 2000). Hunter's attractiveness can also be presented by some natural or common value of a local marriage market. For example, a community can evaluate hunter's skills as natural by three partridges per day. While the *v* value of male attractiveness like the female attractiveness is the relative value, we can take three partridges per day as the unit attractiveness $v=1$. So, the hunter, who brings four partridges, gets the unit attractiveness $v=1.33$, and the hunter, who brings only two partridges, gets the unit attractiveness $v=0.66$.

These considerations give an idea that the $v=w=1=w_e=v_e$ assumption produces the perfect equilibrium solution, or the ideal family. And allocation of time in the ideal family for both mates is described by exceptional properties of golden ratio:

$$(27.1) \quad \frac{T_d}{T_f} = \frac{1}{\varphi} = \Phi = \varphi - 1$$

$$(27.2) \quad \frac{H_w}{L+S} = \Phi^2; \frac{L+S}{H_w} = \varphi^2 = \varphi + 1$$

Part V. Ideal family as a means of survival

However, the ideal family doesn't represent the 'Valley of Eden' despite its divine proportions. It leaves for mates only 6 hours and 40 minutes of leisure. Of course, spouses spend much more time together, when they are doing the homework, which gives them another 6 hours and 40 minutes. But for all other common activities, including eating, sleeping, and mutual care, they get only the same 6 hours and 40 minutes.

This result looks absurdly from the point of view of modern trends in the allocation of time. But it is not so from the point of view of preindustrial societies.

Recent applied studies of the sleeping habits in preindustrial societies in Tanzania, Namibia, and Bolivia discovered similar sleep parameters with average duration of 5.7-7.1 hours. There people go to sleep after sunset and awaken before sunrise (Yetish et al 2015). The same sleeping habits were discovered by historical studies for times just before the industrial revolution in France and in England (Ekirch 2001, 2021, Garnier 2013). Moreover, these studies also discovered a 'broken sleep', when mates awakened in midnight for some household activities, praying, and making love, because at that time it was a strong belief that children conceived at midnight would grow healthier.

These results give an idea that in preindustrial societies the family with its division of labor really played the role of a means of survival. And golden ratio proportions exhibited the parameters of the survival. The model of ideal family gets features of economic viability. Leisure time was limited by vital need in sleeping hours.[3] It is interesting, that modern society sometimes follows this need. For example, the mean total sleeping time in France, measured in 2017, was equal to 6 hours and 42 minutes in a week and 7 hours and 26 minutes in holidays (Léger et al 2019).

Part VI. Ideal family today

The hypothesis of the ideal family as a means of survival enables the understanding of the difference between golden ratio proportions and modern trends in the allocation of time.

Today the society limits working time by 48 hours a week on average. It looks like modern society itself, without help of golden ratio, determines a feasible working time. But the ideal family implicitly takes part in this allocation of time.

Here the difference between economic feasibility and economic viability appears. Let's take 48 hours a week as the given amount of time on hunting, which becomes a 'market work'. The model of ideal family ($v=w=1$) transforms 6.86 hours a day on 'market work' into 4.24 hours for men's 'non-market work' (Table 4):

Table 4. Feasibility vs Economic Viability in Mating of Likes

w=v	Q/(L+S)	Hw/(L+S)	Tf	Td	Q	P	L+S	Hw	T	Hh	δ_{wh}	AddH	Htotal

[3] The optics of vital needs can also explain the quantity of quarry, consumed by the ideal family. Here we can use a specific Big Mac Index. The value 10.73 can represent the quantity of consumption units. The daily intake for a young male who leads an active lifestyle is equal to 3200 kcal and for an active female to 2400 kcal. If we divide the vitally needed total of 5600 kcal by 10.73, we approximately get 522 kcal, which are close to the mean of 504-550 caloric value of one Big Mac. To meet vital needs of the family, the hunter should get 10.73 'Big Macs' of the quarry (S.M.)

0,9	0,57	0,33	6,86	3,92	6,17	1,57	10,78	3,53	14,31	3,53	0,57	9,69	13,22
1,0	0,62	0,38	6,86	4,24	6,86	1,62	11,10	4,24	15,33	4,24	0,62	8,67	12,90
1,1	0,66	0,44	6,86	4,54	7,54	1,66	11,40	4,99	16,39	4,99	0,66	7,61	12,60

As a result, both mates get the same amount of 4.24 hours for common leisure. But in total, with 'market work' and 'non-market work' this common leisure gives the 15.33-hours' time horizon *T*. In the original 'labor-search-leisure' model, which serves as the basis for the analysis of family, the time horizon is determined like the time until next purchase (Malakhov 2021a,b). Here this assumption means, that in preindustrial society the man should start hunting again after 15.33 hours. It gives us the level of economic viability. But when the society takes 6.86 working hours as economically feasible living standard for 24 hours, both mates get 8.67 hours of additional time *AddH* over the requirements of the ideal family. They can use this additional time in different ways – for gardening, childcare, playing cards, etc. But they can also spend this time together and get the total 12.90-hours' leisure time, which corresponds to modern trends in the allocation of time (Aguiar and Hurst 2007).

These results provide the answer to the question how hunter-housewife ideal family manifests itself in modern family when both mates are working.

The solution is very simple. We take a family of two 'hunters'; a mate plays the role of a 'hunter' with respect to his quarry and at the same time he plays the implicit role of a 'housewife' with respect to the quarry from the counterpart.

Here we don't know the allocation housewife's time because the housewife's role is implicit. But its preferences are considered (Table 5):

Table 5. Allocation of Time for Two Hunters

v=w	Tf	Q	Q/(Tf+Td)	Td	H	P	Qtotal	δ_{wh}
0,90	6,86	6,17	0,57	3,92	13,22	1,57	12,34	0,57
1,00	6,86	6,86	0,62	4,24	12,90	1,62	13,71	0,62
1,10	6,86	7,54	0,66	4,54	12,60	1,66	15,09	0,66

Both mates spend 6.86 hours on 'market work' and produce the 'quarry' with respect to their equal productivity. So, the total quarry Q_{total} is doubled, but not the price. Then, Eq. 22 tells us that for the mating of likes partner's marginal rates of substitution of leisure for consumption, i.e., gravitational fields δ_{wh}, are equal. We substitute $Q(L+S)$ value by $Q/(T_f+T_d)$ value in in quadratic equation from Eq. 20 and get the $Q/(T_f+T_d)$ value with respect to equal productivity $v=w$. It gives us for every mate the value T_d of 'non-market work', his/her total working time (T_f+T_d), and leisure time H within 24 hours' time horizon.

We see that for the doubled 'quarry' the allocation of time stays the same as in Table 4. Working 6.86 hours a day, both spouses spend in ideal family 4.24 hours on 'non-market work' and get 12.90 hours of leisure time that can be spent either on pleasurable non-market work, i.e., some dual activity like gardening and pets' care (Aguiar and Hurst 2007), or on leisure itself. However, the economics of ideal family 'washes its hands' at the viable 15.33 hours' time horizon, leaving 8.67 hours of additional leisure to psychology and family science.

Concluding remarks

The paper provides the analytical tool for deeper understanding of classical Beckerian mating of both likes and unlikes (Becker 1991). The mathematical equality of leisure time for mating of likes proves the reliability of the hunter-housewife model. Despite its static nature, the model discovers unequal allocation of time even in local equilibrium exchange that can show up in dynamics and ruin the family. It also gives us the understanding that in the modern family there are some inner limits of economic viability. In general, this idea corresponds to the economic self-sufficiency, or the ability of individuals and families to meet their basic needs. However, the limits imposed by the model of ideal family on the allocation of time don't match to the modern understanding of basic needs, which include today needs in utilities, transportation, social security, and even tax obligations. The model of ideal family discovers the vital need in leisure time and reduces it to sleeping hours. This approach is confirmed by historical and applied studies and sometimes by actual statistics. The model of ideal family also accepts rules of social game people are playing today and leaves them enough freedom to meet needs with respect to modern understanding of leisure.

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